Callan-Symanzik equations for infrared Yang-Mills theory

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Quantization of continuum Yang-Mills theory

- $SU(N)$ Yang-Mills theory in the continuum, Landau gauge
- Faddeev-Popov determinant $\Rightarrow$ ghosts (and BRST symmetry)
- Faddeev-Popov action in D-dimensional Euclidean space-time

$$S = \int d^D x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_\mu \bar{c}^a (D_\mu c)^a + i B^a \partial_\mu A^a_\mu \right)$$

- gauge copies $\Rightarrow$ restriction of the gauge field configurations to the (first) Gribov region Ω (properly to the fundamental modular region) [Gribov 1978]
- Zwanziger's horizon function, breaks the BRST symmetry; local formulation $\Rightarrow$ additional auxiliary fields [Zwanziger 1989]
- condensates of the additional auxiliary fields: “refined Gribov-Zwanziger scenario” $\Rightarrow$ effective mass term for the gluons [Dudal et al. 2008]

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- Dyson-Schwinger equations are not affected by the restriction of the A-integral to Ω (without introducing the horizon function): the contributions from the boundary of Ω vanish because $\det(-\partial_\mu D_\mu^{ab}) = 0$ at the boundary [Zwanziger 2002]

- the perturbative expansion of the correlation functions, obtained from the iterative solution of the Dyson-Schwinger equations, is also unchanged

- what can change are the (re)normalization conditions; and the BRST symmetry is broken

How nonperturbative is IR Yang-Mills theory?

- effective description by a local renormalizable quantum field theory: include a gluonic mass term in the Faddeev-Popov action (in 4-dimensional Euclidean space-time)

$$S = \int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} A^a_\mu m^2 A^a_\mu + \partial_\mu \bar{c}^a D^{ab}_\mu c^b + iB^a \partial_\mu A^a_\mu \right)$$

  (Curci-Ferrari model, perturbatively renormalizable)

- apply straightforward one-loop perturbation theory to this action;
  adjust *two* constants, g, m^2, at some renormalization scale
  (in principle, m^2 is nonperturbatively fixed in terms of Λ_{QCD}, or g)

- result: excellent fit to the lattice data for the propagators in the IR [Tissier, Wschebor 2010]

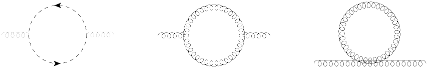
- notations: propagators

$$\langle A^a_\mu(p) A^b_\nu(-q)\rangle = G_A(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}(2\pi)^4\delta(p-q)$$
$$\langle c^a(p)\bar{c}^b(-q)\rangle = G_c(p^2)\,\delta^{ab}(2\pi)^4\delta(p-q)$$

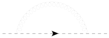
and dressing functions

$$G_A(p^2) = \frac{F_A(p^2)}{p^2}\,, \qquad G_c(p^2) = \frac{F_c(p^2)}{p^2}$$

- one-loop contributions to the gluon self energy



and the ghost self energy



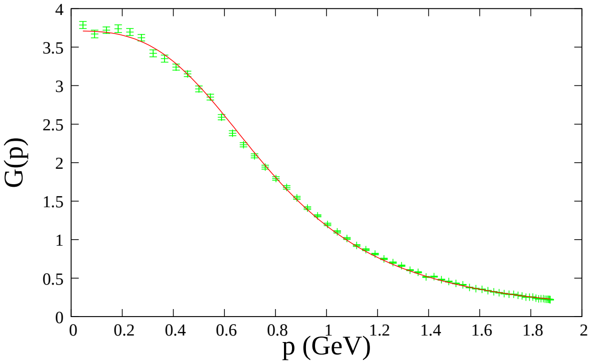
calculated with a massive gluon propagator

$$G_A(p^2) = \frac{1}{m^2 + p^2}$$

gluon propagator $G_A(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(2) lattice data: Cucchieri, Mendes 2008a

