

# **Hamiltonian approach to QCD in Coulomb gauge: Gribov's confinement scenario at work**

**H. Reinhardt**

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TÜBINGEN



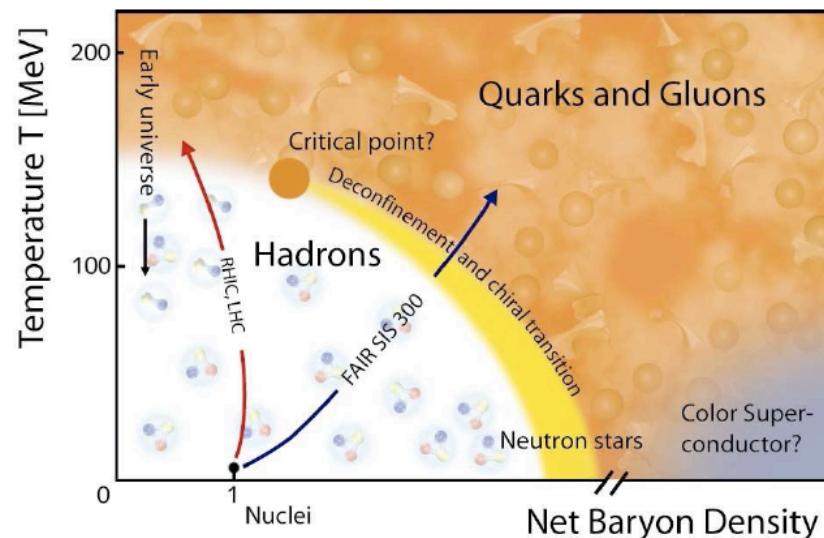
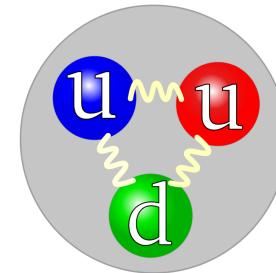
collaborators:

C. Feuchter, D. Epple, W. Schleifenbaum, M. Leder, M. Pak,  
J. Heffner, P. Vastag, H. Vogt, E. Ebadati

G. Burgio, Campagnari, M. Quandt

# *QCD*

- *vacuum*
  - confinement
  - SB chiral symmetry
- *phase diagram*
  - deconfinement
  - rest. chiral symm.
- LatticeMC-fail at large chemical potential  
continuum approaches required  
Hamiltonian approach



# Outline

- introduction
- Hamiltonian approach to QCD in Coulomb gauge at  $T=0$ 
  - variational solution of the Schrödinger equation
    - horizon condition
    - Coulomb string tension
  - Hamiltonian approach to QCD in Coulomb gauge at  $T \neq 0$ 
    - grand canonical ensemble
    - compactification of a spatial dimension
- conclusions

# Canonical Quantization of Yang-Mills theory

cartesian coordinates  $A_{\mu}^a(x)$

momenta  $\Pi_i^a(x) = \delta S / \delta \dot{A}_i^a(x) = E_i^a(x)$

$\Pi_0^a(x) = 0$       Weyl gauge :  $A_0^a(x) = 0$

$$H = \frac{1}{2} \int d^3x (\Pi^2(x) + B^2(x))$$

quantization:  $\Pi_k^a(x) = \delta / i \delta A_k^a(x)$

Gauß law:  $D\Pi\Psi = \rho_m \Psi$

residual gauge invariance  $U(\vec{x})$ :  $\Psi(A^U) = \Psi(A)$

# Coulomb gauge

$$\partial A = 0, \quad A = A^\perp$$

curved space

$$\langle \Psi | \Phi \rangle = \int D A^\perp J(A^\perp) \Psi^*(A^\perp) \Phi(A^\perp)$$

Faddeev-Popov

$$J(A^\perp) = \text{Det}(-D\partial)$$

$$\Pi = \Pi^\perp + \Pi^{\parallel}, \quad \Pi^\perp = \delta / i \delta A^\perp$$

Gauß law:

$$D\Pi\Psi = \rho_m \Psi$$

resolution of  
Gauß' law

$$\Pi^{\parallel} = -\partial(-D\partial)^{-1}\rho, \quad \rho = (-\hat{A}^\perp \Pi^\perp + \rho_m)$$

# Hamiltonian approach to YM<sub>T</sub> in Coulomb gauge $\partial A = 0$

$$H = \frac{1}{2} \int (\mathcal{J}^{-1} \Pi \mathcal{J} \Pi + B^2) + H_C$$

$$\Pi = \delta / i\delta A$$

Christ and Lee

$$J(A^\perp) = \text{Det}(-D\partial) \quad D^{ab} = \delta^{ab} \partial + gf^{abc} A^c$$

$$H_C = \frac{1}{2} \int J^{-1} \rho J (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} \rho$$

Coulomb term

$$\text{color charge density} \quad \rho^a = -f^{abc} A^b \Pi^c + \rho_m^a$$

$$\langle \phi | \dots | \psi \rangle = \int D A \mathcal{J}(A) \phi^*(A) \dots \psi(A)$$

$$H\psi[A] = E\psi[A]$$

# Variational approach

- Gaussian ansatz,

$$\Psi(A) = \exp \left[ -\frac{1}{2} \int dx dy A(x) \omega(x, y) A(y) \right]$$

D. Schütte 1984

.....

A.Szczepaniak & E. Swanson 2002

C. Feuchter & H. R. 2004

-ansatz  
-FP determinant  
-renormalization

- Greensite, Matevosyan,Olejnik,Quandt, Reinhardt, Szczepaniak,PRD83

# Variational approach

- trial ansatz

C.Feuchter & H. R. PRD70(2004)

$$\Psi(A) = \frac{1}{\sqrt{\text{Det}(-D\partial)}} \exp \left[ -\frac{1}{2} \int dx dy A(x) \omega(x, y) A(y) \right]$$

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QM: particle in L=0 state

$$\Psi(r) = \frac{u(r)}{r} \quad r = \sqrt{J} \quad \int dr r^2 |\Psi(r)|^2 = \int dr |u(r)|^2$$

# Variational approach

## ■ trial ansatz

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gluon propagator

$$\langle A(x) A(y) \rangle = (2\omega(x, y))^{-1}$$

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variational kernel

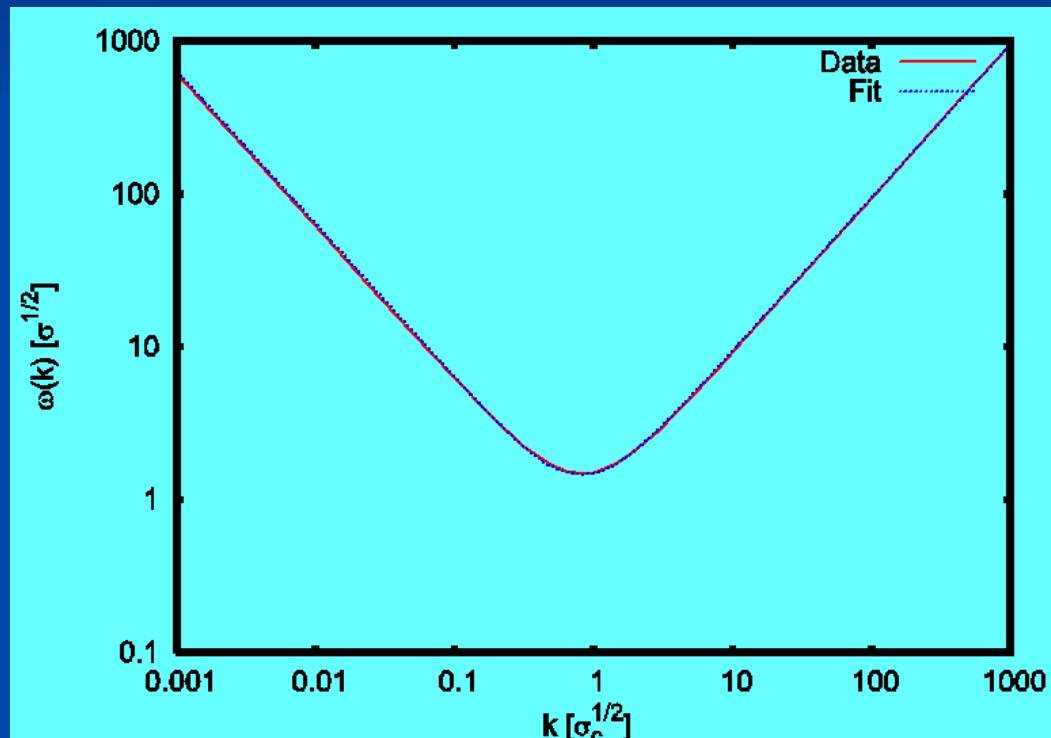
$$\omega(x, x') \quad \text{determined from}$$

$$\langle \Psi | H | \Psi \rangle \rightarrow \min$$

# Numerical results

gluon energy

D. Epple, H. R., W.Schleifenbaum, PRD  
75 (2007)

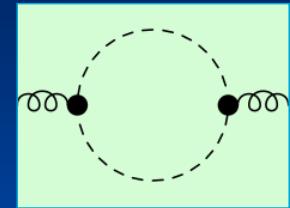


$$IR : \quad \omega(k) \sim 1/k \qquad \qquad UV : \quad \omega(k) \sim k$$

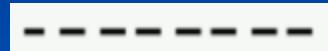
# equation of motion

$$\omega^2(k) = k^2 + \chi^2(k) + \dots \quad \text{---X}$$

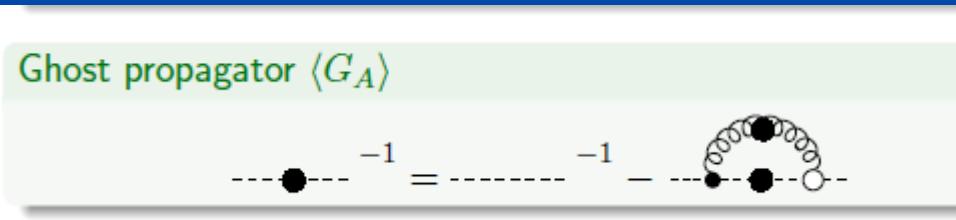
$$\chi(k)$$



*ghost propagator*

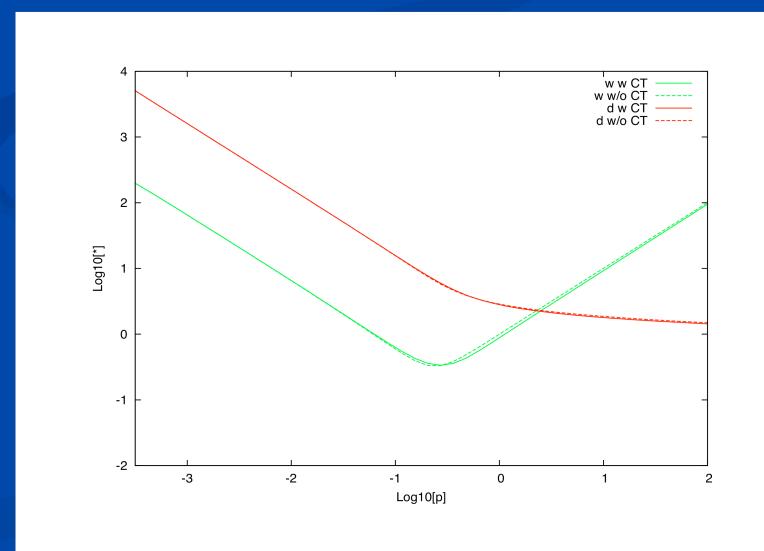


*ghost loop*



$$\left\langle (-D\partial)^{-1} \right\rangle = d / (-\Delta)$$

$$\textit{horizon condition } d^{-1}(0) = 0$$



*horizon condition*  $d^{-1}(0) = 0$  (assumption)

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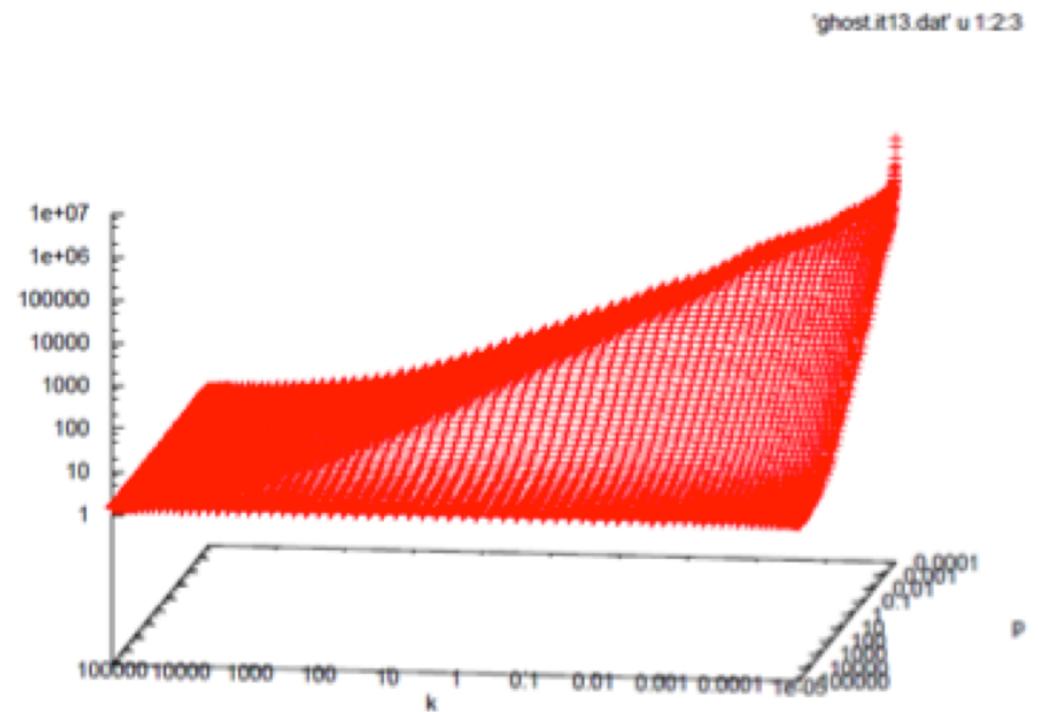
-results from solving the FRG flow eq.

