Modeling Clusterized Nuclear Matter under Stellar Conditions

Igor Mishustin

FIAS, Goethe University, Frankfurt am Main, Germany
and
National Research Center “Kurchatov Institute”, Moscow, Russia
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- Introduction: Macro- and Micro-Superovae
- Uncertainties in Nuclear Statistical Ensemble
- Nuclear structure calculations in stellar environments
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Macro-Supernovae: Numerical simulations

Sketch of the post-collapse stellar core during the neutrino heating and shock revival phase

H.-T. Janka, K. Kifonidis, M. Rampp
Creation of Micro-Supernovae in laboratory

Peripheral AA collision (ALADIN)

$P$ or $T$ spectator

Heating: $T \sim 10$ MeV
$p < p_0$, $P \sim 0$
slow expansion

$Y(A) \sim \ldots \approx$ (liquid-gas phase tr.)

Multifragmentation – creation of micro-supernovae in laboratory,
Power-law mass distributions: $Y(A) \sim \ldots \approx$ (liquid-gas phase tr.)
Can be well understood within the equilibrium statistical approach
developed in 80s (following ideas of N. Bohr and E. Fermi):

Randrup&Koonin, D.H.E. Gross et al, Bondorf-Mishustin-Botvina, Hahn&Stoecker,..
Similarity of conditions in nuclear reactions and supernova explosions

I. Thermal multifragmentation of nuclei:
   • Production of hot fragments at $T \approx 3-8$ MeV $\rho \approx (0.1 - 0.3) \rho_0$
   • A way to investigate properties of hot nuclei in dense environment

II. Collapse of massive stars leading to Supernova Type II explosions:
   • Production of hot nuclei in stellar environments: $T \approx 1-10$ MeV $\rho < 0.3 \rho_0$
   • Characteristic times (milliseconds) are very long for nuclear statistical equilibrium to be established.
   • Properties of hot clustered nuclear matter may influence essentially the neutrino transport in SNe and properties of the NS crust.

Multifragmentation reactions and properties of hot stellar matter at sub-nuclear densities
Statistical Model of Stellar Matter (SMSM)

Statistical Model

Fixed $T, \rho_B, Y_{L(e)}$

Chemical potentials: $\mu_i = B_i \mu_B + Q_i \mu_Q + L_i \mu_L$

nuclear species $(A, Z): \mu_{AZ} = A \mu_B + Z \mu_Q$

electrons $e^- : \mu_{e^-} = -\mu_Q + \mu_L = -\mu_{e^+}^-$

neutrinos $\nu : \mu_\nu = \mu_L = -\mu_i^-$

Baryon number conservation:

$\rho_B = \frac{B}{V} = \sum_{(A,Z)} A \langle n_{AZ} \rangle$ fixed $\rightarrow \mu_B$

Electric neutrality

$\rho_Q = \frac{Q}{V} = \sum_{(A,Z)} Z \langle n_{AZ} \rangle - n_e = 0$ $\rightarrow \mu_Q$

Lepton number conservation

$Y_L = \frac{L}{B} = \frac{n_e + n_\nu}{\rho_B}$ (trapped $\nu$) or $Y_e = \frac{n_e}{\rho_B}$ (no $\nu$)

Calculations are done in a box containing 1000 baryons, density of individual fragments is fixed at $\rho_0 = 0.16$ fm$^3$


Nuclear statistical equilibrium ensembles

All nuclear species are included, not only one "average" nucleus!

Number density of nuclear species \((A,Z)\): \[ n_{AZ} = g_{AZ} \frac{V_f}{V} \frac{A^{3/2}}{\lambda_T^3} \exp \left[ -\frac{1}{T} (F_{AZ} - \mu_{AZ}) \right] \]

Pressure: \[ P_{\text{nuc}} = T \sum_{(A,Z)} n_{AZ} \]

Internal free energy of species \((A,Z)\) for \(A > 4\)

\[ F_{AZ} = F^B_{AZ} + F^S_{AZ} + F^{\text{sym}}_{AZ} + F^C_{AZ} \] liquid drop parametrization

\[ F^B_{AZ}(T) = \left( -w_0 - \frac{T^2}{\varepsilon_0} \right) A, \quad F^S_{AZ} = \beta_0 \left( \frac{T_c^2 - T^2}{T_c^2 + T^2} \right)^{5/4} A^{2/3}, \quad F^{\text{sym}}_{AZ} = \gamma \frac{(A - 2Z)^2}{A} \]

