# A possible imprint of light-speed anisotropy on the CMB

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# Outline

**The modern view is that there exists a preferred frame of reference** related to the cosmic microwave background (CMB), more precisely to the last scattering surface (LSS), and that our galaxy's peculiar motion with respect to the CMB produces Doppler effect responsible for the CMB temperature anisotropies.

It is evident that the existence of a preferred frame of reference is in contradiction with the fundamental hypothesis of the special relativity which Einstein termed the **'principle of relativity'**.

A violation of the relativity principle influences also a validity of the *principle of universality of the speed of light*, in particular its constancy and isotropy, and, in general, implies a **violation of the special relativity**.

Nevertheless, **the special relativity formulas are commonly used in the context of cosmology** when there is a need to relate physical effects in the frames moving with respect to each other.

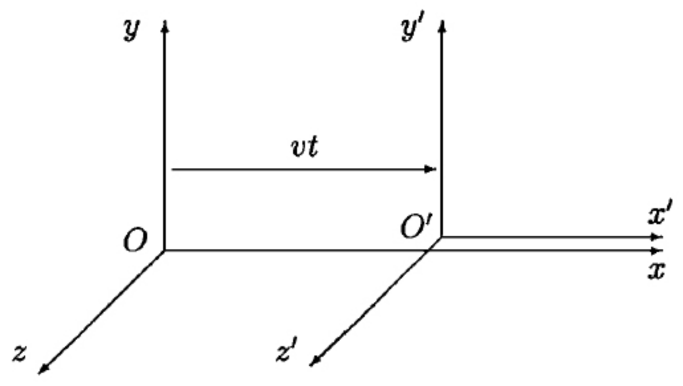
## Example: the CMB temperature anisotropy due to our galaxy's peculiar motion

The CMB temperature angular distribution due to *the Doppler effect* is a pure dipole pattern in terms of the angle between the line of sight and the direction of the observer motion, measured in the frame of the source.

Due to the effect of *light aberration* this angle transforms into the angle measured in the moving frame of the observer and, as the result, the quadrupole and higher moments arise

$$T(\tilde{\theta}) = T_0 \left(1 + + \beta \cos\tilde{\theta} + \frac{\beta^2}{2} \cos 2\tilde{\theta}\right)$$

`Derivation of this formula is based on the Lorentz transformations of the relativity theory.`




Lorentz transformations

$$x = \frac{X - vT}{\sqrt{1 - v^2/c^2}}, \qquad y = Y, \quad z = Z, \qquad t = \frac{T - vX/c^2}{\sqrt{1 - v^2/c^2}}$$

Doppler effect

A source of electromagnetic radiation (light) is in a reference frame $S$ and the observer is in the frame $S'$ moving with velocity $v$ with respect to $S$

$$\nu_r = \nu_e \frac{1 - \beta \cos \Theta}{\sqrt{1 - \beta^2}}$$

where $\Theta$ is the angle between the wave vector and the direction of motion, *measured in the frame of the source* $S$.

The aberration formula

$$\cos \Theta = \frac{\beta - \cos \tilde{\theta}}{1 - \beta \cos \tilde{\theta}}$$

where $\tilde{\theta}$ is the angle between the line of sight and the direction of the observer motion, *measured in the frame of the observer*.

**The frequency shift relation**

$$\nu_r = \nu_e \frac{\left(1 - \beta^2\right)^{\frac{1}{2}}}{1 - \beta \cos \tilde{\theta}}$$

**In the context of the CMB anisotropy**

$$\frac{T(\tilde{\theta})}{\nu_r} = \frac{T_0}{\nu_e}$$

where $T_0$ is the effective temperature measured by the observer, that is at rest relative to the LSS and sees strictly isotropic blackbody radiation and $T(\tilde{\theta})$ is the effective temperature of the blackbody radiation for the moving observer looking in the fixed direction $\tilde{\theta}$.

Using the relation for the frequency shift yields

$$T(\tilde{\theta}) = T_0 \frac{\left(1-\beta^2\right)^{\frac{1}{2}}}{1-\beta\cos\tilde{\theta}}$$

$$T(\tilde{\theta}) = T_0 \left(1 + +\beta\cos\tilde{\theta} + \frac{\beta^2}{2}\cos 2\tilde{\theta}\right)$$

## 'Test theories'

The discovery of the cosmic background radiation, which has shown that cosmologically a preferred system of reference does exist, and the fact, that some of modern theories suggest a violation of special relativity, resulted in renewed interest in sensitive experimental tests of relativity.

