A Model for Soft Interactions based on the CGC/Saturation Approach and BFKL Pomeron

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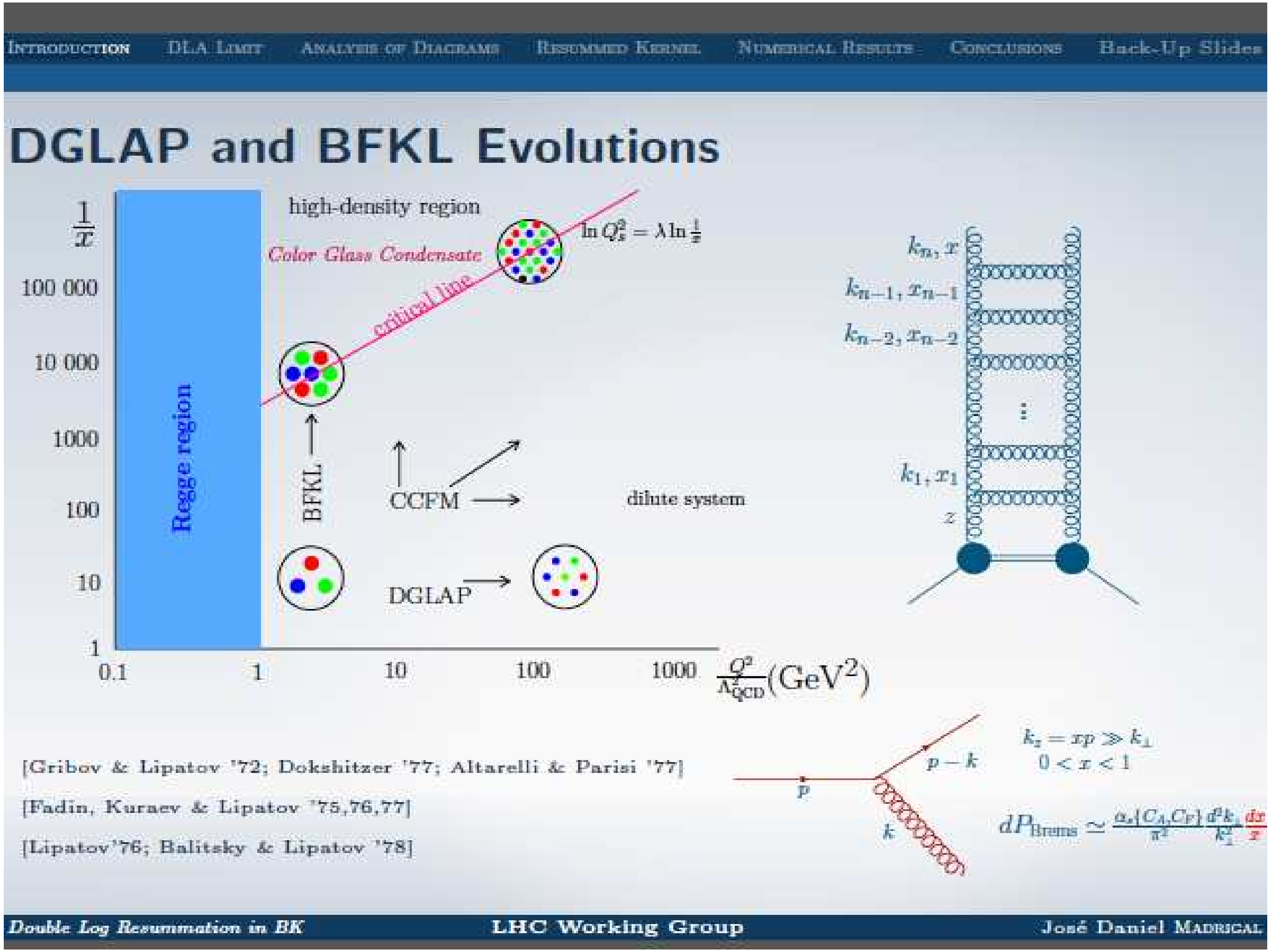
(work done with Genya Levin and Uri Maor)

- Develop a model for SOFT interactions at high energy based on the BFKL Pomeron and the CGC/saturation approach
- Green function for the Pomeron is calculated in framework of the CGC/saturation approach - replacing Pomeron Calculus
- CGC/saturation effective theory for high energy QCD
- BFKL Pomeron describes both hard AND soft interactions at high energy
- For diffractive processes include both low and high mass contributions

Guide to the Various Regions



Scattering near the Unitarity Limit

1. In the Regge limit of pQCD, when $s \gg \Lambda_{hard}$, as the energy increases the parton density becomes more dense, and the scattering amplitude A(s,t) grows.

