Callan-Symanzik equations for infrared Yang-Mills theory

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Quantization of continuum Yang-Mills theory

- $SU(N)$ Yang-Mills theory in the continuum, Landau gauge

- Faddeev-Popov determinant $\Rightarrow$ ghosts (and BRST symmetry)

- Faddeev-Popov action in $D$-dimensional Euclidean space-time

$$S = \int d^D x \left( \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu \bar{c}^a (D_\mu c)^a + i B^a \partial_\mu A_\mu^a \right)$$

- gauge copies $\Rightarrow$ restriction of the gauge field configurations to the (first) Gribov region $\Omega$ (properly to the fundamental modular region) [Gribov 1978]

- Zwanziger’s horizon function, breaks the BRST symmetry; local formulation $\Rightarrow$ additional auxiliary fields [Zwanziger 1989]

- condensates of the additional auxiliary fields: “refined Gribov-Zwanziger scenario” $\Rightarrow$ effective mass term for the gluons [Dudal et al. 2008]
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Dyson-Schwinger equations are not affected by the restriction of the $A$-integral to $\Omega$ (without introducing the horizon function): the contributions from the boundary of $\Omega$ vanish because $\det(-\partial_\mu D_{\mu}^{ab}) = 0$ at the boundary [Zwanziger 2002]

- the perturbative expansion of the correlation functions, obtained from the iterative solution of the Dyson-Schwinger equations, is also unchanged

- what can change are the (re)normalization conditions; and the BRST symmetry is broken
How nonperturbative is IR Yang-Mills theory?

- **Effective description** by a local renormalizable quantum field theory: include a gluonic mass term in the Faddeev-Popov action (in 4-dimensional Euclidean space-time)

\[
S = \int d^4x \left( \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} A^a_\mu m^2 A^a_\mu + \partial_\mu \bar{c}^a D^{ab}_\mu c^b + i B^a A^a_\mu \right)
\]

(Curci-Ferrari model, perturbatively renormalizable)

- Apply straightforward **one-loop perturbation theory** to this action; adjust two constants, \( g, m^2 \), at some renormalization scale (in principle, \( m^2 \) is nonperturbatively fixed in terms of \( \Lambda_{QCD} \), or \( g \))

- Result: excellent fit to the lattice data for the propagators in the IR [Tissier, Wschebor 2010]
notations: propagators

\[
\langle A^a_\mu(p) A^b_\nu(-q) \rangle = G_A(p^2) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \delta^{ab} (2\pi)^4 \delta(p - q)
\]

\[
\langle c^a(p) \bar{c}^b(-q) \rangle = G_c(p^2) \delta^{ab} (2\pi)^4 \delta(p - q)
\]

and dressing functions

\[
G_A(p^2) = \frac{F_A(p^2)}{p^2}, \quad G_c(p^2) = \frac{F_c(p^2)}{p^2}
\]

one-loop contributions to the gluon self energy

and the ghost self energy

calculated with a massive gluon propagator

\[
G_A(p^2) = \frac{1}{m^2 + p^2}
\]
gluon propagator $G_A(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(2) lattice data: Cucchieri, Mendes 2008a

at tree level, $G_A(p^2) \propto \frac{1}{m^2 + p^2}$
ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(2) lattice data: Cucchieri, Mendes 2008b

at tree level, $F_c(p^2) = 1$
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(3) lattice data: Bogolubsky et al. 2009
and Dudal, Oliveira, Vandersickel 2010

at tree level, $F_A(p^2) \propto \frac{p^2}{m^2 + p^2}$

renormalization group improvement necessary for the UV
“IR safe” renormalization scheme proposed by Tissier and Wschebor [Tissier, Wschebor 2011]: normalization conditions for the proper two-point functions

\[ \Gamma_A^\perp(p^2)|_{p^2=\mu^2} = m^2 + p^2 \]

\[ \Gamma_A^\parallel(p^2)|_{p^2=\mu^2} = m^2 \]

\[ \Gamma_c(p^2)|_{p^2=\mu^2} = p^2 \]

\( \Gamma_A^\perp \) and \( \Gamma_A^\parallel \) are the transverse and longitudinal parts of the proper gluonic 2-point function

\[ \Gamma^{(2)}_{A,\mu\nu}(p) = \Gamma_A^\perp(p^2) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \Gamma_A^\parallel(p^2) \frac{p_\mu p_\nu}{p^2} \]

The first two normalization conditions can be rewritten as

\[ \Gamma_A^\perp(p^2) - \Gamma_A^\parallel(p^2)|_{p^2=\mu^2} = p^2 \]

\[ \Gamma_A^\parallel(p^2)|_{p^2=\mu^2} = m^2 \]