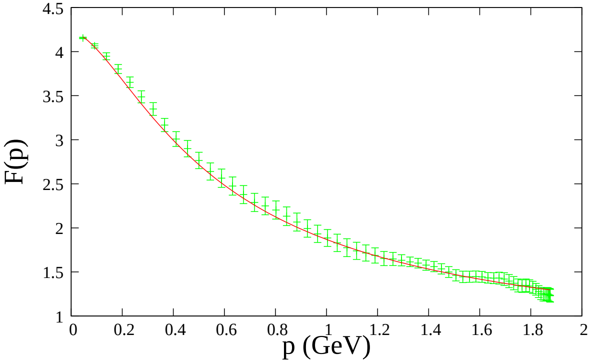


at tree level, $G_A(p^2) \propto \dfrac{1}{m^2 + p^2}$

ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(2) lattice data: Cucchieri, Mendes 2008b



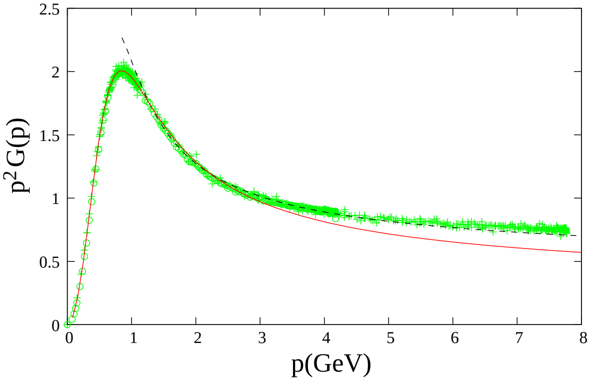
at tree level, $F_c(p^2) = 1$

gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ in 4 dimensions

Tissier, Wschebor 2010

SU(3) lattice data: Bogolubsky et al. 2009

and Dudal, Oliveira, Vandersickel 2010



at tree level, $F_A(p^2) \propto \dfrac{p^2}{m^2 + p^2}$

renormalization group improvement necessary for the UV

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- “IR safe” renormalization scheme proposed by Tissier and Wschebor [Tissier, Wschebor 2011]: normalization conditions for the proper two-point functions

$$\Gamma_A^{\perp}(p^2)\big|_{p^2=\mu^2} = m^2 + p^2$$
$$\Gamma_A^{\parallel}(p^2)\big|_{p^2=\mu^2} = m^2$$
$$\Gamma_c(p^2)\big|_{p^2=\mu^2} = p^2$$

- $\Gamma_A^{\perp}$ and $\Gamma_A^{\parallel}$ are the transverse and longitudinal parts of the proper gluonic 2-point function

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \Gamma_A^{\perp}(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma_A^{\parallel}(p^2)\,\frac{p_\mu p_\nu}{p^2}$$

- the first two normalization conditions can be rewritten as

$$\Gamma_A^{\perp}(p^2) - \Gamma_A^{\parallel}(p^2)\big|_{p^2=\mu^2} = p^2$$
$$\Gamma_A^{\parallel}(p^2)\big|_{p^2=\mu^2} = m^2$$

- these combinations correspond to the decomposition of the 2-point function

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \left(\Gamma_A^{\perp}(p^2) - \Gamma_A^{\parallel}(p^2)\right)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma_A^{\parallel}(p^2)\delta_{\mu\nu}$$

which is analogous to the grouping of terms in the classical action

$$p^2\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + m^2\delta_{\mu\nu}$$

- renormalized coupling constant defined from the renormalized proper ghost-gluon vertex in the Taylor limit (ghost momentum $p \to 0$) where there are no loop corrections to the vertex (alternatively, use the symmetry point $p^2 = q^2 = k^2 = \mu^2$)

