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Modeling Clusterized Nuclear Matter under Stellar Conditions

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FIAS Frankfurt Institute for Advanced Studies

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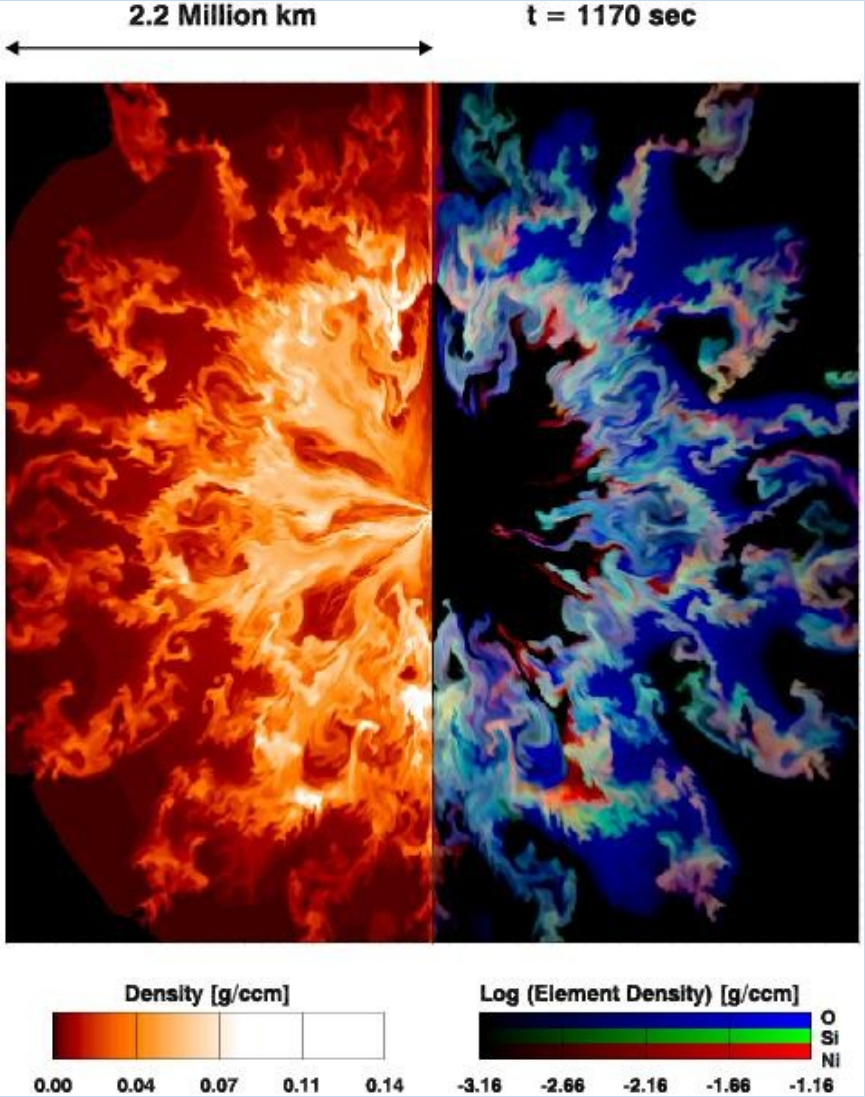
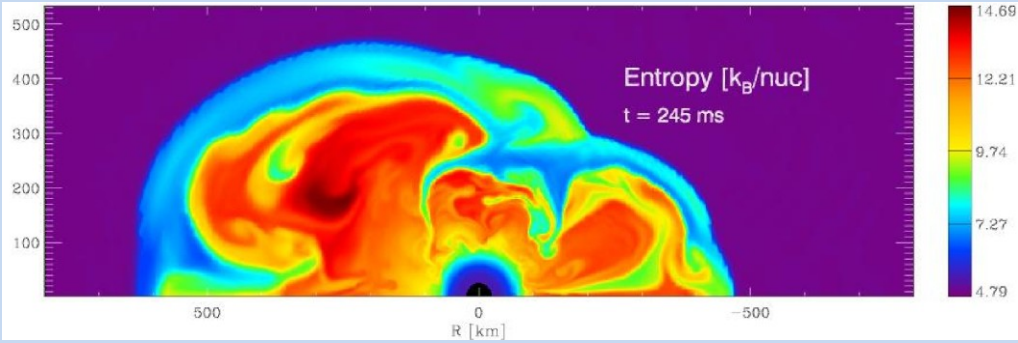
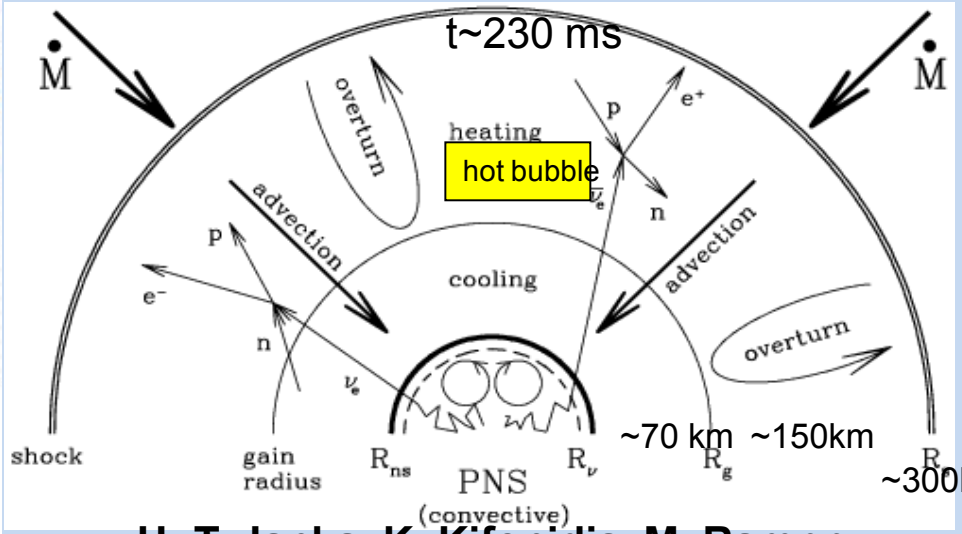
Macro-Supernovae: Numerical simulations





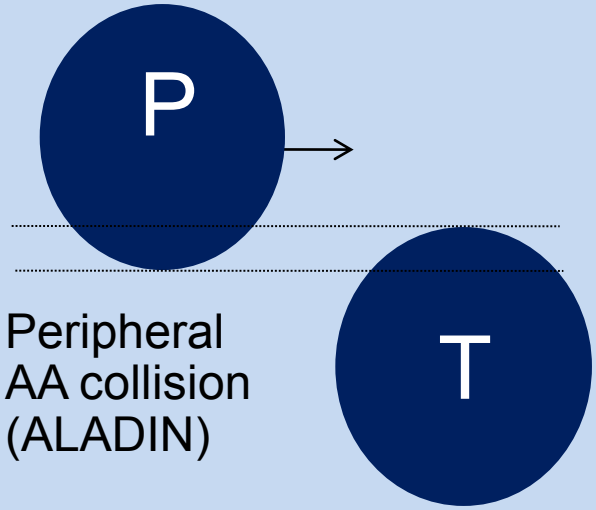
Sketch of the post-collapse stellar core during the neutrino heating and shock revival phase



H.-T. Janka, K. Kifonidis, M. Rampp
Lect.Notes Phys.578:333-363,2001

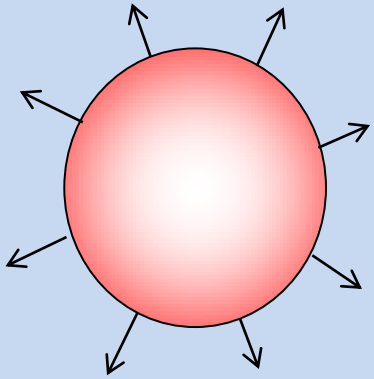
Creation of Micro-Supernovae in laboratory

t=0 fm/c



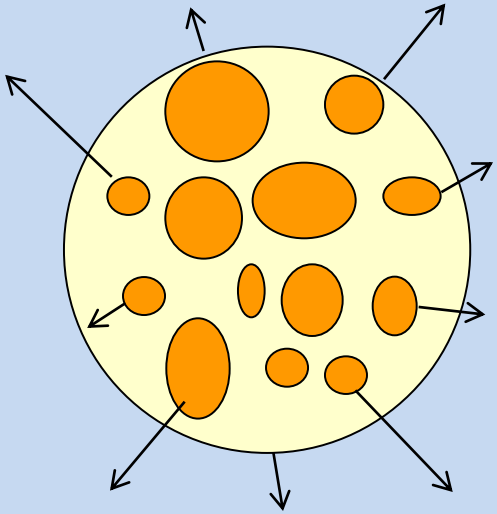
Peripheral
AA collision
(ALADIN)

P or T spectator



Heating : T~10 MeV
ρ<ρ0, P~0
slow expansion

t>100 fm/c

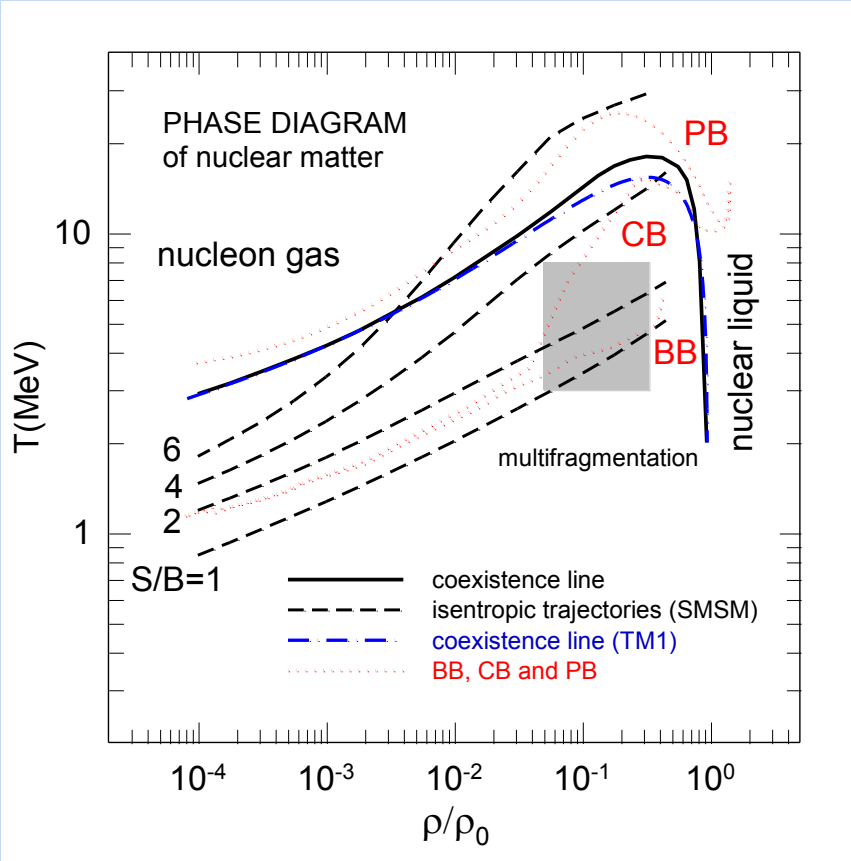


Expanding equilibrated
source made of hot nuclei

Multifragmentation – creation of micro-supernovae in laboratory,
Power-law mass distributions: $Y(A) \sim \qquad \simeq$ **(liquid-gas phase tr.)**
Can be well understood within the equilibrium statistical approach developed in 80s (following ideas of N. Bohr and E. Fermi):

Randrup&Koonin, D.H.E. Gross et al, Bondorf-Mishustin-Botvina, Hahn&Stoecker,..