M. Leder, J. Pawłowski,

H. R & A. Weber

Phys. Rev. D83(2011)

Flow of the ghost form factor  $d_k(p)$

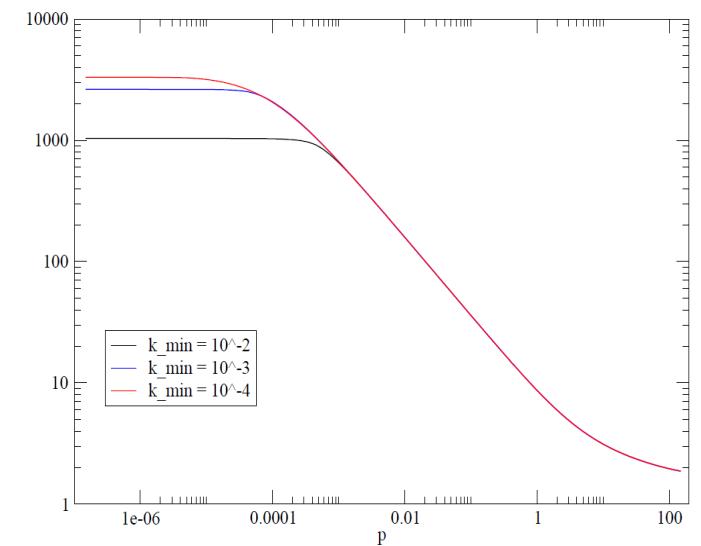
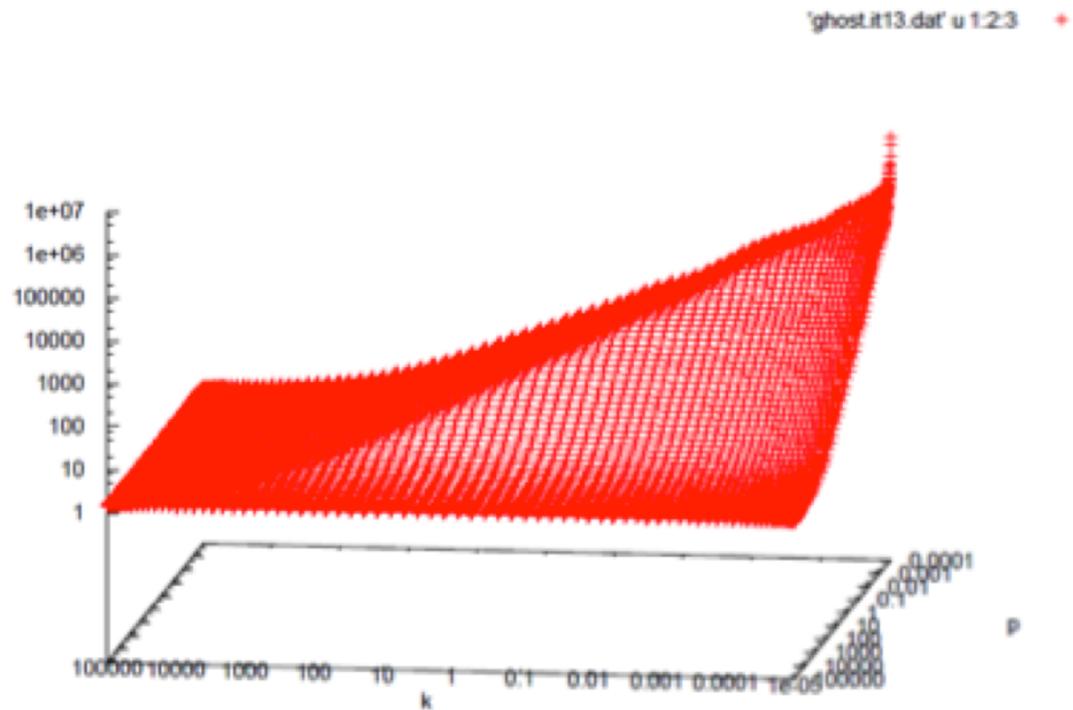


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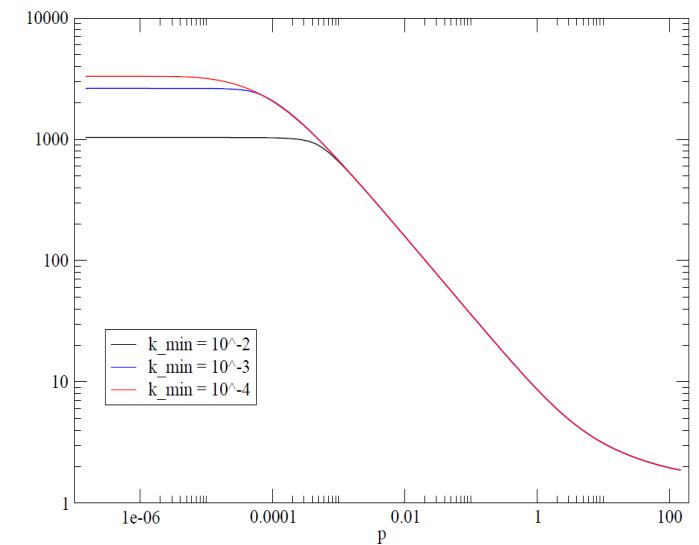
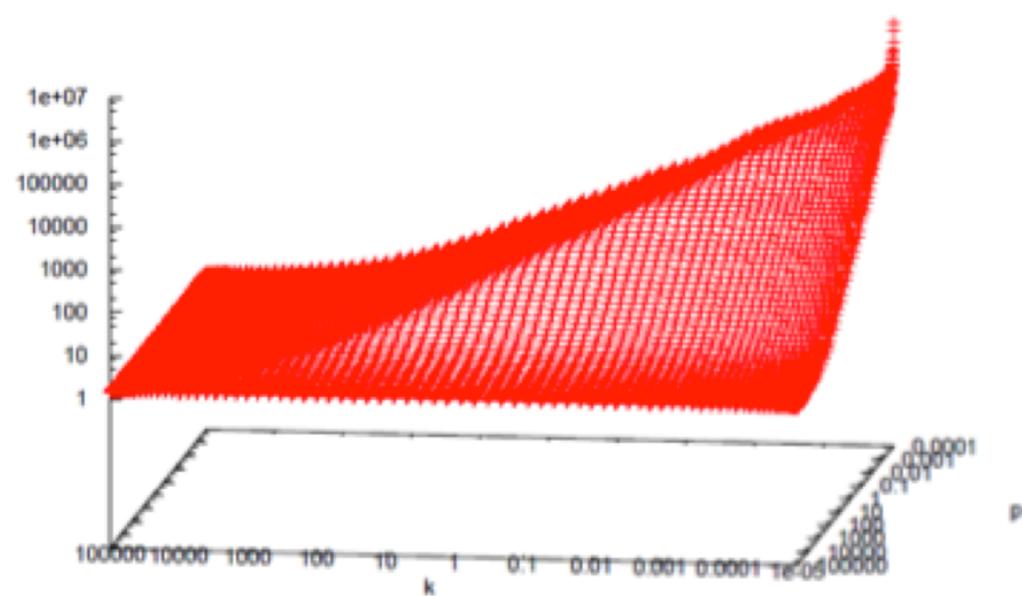


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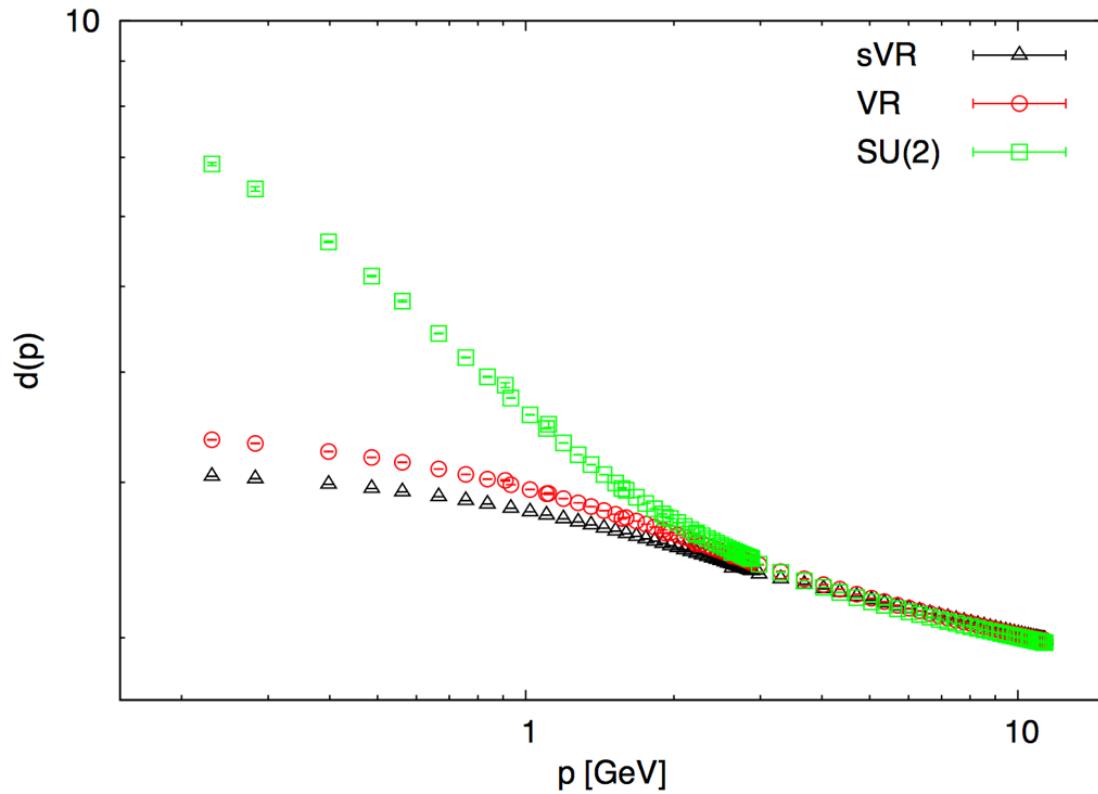
-no assumption in D=2+1

C. Feuchter & H. R Phys. Rev. D77(2008)

-supported by lattice calculation

# Ghost form factor-lattice

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G. Burgio, M. Quandt, H.R. & H.Vogt, Phys. Rev.D92(2015) 034518

*fulfills the horizon condition  $d^{-1}(0)=0$*

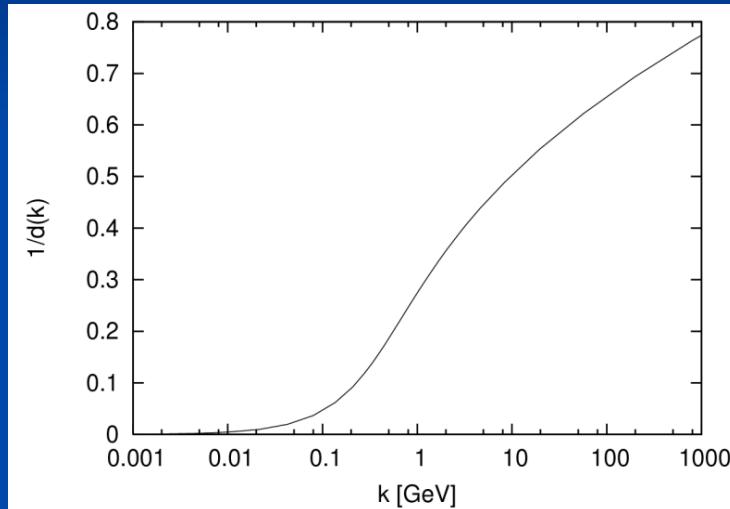
# The color dielectric function of the QCD vacuum

- ghost propagator
- dielectric „constant“

$$\epsilon = d^{-1}$$

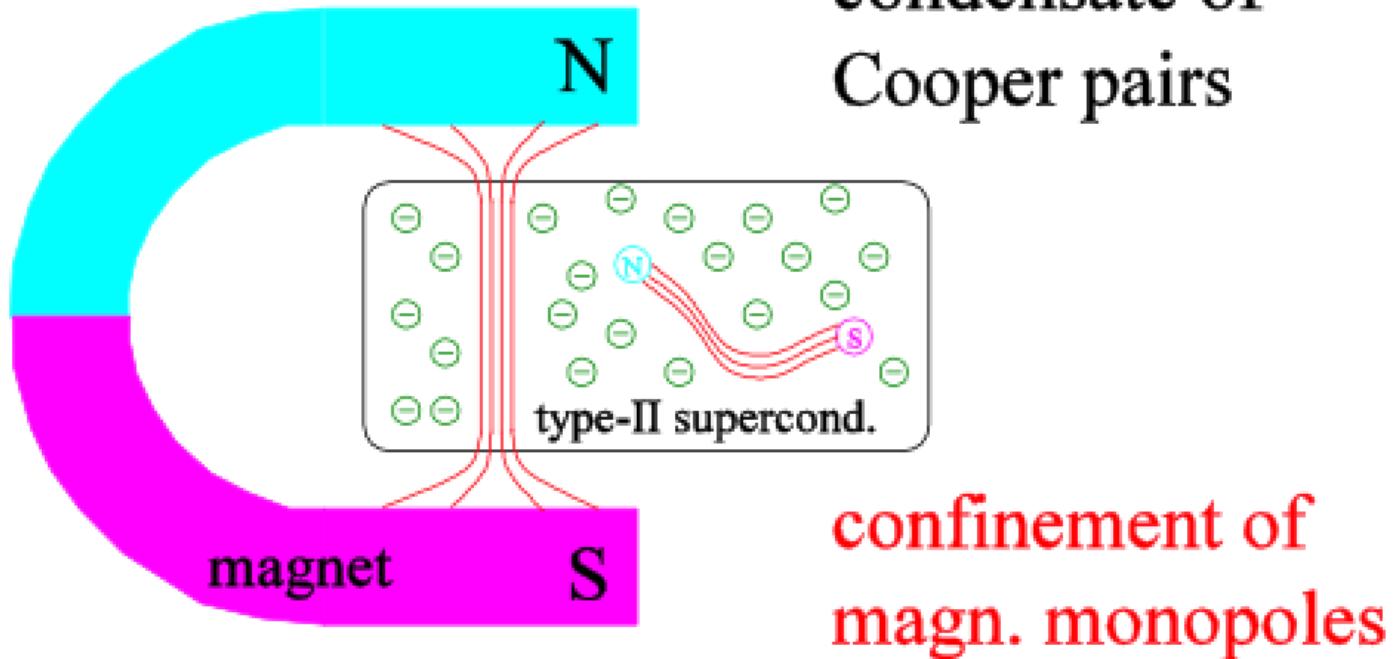
H.R. PRL101 (2008)