2. As long as densities are NOT TOO HIGH, growth is described by BFKL evolution equation.

3. Density becomes higher as A(s,t) $\rightarrow$ 1, and one enters a regime called SATURATION, where the BFKL evolution FAILS.

4. NON LINEARITIES lead to SATURATION + UNITARIZATION of A(s,t).

5. Balitsky-Kovchegov equation is the simplest and most accurate way to describe the saturation regime of QCD. It is non-linear and resums QCD fan diagrams in the LLA.

Phenomenological Input

1. A deficiency that has to be overcome, is the fact that the BFKL Pomeron does NOT lead to shrinkage of the diffractive peak, and has no slope for the Pomeron trajectory.

2. This can be cured by introducing a non-perturbative correction at large impact parameter, which also assures satisfying the Froissart-Martin bound for σ_{tot}.

3. In our model we fix the large b behaviour by assuming that the SATURATION MOMENTUM has the following form:

   $Q_s^2(b,Y) = Q_{0s}^2(b,Y_0)e^{\lambda(Y-Y_0)}$ and $Q_{0s}^2(b,Y_0) = (m^2)^{(1-\frac{1}{\bar{\gamma}})}[S(b,m)]^{\frac{1}{\bar{\gamma}}}$
   $S(b,m) = \frac{m^2}{2\pi}e^{-mb}$ and $\bar{\gamma} = 0.63 = 1 - \gamma_{cr}$

   The parameter $\lambda = \bar{\alpha}_S \chi(\gamma_{cr})/(1-\gamma_{cr})$,
   in leading order of perturbative QCD ($\lambda = 0.2$ to 0.3)

   The parameter m is introduced to describe the large b behaviour, it determines the typical sizes of dipoles inside the hadron.

Phenomenological Input continued

4. Our model includes two additional scales m_1 and m_2, which describe two typical sizes in the proton wave function.

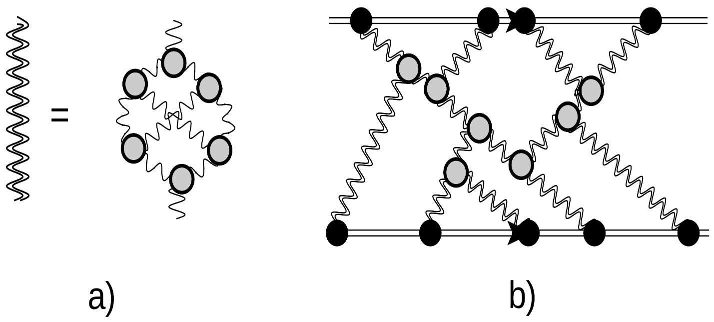
   Can associated these with:

   (i) the distance between the constituent quarks; size of the proton $R_p \approx \frac{1}{m_1}$.

   (ii) m_2 can be associated with the size of the constituent quark; $R_q \approx \frac{1}{m_2}$.

5. Altinoluk et al JHEP 1404, 075 (2014) have proved the equivalence of the CGC/saturation approach and the BFKL Pomeron calculus for a wide range of rapidities
   $Y \;\leq\; \frac{2}{\Delta_{\text{BFKL}}} \ln\left(\frac{1}{\Delta^2_{\text{BFKL}}}\right)$.

Dressed Pomeron in MPSI approximation



a) Dressed Pomeron in MPSI approximation
and
b) Sum of net diagrams
Wavy lines describe BFKL Pomerons.
The grey blobs stand for triple Pomeron vertices,
while black blobs show the hadron-Pomeron vertex g(b).

Since the typical rapidity is $O(Y - Y_i) \approx \frac{1}{\Delta_{BFKL}}$, only large Pomeron loops with rapidity $O(Y)$
contribute at high energies $\rightarrow$ can sum such loops using MPSI approximation.

For the BFKL Pomeron $\lambda = 4.88\bar{\alpha}_s$ while $\Delta_{BFKL} = 4ln2\bar{\alpha}_s \approx 0.2$

Dressed Pomeron in MPSI approximation (continued)

