

Some Developments in Gribov's Approach to QCD

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Personal Recollection

- In 1964, long before most of you were born, I was a post-doc at CEA-Saclay, France, and attended the “Rochester” (High Energy Physics) Conference which that year was held at Dubna Laboratory in the Soviet Union. Also attending from Saclay were Marcel Froissart and Maurice Jacob. Unexpectedly Valodya Gribov invited the three of us to visit him in Leningrad. We took the train from Moscow to Leningrad where we met him and the members of his group. He also invited the three of us and the members of his group to a party at his home in Leningrad where I met him and his first wife.
- I should mention that at that time Gribov and I had no particular common interests in physics. Rather he invited the three of us because he and Froissart shared an interest in analytic properties of scattering amplitudes.
- It says something about the man that at a time when, I believe, he was not allowed to travel abroad, he nevertheless broke through the iron curtain with his hospitality.
- The visit to his home was prophetic because the paper that has inspired my research was written 14 years later, in 1978, but I never met him after 1964.
- However we did speak by telephone many years later. He came to Princeton on a visit, not far from New York where I live. I immediately called him up and invited him to New York University. He declined, saying that it was not a good time, that he had just travelled, that he was a bit tired, that his wife had not yet arrived. I concluded that the great man was happy at Princeton and I did not disturb him further. Later I was told that he said I was angry at him. I did not have an opportunity to tell him that I was not angry, for he passed away not long after.

7. Coulomb gauge

In sect. 6 we discussed the effect of limiting the integration over the fields on the properties of vacuum fluctuations in the invariant Euclidean formulation of the theory. In so doing, we adduced arguments for singularity of the ghost Green function as $k^2 \rightarrow 0$ (for example, $1/k^4$). This certainly is an indication of a substantial long-range effect in the theory that may result in colour confinement, but the ghost Green function in an arbitrary gauge is not connected directly with the Coulomb interaction at large distances. Hence, in this section we shall rewrite the foregoing analysis for the Coulomb gauge [13] where the Green function of the ghost determines directly the Coulomb interaction. We shall show that the situation which involves a restriction on the integration range over fields and a cutoff of the infrared singularity found in perturbation theory is exactly the same as in invariant gauges. The arguments for singularity of the ghost Green function hold here as well. In this case, however, a singularity of the ghost Green function as $k^2 \rightarrow 0$ of the type $1/k^4$ is indicative of a linear increase in the Coulomb interaction with distance.

The most natural way of formulating the Coulomb gauge is the Hamiltonian form which shows up vividly the unitarity of the theory because of the lack of ghosts. To this end, the functional integral W incorporates the fields which satisfy the three-dimensional transversality condition

$$\frac{\partial A_i}{\partial x_i} = 0, \quad (49)$$

and momenta π_i which are canonically conjugated with them and stand for the transverse part of the electric field

$$\pi_i = E_i^\perp = \left(\frac{\partial A_i}{\partial t} - [\nabla_i A_0] \right)^\perp \quad (50)$$

NO CONFINEMENT WITHOUT COULOMB CONFINEMENT

Gribov's insight into the mechanism of confinement is substantiated by the theorem,