$$\langle (-D\partial)^{-1} \rangle = d / (-\Delta)$$

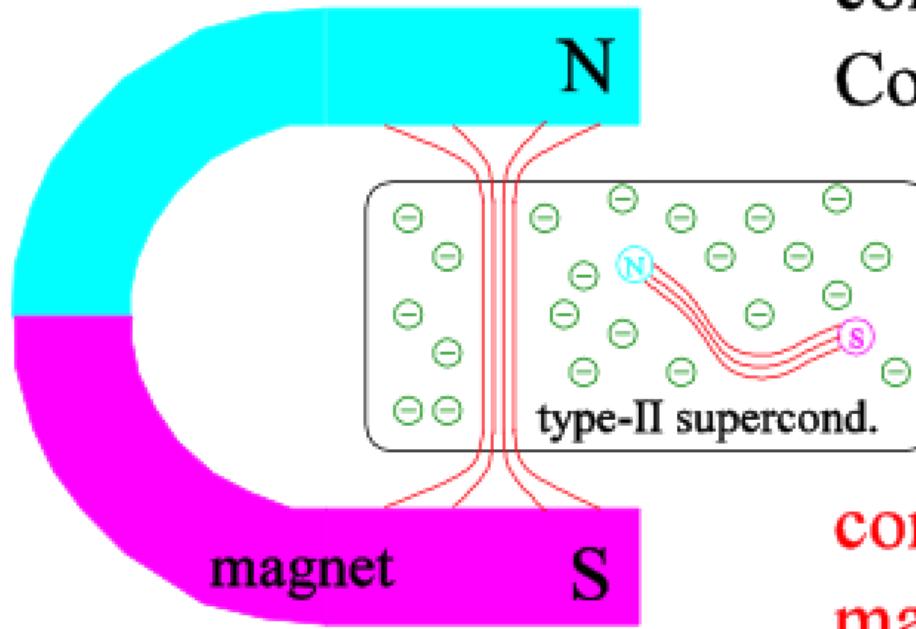


- horizon condition:
  - :  $d^{-1}(k=0)=0$      $\epsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor
  - $\epsilon(k)<1$  anti-screening

# *superconductor*



# *superconductor*



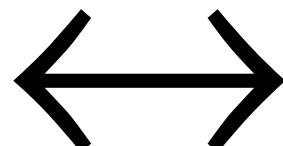
condensate of  
Cooper pairs

confinement of  
magn. monopoles

# *dual superconductor*

*magnetic field*

*magnetic charge*



*electric field*

*electric charge*

‘t Hooft, Mandelstam

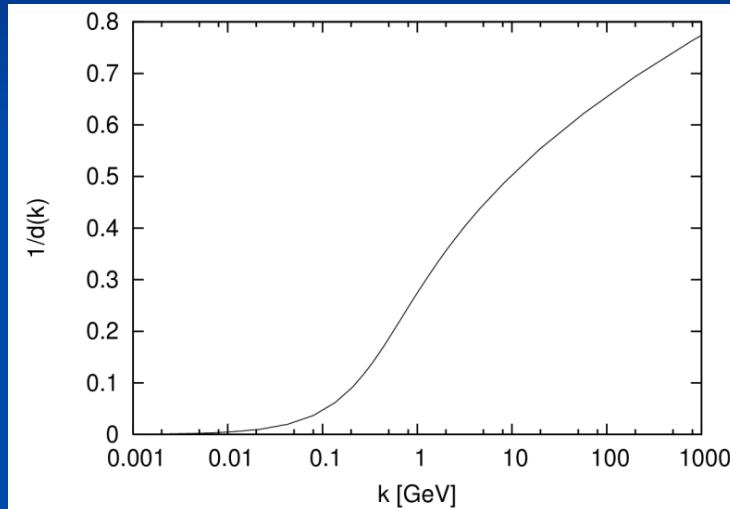
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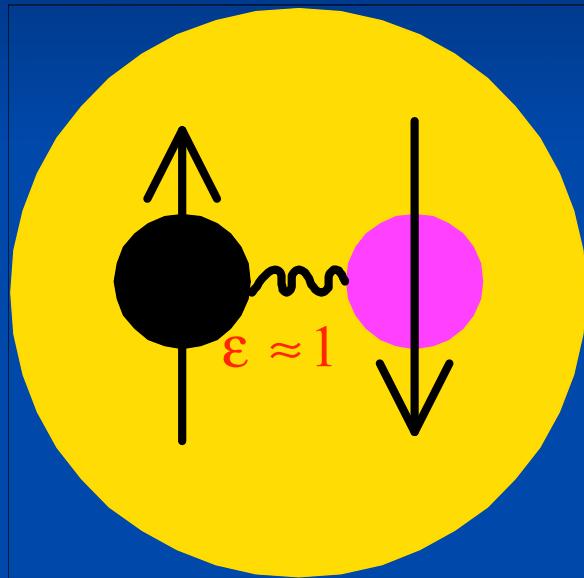
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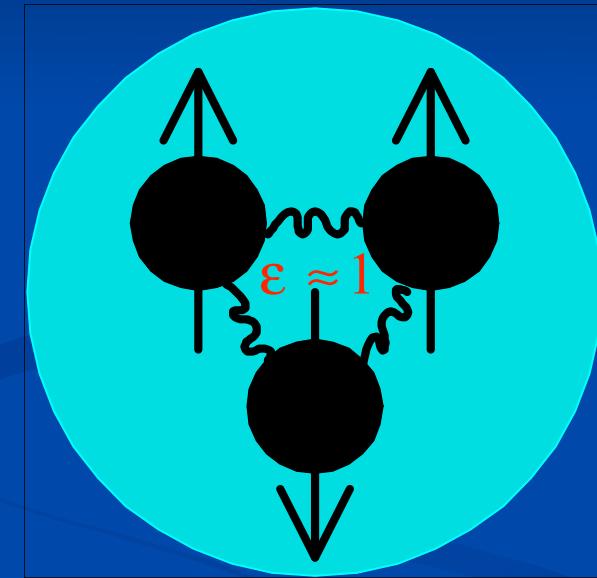
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$$D = \epsilon E$$

$$\partial D = \rho_{free}$$



$$\epsilon = 0$$



no free color charges in the vacuum: confinement

*Gribou's confinement scenario assumes:*

*horizon condition*  $d^{-1}(0) = 0$

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*What are the field configurations  
which induce the horizon condition  
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**-magnetic monopoles**  
*dual superconductor*

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- magnetic monopoles**  
*dual superconductor*
- center vortices**

# Center Vortices

**SU(N)**

**center Z(N)**

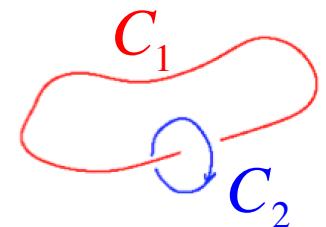
$$z_k = e^{i2\pi k/N} \mathbf{1}_N, \quad k = 0, 1, \dots, N-1,$$

*Wilson loop*

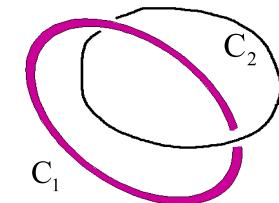
$$W[A](C) = P \exp(i \oint_C A)$$

*center vortex field*

$$W[A(C_1)](C_2) = z^{L(C_1, C_2)}$$



*Gauss linking number L(C1,C2)*



*non-trivial center element*

$$z$$

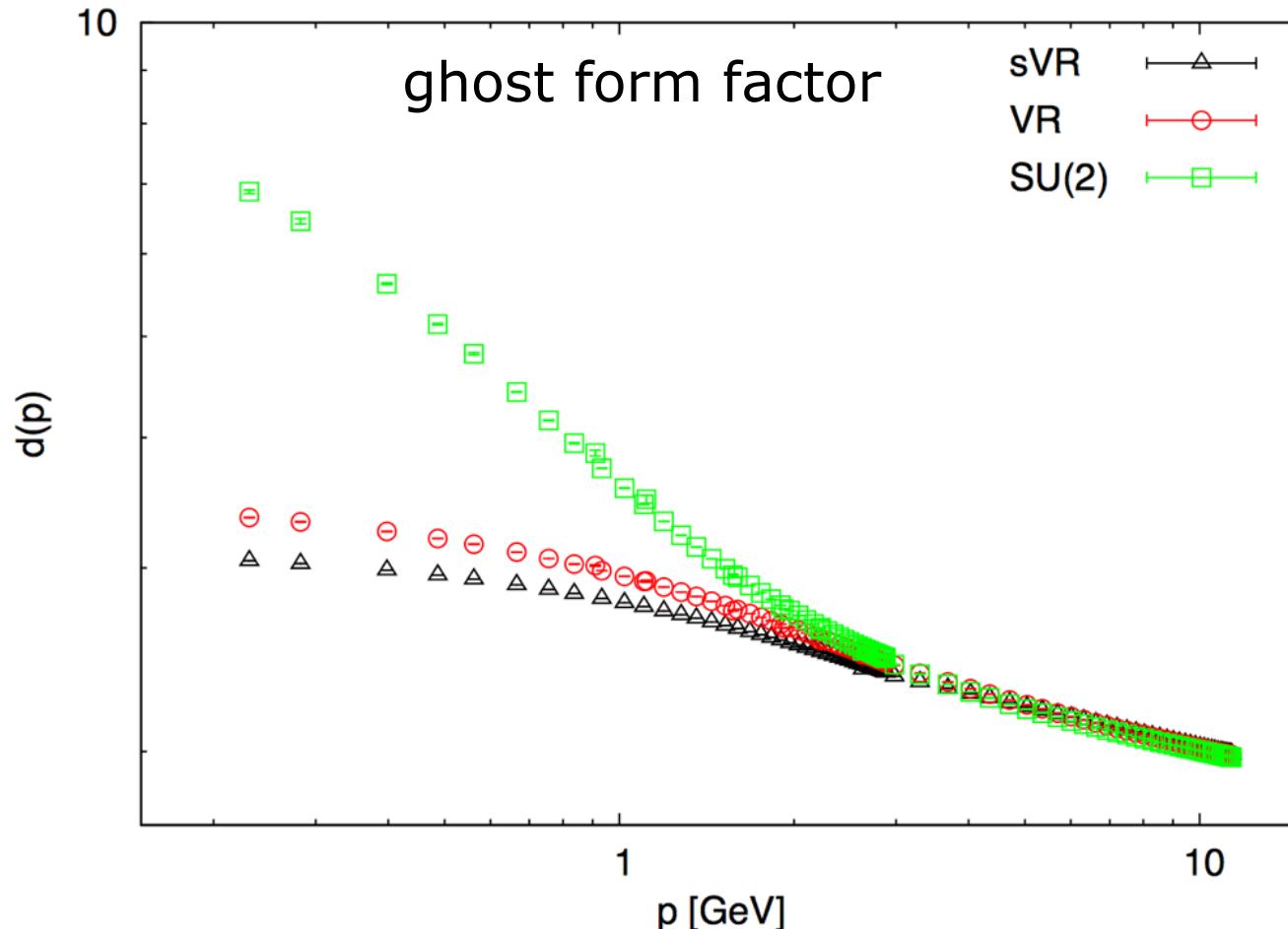
**SU(2)**

**center Z(2)**

$$z = 1, -1$$

$$W[A(C_1)](C_2) = (-1)^{L(C_1, C_2)}$$

# Gribov scenario & center vortex picture



*G. Burgio, M. Quandt,  
H.R. & H.Vogt,  
Phys. Rev.D92(2015)*

- elimination of center vortices: IR enhancement disappears
- horizon condition  $d^{-1}(0)=0$  is lost

*Gribou's confinement scenario assumes:*

*horizon condition  $d^{-1}(0) = 0$*

*What are the field configurations  
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**-magnetic monopoles**

*dual superconductor*

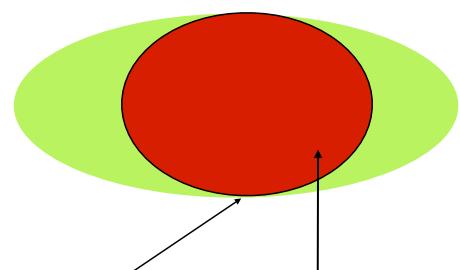
**-center vortices**

*are on the Gribov horizon*

Greensite, Olejnik, Zwanziger

center vortices

FMR



# Static gluon propagator in D=3+1

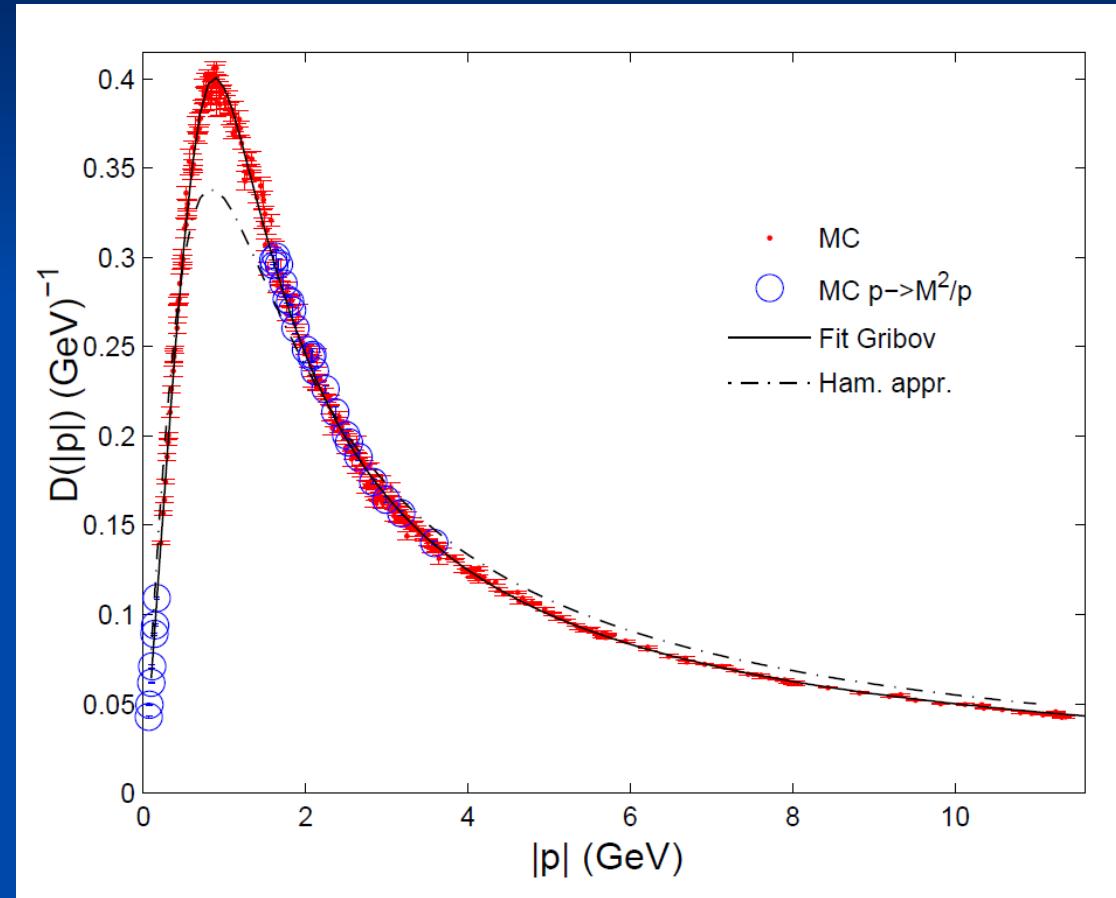
$$D(k) = (2\omega(k))^{-1}$$

*Gribov's formula*

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 \text{ GeV}$$

missing strength in  
mid momentum regime:  
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

# Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R,  
Phys.Rev.D82(2010)

*wave functional*

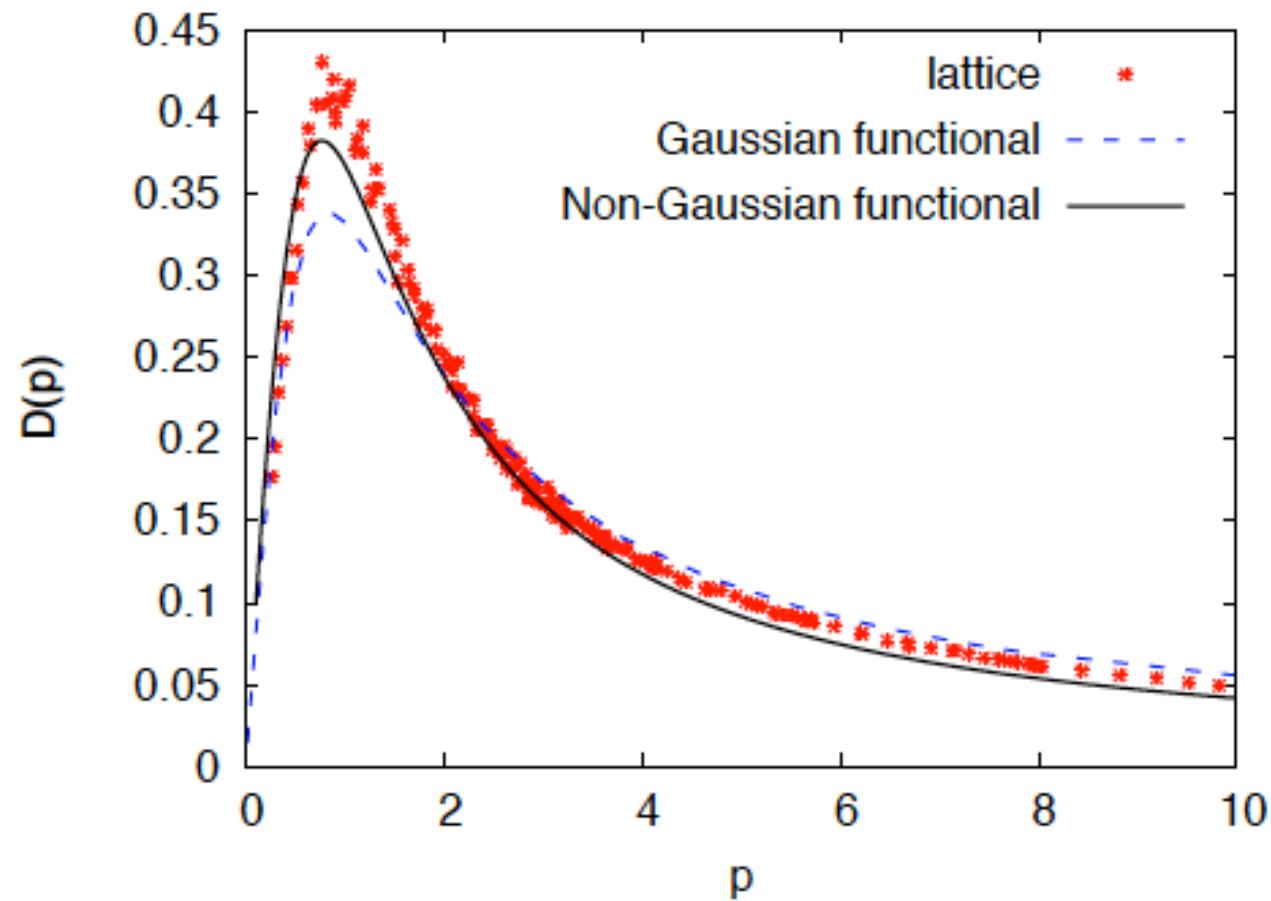
$$|\psi[A]|^2 = \exp(-S[A])$$

*ansatz*

$$S[A] = \int \omega A^2 + \frac{1}{3!} \int \gamma^{(3)} A^3 + \frac{1}{4!} \int \gamma^{(4)} A^4$$

exploit DSE

## Corrections to the gluon propagator



D. Campagnari & H.R, Phys.Rev.D82(2010)

# YM Hamiltonian in $\partial A = 0$

$$H = \frac{1}{2} \int (\mathcal{J}^{-1} \Pi \mathcal{J} \Pi + B^2) + H_C$$

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Christ and Lee

$$J(A^\perp) = \text{Det}(-D\partial) \quad D^{ab} = \delta^{ab} \partial + gf^{abc} A^c$$

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Coulomb term

color charge density  $\rho^a = -f^{abc} A^b \Pi^c + \rho_m^a$

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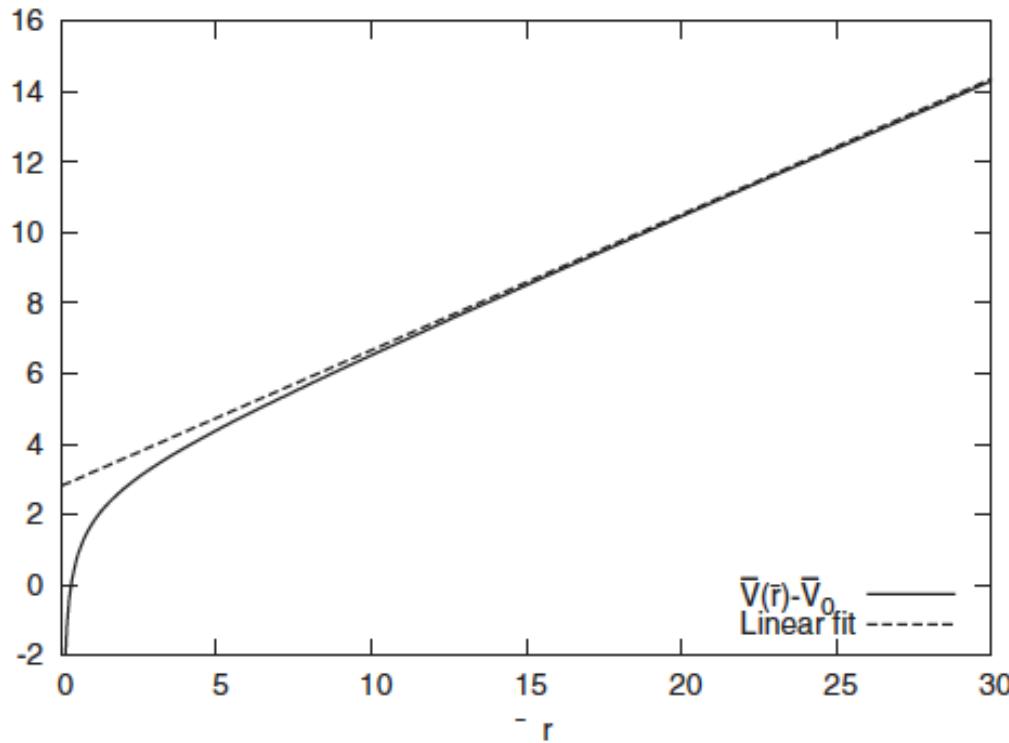
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static quark potential

$$V_C(|\vec{x} - \vec{y}|) = \langle \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle \rangle$$

# Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1} (-\partial^2) (-D\partial)^{-1} | \vec{y} \rangle$$



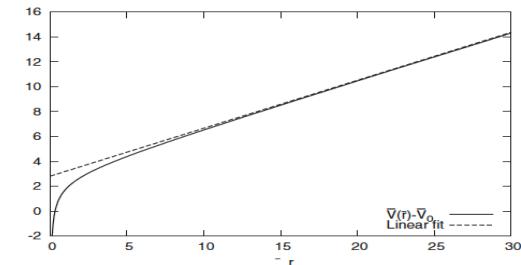
D. Epple, H. Reinhardt  
W. Schleifenbaum,  
PRD 75 (2007)

$$V(r) = \lim_{r \rightarrow 0} \sim 1/r$$

$$V(r) = \lim_{r \rightarrow \infty} \sigma_C r, \quad \text{lattice: } \sigma_C = 2 \dots 3 \sigma_W$$

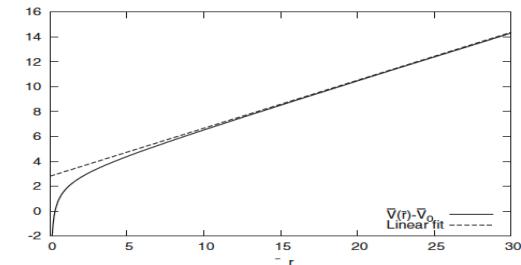
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$$\xrightarrow[|\vec{x}-\vec{y}| \rightarrow \infty]{} \sigma_C |\vec{x} - \vec{y}|$$



# Non-Abelian Coulomb potential

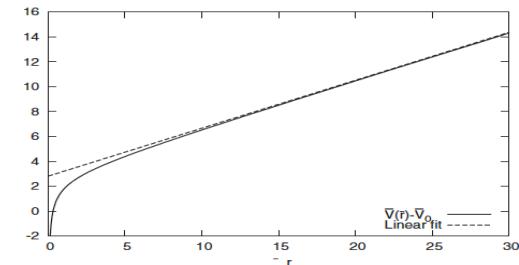
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D. Zwanziger     $\sigma_C \geq \sigma_W$

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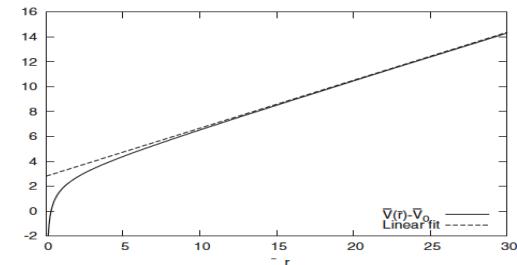
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lattice:  $\sigma_C = 2 \dots 3 \sigma_W$

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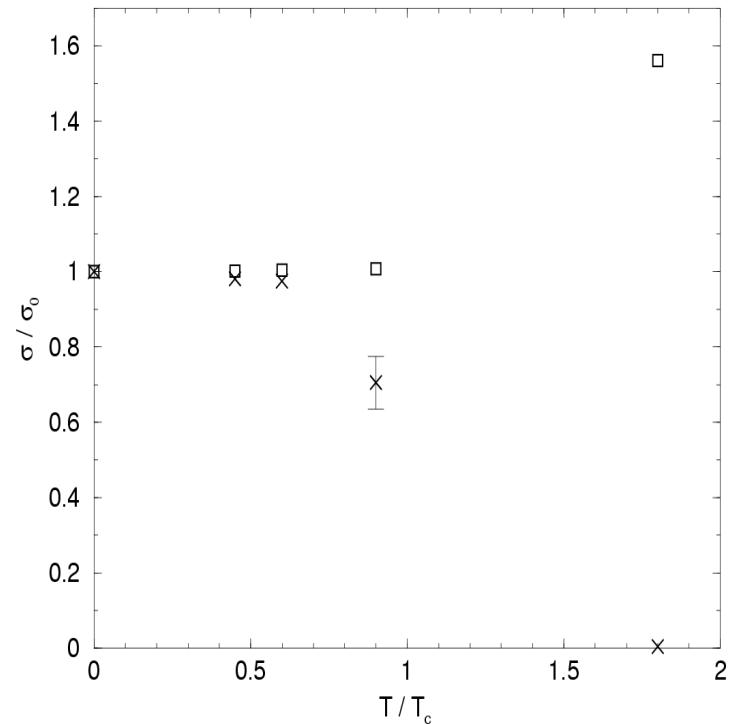
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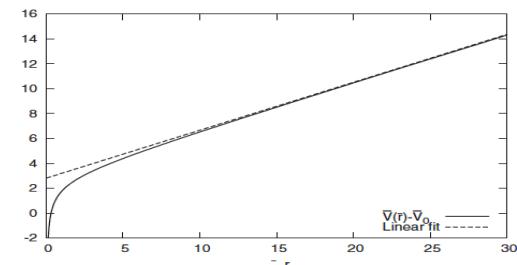
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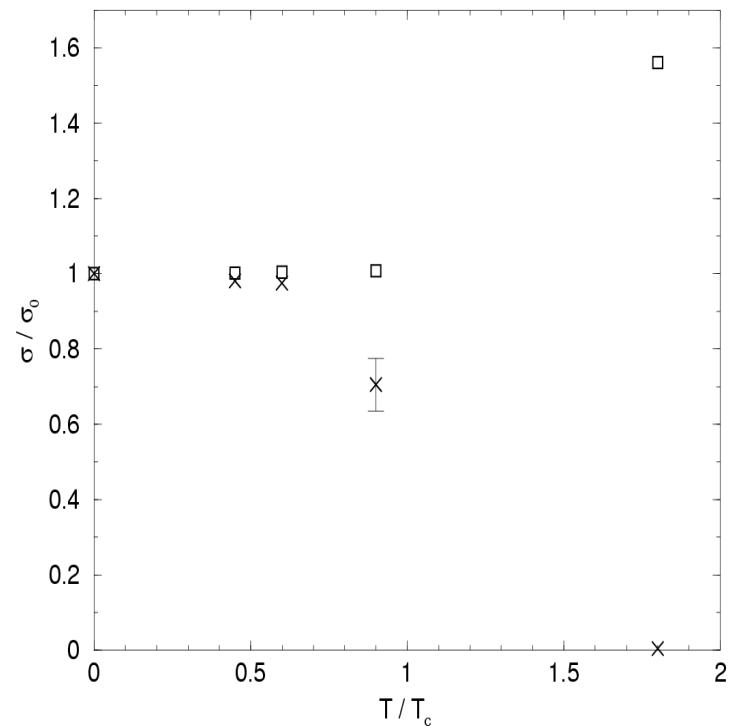