Similarity of conditions in nuclear reactions and supernova explosions



[Multifragmentation reactions and properties of hot stellar matter at sub-nuclear densities](#)
Botvina&Mishustin, PRC72 (2005) 048801;
N. Buyukcizmecl, A. Botvina, I. Mishustin Et al., ***Nucl. Phys.*** *A907 (2013) 540*

I. Thermal multifragmentation of nuclei:

- **Production of hot fragments at T≈ 3-8 MeV $\rho \approx (0.1 - 0.3)\ \rho_0$**
- **A way to investigate properties of hot nuclei in dense environment**

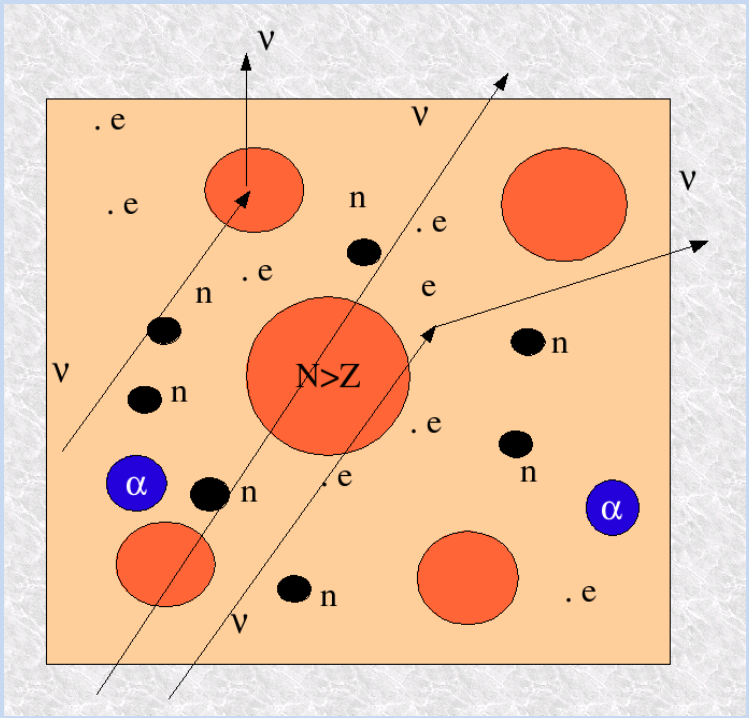
II. Collapse of massive stars leading to Supernova Type II explosions:

- **Production of hot nuclei in stellar environments: T≈ 1-10 MeV $\rho < 0.3\rho_0$**
- **Characteristic times (milliseconds) are very long for nuclear statistical equilibrium to be established.**
- **Properties of hot clustered nuclear matter may influence essentially the neutrino transport in SNe and properties of the NS crust.**

Statistical Model of Stellar Matter (SMSM)

Grand Canonical version of SMM: **Botvina, Iljinov, Mishustin, Sov. J. Nucl. Phys. 42(1985)712**

Fixed $T, \rho_B, Y_{L(e)}$



Calculations are done in a box containing 1000 baryons, density of individual fragments is fixed at $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ Botvina&Mishustin, PLB 584 (2004) 233 584 (2004) 233 584 (2004) 233

Chemical potentials: $\mu_i = B_i\mu_B + Q_i\mu_Q + L_i\mu_L$

nuclear species (A,Z) : $\mu_{AZ} = A\mu_B + Z\mu_Q$

electrons e^- : $\mu_{e^-} = -\mu_Q + \mu_L = -\mu_{e^+}$

neutrinos ν : $\mu_\nu = \mu_L = -\mu_{\tilde{\nu}}$

Baryon number conservation :

$$\rho_B = \frac{B}{V} = \sum_{(A,Z)} A\langle n_{AZ}\rangle \text{ fixed} \quad \rightarrow \mu_B$$

Electric neutrality

$$\rho_Q = \frac{Q}{V} = \sum_{(A,Z)} Z\langle n_{AZ}\rangle - n_e = 0 \quad \rightarrow \mu_Q$$

Lepton number conservation

$$Y_L = \frac{L}{B} = \frac{n_e + n_\nu}{\rho_B} \text{ (trapped } \nu\text{) or } Y_e = \frac{n_e}{\rho_B} \text{ (no } \nu\text{)}$$

Nuclear statistical equilibrium ensembles

All nuclear species are included, not only one "average" nucleus!

Number density of nuclear species (A,Z): Pressure $= P_{\text{nuc}} + P_{\text{Coul}} + P_{electrons}$

$$n_{AZ} = g_{AZ}\frac{V_f}{V}\frac{A^{3/2}}{\lambda_T^3}\exp\left[-\frac{1}{T}\left(F_{AZ}-\mu_{AZ}\right)\right] \qquad P_{nuc} = T\sum_{(A,Z)} n_{AZ}$$

Internal free energy of species (A,Z) for $A>4$

$$F_{AZ} = F_{AZ}^{B} + F_{AZ}^{S} + F_{AZ}^{sym} + F_{AZ}^{C}$$ liquid drop parametrization

$$F_{AZ}^{B}(T) = \left(-w_0 - \frac{T^2}{\varepsilon_0}\right)A, \quad F_{AZ}^{S} = \beta_0\left(\frac{T_c^2 - T^2}{T_c^2 + T^2}\right)^{5/4} A^{2/3}, \quad F_{AZ}^{sym} = \gamma\frac{(A-2Z)^2}{A}$$

Reduced Coulomb energy due to the electron screening

$$F_{AZ}^{C}(n_e) = c(n_e)\frac{3}{5}\frac{(eZ)^2}{r_0 A^{1/3}}, \quad c(n_e) = 1 - \frac{3}{2}\left(\frac{n_e}{n_{0p}}\right)^{1/3} + \frac{1}{2}\left(\frac{n_e}{n_{0p}}\right) < 1$$

Consider box with 10^3 nucleons, μ_B, μ_Q, μ_L are found iteratively

Other versions of Statistical Model

SMSM:Statistical Model for Supernova Matter: A.S. Botvina, I.N. Mishustin, PLB 584 (2004)233,NPA 843, (2010) 98., Buyukcizmeci, Botvina, Mishutin, APJ 789 (2014)33.

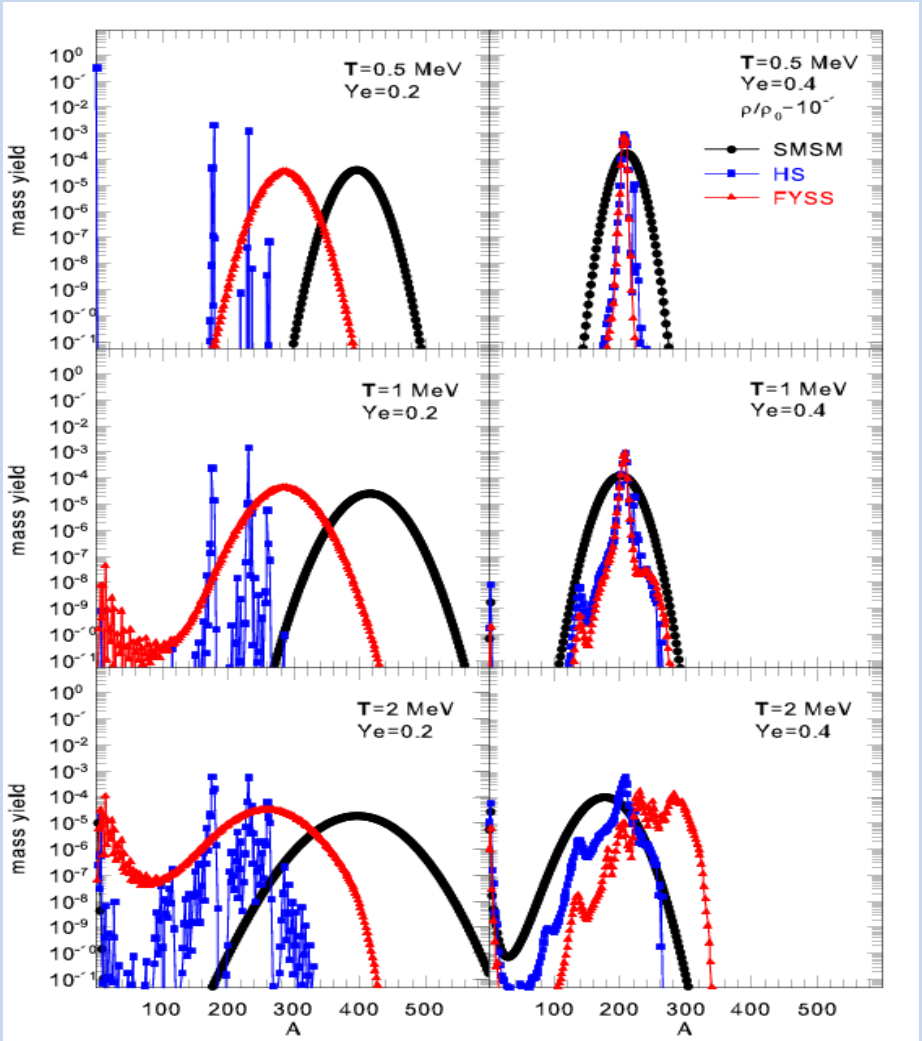
HS: M. Hempel and J.Schaffner-Bielich, NPA 837 (2010) 210.