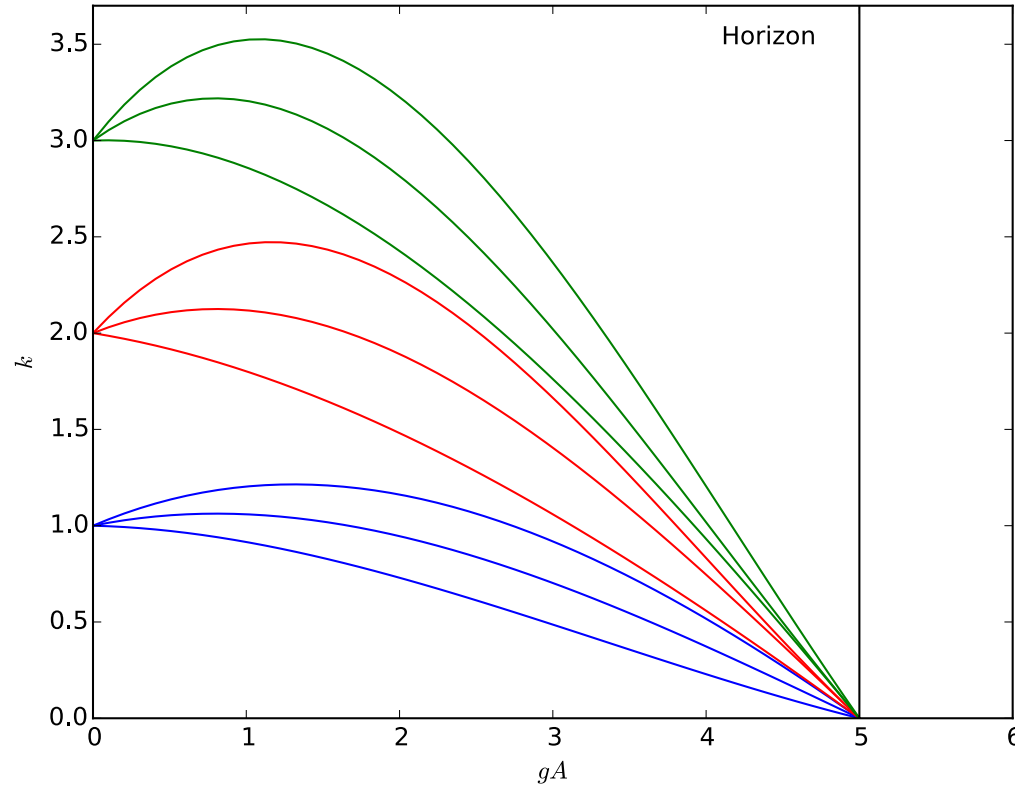
$$V_{\text{coul}}(R) \geq V_{\text{wilson}}(R),$$

for $R \rightarrow \text{infinity}$, where the color-Coulomb potential is the temporal gluon propagator in Coulomb gauge,

$$D_{00}(R, T) = V_{\text{coul}}(R) \delta(T) + \text{N.I.}$$

When the Wilson potential is linearly rising, the color-Coulomb potential is linear or super-linear.

ALL HORIZONS ARE ONE HORIZON



Cartoon of the eigenvalues of the Faddeev-Popov operator

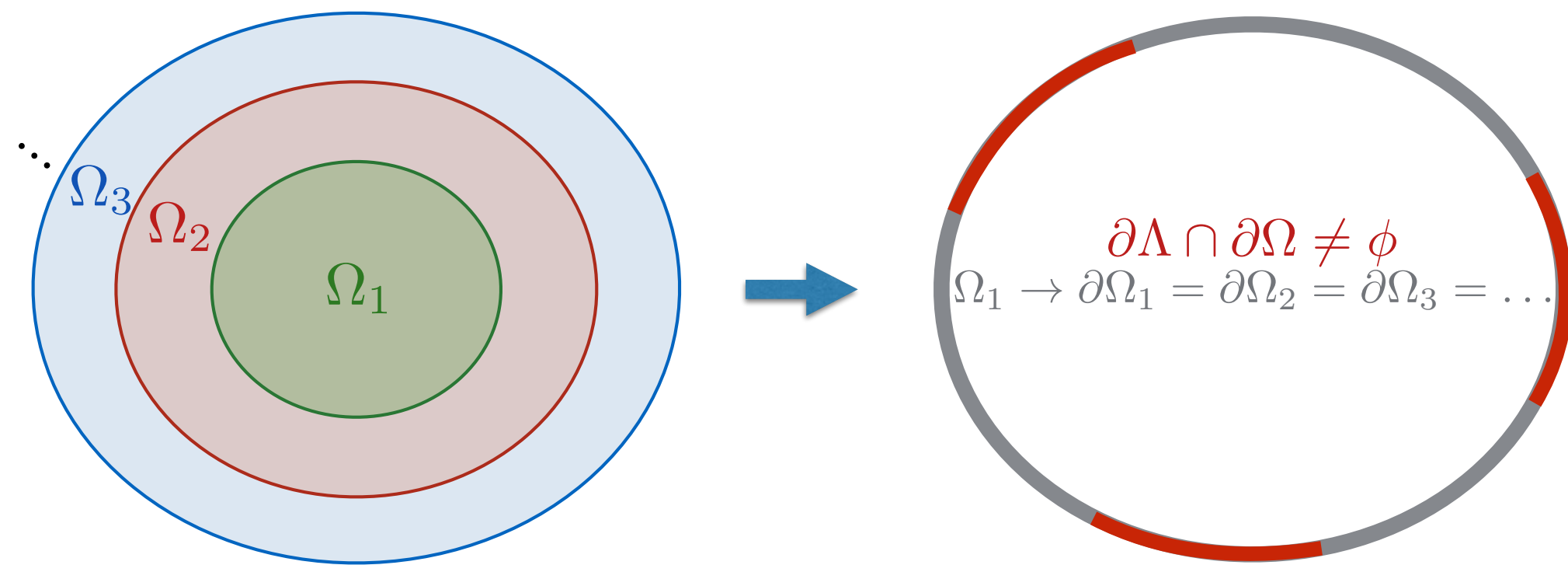
$$M^{ac}(gA) = -\partial_i^2 \delta^{ac} - f^{abc} gA_i^b \partial_i.$$