To describe tests of basic principles underlying a theory and to quantitatively express the degree of agreement between experiments and these principles, a theory which allows violations of these principles is required.

`The Mansouri and Sexl test theory framework`

- It is assumed that there exists a preferred inertial reference frame ("ether frame"), in which the speed of light is isotropic.
- Generalized transformations between a preferred frame and a moving frame are postulated.

Since the only preferred frame one may think of is the cosmological frame, in which the microwave background radiation is isotropic, the preferred frame of reference is identified with the CMB frame.

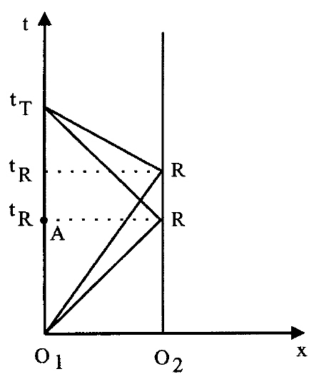
Transformations postulated in the Mansouri and Sexl (MS) test theory

$$t = a(v)T + \epsilon_1(v)x + \epsilon_2(v)y + \epsilon_3(v)z$$
$$x = d(v)X + (b\,(v) - d\,(v)) - b(v)vT$$
$$y = d(v)Y, \quad z = d(v)Z$$

- Abolition of the relativity principle
- Anisotropy of the *two-way speed of light* in the frame moving with respect to the preferred frame.

## Conventionality of simultaneity and anisotropic propagation of light

- Simultaneity at distant space points of an inertial system is defined by a clock synchronization that makes use of light signals.
- We have empirical access only to the round-trip (two-way) average speed of light.
- There exists inescapable entanglement between remote clock synchronization and one-way velocity of light.
- If a light ray is emitted from the master clock and reflected off the remote clock one has a freedom to give the reflection time $t$ at the remote clock any intermediate time in the interval between the emission and reception times



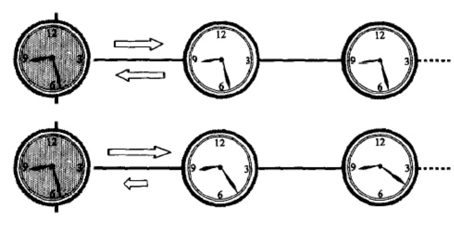

$$\textit{Einstein (standard) synchronization}: \quad t_R^{(s)} = \frac{t_0 + t_A}{2} = t_0 + \frac{1}{2}(t_A - t_0)$$

$$\textit{Non} - \textit{standard synchronization}: \quad t_R = t_0 + \epsilon(t_A - t_0); \quad \left(\epsilon \neq \frac{1}{2}\right)$$

$$\epsilon = \frac{1 + k_\epsilon}{2}; \quad V_+ = \frac{c}{1 + k_\epsilon}. \ V_- = \frac{c}{1 - k_\epsilon}$$

*If the described procedure is used for setting up throughout the frame of a set of clocks using signals from some master clock placed at the spatial origin*

$$t_R = t_0 + \frac{x}{V_+}, \quad t_A = t_R + \frac{x}{V_-}, \Rightarrow \; t_R = \frac{t_0 + t_A}{2} + x\left(\frac{1}{2V_+} - \frac{1}{2V_-}\right)$$



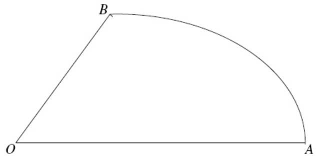

$$t_R = \frac{t_0 + t_A}{2} + \frac{k_\epsilon x}{c} = t_R^{(s)} + \frac{k_\epsilon x}{c}$$

**Thus, a difference in the standard and nonstandard clock synchronization may be reduced to a change of coordinates**

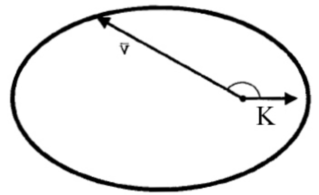
$$t = t^{(s)} + \frac{k_\epsilon x}{c}, \quad x = x^{(s)} \tag{1}$$

where $t^{(s)}$ is the time setting according to Einstein (standard) synchronization procedure.