The resulting Green function of the Dressed Pomeron is given by:

$$G_{I\!P}^{\text{dressed}}\left(Y - Y_0, r, R, b\right) \;=$$

$$a^2 \left\{ 1 \;-\; \exp\left(-T\left(Y - Y_0, r, R, b\right)\right) \;\right\} \;+\; 2a(1-a)\frac{T\left(Y - Y_0, r, R, b\right)}{1 \;+\; T\left(Y - Y_0, r, R, b\right)}$$

$$+\;(1-a)^2 \left\{ 1 - \exp\left(\frac{1}{T\left(Y - Y_0, r, R, b\right)}\right) \frac{1}{T\left(Y - Y_0, r, R, b\right)}\,\Gamma\left(0, \frac{1}{T\left(Y - Y_0, r, R, b\right)}\right) \right\}$$

$$with \;\; T\left(Y - Y_0, r, R, b\right) \;=\; \frac{\bar{\alpha}_S^2}{4\pi}\,G_{I\!P}\left(z \to 0\right) \;=\; \phi_0 \left(r^2 Q_s^2\left(R, Y, b\right)\right)^{1-\gamma_{cr}}$$

$$=\; \phi_0 S\left(b\right) e^{\lambda(1-\gamma_{cr})Y}$$

z $= ln(r^2 Q_s^2(b, Y))$, a $=$ 0.65, $\gamma_{cr} \;\approx\;$ 0.37

Parameters of the Model

We need to introduce four constants: g_i and m_i $(i = 1, 2)$, to describe the vertices of the hadron-Pomeron interaction

$$g_i\left(b\right) \;=\; g_i\, S_{I\!P}\left(b\right) \text{ with } S_{I\!P}\left(b\right) \;=\; \frac{m_i^3\, b}{4\pi}\, K_1\left(m_i b\right)$$

$$S_{I\!P}\left(b\right) \xrightarrow{\text{Fourier image}} \left(\frac{m_i^2}{q^2 + m_i^2}\right)^2$$

$$\Omega_{i,k}\left(Y; b\right) \;=\; \int d^2 b' \; \frac{g_i\left(\vec{b}'\right)\, g_k\left(\vec{b} - \vec{b}'\right)\, \bar{G}^{\text{dressed}}_{I\!P}\left(Y\right)}{1 + 1.29\, \bar{G}^{\text{dressed}}_{I\!P}\left(Y\right)\left[g_i\left(\vec{b}'\right) + g_k\left(\vec{b} - \vec{b}'\right)\right]},$$

where $\bar{G}^{\text{dressed}}_{I\!P}\left(Y\right) \;=\; \int d^2\, b''\, G^{\text{dressed}}_{I\!P}\left(Y; b''\right).$

Basic formalism for Two Channel Model

Following Good-Walker the observed physical hadronic and diffractive states are written

$$\psi_h \,=\, \alpha\,\Psi_1 + \beta\,\Psi_2\,; \qquad \psi_D \,=\, -\beta\,\Psi_1 + \alpha\,\Psi_2; \qquad \text{where} \qquad \alpha^2 + \beta^2 \,=\, 1.$$

Functions ψ_1 and ψ_2 form a complete set of orthogonal functions $\{\psi_i\}$ which diagonalize the interaction matrix $\mathbf{T}$

$$A^{i'k'}_{i,k} =< \psi_i\,\psi_k|\mathbf{T}|\psi_{i'}\,\psi_{k'} >= A_{i,k}\,\delta_{i,i'}\,\delta_{k,k'}.$$

The unitarity constraints can be written as

$$2\,\mathrm{Im}\,A_{i,k}\,(s,b) \,=\, |A_{i,k}\,(s,b)\,|^2 + G^{in}_{i,k}(s,b)$$

At high energies a simple solution to this equation is

$$A_{i,k}(s,b) = i\left(1 - \exp\left(-\frac{\Omega_{i,k}(s,b)}{2}\right)\right)$$

$$G^{in}_{i,k}(s,b) = 1 - \exp\left(-\Omega_{i,k}(s,b)\right).$$

$G^{in}_{i,k}(s,b)$ denotes the contribution of all non-diffractive inelastic processes

Physical Observables for Elastic, and Low Mass Diffraction

elastic amplitude : $a_{el}(s,) = i\left(\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\right);$

elastic observables : $\sigma_{tot} = 2\int d^2b\, a_{el}(s,b); \quad \sigma_{el} = \int d^2b\, |a_{el}(s,b)|^2;$

optical theorem : $2\,\mathrm{Im}A_{i,k}(s,t=0) = 2\int d^2b\,\mathrm{Im}A_{i,k}(s,b) = \sigma_{el} + \sigma_{in} = \sigma_{tot}$

single diffraction : $\sigma^{GW}_{sd} = \int d^2b\, \left(\alpha\beta\{-\alpha^2 A_{1,1} + (\alpha^2 - \beta^2)A_{1,2} + \beta^2 A_{2,2}\}\right)^2;$

double diffraction : $\sigma^{GW}_{dd} = \int d^2b\, \alpha^4\beta^4\{A_{1,1} - 2A_{1,2} + A_{2,2}\}^2.$

'GW' denotes the Good -Walker component, that is responsible for diffraction in the small mass region.