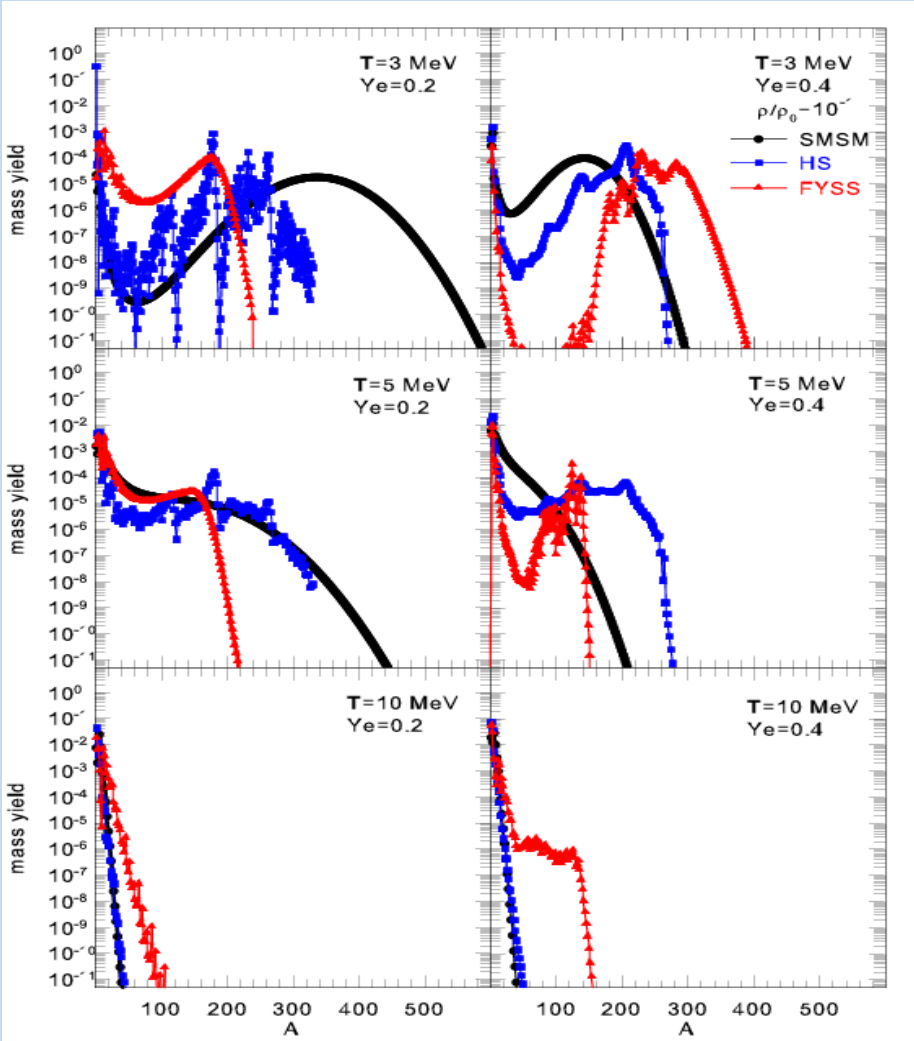
FYSS: S. Furusawa, S. Yamada, K. Sumiyoshi, H. Suzuki, APJ, 738 (2011) 178.

	SMSM	HS	FYSS
Model	NSE +LDM	RMF + NSE+ mass data (Theoretical +Experimental)	RMF+ NSE+ +LDM +mass data(Experimental)
Component heavy nuc.	multi (Z<1000) $1 \leq A \leq 1000$	multi (Z<100) $1 \leq A \leq 331$	multi (Z<1000) $1 \leq A \leq 1000$
Shell term	×	○	△
Nuclear Shape	Droplet only	Droplet only	Droplet +bubble (+other)
$E_{symmetry}$ of nuclei	25 MeV	mass data	38 MeV
$E_{surface}$	△ depending on T	mass data	△ depending on density &symmetry Z/A

N. Buyukcizmeci, A.S. Botvina, I.N. Mishustin, R. Ogul, M. Hempel, J. Schaffner-Bielich, F.-K. Thielemann, S. Furusawa, K. Sumiyoshi, S. Yamada, H. Suzuki, NPA 907 (2013)13-540

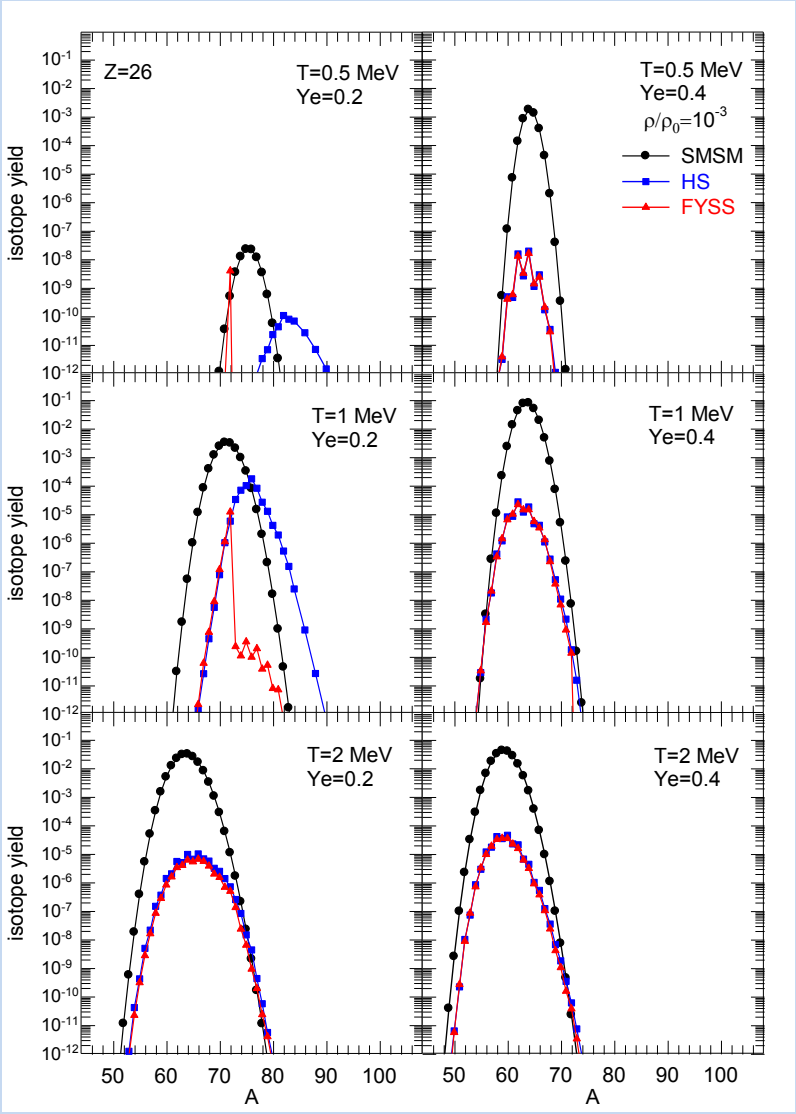
Mass distributions : SMSM, FYSS and HS ($\rho/\rho_0=10^{-1}$)

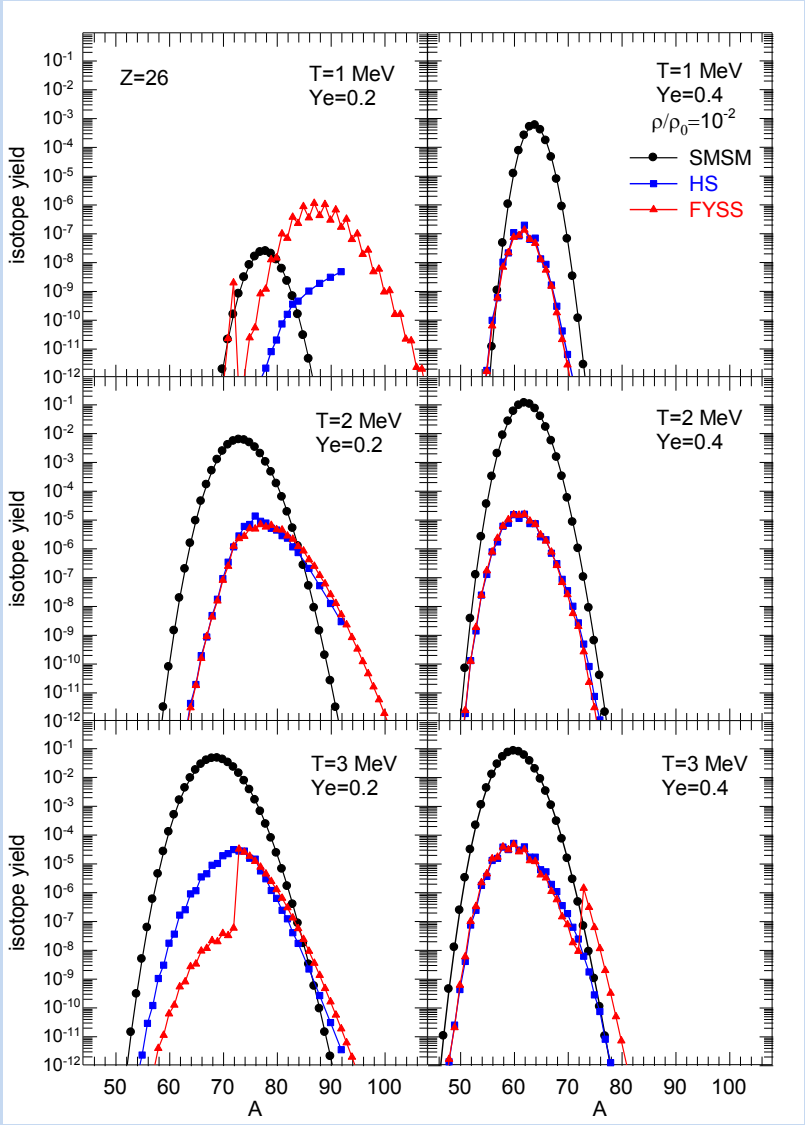




Different versions of the statistical model using "reasonable" assumptions on nuclear properties give very different results!

Isotopic distributions : ^{26}Fe

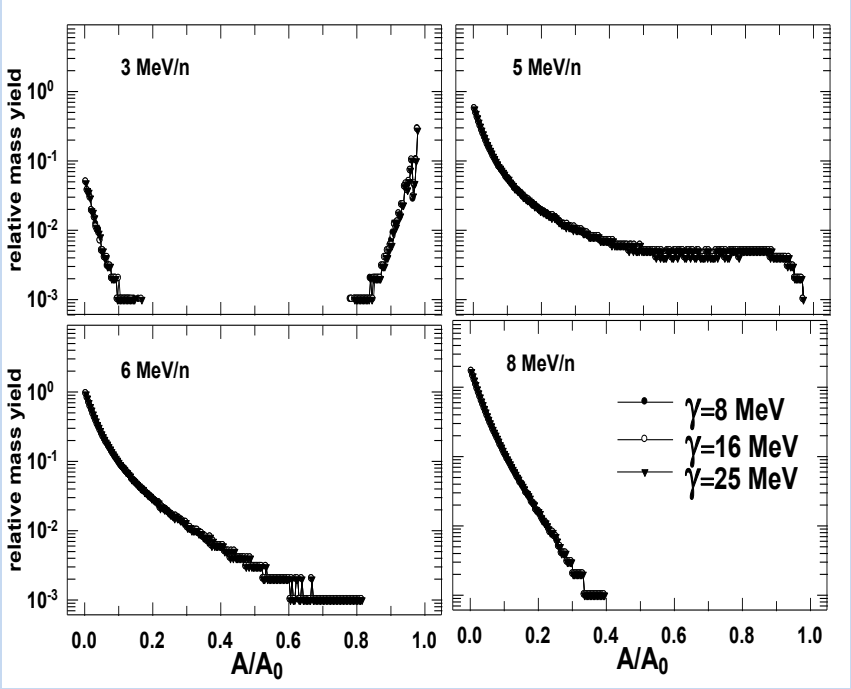




Significant differences are seen at all T and Y_e!