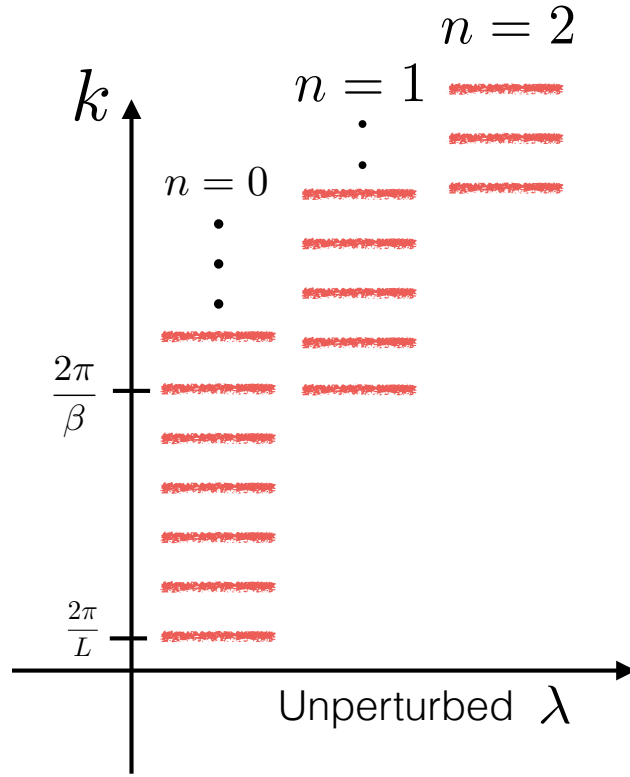
$$\lambda_{|\vec{k}|}(gA) \equiv \vec{k}^2 \left(1 - [d(N^2 - 1)V]^{-1} H(gA) \right) + \text{higher-order in } \vec{k}.$$

There is a common factor \vec{k}^2 , where $k_i = 2\pi n_i/L$.

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A sketch of the spectrum of the unperturbed Faddeev-Popov operator in Landau gauge $M(0) = -\partial_\mu^2$ at finite temperature. The index n labels the finite Matsubara frequencies. When the perturbation is turned on, $M(gA)$, the eigenvalues corresponding to the zero-Matsubara frequency will always be the ones that cross the Gribov horizon first.

HORIZON FUNCTION AND NON-LOCAL ACTION

$$S = S_{\text{FD}} + \gamma H - \gamma \int d^d x \, d(N^2 - 1)$$

Euclidean action, where the horizon function is given by

$$H = \int d^d x d^d y \, D_\mu^a(x) D_\mu^b(y) (M^{-1})^{ab}(x, y; A).$$

The horizon function cuts off the functional integral at the Gribov horizon as one sees from the eigenfunction expansion

$$(M^{-1})^{ce}(x, y; A) = \sum_n \frac{\psi_n^c(x) \psi_n^e(y)}{\lambda_n(A)}$$

HORIZON CONDITION AND KUGO OJIMA CONFINEMENT CONDITION

The Gribov parameter is fixed by the horizon condition

$$\langle H \rangle = d(N^2 - 1) \int d^d x$$

It is a remarkable fact that the horizon condition and the famous Kugo-Ojima confinement condition are the same statement:

$$-i \int d^d x \langle (D_\mu c)^a(x) (D_\mu \bar{c})^a(0) \rangle = d(N^2 - 1).$$

Thus color confinement is assured in this theory.