*Round-trip postulate*




$$V(\mathbf{n}) = \frac{c\mathbf{n}}{1+\mathbf{k}_\epsilon \mathbf{n}} \quad \text{or} \quad V(\Theta) = \frac{c}{1+k_\epsilon \cos\Theta}$$




$$t = t^{(s)} + \frac{\mathbf{k}_\epsilon \mathbf{r}}{c}, \quad \mathbf{r} = \mathbf{r}^{(s)}$$

**Generalized Lorentz transformations** (Edwards (1963), Winnie (1970), Ungar (1986) and others)

$$x = \frac{X - cT\beta}{\sqrt{(1-k_\epsilon\beta)^2 - \beta^2}}, \quad ct = \frac{cT\left(1-2k_\epsilon\beta\right) - X\left(1-k_\epsilon^2\right)\beta}{\sqrt{(1-k_\epsilon\beta)^2 - \beta^2}}; \quad \beta = \frac{v}{c}$$

Applying the transformation from the non-standard synchronization to the standard one

$$t = t^{(s)} + \frac{k_\epsilon x}{c}, \quad x = x^{(s)}; \qquad T = T^{(s)} + \frac{k_\epsilon X}{c}, \quad X = X^{(s)}$$

yields the Lorentz transformations

$$x = \frac{X - cT^{(s)}\beta_s}{\sqrt{1-\beta_s^2}}, \quad ct^{(s)} = \frac{cT^{(s)} - X\beta_s}{\sqrt{1-\beta_s^2}}; \quad \beta_s = \frac{v_s}{c}$$

It could be expected since the derivation is based on the assumption that, in the case of $k_\epsilon = 0$ (standard synchronization), the relations of the special relativity theory are valid.

*From the statement that*

`There exists inescapable entanglement between remote clock synchronization and one-way speed of light`

*the conclusion is made that*

`The one-way speed of light is irreducibly conventional in nature.`

**It is not correct**

`The one-way velocity cannot be defined separately from the synchronization choice but there could be measurable effects which allow to distinguish a specific value of the` *one-way speed of light and the corresponding synchronization* `from others.`

The 'Generalized Lorentz Transformations' (GLT), commonly considered as showing conventionality of the one-way speed of light, *prove the opposite*.

- The GLT are conceptually inconsistent: the one-way speed of light is assumed to be anisotropic but the relations of the standard special relativity based on isotropy of the one-way speed of light are used.
- Thus, the GLT are the Lorentz transformations distorted by a change of variables and all predictions (measurable effects) based on the GLT are the same as in the special relativity theory.
- The inconsistency is illustrated by that the GLT does not satisfy the **Correspondence principle**:
  *in the limit of small velocities, the formula for transformation of the coordinate $x$ turns into that of the Galilean transformation*

  $$x = X - vT = X - \beta cT$$

  For the GLT:

  $$x = \frac{X - cT\beta}{\sqrt{(1 - k_\epsilon \beta)^2 - \beta^2}} \Rightarrow x = X - \beta cT + k_\epsilon \beta X \quad \text{as } \beta = \frac{v}{c} \Rightarrow 0$$

- The correspondence principle **selects** the isotropic one-way speed of light and the corresponding Einstein synchronization from all others
- Thus, both the Lorentz transformations and the GLT describe the situation when *there is no anisotropy in a physical system* and correspondingly no anisotropy of the one-way speed of light.
- All predictions (measurable effects) of such a theory are valid only for the isotropic situation.
- The Generalized Lorentz transformations cannot provide a basis for kinematics of the special relativity if an anisotropy of the one-way speed of light is due to a *real* space anisotropy
- The special relativity kinematics applicable to that situation should be developed based on the **first principles**, without refereeing to the relations of the standard relativity theory.

The basis of the special relativity

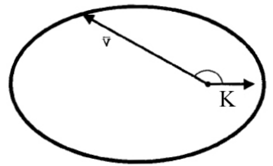
- *Principle of relativity*, which states the equivalence of all inertial frames as regards the formulation of the laws of nature
- *Universality of light propagation* in inertial frames

The two principles imply

**Invariance of the equation of light propagation** with respect to the coordinate transformations between inertial frames. In the present context, it should be invariance of the *equation of propagation of light incorporating the anisotropy of the one-way speed of light*.