Influence of the symmetry energy term on mass distributions

nuclear reactions

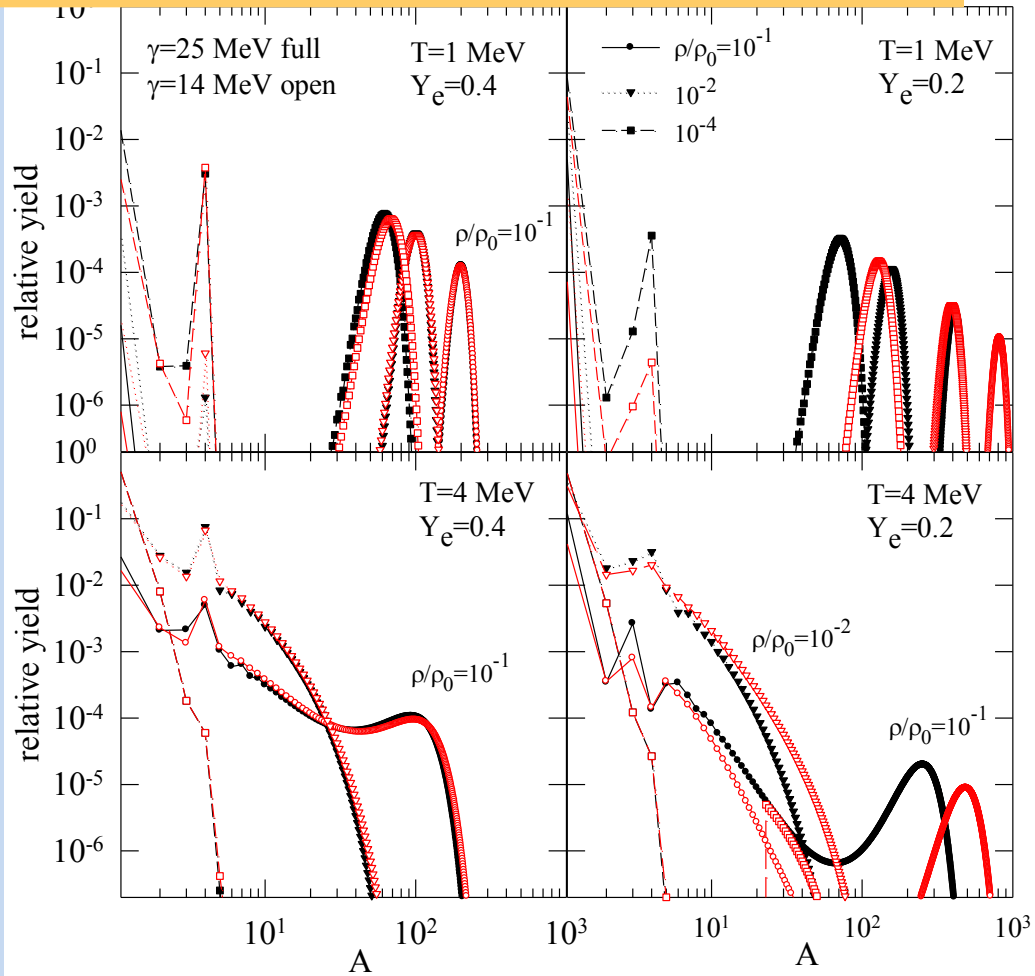


practically no influence

N. Buyukcizmeci, R. Ogul and A. S. Botvina, EPJ A 25, 57(2005).

A.S. Botvina, N. Buyukcizmeci, et al., Phys. Rev. C 74, 044609 (2006)

stellar matter



For the stellar matter, with smaller symmetry energy (γ=14) much more heavy nuclei may be formed, since they can accumulate more neutrons at lower temperatures and Ye.

Buyukcizmeci N., A. S. Botvina, I. N. Mishustin, R. **Ogul**, *Jour. Phys. Con. Ser.* **202 (2010) 012003**

Challenge for nuclear physics community

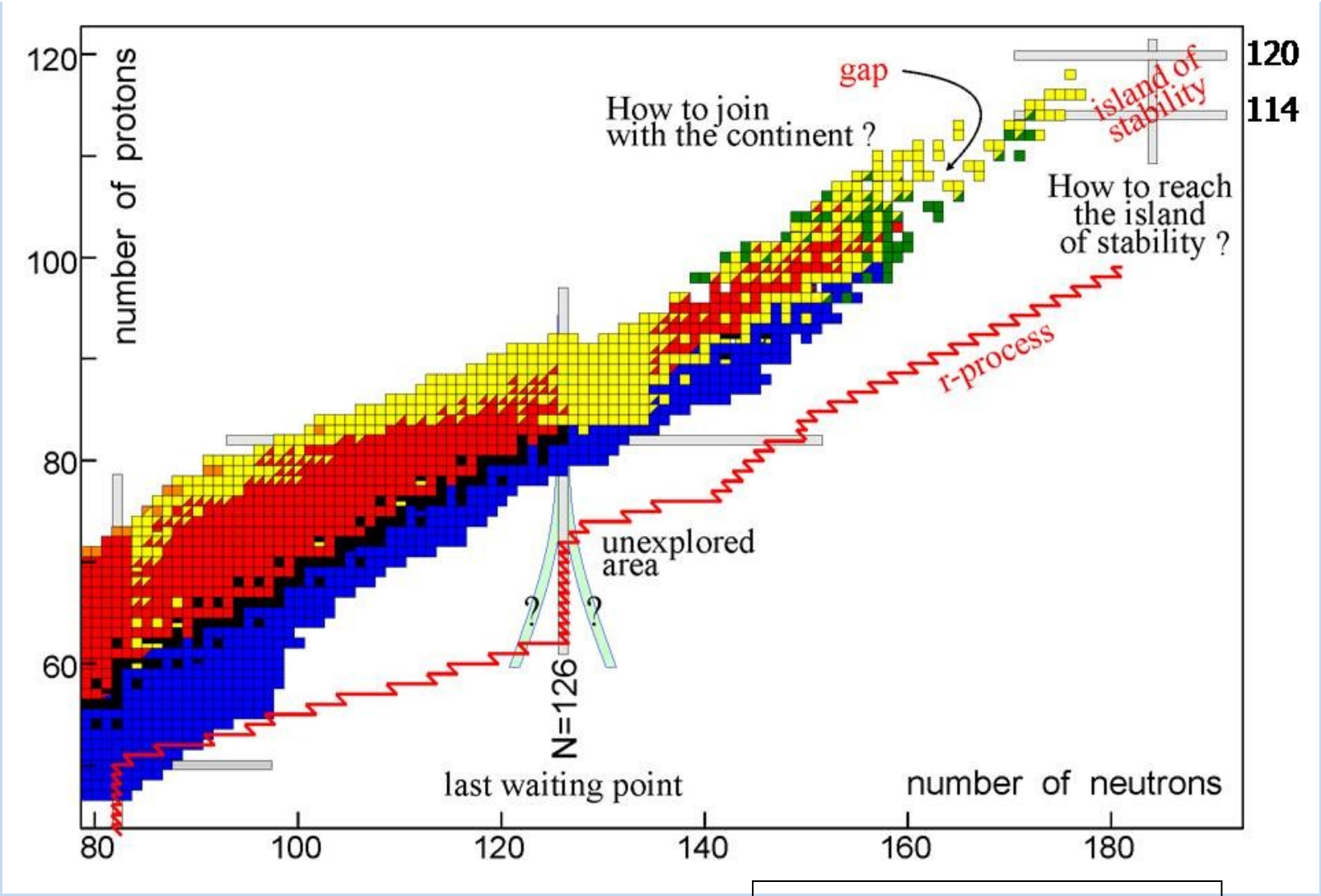
Evaluate nuclear properties in stellar environments, i.e. at finite temperature (T), average baryon density (ρ) and electron or lepton fraction (Y): $0<T<10$ MeV, $10^{-5}<\rho/\rho_0<0.5$, $0.1<Y<0.6$.

One should evaluate a) binding energies, b) excited states, c) decay probabilities of a huge number of nuclei ($1<A<1000$, $0<Z<A$), i.e. construct a "Nuclear Chart" for each set (T,ρ,Y).

Existing nuclei represent only a very small subset around (0,0,0) corner. Altogether up to 10^8 new "data" points should be calculated using any "reasonable" approach (LDM, HFB, RMF, CEFT,…)

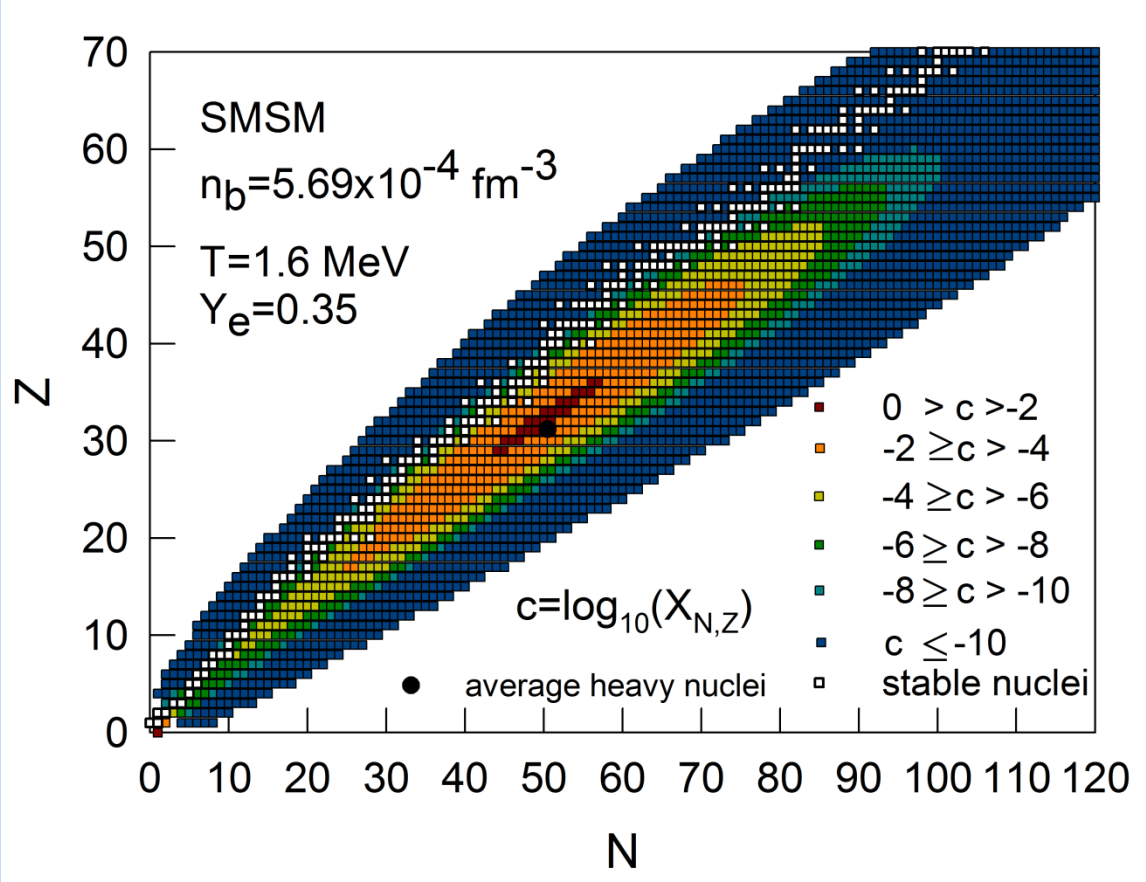
Such data are needed as inputs for a Statistical Model to find the most probable Nuclear Statistical Ensemble (NSE) in a specific stellar environment.

