Mass generation in Yang-Mills theories: Infrared finite gluon propagators

A.C.Aguilar, D.Binosi, C.T. Figueiredo, and JP: arXiv:1604.08456

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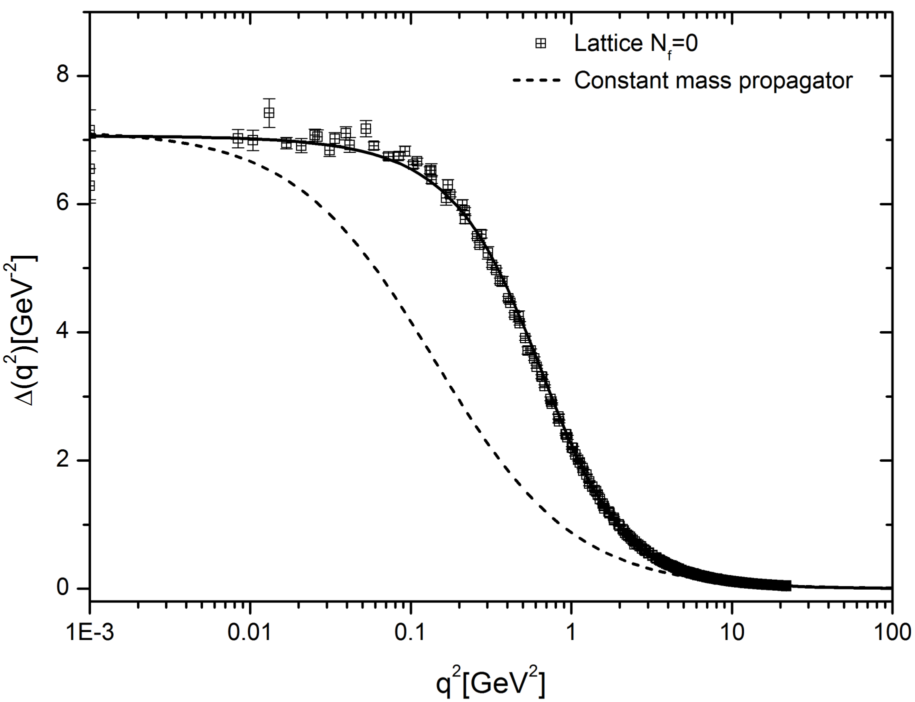
on New Frontiers in Physics, ICNFP2016

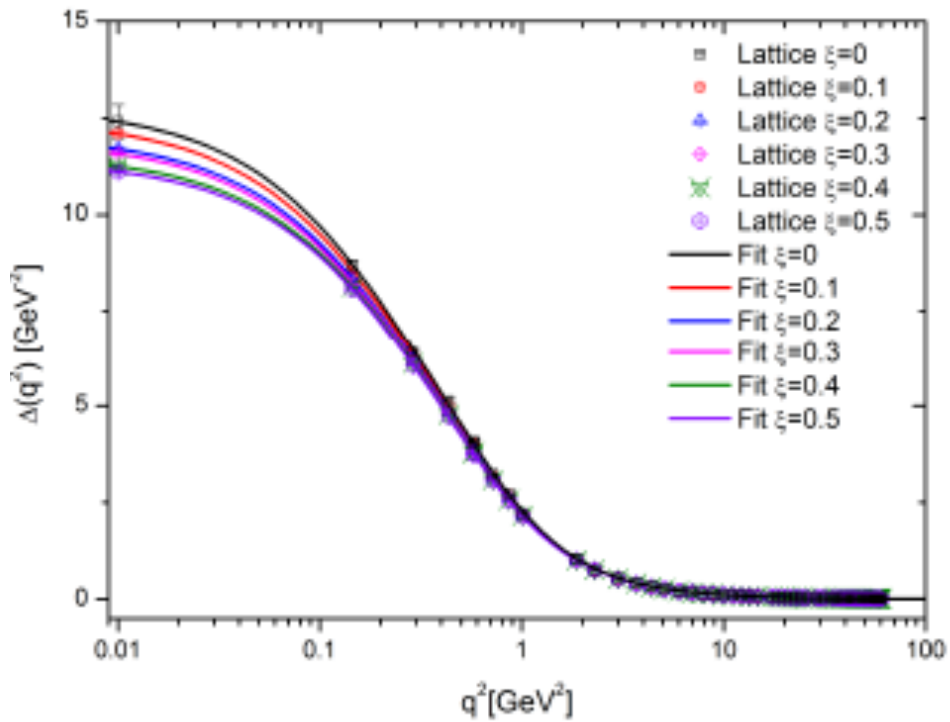
Large volume lattice simulations

Gluon propagator (**in the Landau gauge**) *saturates* in the deep infrared

I.L.Bogolubsky, et al , PoS LAT2007, 290 (2007)