The law of variation of the speed of light with direction should have a form consistent with the experimentally verified round-trip light principle.

# Equation of light propagation with the anisotropy in the light speed




$$V(\mathbf{n}) = \frac{c\mathbf{n}}{1+\mathbf{kn}} \quad \text{or} \quad V(\theta) = \frac{c}{1+k\cos\theta}$$

**Equation for anisotropic light propagation**

$$c^2dt^2 - 2kc\ dtdx - (1-k^2)dx^2 - dy^2 - dz^2 = 0$$

- In the standard special relativity, the **condition of invariance of the equation of light propagation** is replaced by a more general **condition of invariance of the interval between two events**.
- The condition of invariance of the interval does not follow from the basic principles of the theory and is usually preceded by a proof of its validity based on invariance of the equation of light propagation.
- However, some of the arguments used in such proofs **are not valid if the anisotropy is present**. In particular, *the symmetry arguments are not valid*. As a physical phenomenon it influences all the processes so that any effects due to movement of frame $S'$ relative to $S$ in the direction of the anisotropy axis $\mathbf{k}$ are not equivalent to those due to movement of frame $S''$ relative to $S$ in the direction *opposite* to the anisotropy axis $\mathbf{k}$.
- Thus, not the invariance of the interval but invariance of the equation of light propagation, which is a physical law, should be a starting point for derivation of the transformations between inertial frames.

## Invariance of the equation of light propagation

Two cases

- The degree of anisotropy does not depend on the observer motion

$$c^2 dT^2 - 2kc\, dTdX - (1-k^2)dX^2 - dY^2 - dZ^2 = 0,$$
$$c^2 dt^2 - 2kc\, dtdx - \left(1-k^2\right) dx^2 - dy^2 - dz^2 = 0$$

Burde, *Foundations of Physics*, 2016.

- The anisotropy is due to the observer motion with respect to a preferred frame

$$c^2 dT^2 - 2Kc\, dTdX - (1-K^2)dX^2 - dY^2 - dZ^2 = 0,$$
$$c^2 dt^2 - 2kc\, dtdx - \left(1-k^2\right) dx^2 - dy^2 - dz^2 = 0$$

# First principles

- **Invariance of the equation of light propagation**
- **Group property** The transformations between inertial frames form a one-parameter group with the group parameter $a = a(v)$ (such that $v \ll 1$ corresponds to $a \ll 1$).
  Based on the symmetry arguments it is assumed that the transformations of the variables $x$ and $t$ do not involve the variables $y$ and $z$ and vice versa:

$$x = f(X, T, K; a), \; t = q(X, T, K; a),$$
$$y = g(Y, Z, K; a), \; z = h(Y, Z, K; a); \quad k = p(K; a)$$

- **Correspondence principle**. In the limit of small velocities $v \ll c$ (small values of the group parameter $a \ll 1$), the formula for transformation of the coordinate $x$ turns into that of the Galilean transformation:

$$x = X - vT \tag{2}$$

The *group property* and the requirement of *invariance of the equation of light propagation* suggest applying the **infinitesimal Lie technique**.
The infinitesimal transformations are introduced, as follows

$$x \approx X + \xi(X, T, K)a, \quad t \approx T + \tau(X, T, K)a,$$
$$y \approx Y + \eta(Y, Z, K)a, \quad z \approx Z + \zeta(Y, Z, K)a, \quad k \approx K + a\chi(K)$$

**The procedure**

- Using condition of invariance to derive determining equations for the group generators $\tau(X, T, K)$, $\xi(X, T, K)$, $\eta(Y, Z, K)$, $\zeta(Y, Z, K)$ and $\chi(K)$
- Solving the determining equations
- Specifying the solutions using the correspondence principle to calculate the group generator $\xi(X, T)$, as follows

$$\xi = \left(\frac{\partial x}{\partial a}\right)_{a=0} = \left(\frac{\partial\left(X - v(a)T\right)}{\partial a}\right)_{a=0} = -bT; \qquad b = v'(0)$$

- Defining the finite transformations by solving the Lie equations
- Relating the group parameter to physical parameters