Nuclear Chart is not fully explored even at laboratory conditions but we need much more!



This slide is borrowed from **Valery Zagrebaev**

Example of "Nuclear chart" for supernova conditions



N. Buyukcizmecl et al. to be published

Possible tool: RMF model + electrons

T. Buervenich, I. Mishustin and W. Greiner, Phys. Rev. C76 (2007) 034310;
C. Ebel, U. Heinzmann, I. Mishustin, S. Schramm, work in progress

$+$ BCS δ – force pairing

parameter set: NL3

$$\mathcal{L} = \mathcal{L}^{\text{free}}_{\text{nucleon}} + \mathcal{L}^{\text{free}}_{\text{meson}} + \mathcal{L}^{\text{lin}}_{\text{coupl}} + \mathcal{L}^{\text{nonlin}}_{\text{coupl}}$$

$$\mathcal{L}^{\text{free}}_{\text{nucleon}} = \bar{\psi}(i\gamma_\mu\partial^\mu - m_n)\psi$$

$$\begin{aligned}\mathcal{L}^{\text{free}}_{\text{meson}} &= \frac{1}{2}(\partial_\mu\Phi\partial^\mu\Phi - m_\sigma^2\Phi^2) - \frac{1}{2}(\frac{1}{2}G_{\mu\nu}G^{\mu\nu} - m_\omega^2 V_\mu V^\mu) \\ &- \frac{1}{2}(\frac{1}{2}\vec{B}_{\mu\nu}\cdot\vec{B}^{\mu\nu} - m_\rho^2\vec{R}_\mu\cdot\vec{R}^\mu) - \frac{1}{4}F_{\mu\nu}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\text{lin}}_{\text{coupl}} &= -g_\sigma\Phi\bar{\psi}\psi - g_\omega V_\mu\bar{\psi}\gamma^\mu\psi \\ &- g_\rho\vec{R}_\mu\cdot\bar{\psi}\vec{\tau}\gamma^\mu\psi - eA_\mu\bar{\psi}\frac{1+\tau_3}{2}\gamma^\mu\psi\end{aligned}$$

$$\mathcal{L}^{\text{nonlin}}_{\text{coupl}} = \frac{1}{2}m_\sigma^2\Phi^2 - U_\sigma[\Phi]$$

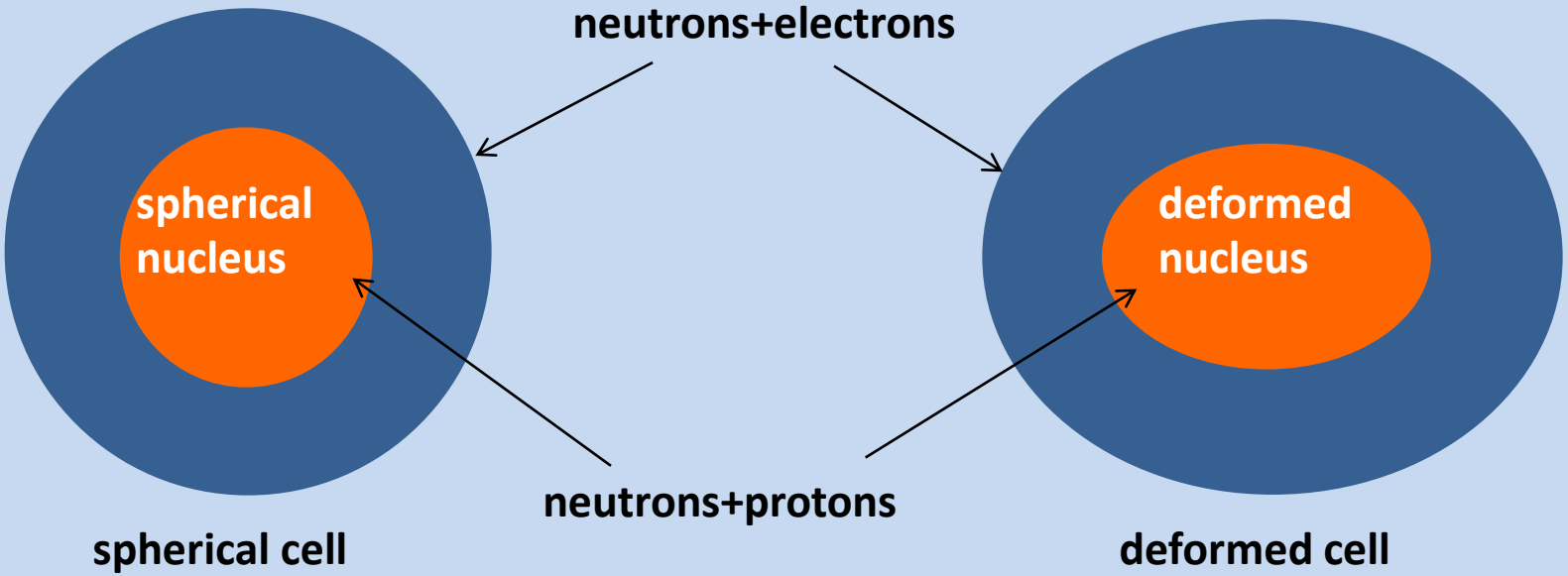
$$-\Delta A_0 = e\rho_p + e\rho_e$$

First step: constant electron density

Second step: self-consistent calculation

Wigner-Seitz approximation

The whole system is subdivided into individual cells each containing one nucleus, free neutrons and electron cloud



Requirements on the cells: 1) electroneutrality, 2) fixed average barion density and Z/A

Nuclear Coulomb energy is reduced due to the electron screening:

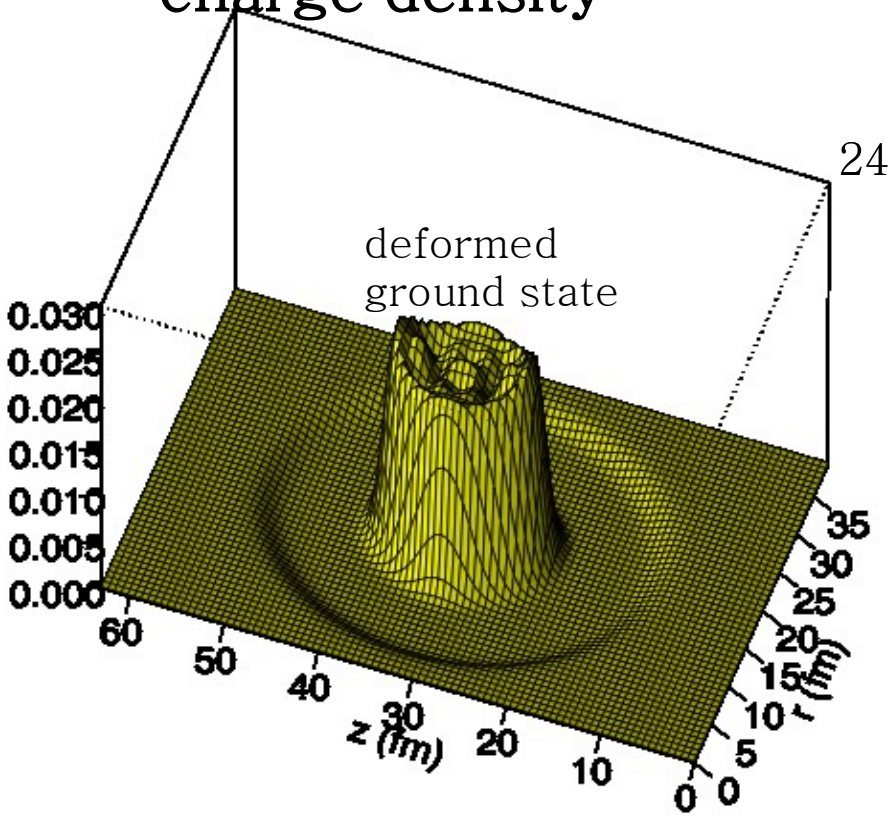
$$F^{C}_{AZ}(n_e)=\frac{3}{5}\frac{(eZ)^2}{R_A}c(n_e),\quad c(n_e)=1-\frac{3}{2}\left(\frac{n_e}{n_p}\right)^{1/3}+\frac{1}{2}\left(\frac{n_e}{n_p}\right)<1$$