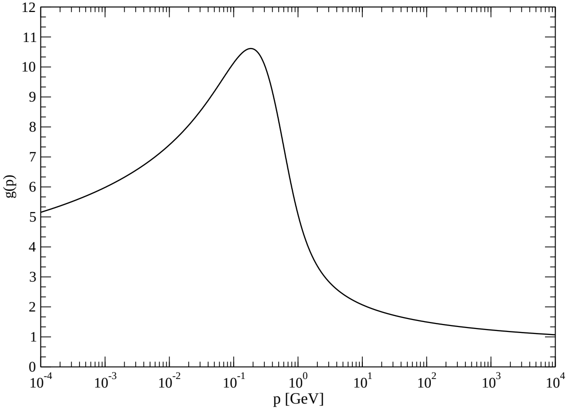
- calculate the flow functions at one-loop order; then, solve the Callan-Symanzik renormalization group equations for the propagators

- adjust $g(\mu_0), m^2(\mu_0)$ at some renormalization scale to fit the lattice data; note that the lattice propagators are not normalized and thus can be arbitrarily rescaled (field rescalings)

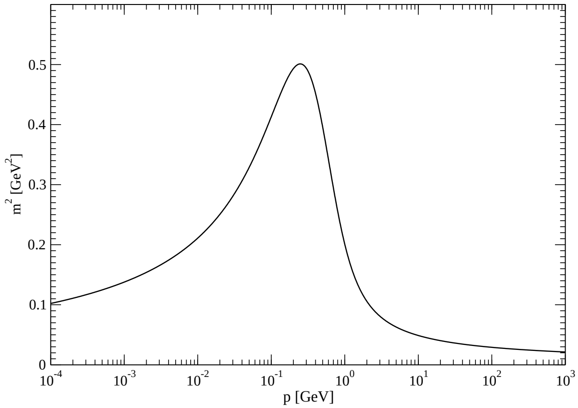
- fitting strategy: fix $g(\mu_0), m^2(\mu_0)$ by adjusting to the data for the ghost propagator and the ghost dressing function; comparison to the data for the gluon propagator and the gluon dressing function then shows how successful the renormalization scheme is in reproducing the lattice data

- in all of the following, the SU(2) lattice data are from Cucchieri and Mendes [Cucchieri, Mendes 2008a, 2008b]

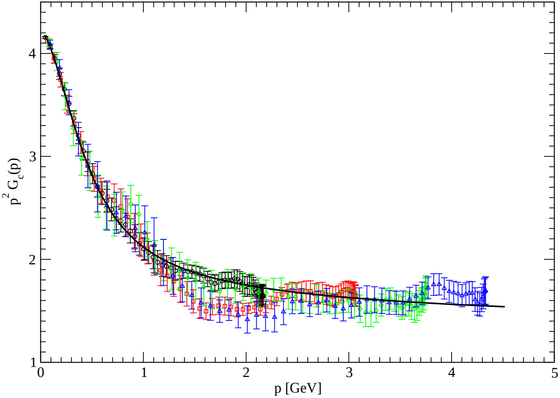
running coupling constant $g(\mu)$ in $D = 4$ dimensions



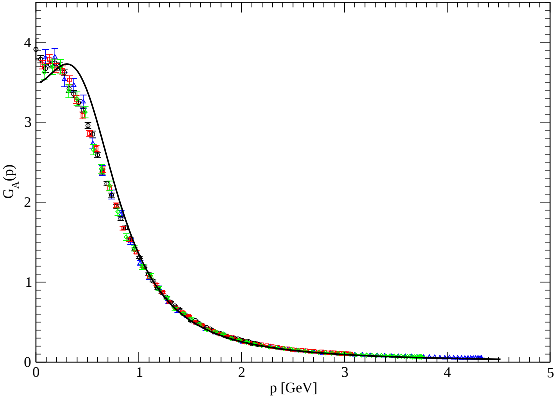
running mass parameter $m^2(\mu)$ in $D = 4$ dimensions