These combinations correspond to the decomposition of the 2-point function

\[ \Gamma^{(2)}_{A,\mu\nu}(p) = \left( \Gamma_A^\perp(p^2) - \Gamma_A^\parallel(p^2) \right) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \Gamma_A^\parallel(p^2) \delta_{\mu\nu} \]

which is analogous to the grouping of terms in the classical action

\[ p^2 \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + m^2 \delta_{\mu\nu} \]
renormalized coupling constant defined from the renormalized proper ghost-gluon vertex in the Taylor limit (ghost momentum $p \to 0$) where there are no loop corrections to the vertex (alternatively, use the symmetry point $p^2 = q^2 = k^2 = \mu^2$)

calculate the flow functions at one-loop order; then, solve the Callan-Symanzik renormalization group equations for the propagators

adjust $g(\mu_0), m^2(\mu_0)$ at some renormalization scale to fit the lattice data; note that the lattice propagators are not normalized and thus can be arbitrarily rescaled (field rescalings)

fitting strategy: fix $g(\mu_0), m^2(\mu_0)$ by adjusting to the data for the ghost propagator and the ghost dressing function; comparison to the data for the gluon propagator and the gluon dressing function then shows how successful the renormalization scheme is in reproducing the lattice data

in all of the following, the SU(2) lattice data are from Cucchieri and Mendes [Cucchieri, Mendes 2008a, 2008b]
running coupling constant $g(\mu)$ in $D = 4$ dimensions
running mass parameter $m^2(\mu)$ in $D = 4$ dimensions
ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$, Taylor scheme
gluon propagator function $G_A(p^2)$, Taylor scheme
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$, Taylor scheme
Derivative schemes

- in the decomposition of the gluonic 2-point function

\[ \Gamma^{(2)}_{A,\mu\nu}(p) = \left( \Gamma^\perp_A(p^2) - \Gamma^\parallel_A(p^2) \right) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \Gamma^\parallel_A(p^2) \delta_{\mu\nu} \]

replace the normalization condition

\[ \Gamma^\perp_A(p^2) - \Gamma^\parallel_A(p^2) \bigg|_{p^2=\mu^2} = p^2 \]

with

\[ \frac{d}{dp^2} \left( \Gamma^\perp_A(p^2) - \Gamma^\parallel_A(p^2) \right) \bigg|_{p^2=\mu^2} = 1 \]

- complement with the normalization conditions

\[ \Gamma^\parallel_A(p^2) \bigg|_{p^2=\mu^2} = m^2 \]

\[ \frac{d}{dp^2} \Gamma_c(p^2) \bigg|_{p^2=\mu^2} = 1 \]

\[ \Rightarrow \text{quantitatively, almost no change} \]

- generalize to the decomposition

\[ \Gamma^{(2)}_{A,\mu\nu}(p) = \left( \Gamma^\perp_A(p^2) - \zeta \Gamma^\parallel_A(p^2) \right) \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + \Gamma^\parallel_A(p^2) \left( \zeta \delta_{\mu\nu} + (1 - \zeta) \frac{p_\mu p_\nu}{p^2} \right) \]

and impose the normalization condition

\[ \frac{d}{dp^2} \left( \Gamma^\perp_A(p^2) - \zeta \Gamma^\parallel_A(p^2) \right) \bigg|_{p^2=\mu^2} = 1 \]
in the IR limit $\mu^2 \ll m^2$, 

$$\beta_g = \mu^2 \frac{d}{d\mu^2} g = \frac{g}{2} (\gamma_A + 2\gamma_c) \approx \frac{g}{2} \gamma_A = \frac{g}{2} \mu^2 \frac{d}{d\mu^2} \ln Z_A$$

and to 1-loop order

$$\mu^2 \frac{d}{d\mu^2} \ln Z_A = \frac{Ng^2}{(4\pi)^2} \left( -\frac{1}{12} + \frac{\zeta}{4} \right)$$

IR safety ($\beta_g > 0$) for $\zeta > 1/3$, the simple derivative scheme corresponds to $\zeta = 1$; the positivity of the beta function arises from the momentum dependence of the longitudinal part $\Gamma_A^{\|} (p^2)$.

in the following, consider only the critical case $\zeta = 1/3$
running coupling constant $g(\mu)$
running mass parameter $m^2(\mu)$
gluon propagator function $G_A(p^2)$, critical derivative scheme
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$, critical derivative scheme
Independent running couplings

- breaking of BRST symmetry destroys the relation between the different renormalized coupling constants

- define the renormalized ghost-gluon coupling constant defined from the renormalized proper ghost-gluon vertex as before, define the renormalized three-gluon coupling constant from the renormalized proper three-point vertex at the symmetry point $p_1^2 = p_2^2 = p_3^2 = \mu^2$

- renormalized four-gluon coupling constant set equal to the renormalized three-gluon coupling constant for the time being

- integration of the Callan-Symanzik equations with two independently running coupling constants and a running mass parameter: BRST symmetry and usual non-massive behavior recovered in the UV, only two adjustable parameters (fine tuning condition)
gluon propagator $G_A(p^2)$ in 4 dimensions, 
two coupling constants
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ in 4 dimensions, two coupling constants
ghost-gluon vertex function at the symmetry point $p^2 = q^2 = k^2$

in 4 dimensions, two coupling constants,

compared to an approximate solution of the Dyson-Schwinger equations

SU(2) lattice data: Cucchieri, Maas, Mendes 2008

at tree level, the vertex function is equal to one
three-gluon vertex function at the symmetry point $p_1^2 = p_2^2 = p_3^2$
in 4 dimensions, two coupling constants,
compared to an approximate solution of the Dyson-Schwinger equations

SU(2) lattice data: Cucchieri, Maas, Mendes 2008
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Summary

- quasi-analytic and systematic description of the correlation functions of Landau gauge Yang-Mills theory: introduce a gluonic mass term, solve the Callan-Symanzik equations

Outlook

- current calculations in 3 space-time dimensions
- continue work on the vertex functions, compare to new lattice and Dyson-Schwinger results
- proceed to two-loop level (with renormalization group improvement)
- extend the formalism to 2 space-time dimensions (scaling solutions)
- include quarks, describe dynamical chiral symmetry breaking
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