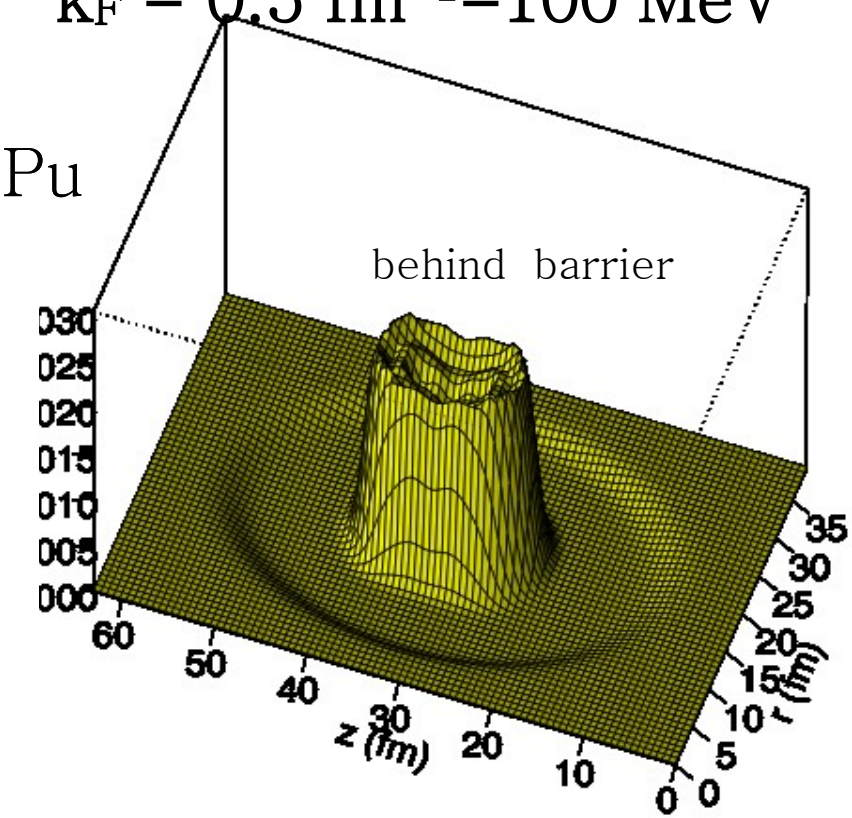
Nuclear structure calculations in uniform electron background