HORIZON CONDITION AND DUAL MEISSNER EFFECT

It is a remarkable fact that, the horizon condition is equivalent to the statement that the QCD vacuum is a perfect color-electric superconductor, which is dual Meissner effect (H. Reinhardt, Phys. Rev. Lett. 101, 061602 (2008), 08030504.)

$$G(\vec{k}) = \frac{d(\vec{k})}{\vec{k}^2} = \frac{1}{\epsilon(\vec{k}) \vec{k}^2}$$

$$d^{-1}(\vec{k} = 0) = 0 \iff \epsilon(\vec{k} = 0)$$

where $G(k)$ is the ghost propagator.

AUXILIARY GHOSTS

Just as the Faddeev-Popov determinant is localized by introducing ghosts

$$\det M = \int dcd\bar{c} \exp \left(- \int d^d x \bar{c} M c \right),$$

Likewise, the horizon function in the action may be localized by introducing “auxiliary” ghosts,

$$\exp(-\gamma H) = \int d\varphi d\bar{\varphi} d\omega d\bar{\omega} \exp \left(- \int d^d x \left[\bar{\varphi} M \varphi - \bar{\omega} M \omega + \gamma^{1/2} D \cdot (\varphi - \bar{\varphi}) \right] \right)$$

LOCAL ACTION AND PHYSICAL DEGREES OF FREEDOM IN COULOMB GAUGE

$$\begin{aligned}
 S = \int d^{d+1}x \Big\{ & i\tau_i \partial_0 A_i + (1/2)[\tau^2 + (\partial_i \lambda)^2] + (1/4)F_{ij}^2 - i\tau_i (A_i \times A_0) \\
 & + i\partial_i \lambda D_i A_0 - \partial_i \bar{c} \cdot D_i c + \partial_i \bar{\varphi}_j \cdot D_i \varphi_j - \partial_i \bar{\omega}_j \cdot (D_i \omega_j + D_i c \times \varphi_j) \\
 & + \gamma^{1/2} \text{Tr}[D_j(\varphi_j - \bar{\varphi}_j) - D_j c \times \bar{\omega}_j] - \gamma d(N^2 - 1) \Big\}.
 \end{aligned}$$

τ_i is the momentum conjugate to A_i and both are transverse

$$\partial_i A_i = \partial_i \tau_i = 0.$$

Only the first term $i\tau_i \partial_0 A_i$ contains a time derivative and it acts on the two transverse degrees of freedom of the gluon which are the physical degrees of freedom. All the remaining terms impose constraints.

MAGGIORE-SCHADEN SHIFT

It is another remarkable fact that the same action may be derived by a completely different method developed by Maggiore and Schaden

Start with the usual Faddeev-Popov fields and action,

$$sA_\mu = D_\mu c, \quad sc = -(1/2)(c \times c),$$

$$s\hat{c} = i\hat{b}, \quad s\hat{b} = 0,$$

$$s\pi_i = \pi_i \times c,$$

Introduce auxiliary quartets of bose and fermi ghosts whose determinants cancel when they are integrated out,

$$s\phi_B = \omega_B$$

$$s\omega_B = 0$$

$$s\bar{\omega}_B = \bar{\phi}_B$$

$$s\bar{\phi}_B = 0.$$

MAGGIORE-SCHADEN SHIFT

The resulting action is BRST-invariant by construction,

$$\mathcal{L} = \mathcal{L}^{\text{YM}} + \mathcal{L}^{\text{gf}} = \mathcal{L}^{\text{YM}} + s\Psi \quad s\mathcal{L} = 0$$

$$\Psi \equiv \partial_i \hat{\bar{c}} \cdot A_i + \partial_i \bar{\omega}_j \cdot D_i \phi_j,$$

$$s\Psi = i\partial_i \hat{b} \cdot A_i - \partial_i \hat{\bar{c}} \cdot D_i c + \partial_i \bar{\phi}_j \cdot D_i \phi_j - \partial_i \bar{\omega}_j \cdot (D_i \omega_j + D_i c \times \phi_j) .$$

Now make the change of variables

$$\phi_{jb}^a(x) = \varphi_{jb}^a(x) - \gamma^{1/2} x_j \delta_b^a$$

$$\bar{\phi}_{jb}^a(x) = \bar{\varphi}_{jb}^a(x) + \gamma^{1/2} x_j \delta_b^a$$

$$\hat{b}^a(x) = b^a(x) + i\gamma^{1/2} x_j f^{abc} \bar{\varphi}_{jc}^b(x)$$

$$\hat{\bar{c}}^a(x) = \bar{c}^a(x) - \gamma^{1/2} x_j f^{abc} \bar{\omega}_{jc}^b(x)$$

This procedure, by a completely different derivation, yields precisely the same x-independent Lagrangian density that was derived above from a cut-off at the Gribov horizon!