P. Bicudo et al, Phys. Rev.D 92, 114514 (2015)



$$\Delta^{-1}(0) = m^2(0)$$

Saturation explained through dynamical gluon mass generation

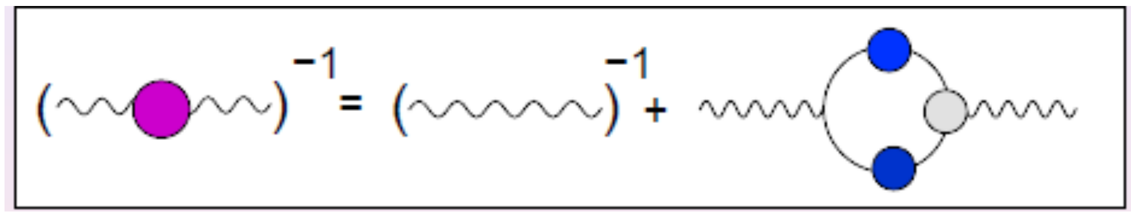
A.C.Aguilar. D.Binosi, and JP: Phys.Rev.D 78 (2008) 025010

Non-perturbative explanation in the continuum : Schwinger-Dyson equations

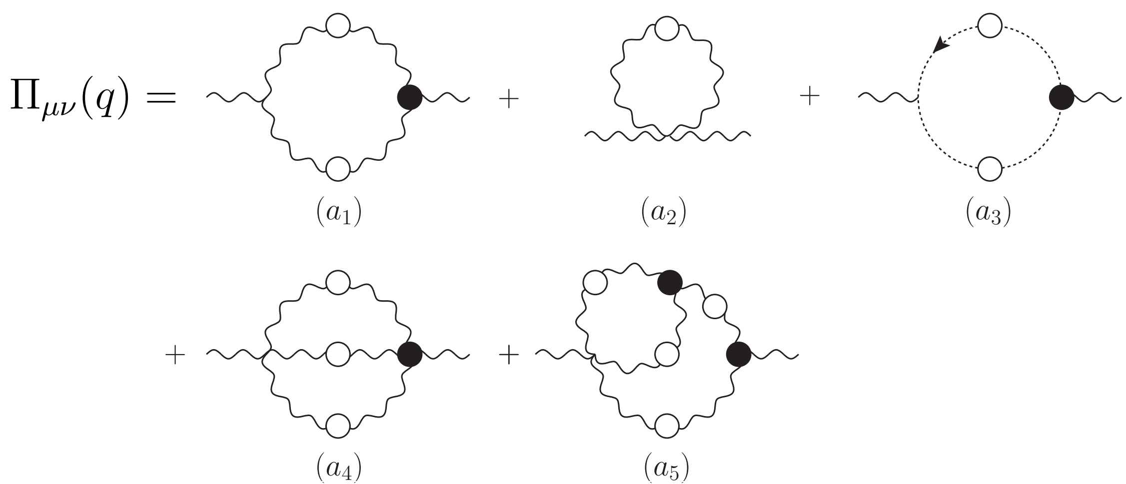
Dynamical equations for off-shell Green's functions

Infinite system of coupled non-linear integral equations

Photon propagator



Gluon self-energy
in linear covariant gauges



Switch from the linear covariant gauges to Background Field Method

BFM reminder: Basic concepts

- The BFM is a special quantization scheme: Split the gauge field

$$A_\mu^a \to \widetilde{A}_\mu^a + A_\mu^a$$

$\widetilde{A}_\mu^a$ → background field; A_μ^a → quantum (fluctuating) field;