Reduced Coulomb energy due to the electron screening

\[ F^C_{AZ}(n_e) = c(n_e) \frac{3}{5} \frac{(eZ)^2}{r_0 A^{1/3}}, \quad c(n_e) = 1 - \frac{3}{2} \left( \frac{n_e}{n_{0p}} \right)^{1/3} + \frac{1}{2} \left( \frac{n_e}{n_{0p}} \right) \]

Consider box with \(10^3\) nucleons, \(\mu_B, \mu_Q, \mu_L\) are found iteratively
## Other versions of Statistical Model

<table>
<thead>
<tr>
<th></th>
<th>SMSM</th>
<th>HS</th>
<th>FYSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>NSE + LDM</td>
<td>RMF + NSE+ mass data</td>
<td>RMF+ NSE+ LDM + mass data</td>
</tr>
<tr>
<td></td>
<td>(Theoretical + Experimental)</td>
<td></td>
<td>(Experimental)</td>
</tr>
<tr>
<td>Component heavy nuc.</td>
<td>multi (Z&lt;1000) 1 ≤ A ≤ 1000</td>
<td>multi (Z&lt;100) 1 ≤ A ≤ 331</td>
<td>multi (Z&lt;1000) 1 ≤ A ≤ 1000</td>
</tr>
<tr>
<td>Shell term</td>
<td>×</td>
<td>○</td>
<td>△</td>
</tr>
<tr>
<td>Nuclear Shape</td>
<td>Droplet only</td>
<td>Droplet only</td>
<td>Droplet + bubble (+other)</td>
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<tr>
<td>$E_{\text{symmetry of nuclei}}$</td>
<td>25 MeV</td>
<td>mass data</td>
<td>38 MeV</td>
</tr>
<tr>
<td>$E_{\text{surface}}$</td>
<td>△ depending on T</td>
<td>mass data</td>
<td>△ depending on density &amp; symmetry Z/A</td>
</tr>
</tbody>
</table>

Mass distributions: SMSM, FYSS and HS ($\rho/\rho_0=10^{-1}$)

Different versions of the statistical model using “reasonable” assumptions on nuclear properties give very different results!
Isotopic distributions: $^{26}$Fe

Significant differences are seen at all $T$ and $Y_e$!
Influence of the symmetry energy term on mass distributions

For the stellar matter, with smaller symmetry energy ($\gamma=14$) much more heavy nuclei may be formed, since they can accumulate more neutrons at lower temperatures and $Y_e$.

Challenge for nuclear physics community

Evaluate nuclear properties in stellar environments, i.e. at finite temperature (T), average baryon density (ρ) and electron or lepton fraction (Y): 0<T<10 MeV, 10^{-5}<ρ/ρ_0<0.5, 0.1<Y<0.6.

One should evaluate a) binding energies, b) excited states, c) decay probabilities of a huge number of nuclei (1<A<1000, 0<Z<A), i.e. construct a “Nuclear Chart” for each set (T,ρ,Y).

Existing nuclei represent only a very small subset around (0,0,0) corner. Altogether up to 10^8 new “data” points should be calculated using any “reasonable” approach (LDM, HFB, RMF, CEFT,...)

Such data are needed as inputs for a Statistical Model to find the most probable Nuclear Statistical Ensemble (NSE) in a specific stellar environment.
Nuclear Chart is not fully explored even at laboratory conditions but we need much more!
Example of “Nuclear chart” for supernova conditions