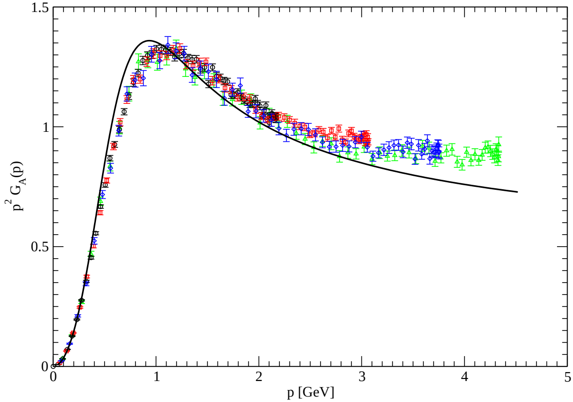
ghost dressing function $F_c(p^2) = p^2 G_c(p^2)$, Taylor scheme



gluon propagator function $G_A(p^2)$, Taylor scheme



gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$, Taylor scheme



Derivative schemes

- in the decomposition of the gluonic 2-point function

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \left(\Gamma^{\perp}_{A}(p^2) - \Gamma^{\parallel}_{A}(p^2)\right)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma^{\parallel}_{A}(p^2)\delta_{\mu\nu}$$

  replace the normalization condition

$$\Gamma^{\perp}_{A}(p^2) - \Gamma^{\parallel}_{A}(p^2)\big|_{p^2=\mu^2} = p^2$$

  with

$$\frac{d}{dp^2}\left(\Gamma^{\perp}_{A}(p^2) - \Gamma^{\parallel}_{A}(p^2)\right)\Big|_{p^2=\mu^2} = 1$$

- complement with the normalization conditions

$$\Gamma^{\parallel}_{A}(p^2)\big|_{p^2=\mu^2} = m^2$$

$$\frac{d}{dp^2}\Gamma_c(p^2)\Big|_{p^2=\mu^2} = 1$$

  $\Rightarrow$ quantitatively, almost no change

- generalize to the decomposition

$$\Gamma^{(2)}_{A,\mu\nu}(p) = \left(\Gamma^{\perp}_{A}(p^2) - \zeta\Gamma^{\parallel}_{A}(p^2)\right)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \Gamma^{\parallel}_{A}(p^2)\left(\zeta\delta_{\mu\nu} + (1-\zeta)\frac{p_\mu p_\nu}{p^2}\right)$$

  and impose the normalization condition

$$\frac{d}{dp^2}\left(\Gamma^{\perp}_{A}(p^2) - \zeta\Gamma^{\parallel}_{A}(p^2)\right)\Big|_{p^2=\mu^2} = 1$$

- in the IR limit $\mu^2 \ll m^2$,

$$\beta_g = \mu^2 \frac{d}{d\mu^2} g = \frac{g}{2}(\gamma_A + 2\gamma_c) \approx \frac{g}{2}\gamma_A = \frac{g}{2}\mu^2 \frac{d}{d\mu^2} \ln Z_A$$

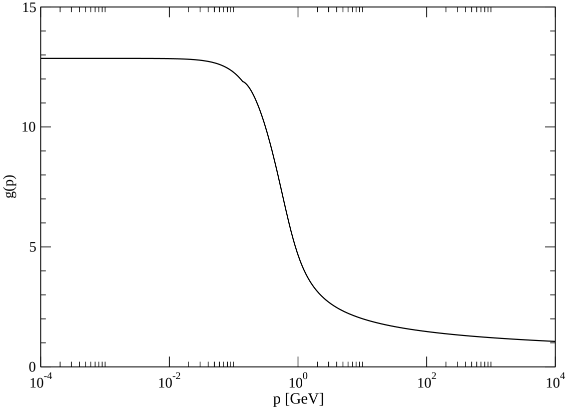
and to 1-loop order

$$\mu^2 \frac{d}{d\mu^2} \ln Z_A = \frac{Ng^2}{(4\pi)^2}\left(-\frac{1}{12} + \frac{\zeta}{4}\right)$$