- In the generating functional integrate only over A_μ^a
- The gauge fixing condition $G(A)$ is chosen to be → $G(A) = D^\mu A_\mu^a$

  where $D_\mu = \partial_\mu - i\widetilde{A}_\mu^a t^a$ instead of the "standard"→ $G(A) = \partial^\mu A_\mu^a$

- The gauge-fixed $\mathcal{L}$ is invariant under the transformations:

$$\widetilde{A}_\mu^a \to \widetilde{A}_\mu^a + D_\mu \theta^a \qquad\qquad A_\mu^a \to A_\mu^a - f^{abc} A_\mu^a \theta^b$$

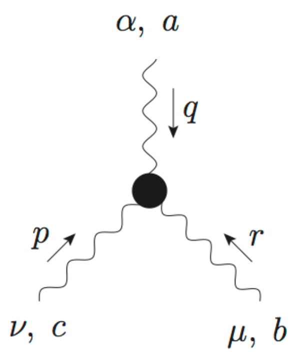
- $\widetilde{A}_\mu^a$ carries the local gauge transformation $\partial^\mu \theta^a$
- A_μ^a transforms as matter field in the adjoint.

B. S. DeWitt, Phys. Rev. 162 (1967) 195—1239

G. 't Hooft, In *Karpacz 1975, Proceedings, Acta Universitatis Wratislaviensis No.368, 1976, 345-369

L. F. Abbott, Nucl. Phys. B185 (1981) 189

Abelian-like Slavnov-Taylor identities



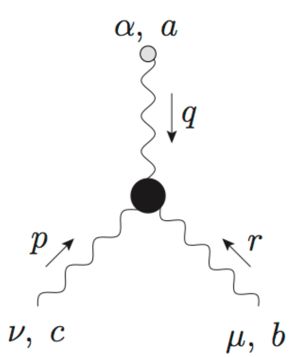
$F(q^2)$: Ghost dressing function

$D(q^2) = F(q^2)/q^2$: Ghost propagator

$H_{\mu\nu}(q,r,p)$: Ghost-gluon kernel

$$q^\alpha \Gamma_{\alpha\mu\nu}(q,r,p) = iF(q)\left[\Delta^{-1}_{\sigma\nu}(r)H^\sigma_\mu(q,r,p) - \Delta^{-1}_{\sigma\mu}(p)H^\sigma_\nu(q,r,p)\right]$$

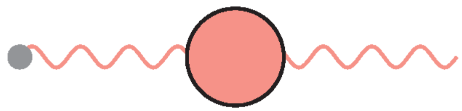
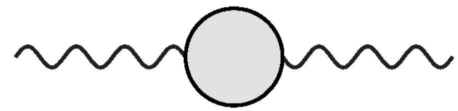
INSTEAD



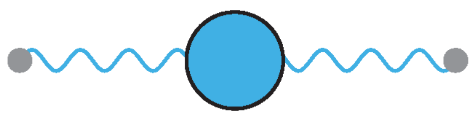
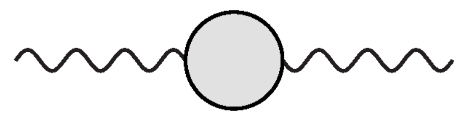
$$q^\alpha \widetilde{\Gamma}_{\alpha\mu\nu}(q,r,p) = i\Delta^{-1}_{\mu\nu}(r) - i\Delta^{-1}_{\mu\nu}(p)$$

Abelian-like (“ghost-free”)

Background and quantum propagators are related !

 $= (\mathbf{1}+\mathbf{G})\otimes$ 

$$\widetilde{\Delta}(q) = [\mathbf{1}+\mathbf{G}(\mathbf{q})]\Delta(q)$$

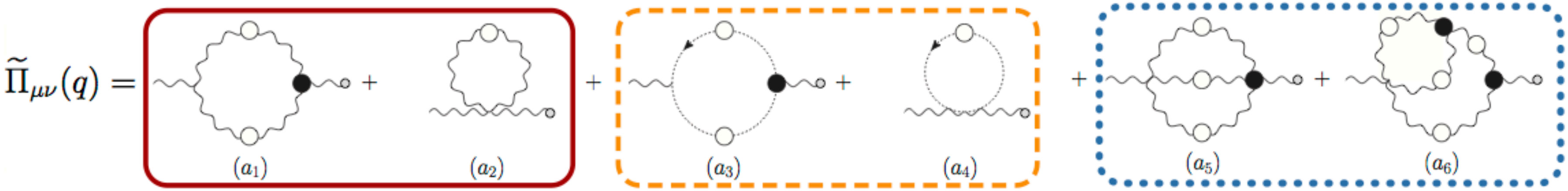
 $= (\mathbf{1}+\mathbf{G})^{\mathbf{2}}\otimes$ 