Lie equations

$$
\begin{aligned}
&\frac{dk(a)}{da} = \chi\left(k\left(a\right)\right); \quad k(0) = K,\\
&\frac{dx(a)}{da} = -ct(a), \quad \frac{d\left(ct\left(a\right)\right)}{da} = -\left(1 - k\left(a\right)^2 - \chi\left(k\left(a\right)\right)\right)x(a) - 2k(a)ct\left(a\right),\\
&\frac{dy(a)}{da} = -k(a)y(a), \quad \frac{dz(a)}{da} = -k(a)z(a);\\
&x(0) = X,\ t(0) = T,\ y(0) = Y,\ z(0) = Z.
\end{aligned}
$$

Solutions

$$
\begin{aligned}
&x = e^{-\varphi(a)}\left(X\left(\cosh a + K \sinh a\right) - cT \sinh a\right),\\
&ct = e^{-\varphi(a)}\Big(cT\left(\cosh a - k\left(a\right)\sinh a\right)\\
&-X\left(\left(1 - Kk\left(a\right)\right)\sinh a + \left(K - k\left(a\right)\right)\cosh a\right)\Big)
\end{aligned}
$$

$$
\varphi(a) = \int_0^a k(\alpha)d\alpha
$$

To complete the derivation of the transformations the group parameter $a$ is to be related to the velocity $v$ using the condition

$$x = 0 \quad \text{for} \quad X = vT$$

which yields

$$a = \frac{1}{2} \ln \frac{1 + \beta - K\beta}{1 - \beta - K\beta}; \qquad \beta = \frac{v}{c}$$

The resulting transformations

$$x = \frac{e^{-\varphi(a)}}{\sqrt{(1 - K\beta)^2 - \beta^2}} \left(X - cT\beta\right),$$

$$ct = \frac{e^{-\varphi(a)}}{\sqrt{(1 - K\beta)^2 - \beta^2}} \left(cT\left(1 - K\beta - k\beta\right) - X\left(\left(1 - K^2\right)\beta + K - k\right)\right)$$

$$y = e^{-\varphi(a)}Y, \quad z = e^{-\varphi(a)}Z; \quad \varphi(a) = \int_0^a k(\alpha)d\alpha$$

# Conformal invariance

Calculating the **interval**

$$ds^2 = c^2dt^2 - 2kc\ dtdx - (1-k^2)dx^2 - dy^2 - dz^2$$

with the transformations defined yields

$$ds^2 = e^{-2\varphi(a)}dS^2, \quad dS^2 = c^2dT^2 - 2Kc\ dTdX - (1-K^2)dX^2 - dY^2 - dZ^2$$

$$\varphi(a) = \int_0^a k(\alpha)d\alpha$$

Thus, in the case when the anisotropy exists, the **interval invariance** is replaced by **conformal invariance** with the conformal factor dependent on the relative velocity of the frames and the anisotropy degree.

# Specifying the transformations

- In all the derivations above, the fact, that there exists a (preferred) frame in which the light speed is isotropic, have not been used.
- Thus, the theory developed above is a counterpart of the standard special relativity kinematics which incorporates an anisotropy of the light propagation, with the anisotropy parameter varying from frame to frame.
- Nevertheless, the fact of the existence of a frame with the anisotropy parameter $k = 0$, being incorporated into the analysis, allows to specify the formulas for transformations and their consequences.

# Specified transformations

With the approximation

$$k_s = F\left(\bar{\beta}_s\right) \approx q\bar{\beta}_s$$

$$x = \frac{R}{\sqrt{\left(1 - K\beta\right)^2 - \beta^2}}\left(X - cT\beta\right),$$

$$ct = \frac{R}{\sqrt{\left(1 - K\beta\right)^2 - \beta^2}}\left(cT\left(1 - K\beta - k\beta\right) - X\left(\left(1 - K^2\right)\beta + K - k\right)\right)$$

$$y = RY, \quad z = RZ$$

$$k = \frac{q\left(K + \beta\left(q - K^2\right)\right)}{q + \beta K\left(1 - q\right)}$$

$$R = \left(\frac{q^2\left(1 + \beta\left(1 - K\right)\right)\left(1 - \beta\left(1 + K\right)\right)}{\left(q + \beta K\left(1 - q\right)\right)^2}\right)^{\frac{q}{2}}$$