charge density

$k_F = 0.5\ fm^{-1} = 100\ MeV$

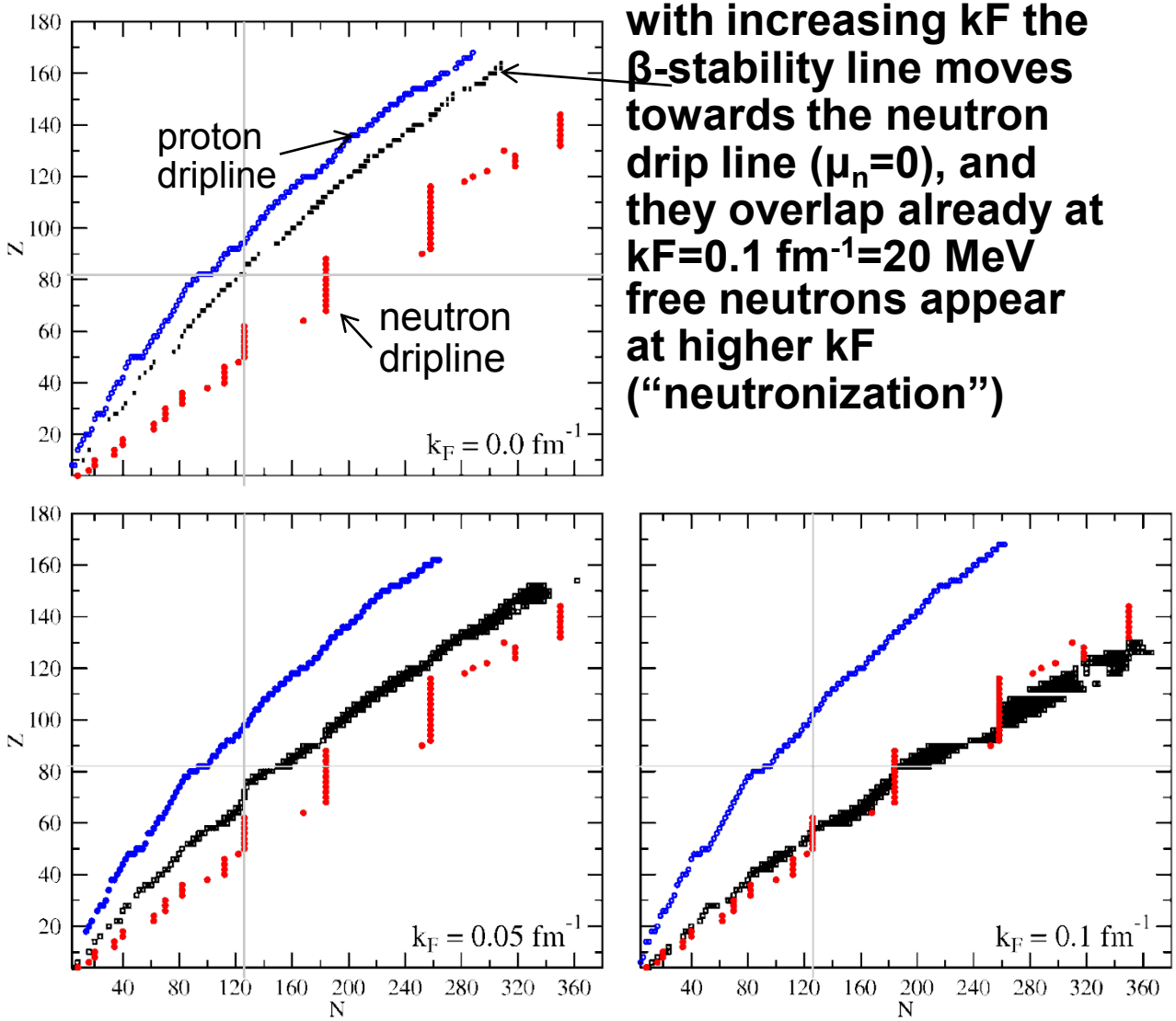
^{240}Pu





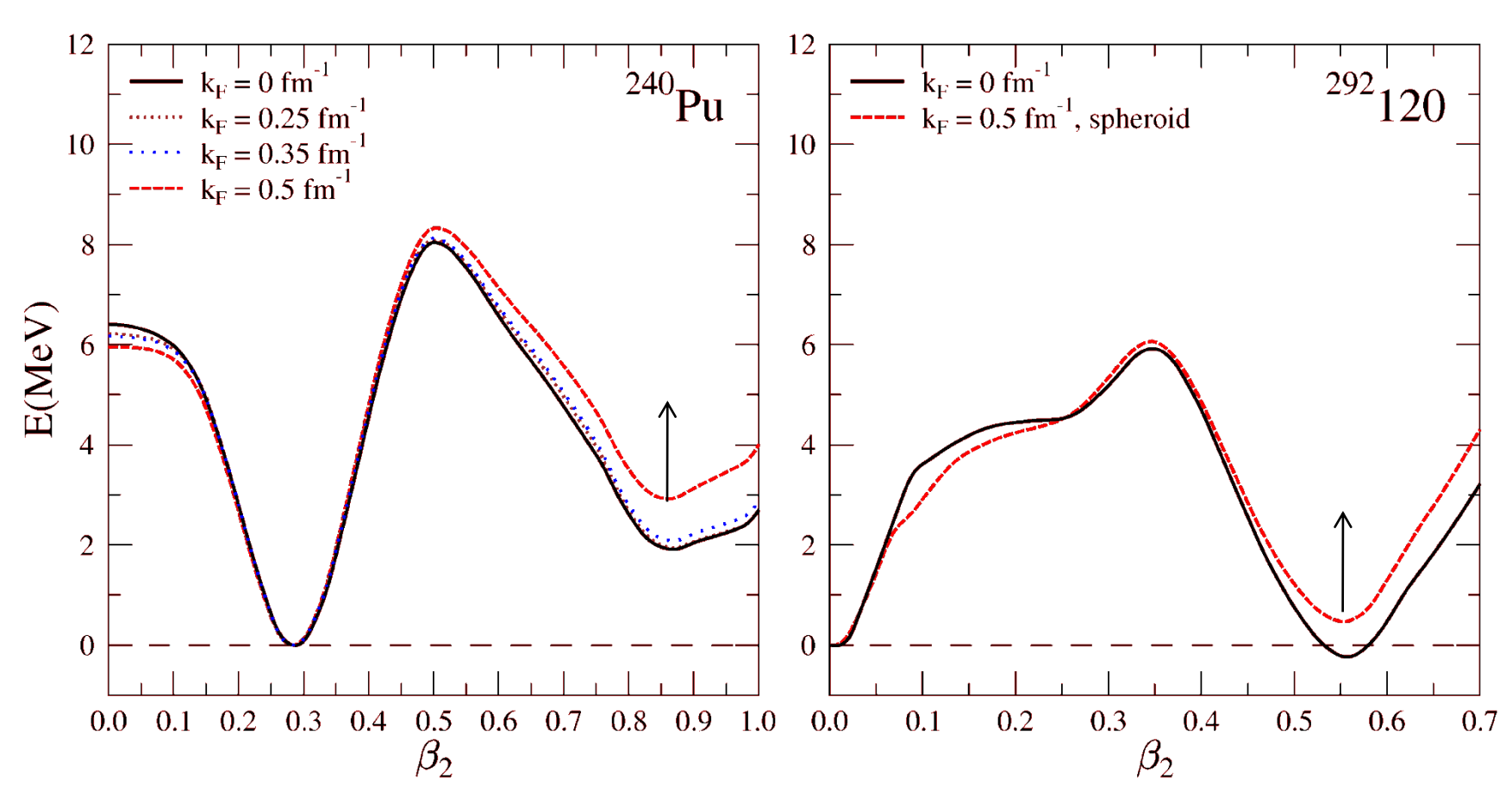
β_2 0.28 → 0.60

Nuclear chart in supernova interiors



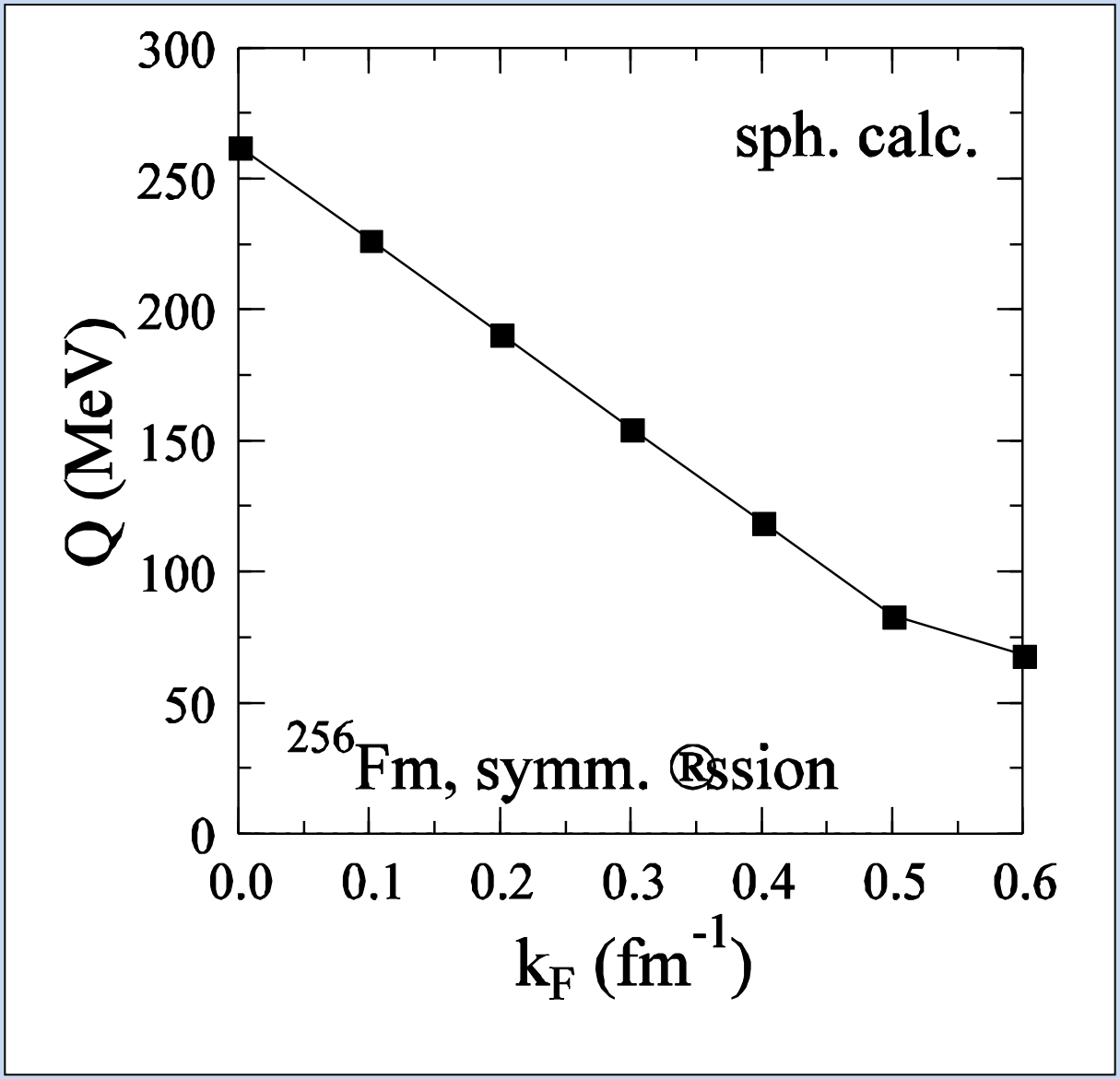
with increasing kF the β-stability line moves towards the neutron drip line (μ_n=0), and they overlap already at kF=0.1 fm^{-1}=20 MeV free neutrons appear at higher kF ("neutronization")

Energy of ellipsoidal deformation



1) Deformation becomes less favourable because of reduced Coulomb energy
2) Energy of isomeric state (or saddle point) goes up with n_e

Suppression of spontaneous fission



Decreasing Q-values disfavor fission mode

Fissility parameter

$$\frac{Z^2}{A} \geq \frac{2a_S}{a_C} \simeq$$

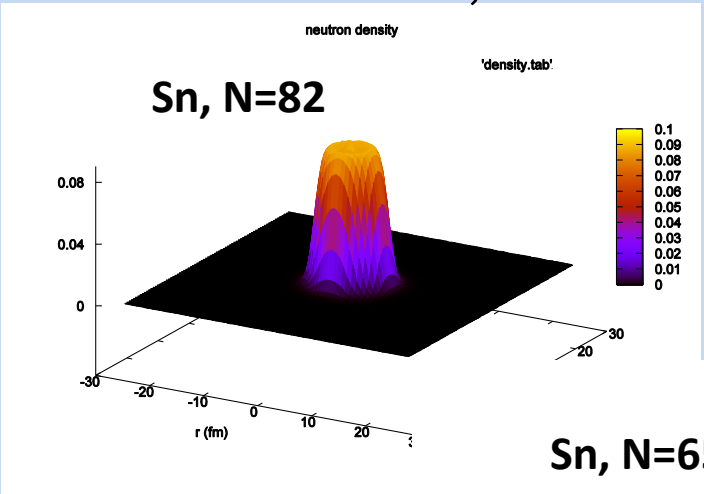
increases with kF due to reduced Coulomb energy