$$\widehat{\Delta}(q) = [\mathbf{1}+\mathbf{G}(\mathbf{q})]^{\mathbf{2}}\Delta(q)$$

Known function in the Landau gauge

P. A. Grassi, T. Hurth and M. Steinhauser, Annals Phys. 288 , 197 (2001)
D. Binosi and J. P., Phys. Rev. D66, 025024 (2002)

SDEs in the BFM framework



$$\Delta^{-1}(q^2)P_{\mu\nu}(q) = \frac{q^2 P_{\mu\nu}(q) + i\sum_{i=1}^{6}(a_i)_{\mu\nu}}{1+G(q^2)},$$

Transversality is enforced separately for gluon and ghost loops, and order by order in the "dressed-loop" expansion!

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p) \quad \Rightarrow \quad q^\mu[(a_1)+(a_2)]_{\mu\nu} = 0$$

$$q^\mu \widetilde{\Gamma}_{\mu}(q,r,-p) = D^{-1}(p) - D^{-1}(r) \quad \Rightarrow \quad q^\mu[(a_3)+(a_4)]_{\mu\nu} = 0$$

$$q^\mu \widetilde{\Gamma}^{mnrs}_{\mu\alpha\beta\gamma} = f^{mse}f^{ern}\Gamma_{\alpha\beta\gamma} + f^{mne}f^{esr}\Gamma_{\beta\gamma\alpha} + f^{mre}f^{ens}\Gamma_{\gamma\alpha\beta} \quad \Rightarrow \quad q^\mu[(a_5)+(a_6)]_{\mu\nu} = 0$$

A.C. Aguilar and J.P. ,JHEP 0612, 012 (2006)
D. Binosi and J. P. , Phys.Rev. D 77, 061702 (2008); JHEP 0811:063,2008.

From Takahashi to Ward identities

$$q^{\mu}\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

Taylor expansion around $q=0$

assuming no poles $1/q^2$!

↓

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^{\mu}}$$

Seagull identity

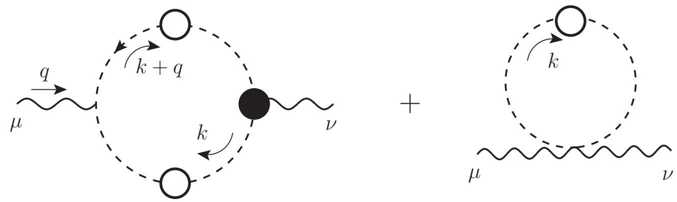
In dimensional regularization, any function that satisfies the criterion of Wilson

$$\int_k f(k^2) = \frac{1}{(4\pi)^{\frac{d}{2}}\Gamma(\frac{d}{2})}\int_0^\infty dy\, y^{\frac{d}{2}-1} f(y) = \text{finite}, \quad \text{for } 0 < d < d^*$$

Seagull identity

$$\int_k k^2 \frac{\partial f(k^2)}{\partial k^2} + \frac{d}{2}\int_k f(k^2) = 0$$

One-loop example: $\Pi_{\mu\nu}(q) =$



Scalar QED

$$\int_k \frac{k^2}{(k^2+m^2)^2} = -(4\pi)^{\frac{d}{2}}\left(\frac{d}{2}\right)\Gamma\left(1-\frac{d}{2}\right)(m^2)^{\frac{d}{2}-1}$$

$$\int_k \frac{1}{k^2+m^2} = (4\pi)^{\frac{d}{2}}\Gamma\left(1-\frac{d}{2}\right)(m^2)^{\frac{d}{2}-1}$$