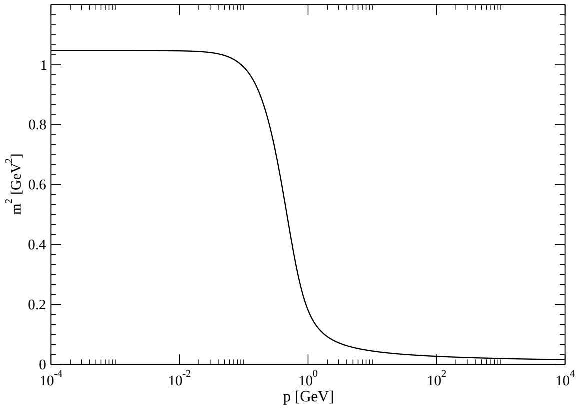
- IR safety ($\beta_g > 0$) for $\zeta > 1/3$, the simple derivative scheme corresponds to $\zeta = 1$; the positivity of the beta function arises from the momentum dependence of the longitudinal part $\Gamma_A^{\parallel}(p^2)$!

- in the following, consider only the critical case $\zeta = 1/3$

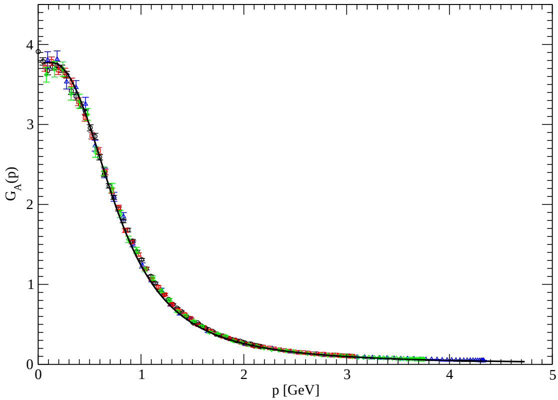
running coupling constant $g(\mu)$



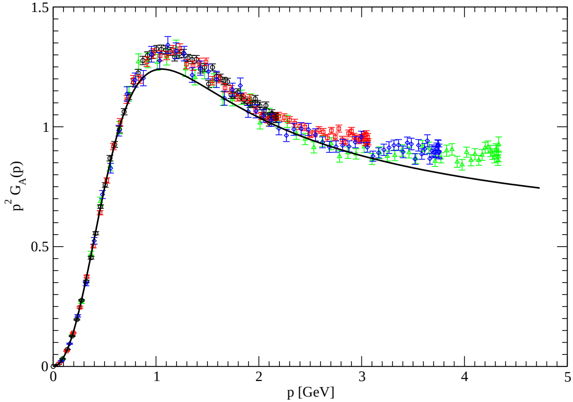
running mass parameter $m^2(\mu)$



gluon propagator function $G_A(p^2)$, critical derivative scheme



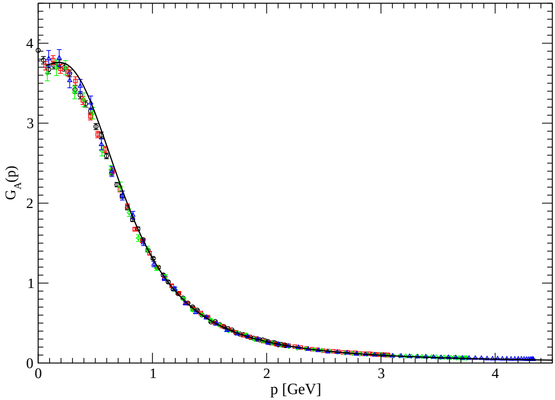
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$, critical derivative scheme