Single Diffractive Scattering in the region of Large Mass

The large Mass contribution for single diffraction is:

$$\sigma_{sd}^{\text{large mass}} = 2\int d^2b$$

$$\Big\{ \alpha^6\, A^{sd}_{1;1,1}\, e^{-2\,\Omega^D_{1,1}(Y;b)} + \alpha^2\beta^4 A^{sd}_{1;2,2}\, e^{-2\,\Omega^D_{1,2}(Y;b)} + 2\,\alpha^4\,\beta^2\, A^{sd}_{1;1,2}\, e^{-\left(\Omega^D_{1,1}(Y;b)+\Omega^D_{1,2}(Y;b)\right)}$$

$$+ \beta^2\,\alpha^4\, A^{sd}_{2;1,1}\, e^{-2\,\Omega^D_{1,2}(Y;b)} + 2\,\beta^4\alpha^2\, A^{sd}_{2;1,2}\, e^{-\left(\Omega^D_{1,2}(Y;b)+\Omega^D_{2,2}(Y;b)\right)} + \beta^6\; A^{sd}_{2;2,2}\, e^{-2\,\Omega_{2,2}(Y;b)} \Big\}$$

$$\Omega^D_{i,k}(Y;b) = \int d^2b' \; \frac{g_i\left(\vec{b}'\right)\; g_k\left(\vec{b}-\vec{b}'\right)\; \bar{G}^{\text{ dressed}}(T)}{\left(1 + 1.29\,\bar{G}^{\text{ dressed}}(T)\left[g_i\left(\vec{b}'\right) + g_k\left(\vec{b}-\vec{b}'\right)\right]\right)^2}$$

$$A^{sd}_{i;k,l}(Y, Y_{max}, Y_{min}; b) = \int d^2b'\,\sigma_{diff}(Y, Y_{max}, Y_{min}, 1/m)\,.$$

$$g_i g_k g_l S_{I\!P}(b', m_i)\; S_{I\!P}\left(\vec{b}-\vec{b}', m_k\right)\; S_{I\!P}\left(\vec{b}-\vec{b}', m_l\right),$$

and $S_{I\!P}(b', m_i) = \frac{1}{4\pi}\, m_i^3\, b'\, K_1(b', m_i)$

Double Diffractive Scattering in the region of Large Mass

Unitarity constraints for the dressed Pomeron takes the form:

$$2\,G^{\text{dressed}}\left(T\left(Y,b\right)\right) \;=\; G^{\text{dressed}}\left(2\,T\left(Y;b\right)\right) \;+\; N_{DD}\left(Y;b\right)$$

where $G^{\text{dressed}}\left(2\,T\left(Y;b\right)\right)$ describes all inelastic processes that are generated by the exchange of the dressed Pomeron.

$$\sigma_{dd} \;=\; \int d^2b\left\{2G^{\text{dressed}}\left(T\left(Y,b\right)\right) \;-\; G^{\text{dressed}}\left(2\,T\left(Y;b\right)\right)\right\}$$

For the double diffraction production at large mass we have

$$\sigma_{dd}^{\text{large mass}} \;=\; \int d^2b\,\left\{\alpha^4\,A^{dd}_{1,1}\,e^{-2\Omega^D_{1,1}(Y;b)} \;+\; 2\alpha^2\,\beta^2 A^{dd}_{1,2}\,e^{-2\Omega^D_{1,2}(Y;b)} \;+\; \beta^4\,A^{dd}_{2,2}\,e^{-2\Omega^D_{2,2}(Y;b)}\right\}.$$

where

$$A^{dd}_{i,k} \;=\; \int d^2\,b\,g_i\,g_k\,S^{i,k}_{DD}\left(b\right)\sigma_{dd}\left(Y\right)$$

$$S^{i,k}_{DD}\left(b\right) \;=\; \int d^2b'\,S_{I\!P}\left(b',m_i\right)\;S_{I\!P}\left(\vec{b}-\vec{b}',m_k\right)$$