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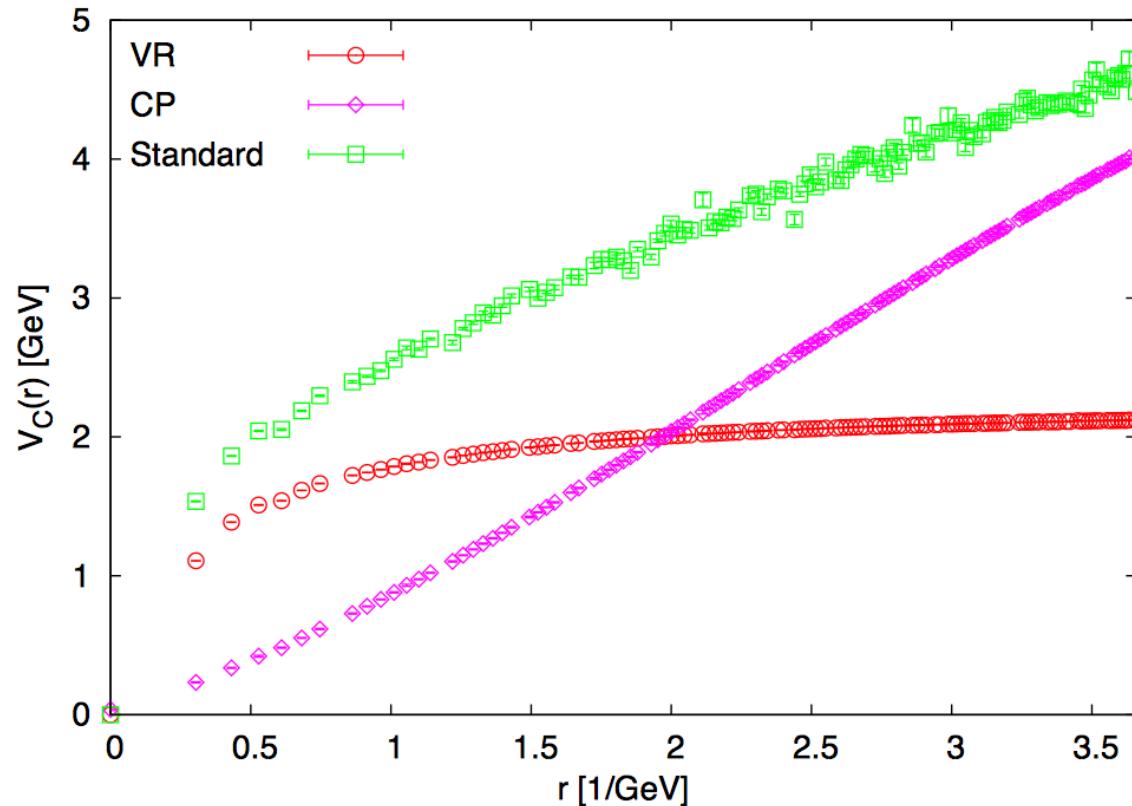
*G. Burgio, M. Quandt,  
H. R. & H. Vogt  
Phys.Rev.D92(2015)*

$\sigma_C \neq \sigma_T$        $\sigma_C \sim \sigma_S$



# Gribov scenario & center vortex picture

- Coulomb potential



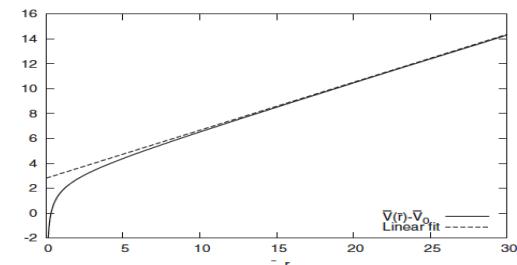
*G. Burgio, M. Quandt,  
H. R. & H. Vogt,  
Phys.Rev.D92(2015)*

- Coulomb string tension disappears after elimination of spatial center vortices

# Non-Abelian Coulomb potential

$$V_C(\vec{x}, \vec{y}) = \langle \vec{x} | (-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1} | \vec{y} \rangle$$

$$\xrightarrow[|\vec{x}-\vec{y}| \rightarrow \infty]{} \sigma_C |\vec{x} - \vec{y}|$$

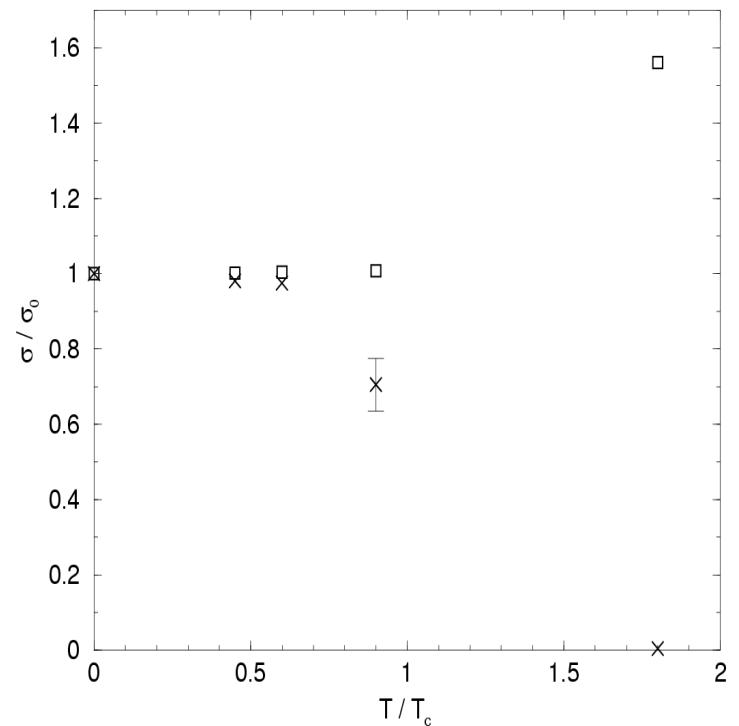


*D. Zwanziger*     $\sigma_C \geq \sigma_W$

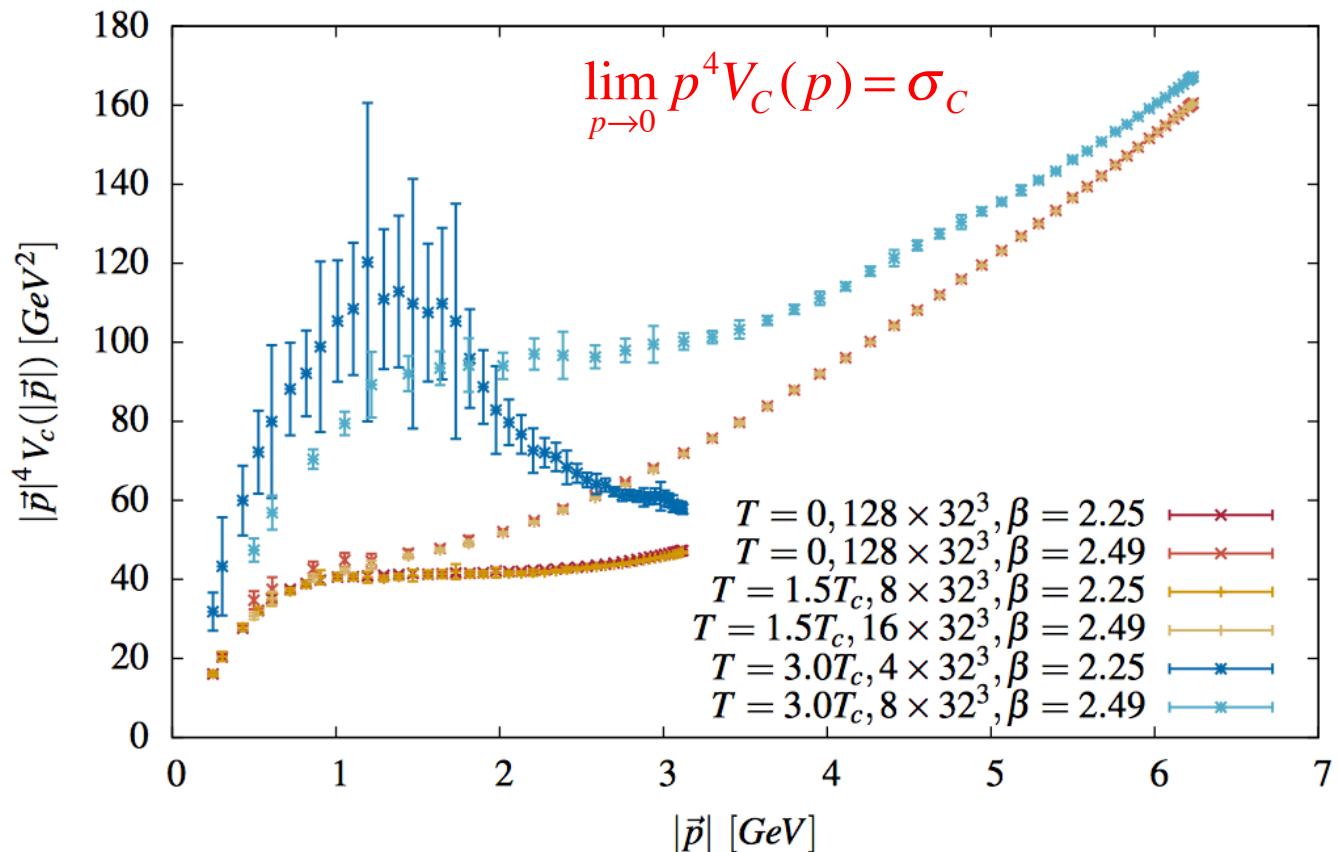
lattice:  $\sigma_C = 2 \dots 3 \sigma_W$

*G. Burgio, M. Quandt,  
H. R. & H. Vogt  
Phys.Rev.D92(2015)*

$\sigma_C \neq \sigma_T$        $\sigma_C \sim \sigma_S$



# non-Abelian Coulomb potential at finite T

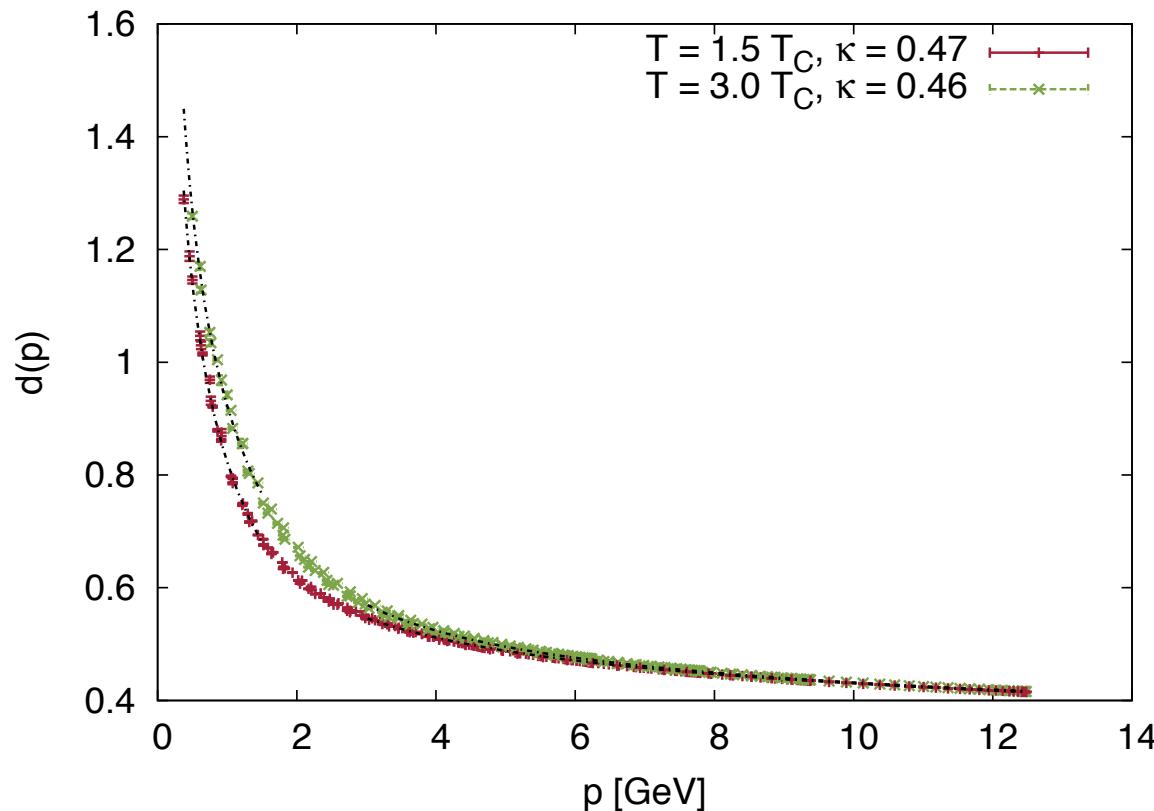


*H. Vogt, G. Burgio, M. Quandt & H. R.  
PoS Lattice(2014)363*

*Y. Nakagawa, A. Nakamura, T. Saito, H. Toki, and D. Zwanziger,  
Phys. Rev., D73:094504, 2006.*