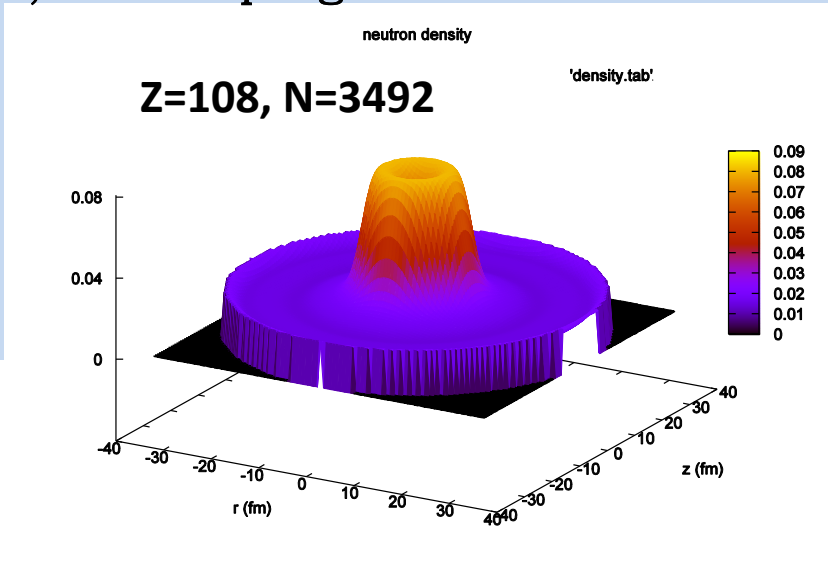
At kF=0.25 fm^{-1} =50 MeV

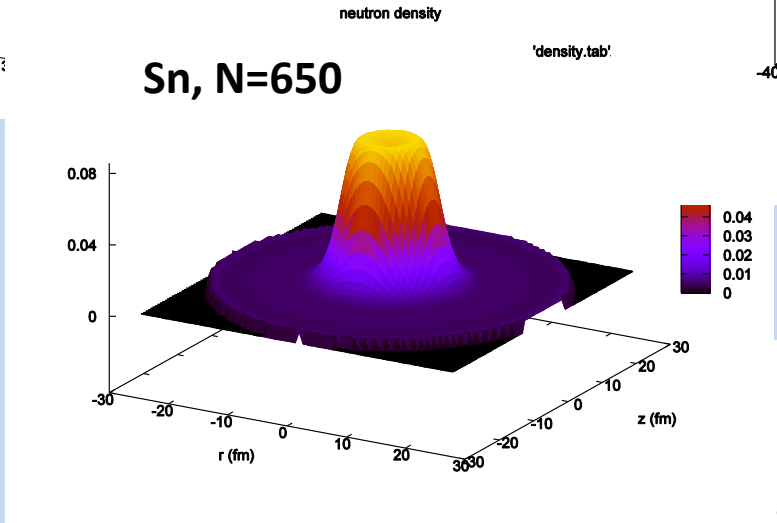
$$\frac{Z^2}{A} \simeq$$

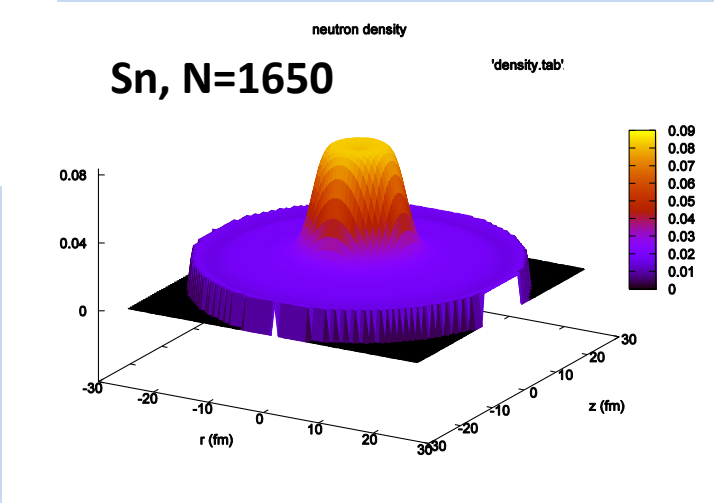
Adding neutrons into the WS cell 1

Th. Buervenich, I. Mishustin, C. Ebel et al., work in progress

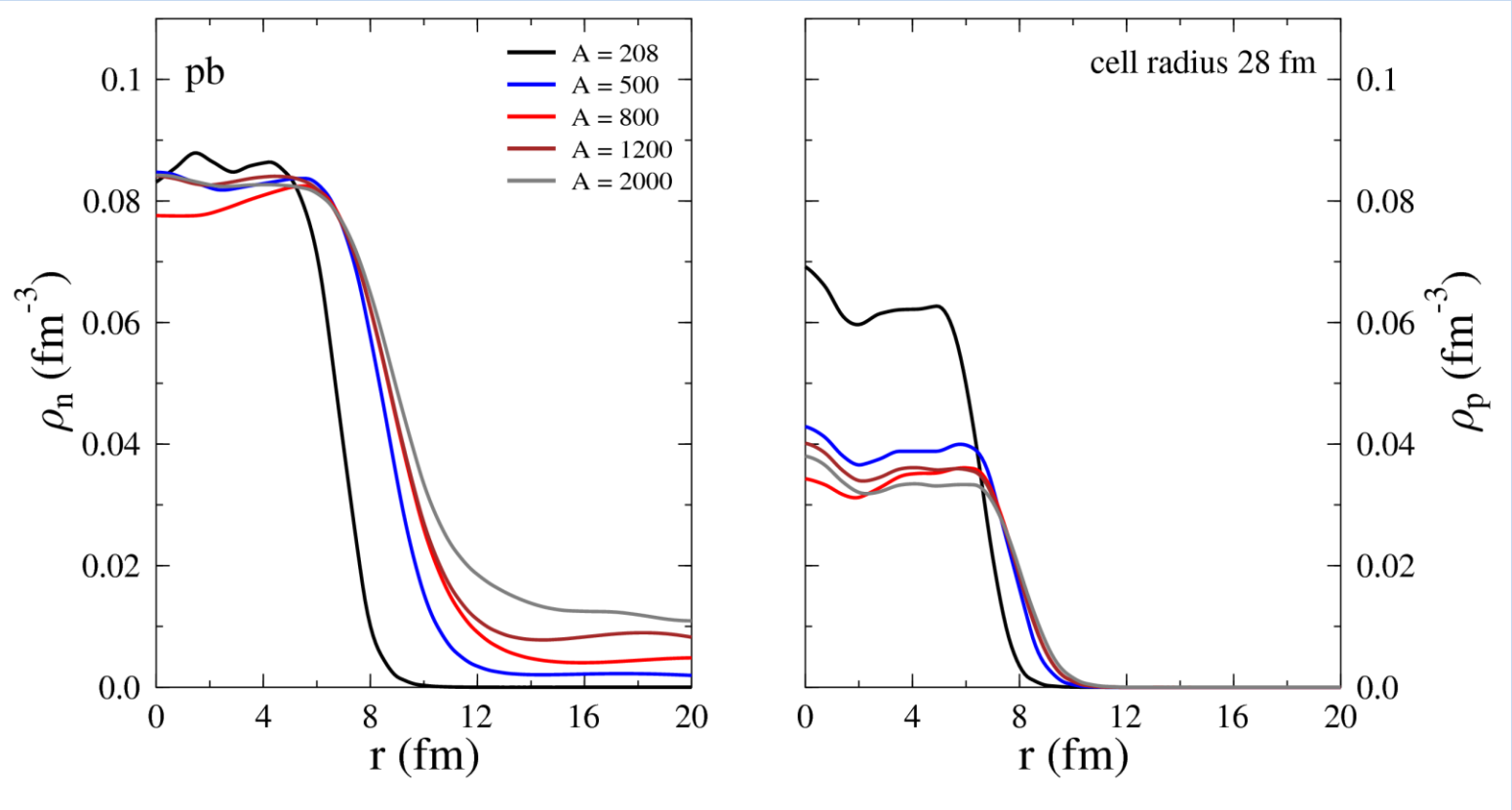






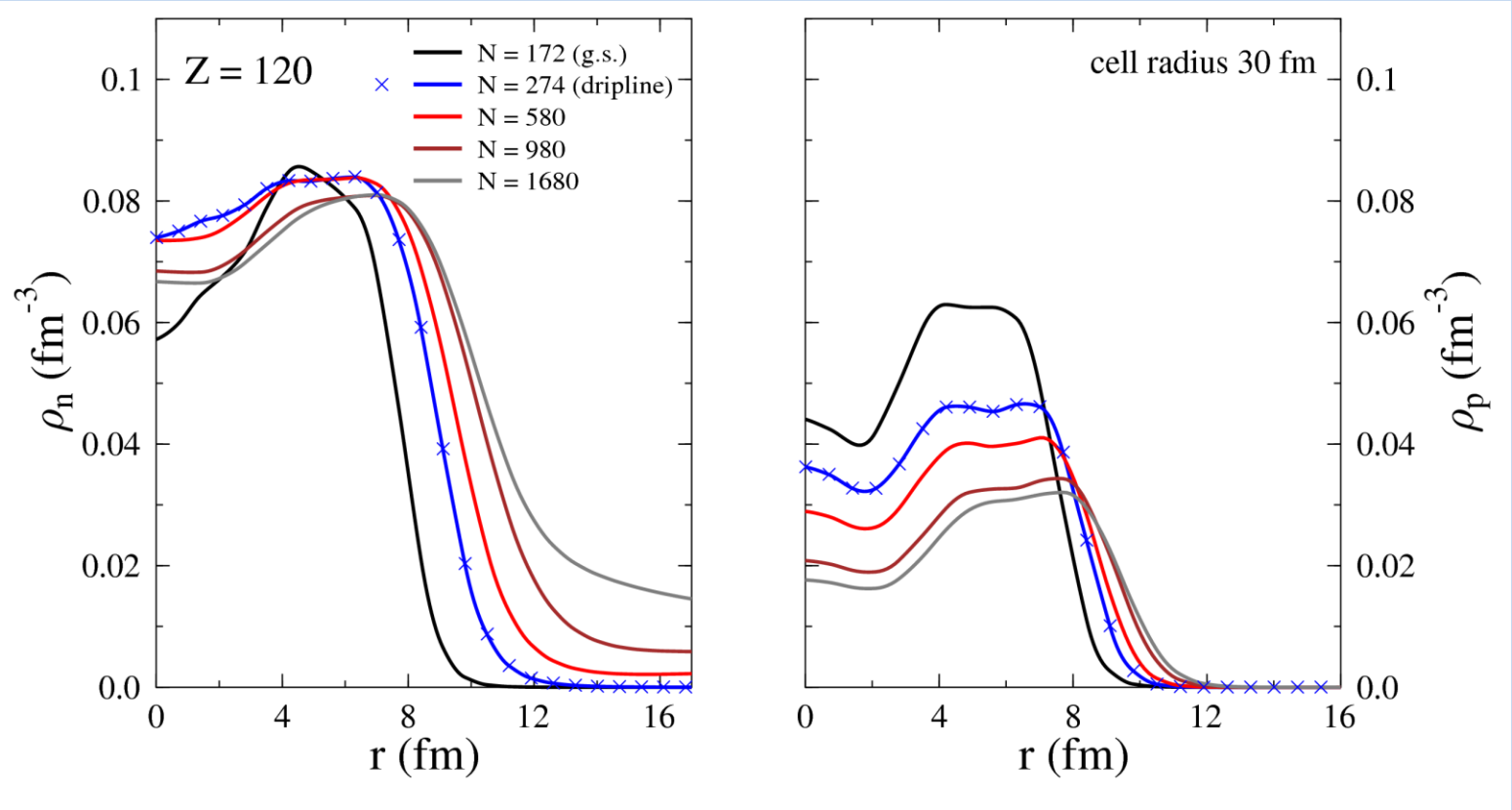


Adding neutrons into the WS cell 2



1) Dripping neutrons are spread rather uniformly outside the nucleus
2) Protons are distributed rather uniformly inside the nucleus
3) With increasing A the surface tension decreases (smaller density gradients)

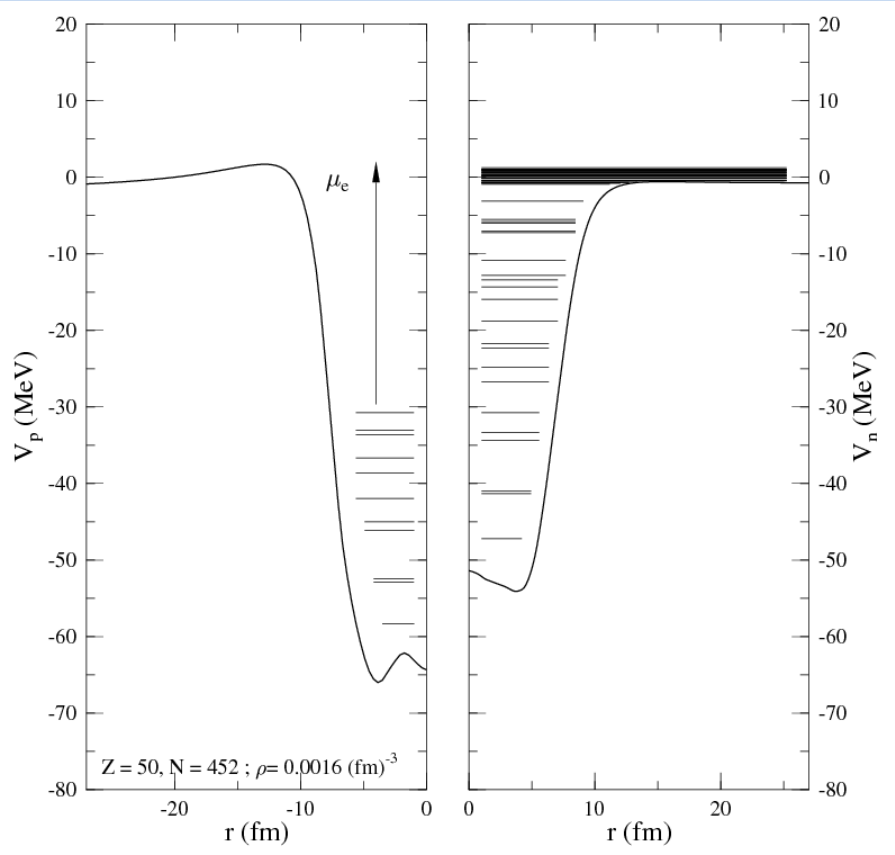
Adding neutrons into the WS cell 3

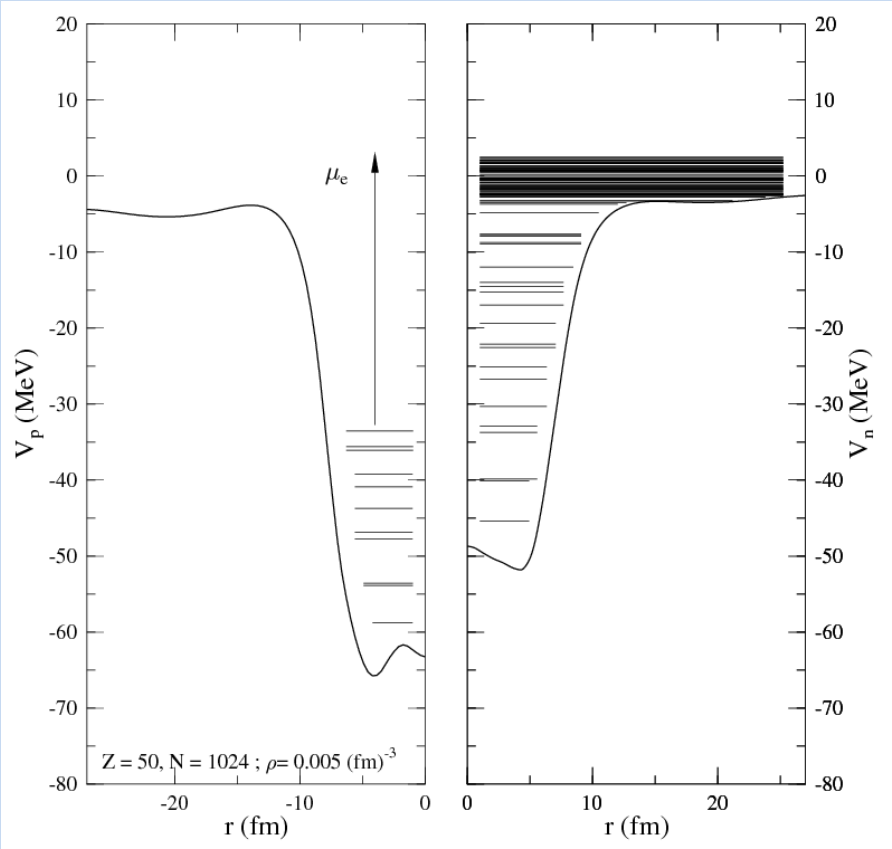


1) Neutrons as well as protons develop a hole at the center
2) Central proton density drops gradually with increasing nucleus size

Single-particle levels in β-equilibrium

$$\mu_n - \mu_p = \mu_e$$





1) Protons are shifted down due to the attractive potential generated by electrons.
2) Neutrons have attractive mean field inside and outside the nucleus.
3) Neutron level density in the continuum is very high.

Conclusions

- **Microscopic (HFB, RMF, CEFT,...) calculations are needed to obtain information about nuclear properties (binding energies, level densities etc) in dense and hot stellar environments.**
- **Partly such information can be obtained also from experimental studies of nuclear reactions at low and intermediate energies (ISOLDE@CERN, SPIRAL2@GANIL, NUSTAR@GSI, FRIB@MSU,...).**
- **This information is crucial for calculating realistic NEOS and nuclear composition of supernova matter within the Statistical equilibrium approach.**
- **Survival of (hot) nuclei may significantly influence the explosion dynamics through both the energy balance and modified weak reaction rates.**