The lagrangian density is BRST-invariant by construction

$$s\mathcal{L} = 0$$

But the form of the BRST operator s acting on the new fields is changed.

(For an alternative approach and further references, see M. A. L. Capri et al, arXiv:1606.06601.)

SPONTANEOUS BREAKING OF BRST

s acts on the shifted variables just like its action on the unshifted variables except that

$$s\bar{\omega}_{jb}^a = \bar{\varphi}_{jb}^a + \gamma^{1/2}x_j\delta_b^a$$

We have regained the standard form of BRST quantization with s -invariant Lagrangian density, but BRST is spontaneously broken,

$$\langle \{Q_B, \bar{\omega}_{ib}^a(x)\} \rangle = \langle s\bar{\omega}_{ib}^a(x) \rangle = \gamma^{1/2}x_i\delta_b^a.$$

How should we define physical states?!

We cannot say they are the cohomology of s because that would exclude the vacuum state.

BRST IN FADDEEV-POPOV THEORY

Let us recall how Faddeev-Popov theory works. The Faddeev-Popov determinant is non-local. It is localized at the cost of introducing the unphysical Faddeev-Popov ghosts, which are unphysical states.

$$\mathcal{L}_{\text{FP}} = \mathcal{L}_{\text{YM}} + s\Psi$$

However the local action has a new symmetry, the BRST-symmetry. The physical observables and physical states are required to be s -invariant under this new symmetry, and this eliminates the unphysical states.

$$\mathcal{W}_{\text{phys}} \equiv \{G : sG = 0\}.$$

POSSIBLE SOLUTION

Localization of the cut-off at the Gribov horizon yields

$$\mathcal{L} = \mathcal{L}_{\text{FP}} + s\chi = \mathcal{L}_{\text{FP}} + \mathcal{L}_{\text{aux.gh}}.$$

The new action with new auxiliary ghosts possesses many new “phantom” symmetries, with generators that leave ordinary physical observables invariant

$$[Q_Y, \mathcal{L}] = [Q_Y, F^2] = [Q_Y, \bar{\psi}\psi] = 0$$

It is natural to require that all physical observables be invariant under these new phantom symmetries (M. Schaden and DZ, arXiv:1412.4823),

$$\mathcal{W}_{\text{phys}} \equiv \{G : sG = 0; [Q_Y, G] = 0\}.$$

CONJECTURE

On the other hand unphysical operators such as $s\bar{\omega}_{ib}^a$ are not invariant under the phantom symmetries,

$$[Q_Y, s\bar{\omega}_{ib}^a] \neq 0$$

We offer the **conjecture** that BRST symmetry is not broken by s-exact observables,

$$\langle sY \rangle = 0 \text{ for all } sY \in \mathcal{W}_{\text{phys}}.$$

In other words, BRST symmetry is preserved where it is needed, namely in the physical subspace.

Explicit calculation of several examples reveals that BRST symmetry breaking apparently afflicts the unphysical sector of the theory, but may be unbroken where needed, in cases of physical interest. For example, the BRST-exact part of the conserved energy-momentum tensor has vanishing expectation value,

$$T_{\mu\nu} = T_{\mu\nu}^{\text{YM}} + s\Xi_{\mu\nu}$$

$$\langle s\Xi_{\mu\nu} \rangle = 0$$

CONCLUSION

We have a local, renormalizable quantum field theory with the following interesting properties:

- It provides a cut-off at the Gribov horizon.
- The Kugo-Ojima color confinement condition is satisfied.

The vacuum is a perfect dielectric

- There is an alternate derivation by the Maggiore-Schaden method that provides a BRST-invariant Lagrangian.
- BRST-symmetry is spontaneously broken, but perhaps only in the unphysical sector. Further work is required here.
- In Coulomb gauge the color-Coulomb potential is linear or super-linear when the Wilson potential is linearly rising, but dynamical calculations of confinement are still out of reach.