# ghost form factor at finite T

---



*H. Vogt, G. Burgio, M. Quandt & H. R.  
PoS Lattice(2014)363*

# Hamiltonian approach to QCD in Coulomb gauge

*M. Pak & H.R. Phys.Rev.D88(2013)*

*P. Vastag, H. R. & D.Campagnari  
Phys.Rev.D93(2016)*

# Hamiltonian approach to YMT at finite T

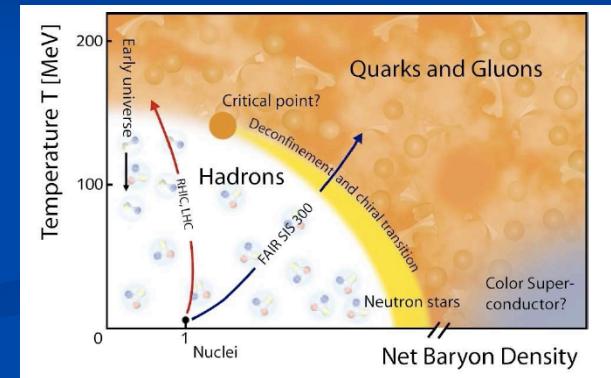
Reinhardt, Campagnari, Szczeplaniak, PRD84(2011)  
Heffner, Reinhardt, Campagnari, Phys. Rev D85(2012)

## ■ Grand canonical ensemble with $\mu = 0$

- quasi-particle variational ansatz for the density matrix

$$D = \exp(-\tilde{H} / T)$$

- minimization of the free energy

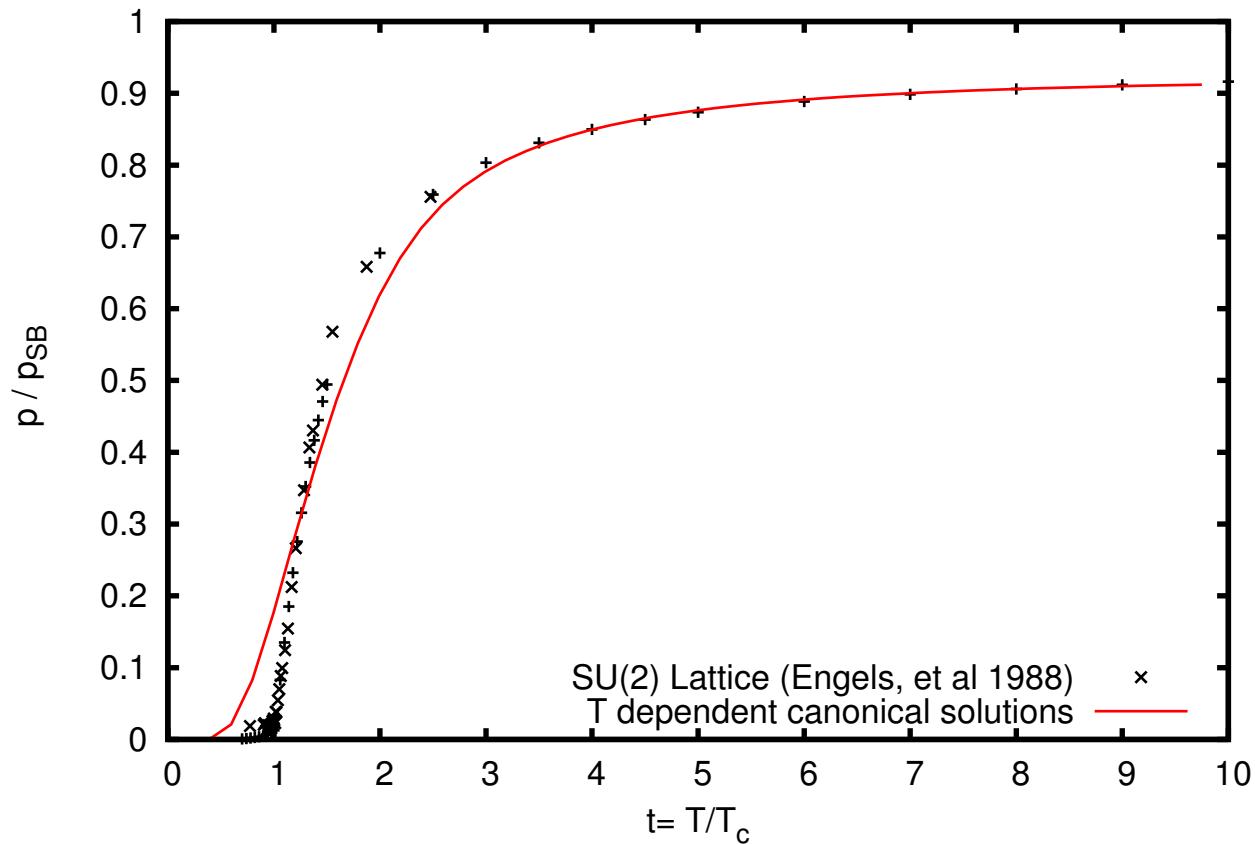


$$F = \langle H \rangle_T - TS \rightarrow \min \quad \langle \dots \rangle_T = \frac{\text{Tr}(D\dots)}{\text{Tr}D}$$

- entropy

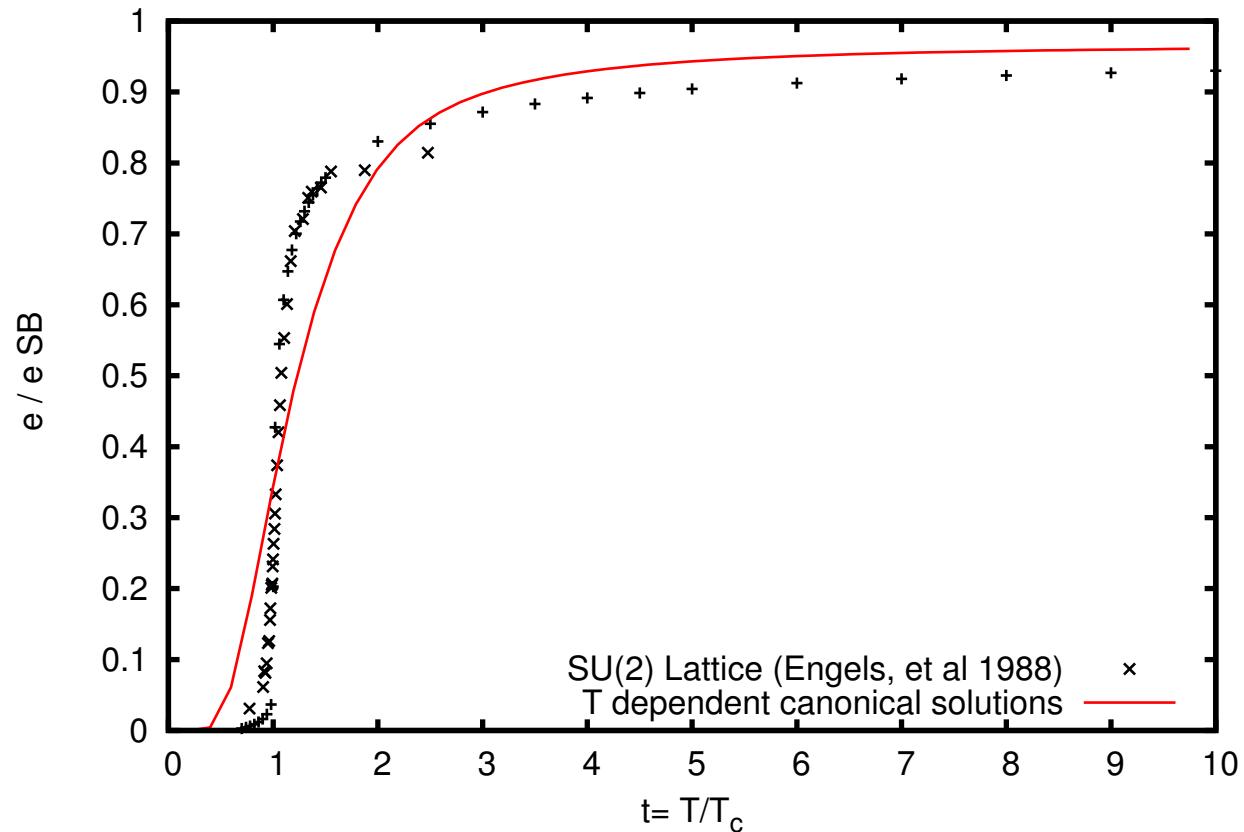
$$S = -\text{Tr}(D \ln D) / \text{Tr}D = -\langle \ln D \rangle_T$$

# pressure-grand canonical (self-consistent)

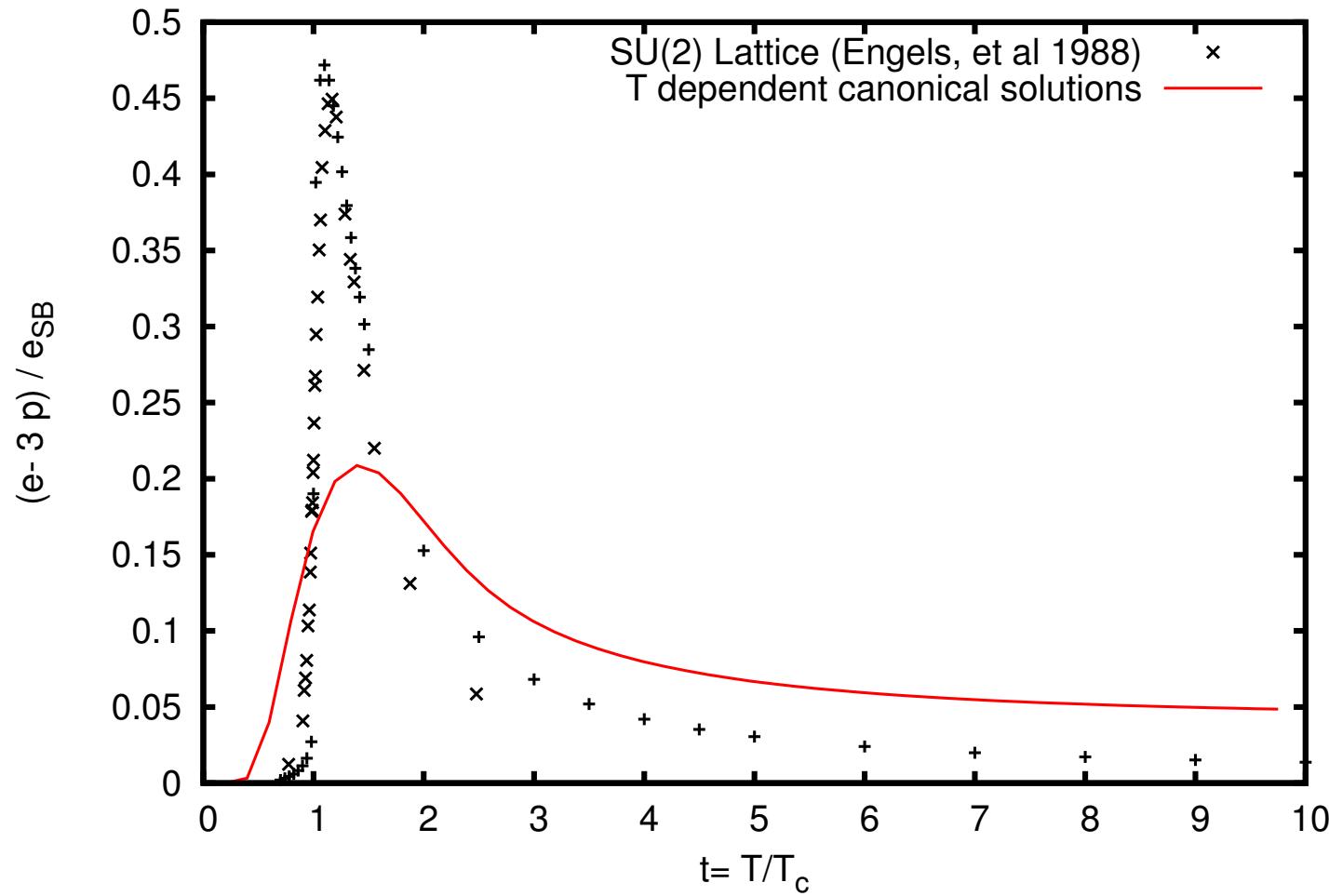


J.Heffner & H.Reinhardt, to be published

# energy density-grand canonical (self-consistent)



# interaction measure-grand canonical (self-consistent)



# quasi-gluon gas

D.Zwanziger  
PRL94(2005)

*Gribov's formula*

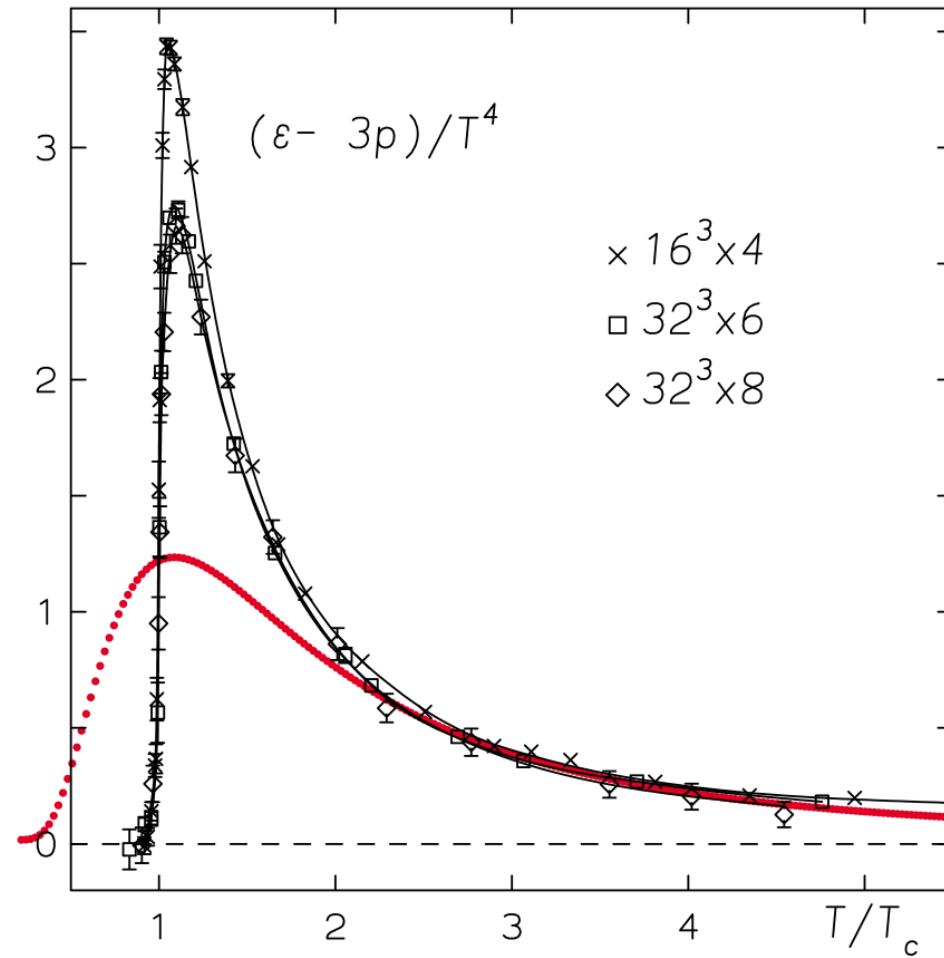
$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