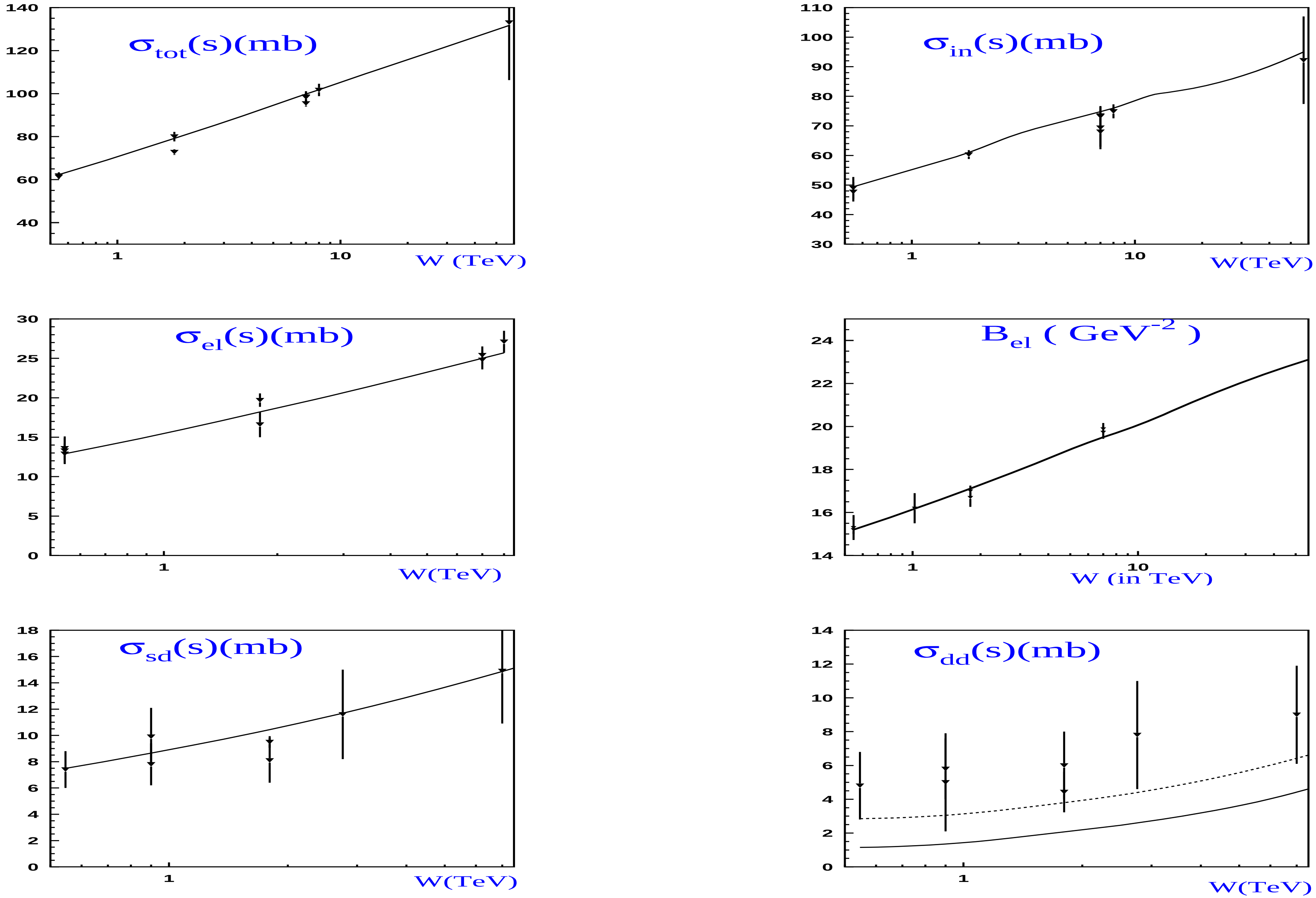
and $S_{I\!P}\left(b',m_i\right) \;=\; \frac{1}{4\pi}\,m_i^3\,b'\,K_1\left(b',m_i\right)$

Parameters and Predictions for the Model

model	λ	ϕ_0	$g_1\ (GeV^{-1})$	$g_2\ (GeV^{-1})$	$m(GeV)$	$m_1(GeV)$	$m_2(GeV)$
2 channel	0.38	0.0019	110.2	11.2	5.25	0.92	1.9

W (TeV)	σ_{tot} (mb)	σ_{el}(mb) (mb)	B_{el} (GeV^{-2})	single σ_{sd}^{LM} (mb)	diffraction σ_{sd}^{HM} (mb)	double σ_{dd}^{LM} (mb)	diffraction σ_{dd}^{HM} (mb)
0.546	62.3	12.9	15.2	5.64	1.85	0.7	0.46
0.9	69.2	15	16	6.25	2.39	0.77	0.67
1.8	79.2	18.2	17.1	7.1	3.35	0.89	1.17
2.74	85.5	20.2	17.8	7.6	4.07	0.97	1.62
7	99.8	25	19.5	8.7	6.2	1.15	3.27
8	101.8	25.7	19.7	8.82	6.55	1.17	3.63
13	109.3	28.3	20.6	9.36	8.08	1.27	5.11
14	110.5	28.7	20.7	9.44	8.34	1.27	5.4
57	131.7	36.2	23.1	10.85	15.02	1.56	13.7

Results for Two channel Model



Application of CGC/saturation approach for INCLUSIVE PRODUCTION

[*Phys. Letts.* **B746**, 154 (2015)]

INCLUSIVE PRODUCTION occurs in two stages:

- First stage: Production of a mini-jet with typical transverse momentum Q_s: Q_s (saturation scale)$\gg$ soft scale.
- Second stage: Decay of minijets into hadrons, which is treated phenomenologically.