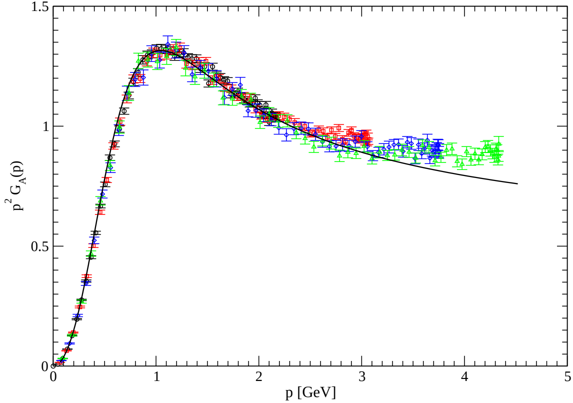
Independent running couplings

- breaking of BRST symmetry destroys the relation between the different renormalized coupling constants
- define the renormalized ghost-gluon coupling constant defined from the renormalized proper ghost-gluon vertex as before, define the renormalized three-gluon coupling constant from the renormalized proper three-point vertex at the symmetry point $p_1^2 = p_2^2 = p_3^2 = \mu^2$
- renormalized four-gluon coupling constant set equal to the renormalized three-gluon coupling constant for the time being
- integration of the Callan-Symanzik equations with two independently running coupling constants and a running mass parameter: BRST symmetry and usual non-massive behavior recovered in the UV, only two adjustable parameters (fine tuning condition)

gluon propagator $G_A(p^2)$ in 4 dimensions,
two coupling constants

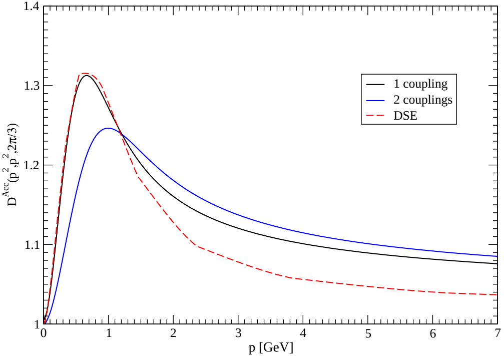


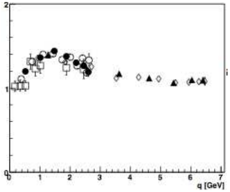
gluon dressing function $F_A(p^2) = p^2 G_A(p^2)$ in 4 dimensions, two coupling constants



ghost-gluon vertex function at the symmetry point $p^2 = q^2 = k^2$
in 4 dimensions, two coupling constants,
compared to an approximate solution of the Dyson-Schwinger equations

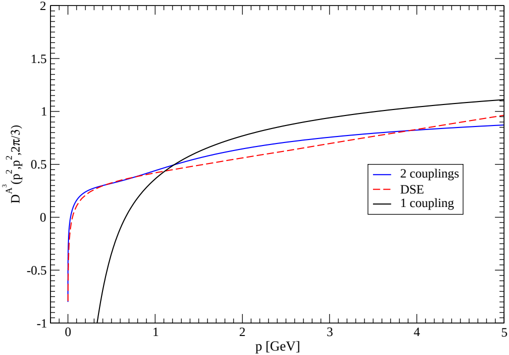
SU(2) lattice data: Cucchieri, Maas, Mendes 2008

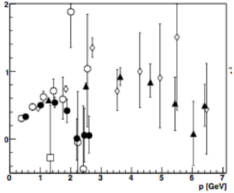




at tree level, the vertex function is equal to one

three-gluon vertex function at the symmetry point $p_1^2 = p_2^2 = p_3^2$
in 4 dimensions, two coupling constants,
compared to an approximate solution of the Dyson-Schwinger equations
SU(2) lattice data: Cucchieri, Maas, Mendes 2008





Contents

Summary

- quasi-analytic and systematic description of the correlation functions of Landau gauge Yang-Mills theory: introduce a gluonic mass term, solve the Callan-Symanzik equations

Outlook

- current calculations in 3 space-time dimensions
- continue work on the vertex functions, compare to new lattice and Dyson-Schwinger results
- proceed to two-loop level (with renormalization group improvement)
- extend the formalism to 2 space-time dimensions (scaling solutions)
- include quarks, describe dynamical chiral symmetry breaking

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