*fit M to  $T_c$ :*

$$M = 2.6T_c = 705\text{ MeV}$$

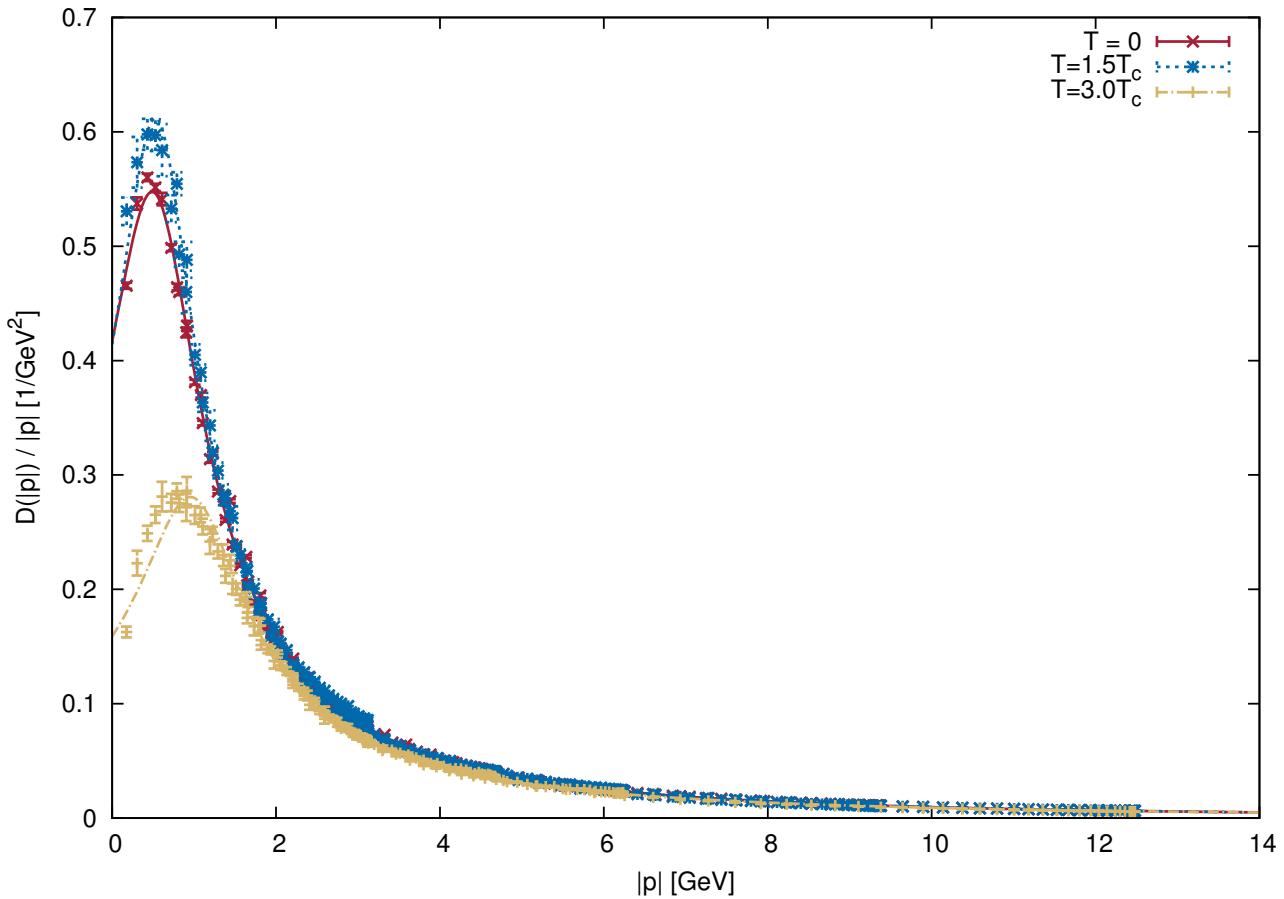
$$\text{lattice: } M = 880\text{ MeV}$$

G.Burgio, M.Quandt , H.R.  
PRL102(2009)



# gluon propagator at finite T

$$\begin{aligned} D(p)/p &= \frac{1}{2\omega(p)p} \\ &= \frac{1}{2\sqrt{p^4 + M^4}} \end{aligned}$$



*H. Vogt, G. Burgio, M. Quandt & H. R.  
PoS Lattice(2014)363*

# Alternative Hamiltonian approach to finite temperature QFT

H. Reinhardt arXiv:1604.06273

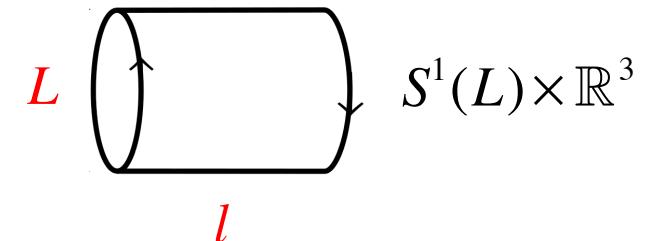
- no ansatz for the density matrix required
- motivation: Polyakov loop

$$P[A_0](\vec{x}) = \frac{1}{d_r} \text{tr} P \exp \left[ i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

- $\langle P[A_0] \rangle$  order parameter of confinement
- Hamiltonian approach
  - Weyl gauge  $A_0=0$
- How to calculate the Polyakov loop in the Hamiltonian approach?

# Finite temperature QFT

- compactification of (Euclidean) time
- bc:  $A(x^0 = L/2) = A(x^0 = -L/2)$  Bose fields  
 $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$  Fermi fields
- temperature  $T = L^{-1}$   $l \rightarrow \infty$



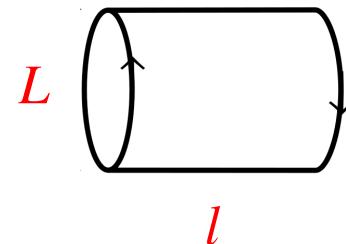
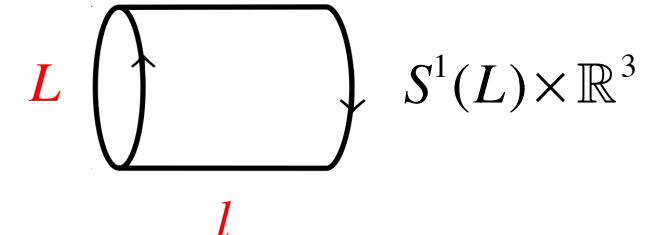
# Finite temperature QFT

- compactification of (Euclidean) time
- bc:  $A(x^0 = L/2) = A(x^0 = -L/2)$  Bose fields  
 $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$  Fermi fields
- temperature  $T = L^{-1}$   $l \rightarrow \infty$
- exploit the  $O(4)$ -invariance of the Euclidean Lagrangian

- $O(4)$ -rotation  $x^0 \rightarrow x^3$   $A^0 \rightarrow A^3$   $\gamma^0 \rightarrow \gamma^3$   
 $x^1 \rightarrow x^0$   $A^1 \rightarrow A^0$   $\gamma^1 \rightarrow \gamma^0$

- one compactified spatial dimension
- bc:  $A(x^3 = L/2) = A(x^3 = -L/2)$  Bose fields  
 $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$  Fermi fields

- spatial manifold:  $\mathbb{R}^2 \times S^1(L)$



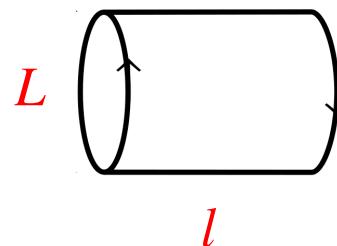
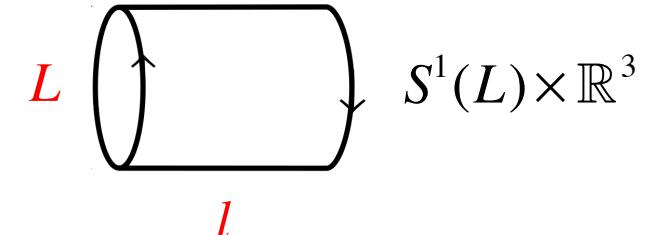
# Finite temperature QFT

- compactification of (Euclidean) time
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- one compactified spatial dimension
- bc:  $A(x^3 = L/2) = A(x^3 = -L/2)$  Bose fields  
 $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$  Fermi fields
- spatial manifold:  $\mathbb{R}^2 \times S^1(L)$

*Hamiltonian approach*



- *temperature is now encoded in one „spatial“ dimension while „time“ has infinite extension independent of the temperature*

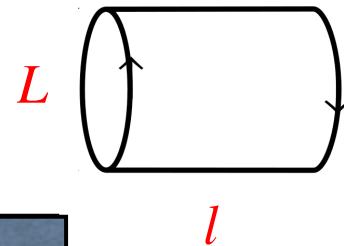
# Finite temperature QFT

- partition function

$$Z(L) = \lim_{l \rightarrow \infty} \text{Tr} \exp(-lH(L)) = \lim_{l \rightarrow \infty} \sum_n \exp(-lE_n(L)) = \lim_{l \rightarrow \infty} \exp(-lE_0(L))$$

- ground state energy  $E_0(L) = l^2 L e(L)$

- on the spatial manifold:  $\mathbb{R}^2 \times S^1(L)$



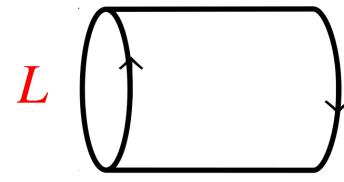
# Finite temperature QFT

- partition function

$$Z(L) = \lim_{l \rightarrow \infty} Tr \exp(-lH(L)) = \lim_{l \rightarrow \infty} \sum_n \exp(-lE_n(L)) = \lim_{l \rightarrow \infty} \exp(-lE_0(L))$$

- ground state energy  $E_0(L) = l^2 L e(L)$

- on the spatial manifold:  $\mathbb{R}^2 \times S^1(L)$



- pressure:

$$p = -e(L)$$

- energy density:

$$\varepsilon = \partial [Le(L)] / \partial L - \mu \partial e / \partial \mu$$

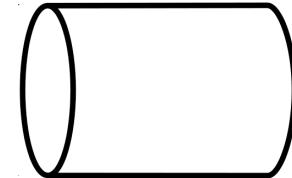
- Dirac fermions with finite chemical potential

$$h = \vec{\alpha} \cdot \vec{p} + \beta m \rightarrow h + i\mu\alpha^3$$

# The Polyakov loop - order parameter of confinement

$$P[A_0](\vec{x}) = \frac{1}{d_r} \text{tr} P \exp \left[ i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$$T^{-1} = L$$



Polyakov gauge  $\partial_0 A_0 = 0, A_0 = \text{diagonal}$

$$SU(2): P[A_0](\vec{x}) = \cos(\tfrac{1}{2} A_0(\vec{x}) L)$$

$P[A_0]$  – unique function of  $A_0$

## alternative order parameters of confinement

$$\langle P[A_0](\vec{x}) \rangle \quad P[\langle A_0 \rangle](\vec{x}) \quad \langle A_0(\vec{x}) \rangle$$

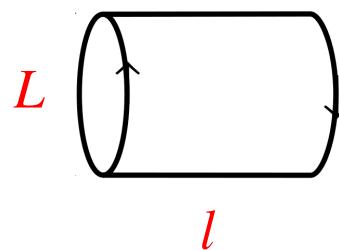
- J. Braun, H. Gies, J. M. Pawłowski, Phys. Lett. B684(2010)262

# Effective potential of the order parameter for confinement

- background field calculation  $a_0 = \langle A_0(\vec{x}) \rangle - \text{const}$ , diagonal (Polyakov gauge)
- effective potential  $e[a_0] \rightarrow \min \Rightarrow a_0 = \bar{a}_0$
- order parameter  $\langle P[A_0] \rangle \approx P[\bar{a}_0]$
- ordinary Hamiltonian approach assumes Weyl gauge  $A_0 = 0$
- $O(4)$ -invariance

▪ compactify (instead of time) one spatial axis to a circle of circumference  $L$  and interpret  $L^{-1}$  as temperature