N. Buyukcizmecl et al. to be published
Possible tool: RMF model + electrons


+ BCS $\delta$ – force pairing

parameter set: NL3

$$\mathcal{L} = \mathcal{L}_{\text{nucleon}}^{\text{free}} + \mathcal{L}_{\text{meson}}^{\text{free}} + \mathcal{L}_{\text{coupl}}^{\text{lin}} + \mathcal{L}_{\text{coupl}}^{\text{nonlin}}$$

$$\mathcal{L}_{\text{nucleon}}^{\text{free}} = \bar{\psi}(i\gamma_\mu \partial^\mu - m_n)\psi$$

$$\mathcal{L}_{\text{meson}}^{\text{free}} = \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - m_\sigma^2 \Phi^2) - \frac{1}{2}(\frac{1}{2}G_{\mu\nu}G^{\mu\nu} - m_\omega^2 V_\mu V^\mu)$$

$$- \frac{1}{2}(\frac{1}{2}B_{\mu\nu} \cdot B^{\mu\nu} - m_\rho^2 R_\mu \cdot R^\mu) - \frac{1}{4}F_{\mu\nu}$$

$$\mathcal{L}_{\text{coupl}}^{\text{lin}} = -g_\sigma \Phi \bar{\psi} \psi - g_\omega V_\mu \bar{\psi} \gamma^\mu \psi$$

$$- g_\rho \bar{R}_\mu \gamma\gamma^\mu \psi - eA_\mu \bar{\psi} \frac{1 + \tau_3}{2} \gamma^\mu \psi$$

$$\mathcal{L}_{\text{coupl}}^{\text{nonlin}} = \frac{1}{2}m_\sigma^2 \Phi^2 - U_\sigma[\Phi]$$

$$-\Delta A_0 = e\rho_p + e\rho_e$$

First step: constant electron density

Second step: self-consistent calculation
Wigner-Seitz approximation

The whole system is subdivided into individual cells each containing one nucleus, free neutrons and electron cloud.

Requirements on the cells: 1) electroneutrality, 2) fixed average barion density and Z/A.

Nuclear Coulomb energy is reduced due to the electron screening:

\[ F_{AZ}^C(n_e) = \frac{3(eZ)^2}{5R_A} c(n_e), \quad c(n_e) = 1 - \frac{3}{2}\left(\frac{n_e}{n_p}\right)^{1/3} + \frac{1}{2}\left(\frac{n_e}{n_p}\right) < 1 \]
Nuclear structure calculations in uniform electron background

$\beta_2 \quad 0.28 \quad \rightarrow \quad 0.60$

$k_F = 0.5 \text{ fm}^{-1} = 100 \text{ MeV}$
with increasing $k_F$ the $\beta$-stability line moves towards the neutron drip line ($\mu_n=0$), and they overlap already at $k_F=0.1 \text{ fm}^{-1}=20$ MeV free neutrons appear at higher $k_F$ (“neutronization”)}
1) Deformation becomes less favourable because of reduced Coulomb energy
2) Energy of isomeric state (or saddle point) goes up with $n_e$
Suppression of spontaneous fission

Decreasing Q-values disfavor fission mode

Fissility parameter

\[ \frac{Z^2}{A} \geq \frac{2a_s}{a_C} \leq e(k_F) \]

increases with k_F due to reduced Coulomb energy

At k_F=0.25 fm\(^{-1}\) =50 MeV

256 Fm, symm. fission
Adding neutrons into the WS cell

Th. Buervenich, I. Mishustin, C. Ebel et al., work in progress
1) Dripping neutrons are spread rather uniformly outside the nucleus
2) Protons are distributed rather uniformly inside the nucleus
3) With increasing A the surface tension decreases (smaller density gradients)
1) Neutrons as well as protons develop a hole at the center
2) Central proton density drops gradually with increasing nucleus size
Single-particle levels in $\beta$-equilibrium

\[ \mu_n - \mu_p = \mu_e \]

1) Protons are shifted down due to the attractive potential generated by electrons.
2) Neutrons have attractive mean field inside and outside the nucleus.
3) Neutron level density in the continuum is very high.
Conclusions

- Microscopic (HFB, RMF, CEFT,...) calculations are needed to obtain information about nuclear properties (binding energies, level densities etc) in dense and hot stellar environments.

- Partly such information can be obtained also from experimental studies of nuclear reactions at low and intermediate energies (ISOLDE@CERN, SPIRAL2@GANIL, NUSTAR@GSI, FRIB@MSU,...).

- This information is crucial for calculating realistic NEOS and nuclear composition of supernova matter within the Statistical equilibrium approach.

- Survival of (hot) nuclei may significantly influence the explosion dynamics through both the energy balance and modified weak reaction rates.