**In these equations, $q$ is a universal constant.**

# Cosmological implications

The coordinate transformations from a preferred frame to the frame of an observer

$$
\begin{aligned}
x &= (X - cT\beta)\left(1-\beta^2\right)^{\frac{q-1}{2}}, \\
ct &= \left(cT\left(1-q\beta^2\right) - Z\beta\,(1-q)\right)\left(1-\beta^2\right)^{\frac{q-1}{2}} \\
y &= Y\left(1-\beta^2\right)^{\frac{q}{2}}, \quad z = Z\left(1-\beta^2\right)^{\frac{q}{2}}
\end{aligned}
$$

Equation relating the frequency $\nu_e$ of the light emitted at the LSS to the frequency $\nu_r$ measured by an observer moving with respect to the LSS becomes

$$
\nu_r = \nu_e \frac{\left(1-\beta^2\right)^{\frac{1}{2}-\frac{q}{2}}}{1-\beta\cos\tilde{\theta}}
$$

**In the context of the CMB anisotropy**

$$\frac{T(\tilde{\theta})}{\nu_r} = \frac{T_0}{\nu_e} \tag{3}$$

where $T_0$ is the effective temperature measured by the observer, that is at rest relative to the LSS and sees strictly isotropic blackbody radiation, and $T(\tilde{\theta})$ is the effective temperature of the blackbody radiation for the moving observer looking in the fixed direction $\tilde{\theta}$.

$$T(\tilde{\theta}) = T_0 \frac{\left(1-\beta^2\right)^{\frac{1}{2}-\frac{q}{2}}}{1-\beta\cos\tilde{\theta}} \tag{4}$$

Thus, the angular distribution of the CMB effective temperature seen by an observer moving with respect to the CMB is not altered by the light speed anisotropy. However, the anisotropy influences the mean temperature.

$$T(\tilde{\theta}) = T_0\left(1 + q\frac{\beta^2}{2} + \beta\cos\tilde{\theta} + \frac{\beta^2}{2}\cos 2\tilde{\theta}\right) \tag{5}$$

which implies that, up to the order $\beta^2$, the amplitudes of the dipole and quadrupole patterns remain the same, only the constant term is modified.

# Cosmological implications

The Doppler frequency shift in the case when an observer in the frame moving with respect to the CMB (Earth) receives light from an object (galaxy) which is also moving with respect to that preferred frame.

$$\nu_r = \nu_e \left( 1 + \left(1 + q\bar{\beta}\right) \beta_g + \frac{1}{2} \left(1 - q\right) \beta_g^2 \right)$$

where

$\beta_g = v_g/c$ is velocity of the object relative to the observer

$\bar{\beta}$ is velocity of the observer with respect to the preferred frame.

Thus, corrections to the Doppler shift due to the presence of the anisotropy (the terms multiplied by $q$) are of the second order in velocities.

## Conclusions

- The theory developed is a counterpart of the standard special relativity kinematics which allows the existence of a preferred frame but does not abolish the relativity principle.
- The theory incorporates an anisotropy of the light propagation (of the one-way speed of light) arising due to motion with respect to the preferred frame.
- The transformations between inertial frames (counterparts of the Lorentz transformations) obey a group property and leave the equation of anisotropic light propagation invariant, which provides validity of the relativity principle.
- In this framework, the preferred frame naturally arises as the frame, in which the light propagation is isotropic, but this does not violate the relativity principle as the transformations to that frame are not distinguished from other members of the group.

- The Lie group theory apparatus is applied for defining groups of space-time transformations between inertial frames. The correspondence principle is used to fix the form of the transformations.
- The transformations derived this way do not leave the interval between two events invariant -- the strict invariance is replaced by conformal invariance.
- The transformations between inertial frames in such "anisotropic special relativity" cannot be converted into the isotropic forms (Lorentz transformations) by a synchronization change.

- Measurable effects that arise as the consequences of the transformations derived within the present framework allow, in principle, to determine the size of the anisotropy and fix the arbitrary constant that is present in the transformations.
- The angular distribution of the CMB effective temperature seen by an observer moving with respect to the CMB is not altered by the light speed anisotropy. However, the anisotropy influences the mean temperature which now does not coincide with the temperature $T_0$ measured by the observer, that is at rest relative to the LSS, and differs from it by the factor $\left(1-\beta^2\right)^{-\frac{q}{2}}$.