- Hamiltonian approach on  $\mathbb{R}^2 \times S^1(L)$



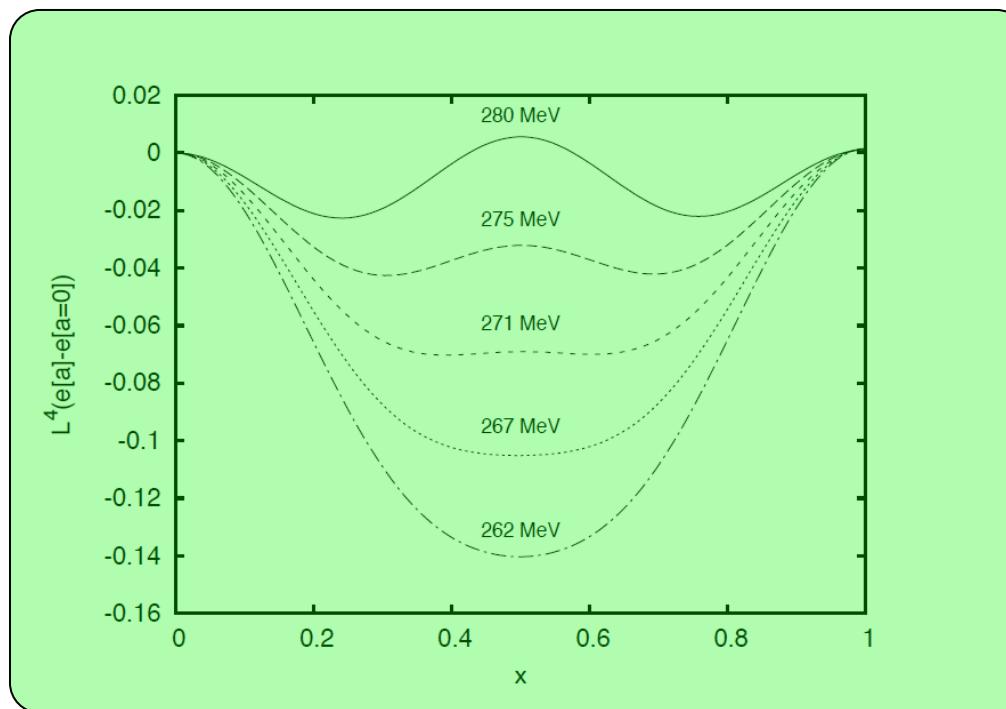
- compactify  $x_3$  – axis  $\vec{a} = a\vec{e}_3$

- calculate the effective potential

$e[a]$

# The gluon effective potential $SU(2)$

variational calculation in Coulomb gauge



$$x = \frac{aL}{2\pi}$$

second order phase transition:

$$\text{input : } M = 880 \text{ MeV} \quad T_c \simeq 269 \text{ MeV}$$

## The effective potential for SU(3)

SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive roots

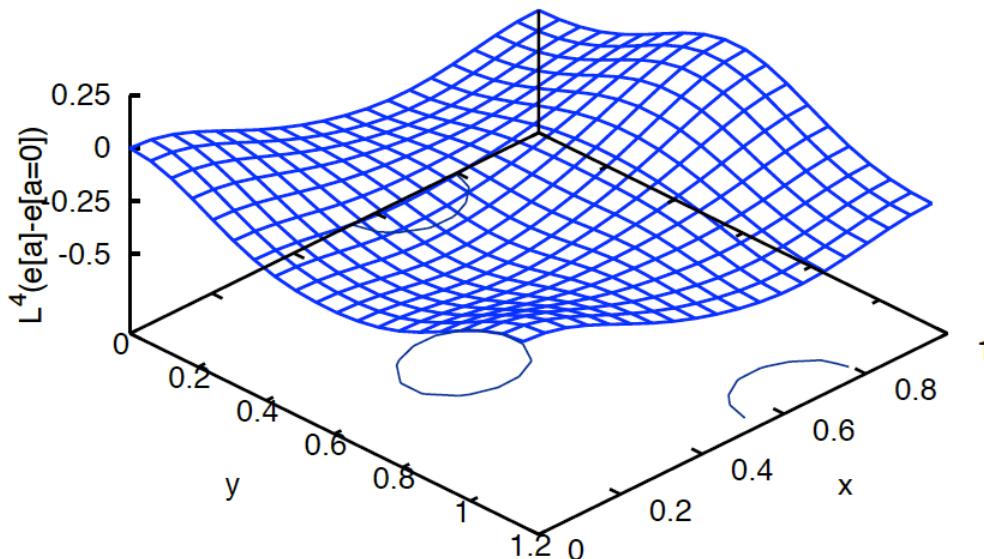
$$\sigma = (1, 0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

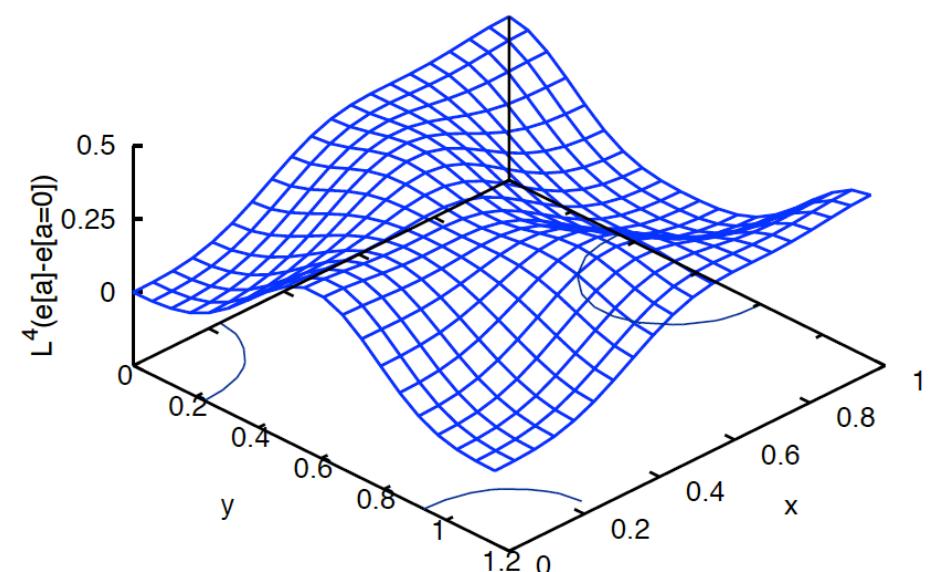
# The full effective potential for SU(3)

variational calculation in Coulomb gauge

$T < T_c$

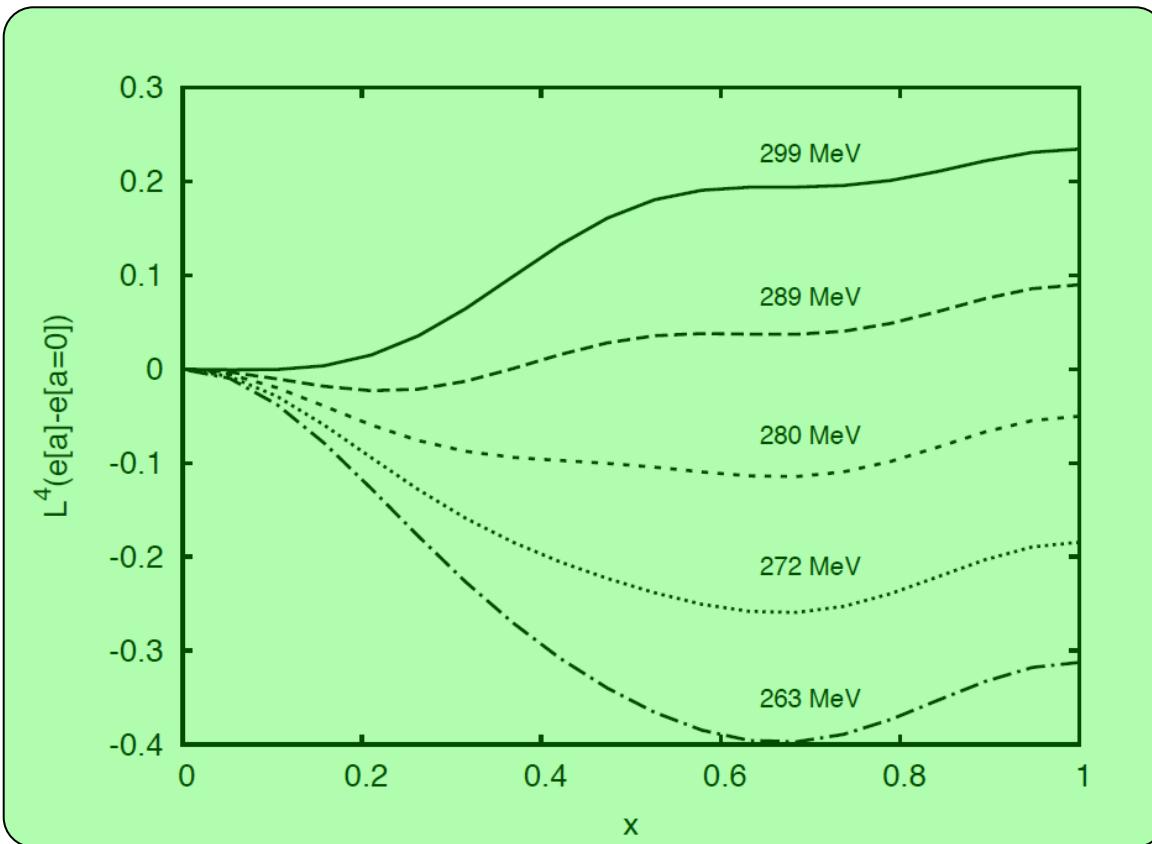


$T > T_c$



$$x = \frac{a_3 L}{2\pi}, \quad y = \frac{a_8 L}{2\pi}$$

# Polyakov loop potential for SU(3)



$$x = \frac{a_3 L}{2\pi}, \quad y = \frac{a_8 L}{2\pi} = 0$$

*input :  $SU(2)$  – data :*  
 $M = 880 \text{ MeV}$

$T_c = 283 \text{ MeV}$

# critical temperature

*lattice:*

$$T_c^{SU(2)} = 312 \text{ MeV} \quad T_c^{SU(3)} = 284 \text{ MeV}$$

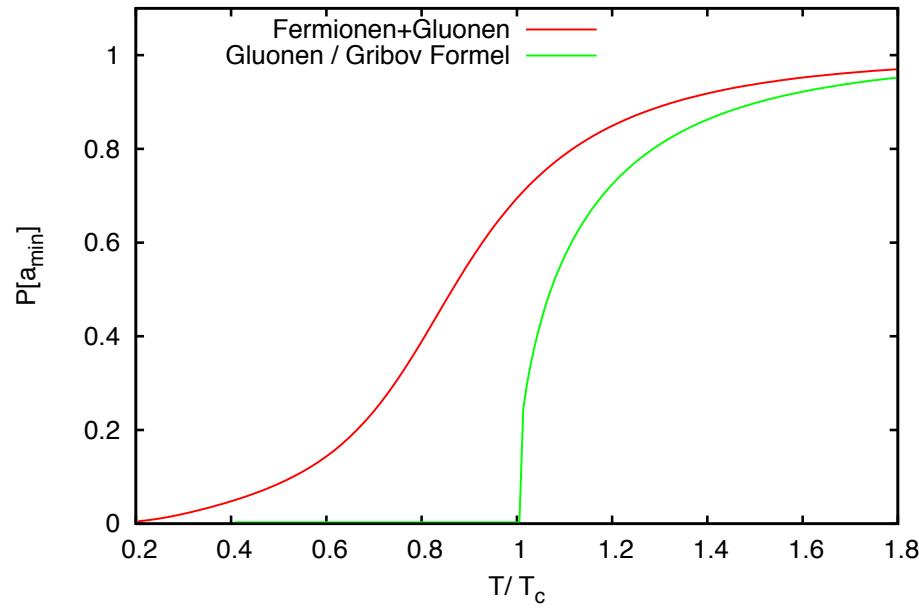
*this work:*

$$T_c^{SU(2)} = 269 \text{ MeV} \quad T_c^{SU(3)} = 283 \text{ MeV}$$

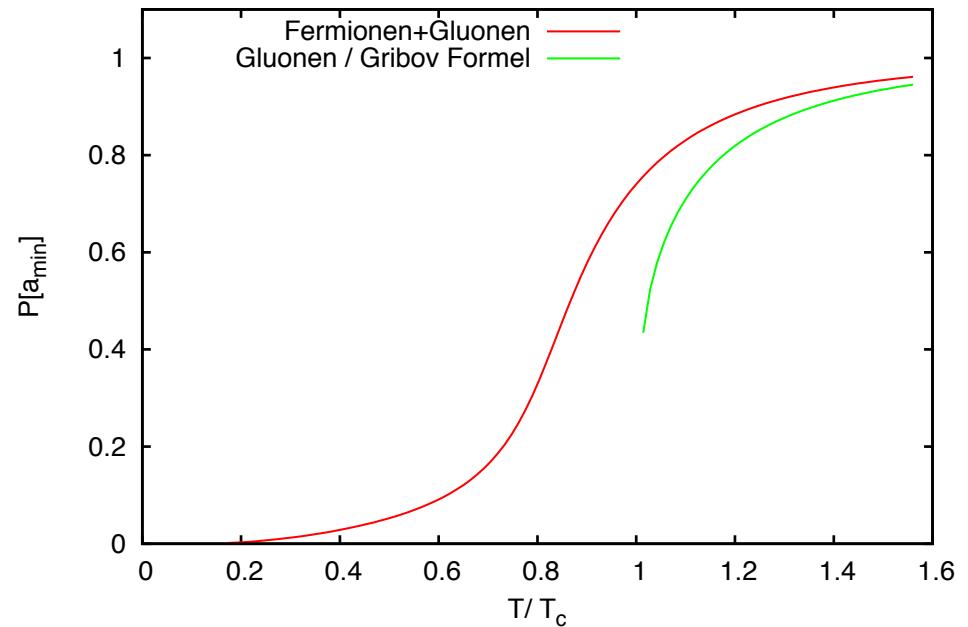
*FRG(Fister & Pawłowski):*  $T_c^{SU(2)} = 230 \text{ MeV}$   $T_c^{SU(3)} = 275 \text{ MeV}$

*lattice: B. Lucini, M. Teper, U. Wenger, JHEP01(2004)061*

# The Polyakov loop



$SU(2)$



$SU(3)$

*J. Heffner, H. Reinhardt & P. Vastag, to be published*

# Conclusions

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- variational approach to the Hamiltonian formulation of YMT in Coulomb gauge
- Gribov-Zwanziger confinement scenario at work
  - inverse ghost form factor = dielectric function
  - Coulomb string tension - spatial string tension
- connection to the alternative confinement pictures:
  - magnetic monopoles
  - center vortices
- Hamiltonian approach to finite temperature YMT
  - grand canonical ensemble
  - compactification of a spatial dimension  $R^2 \times S^1$ 
    - effective potential of the Polyakov loop
      - deconfinement phase transition
      - SU(2): 2.order
      - SU(3): 1.order
    - inclusion of quarks:
      - deconfinement phase transition is turned into a crossover
  - pressure

# Thanks for your attention