For mini-jet production we use the k_T factorization formula:

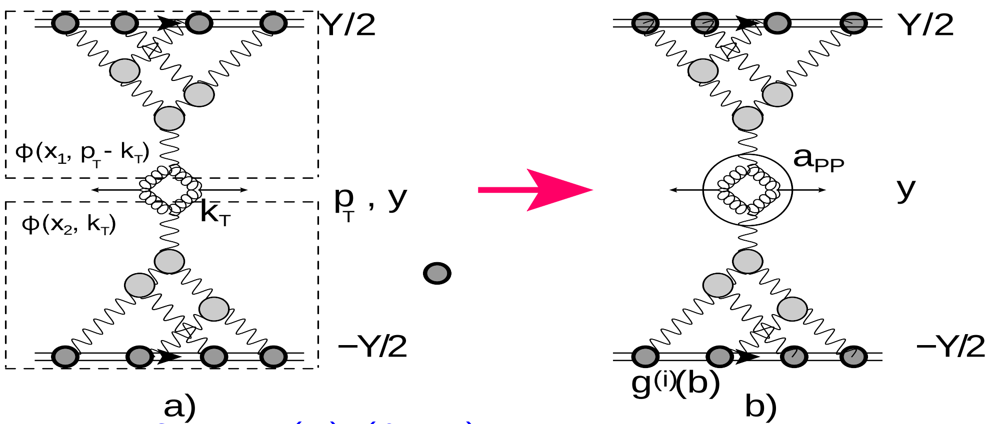
$$\frac{d\sigma}{dy\, d^2 p_T} = \frac{2\pi\bar{\alpha}_S}{p_T^2} \int d^2 k_T\ \phi_G^{h_1}\left(x_1; \vec{k}_T\right)\ \phi_G^{h_2}\left(x_2; \vec{p}_T - \vec{k}_T\right) \tag{1}$$

where $\phi_G^{h_i}$ denotes the probability to find a gluon that carries the fraction x_i of energy with $k_\perp$ transverse momentum.

$\frac{1}{2}Y + y = \ln(1/x_1)$ and $\frac{1}{2}Y - y = \ln(1/x_2)$.

$\phi_G^{h_i}$ is the solution of the Balitsky-Kovchegov(BK) non-linear evolution equation, and can be viewed as the sum of 'fan' diagrams of the BFKL Pomeron interactions, shown in figure.

INCLUSIVE PRODUCTION 2



The graphic representation of Eqn. (1) (fig-a). Wavy lines denote the BFKL Pomerons, while the helical lines illustrate the gluons.

In fig-b the Mueller diagram for the inclusive production is shown.

Eqn.(1) can be rewritten as a Mueller diagram fig-b, and the inclusive cross section is given by:

$$\begin{aligned} \frac{d\sigma}{dy} &= \int d^2 p_T \frac{d\sigma}{dy\, d^2 p_T} = a_{I\!P I\!P} \ln\left(W/W_0\right) \Big\{ \alpha^4\, In^{(1)}\left(\frac{1}{2}Y + y\right)\, In^{(1)}\left(\frac{1}{2}Y - y\right) \\ &+ \alpha^2 \beta^2 \left(In^{(1)}\left(\frac{1}{2}Y + y\right)\, In^{(2)}\left(\frac{1}{2}Y - y\right) + In^{(2)}\left(\frac{1}{2}Y + y\right)\, In^{(1)}\left(\frac{1}{2}Y - y\right) \right) \\ &+ \beta^4\, In^{(2)}\left(\frac{1}{2}Y + y\right)\, In^{(2)}\left(\frac{1}{2}Y - y\right) \Big\} \end{aligned}$$

$$In^{(i)}\left(y\right) \;=\; \int d^2b\;N^{BK}\left(g^{(i)}\,S\left(m_i,b\right)\;\tilde{G}_{I\!P}\left(y\right)\right)$$

$$\text{where } \tilde{G}_{I\!P}\left(y\right) \;=\; \phi_0\exp\left(\lambda\left(1-\gamma_{cr}\right)y\right)$$

The mass of mini jet is given by $m^2_{jet} = 2m_{soft}p_T$.