$$\int_k \frac{k^2}{(k^2+m^2)^2} + \frac{d}{2}\int_k \frac{1}{k^2+m^2} = 0$$

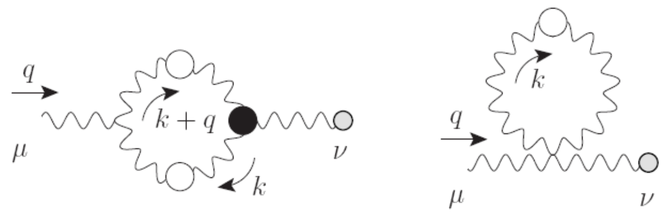
Seagull identity

$$f(k^2) = \frac{1}{k^2+m^2}$$

Gluon propagator at the origin

$$\widetilde{\Gamma}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial\Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu} \quad \Rightarrow \quad \Delta^{\alpha\rho}(k)\Delta^{\beta\sigma}(k)\widetilde{\Gamma}^{\mu}_{\sigma\rho}(0,k,-k) = \frac{\partial\Delta^{\alpha\beta}(k)}{\partial k^\mu}$$

$$\Delta^{-1}(0) = \lim_{q\to 0}\mathrm{Tr}\Big\{$$



$$\Big\} \sim \int_k k^2\frac{\partial\Delta(k^2)}{\partial k^2} + \frac{d}{2}\int_k \Delta(k) = 0$$

Ward Identities (No poles)

Seagull identity

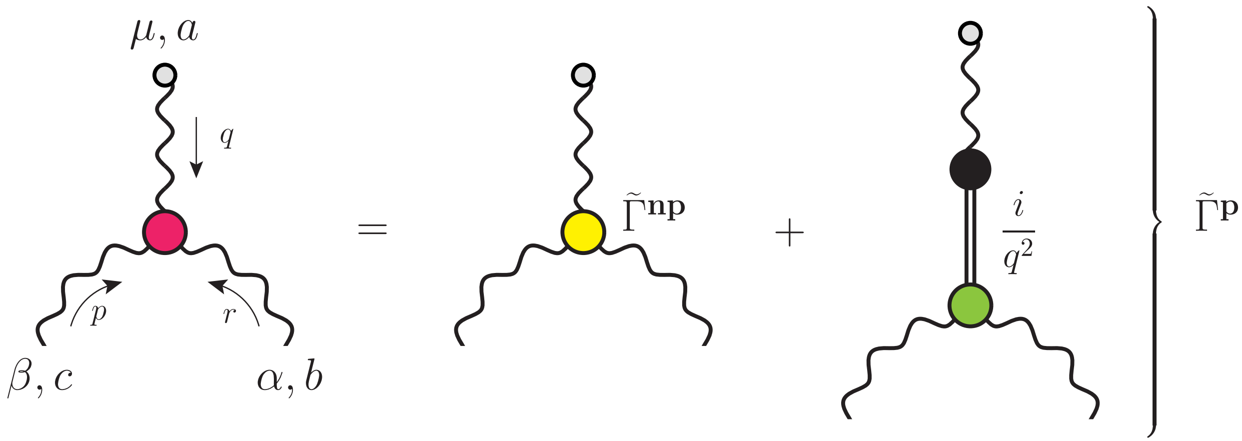
BFM Ward identites + Seagull identity $\Rightarrow$ $\Delta^{-1}(0) = 0$

No "gluon mass"

Exact result from BFM Schwinger-Dyson equation !

Vertices with massless poles

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p)$$



$$\widetilde{\Gamma}^{\mathbf{p}}_{\mu\alpha\beta}(q,r,p) = \frac{q_\mu}{q^2}\widetilde{C}_{\alpha\beta}(q,r,p)$$

Explicit implementation of the famous Schwinger mechanism in Yang-Mills theories

J.S. Schwinger
Phys. Rev.125, 397 (1962);
Phys.Rev.128, 2425 (1962).

R. Jackiw and K. Johnson, Phys. Rev. D8, 2386 (1973)

E. Eichten and F. Feinberg, Phys. Rev. D10, 3254 (1974)

Ward identities in the presence of poles

$$\widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \frac{q_\mu}{q^2}\widetilde{C}(q,r,p)$$

$$q^\mu \widetilde{\Gamma}_{\mu\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p)$$

Same ST identity !

$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(q,r,p) + \widetilde{C}_{\alpha\beta}(q,r,p) = i\Delta^{-1}_{\alpha\beta}(r) - i\Delta^{-1}_{\alpha\beta}(p),$$

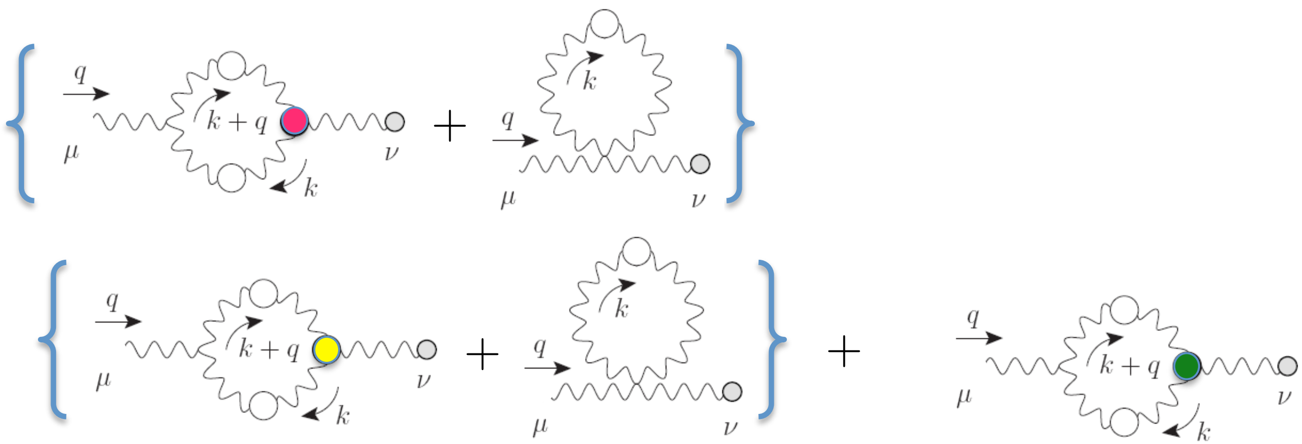
$$q^\mu \widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) + \widetilde{C}_{\alpha\beta}(0,r,-r) + q^\mu \left\{ \frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p) \right\}_{q=0} = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu}$$

$$\widetilde{C}_{\alpha\beta}(0,r,-r) = 0$$

$$\widetilde{\Gamma}^{\mathbf{np}}_{\mu\alpha\beta}(0,r,-r) = -i\frac{\partial \Delta^{-1}_{\alpha\beta}(r)}{\partial r^\mu} - \left\{ \frac{\partial}{\partial q^\mu}\widetilde{C}_{\alpha\beta}(q,r,p) \right\}_{q=0}$$

Evading the seagull identity

$$\Delta^{-1}(0) = \lim_{q\to 0} \mathrm{Tr}$$



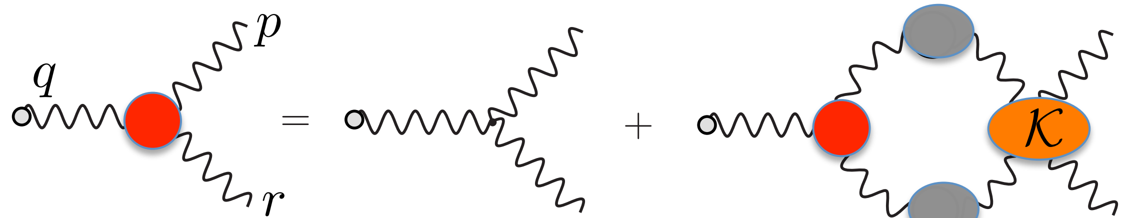
Triggers seagull identity exactly as before