Since the typical transverse momentum is equal to the saturation scale, we have

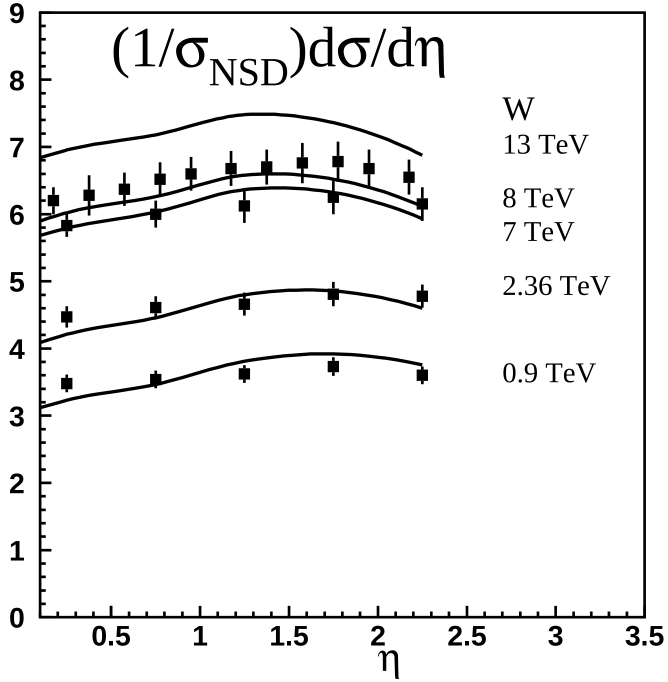
$$\frac{m^2_{jet}}{p^2_T} = \frac{2\,m_{soft}}{Q_s\left(W\right)} \;=\; r^2_0\left(\frac{W}{W_0}\right)^{-\frac{1}{2}\lambda}$$

Values of parameters have been extracted from the diffractive and elastic data.

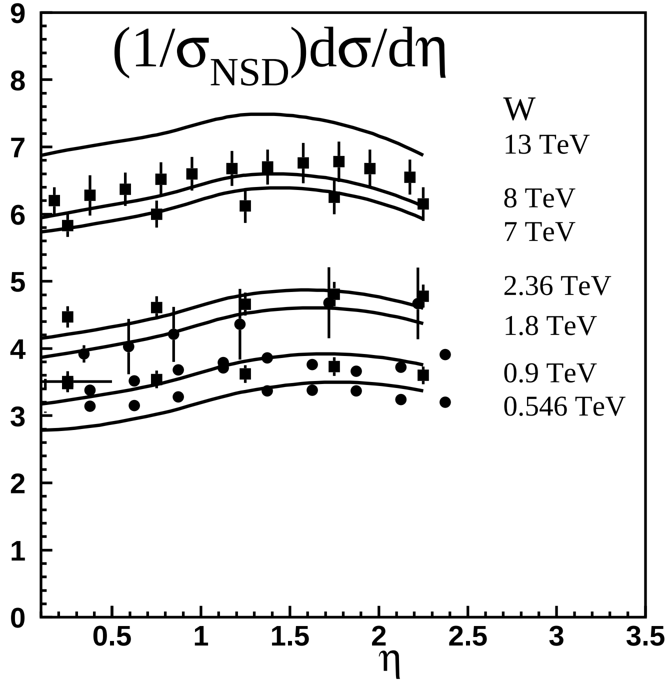
The only free parameters are $a_{I\!PI\!P}$ and r^2_0.

Our curves are calculated for $a_{I\!PI\!P} = 0.21$ and $r^2_0 \;=\; 8$, which have been determined from the experimental data.

Our Model Results for Inclusive Production



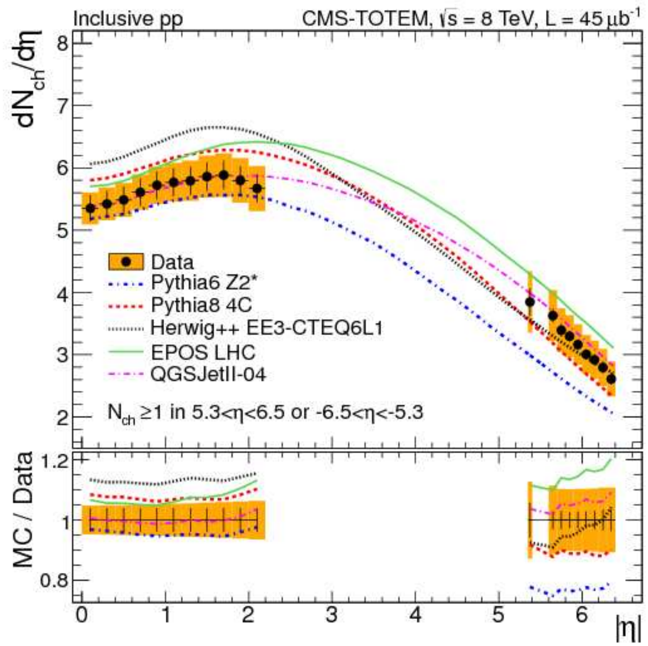
fig(a)