Vanishes identically

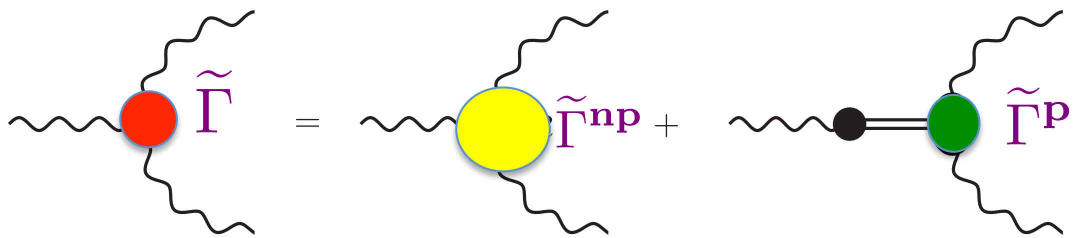
$$\Delta^{-1}(0) \sim \int_k k^2 \Delta^2(k^2)\widetilde{C}_1'(k^2)$$

Dynamical formation of massless poles

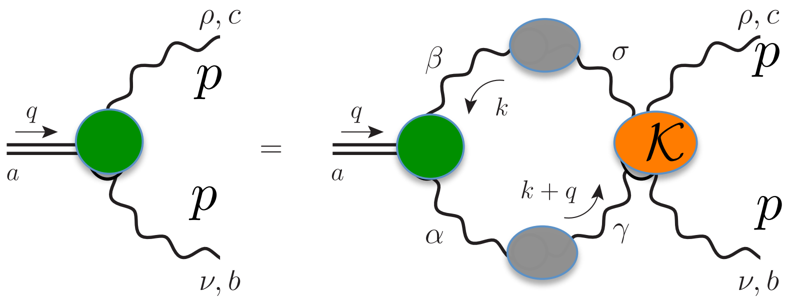
Bethe Salpeter equation for the full vertex



Substitute:



$q \to 0$

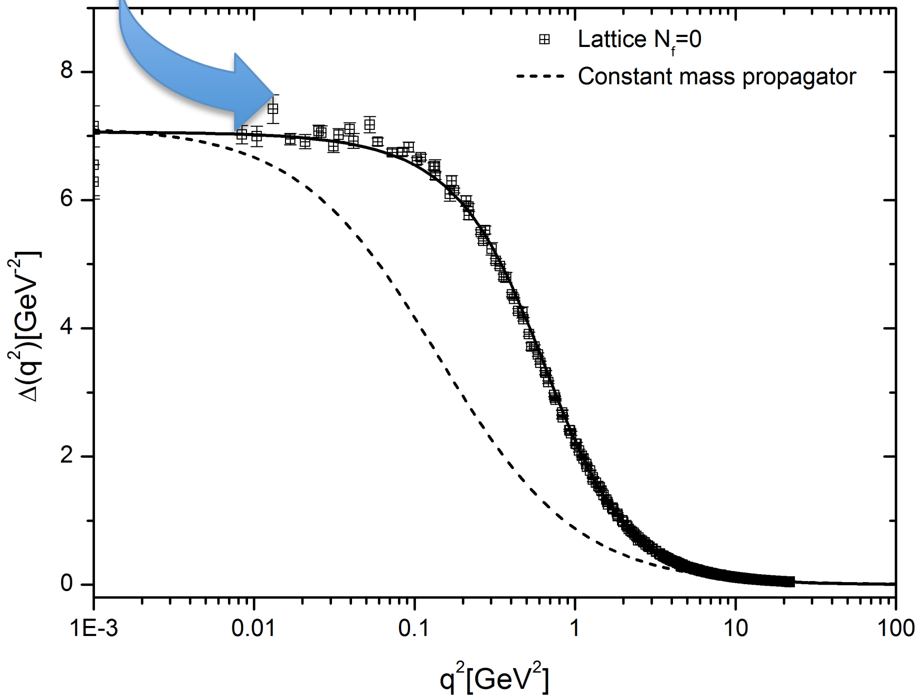


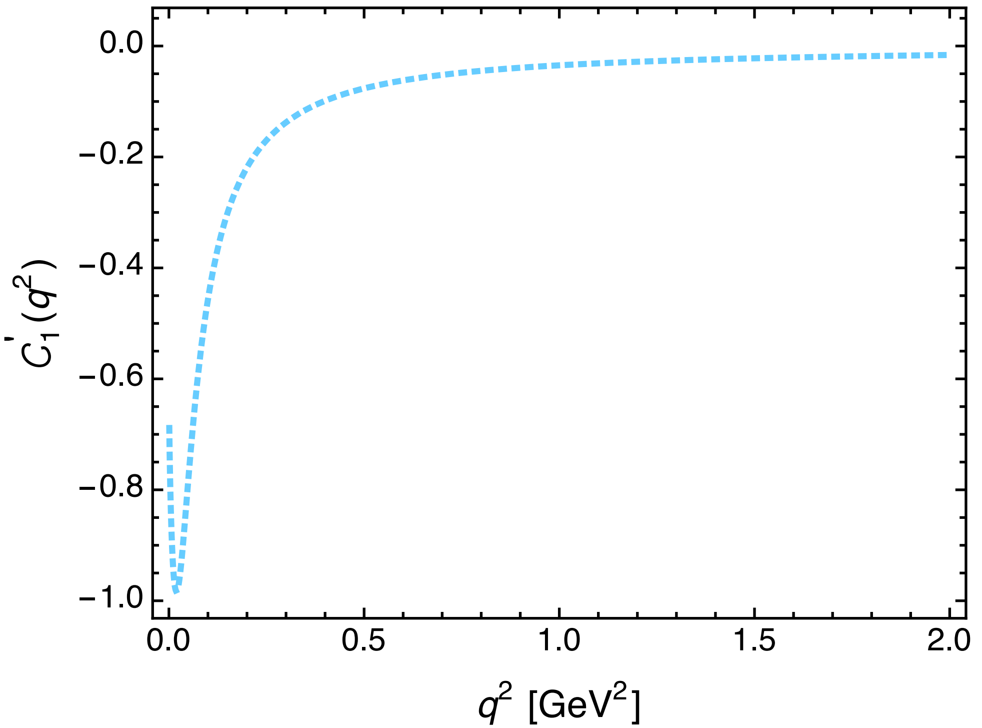
Dynamical equation for massless pole formation

$$\widetilde{C}_1'(p^2) = \lambda \int_k \widetilde{C}_1'(k^2)\Delta^2(k)\Delta(k+p)\,\mathcal{K}(k,p)$$

Homogeneous integral equation coupled with

$$\Delta^{-1}(0) = -\rho \int_k k^2\Delta^2(k^2)\widetilde{C}_1'(k^2)$$





Falsifiable mechanism: can be tested on lattice

$$\widetilde{\Gamma}^{\mu\alpha\beta}(q,r,p) = \sum_{i=1}^{14} \widetilde{A}_i(q^2, r^2, q\cdot r)\, b_i^{\mu\alpha\beta}$$

poles

$$b_1^{\mu\alpha\beta} = q^\mu g^{\alpha\beta}; \quad b_2^{\mu\alpha\beta} = q^\mu q^\alpha q^\beta; \quad b_3^{\mu\alpha\beta} = q^\mu q^\alpha r^\beta; \quad b_4^{\mu\alpha\beta} = q^\mu r^\alpha q^\beta; \quad b_5^{\mu\alpha\beta} = q^\mu r^\alpha r^\beta,$$

No poles

$$b_6^{\mu\alpha\beta} = r^\mu g^{\alpha\beta}; \quad b_7^{\mu\alpha\beta} = r^\mu q^\alpha q^\beta; \quad b_8^{\mu\alpha\beta} = r^\mu q^\alpha r^\beta; \quad b_9^{\mu\alpha\beta} = r^\mu r^\alpha q^\beta; \quad b_{10}^{\mu\alpha\beta} = r^\mu r^\alpha r^\beta,$$

$$b_{11}^{\mu\alpha\beta} = q^\alpha g^{\beta\mu}; \quad b_{12}^{\mu\alpha\beta} = q^\beta g^{\alpha\mu}; \quad b_{13}^{\mu\alpha\beta} = r^\alpha g^{\beta\mu}; \quad b_{14}^{\mu\alpha\beta} = r^\beta g^{\alpha\mu}.$$

$$A_1(q,r,p) = A_1^{\mathbf{np}}(q,r,p) + \frac{C_1(q,r,p)}{q^2}$$

$q \to 0$ + WI

$$\widetilde{A}_6^{\mathbf{np}}(r^2) = 2\left[\frac{\partial \Delta^{-1}(r^2)}{\partial r^2} - \widetilde{C}_1'(r^2)\right].$$