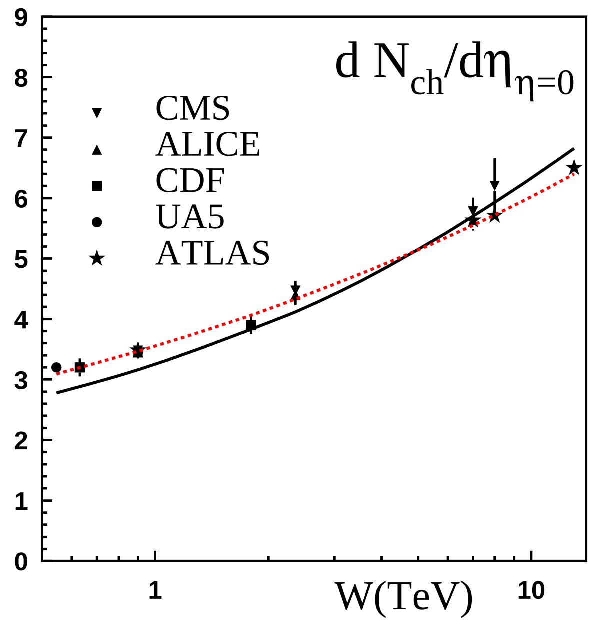
fig(b)

The single inclusive density ($(1/\sigma_{NSD})\, d\sigma/d\eta$) versus energy. The data were taken from ALICE,CMS,ATLAS and from PDG. The description of the CMS data is plotted in fig(a), while fig(b) presents the comparison with all inclusive spectra with $W \geq 0.546\, TeV$.

Our Model Results for Inclusive Production continued



fig(a)



fig(b)

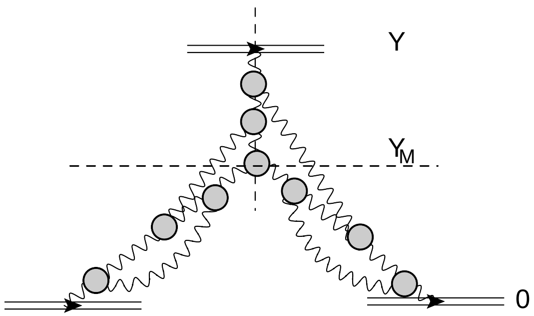
The comparison of the inclusive production at $W = 8\,TeV$ with the Monte Carlo models is shown in fig (a). The figure is taken from [CMS and TOTEM].

In fig.(b) $dN_{ch}/d\eta$ at $\eta = 0$ versus energy W is displayed. Our estimates are shown by the solid line. The dotted line corresponds to fit: $0.725\,(W/W_0)^{0.23}$ with $W_0 = 1\,GeV$. The data are taken from [CMS,ALICE,CDF,UA5, ATLAS*].

Conclusions

- Constructed a model based on the BFKL Pomeron and the CGC/saturation approach, which successfully describes data in the Regge region, for high energy hadron scattering.
- Do not require that the soft Pomeron to appear as a Regge pole.
- Suggest a procedure where the matching with long distance physics (where confinement of quarks and gluons is essential) can be reached within the CGC/saturation approach.
- Model for soft (long distance) interactions, is able to describe inclusive production.
- Model also successfully describes:
  Long range rapidity correlations [EPJ C75,(2015) 518]
  Survival Probability of central exclusive production [EPJ C76 (2016) 177]
  Long-range elliptic anistropies (ridge structure) in proton -proton collisions [Phys. Rev. D93 (2016) 074029]
  Bose-Einstein correlations in hadron and nucleus collisions [arXiv:1604.04461]

Diffractive Scattering based on the Levin-Kovchegov Equation



MPSI approximation: the simplest diagram for single diffraction production. The wavy lines describe BFKL Pomerons. The blobs stand for triple Pomeron vertices. The dashed line denotes the cut Pomeron. $Y_M = \ln\left(M^2/s_0\right)$, where M is the mass of produced particles and s_0 is the scale taken to be of the order of $1\,GeV^2$.

L-K equation has same form as the B-K equation (Nucl.Phys. B577 (2000) 221) for the function

$$G\left(Y, Y_0, r, b\right) \;=\; 2\,N\left(Y, Y_0, r, b\right) \;-\; N_{SD}\left(Y, Y_0, r, b\right),$$

$N\left(Y, Y_0, r, b\right)$ is the imaginary part of the elastic amplitude

The cross section for diffraction production is:

$$\sigma_{diff}\left(Y, Y_0, r\right) \;=\; \int d^2b\, N_{SD}\left(Y, Y_0, r, b\right)$$