

Relativistic Quark-Model Description of Baryon Spectroscopy and Reactions

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5th International Conference on New Frontiers in Physics
OAC Kolymbari, July 8th, 2016

Hadrons at Low Energies

Common methods for dealing with low-energy hadronic phenomena on the basis of QCD nowadays:

- ▶ Lattice QCD (works but faces computational limitations)
- ▶ Chiral perturbation theory (works at low energies, as a systematic expansion but is limited to a few terms)
- ▶ Effective field theories or functional methods (depend on assumptions and regularizations)
- ▶ Effective models, e.g. **constituent-quark models**
(depend on assumptions and input parameters)

Here, results for the **spectroscopy** and **structure** (electromagnetic, weak, scalar, gravitational form factors) of **baryons** from the **relativistic constituent-quark model** (RCQM)

Aspects Suggestive for Quark Models

Search for a non-perturbative tool to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD such phenomena like

- ▶ **hadron spectra**: ground states & excitations
- ▶ **hadron structure**: $r_E, \mu, g_A ; G_E, G_M, G_A, G_P, \dots$
i.e. electroweak form factors etc.
- ▶ **resonance excitations**: $\gamma N \rightarrow N^*, e^- N \rightarrow N^*, \dots$
- ▶ **resonance decays**:
 $\rho \rightarrow \pi\pi, \omega \rightarrow \pi\pi\pi, N^* \rightarrow N\pi, \Delta \rightarrow N\pi, \Lambda^* \rightarrow KN, \dots$
- ▶ **meson-baryon interactions**: $\pi - N, K - N, \dots$
- ▶ **hyperon-hyperon interactions**: $N - N, N - Y, \dots$
etc. etc.

Outline

Relativistic Constituent-Quark Model (RCQM)

Relativistically invariant mass operator

Effective quark interactions

Precision solution of mass-operator eigenvalue problem

Baryon Spectroscopy

Light, strange, charm, bottom

Baryon Structure

Nucleon e.m. form factors, including flavor analysis

Baryon electromagnetic form factors

Nucleon and baryon axial form factors / charges

Strong πNN and $\pi N\Delta$ form factors

Nucleon gravitational form factors

Summary and Conclusions

Baryons

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

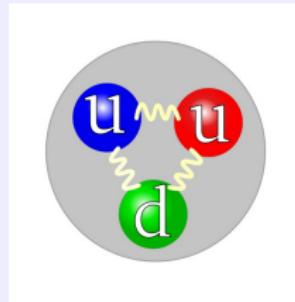
πNN , $\pi N\Delta$

N Gravitational FF

Summary

Baryons are considered as colorless bound states of three constituent quarks Q .

Here the proton:



- ▶ 'Constituent' quarks Q are quasiparticles with dynamical mass, NOT the original QCD d.o.f. (i.e. 'current' quarks q).
- ▶ 'Constituent' quarks Q are subject to effective forces (confinement and hyperfine interactions).

Relativistic quantum mechanics (RQM)

i.e. **Hamiltonian quantum theory** respecting
Poincaré invariance

(theory on a Hilbert space \mathcal{H} corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$$

Eigenvalue equations

$$\hat{M} |P, J, \Sigma\rangle = M |P, J, \Sigma\rangle , \quad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu |P, J, \Sigma\rangle = P^\mu |P, J, \Sigma\rangle , \quad \hat{P}^\mu = \hat{M} \hat{V}^\mu$$

Interacting mass operator

$$\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$$

$$\hat{M}_{\text{free}} = \sqrt{\hat{H}_{\text{free}}^2 - \hat{\vec{P}}_{\text{free}}^2}$$

$$\hat{M}_{\text{int}}^{\text{rest frame}} = \sum_{i < j}^3 \hat{V}_{ij} = \sum_{i < j}^3 [\hat{V}_{ij}^{\text{conf}} + \hat{V}_{ij}^{\text{hf}}]$$

fulfilling the **Poincaré algebra**

$$[\hat{P}_i, \hat{P}_j] = 0, \quad [\hat{J}_i, \hat{H}] = 0, \quad [\hat{P}_i, \hat{H}] = 0,$$

$$[\hat{K}_i, \hat{H}] = -i\hat{P}_i, \quad [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k, \quad [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k,$$

$$[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, \quad [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, \quad [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}$$

\hat{H}, \hat{P}_i ... time and space translations,
 \hat{J}_i ... rotations, \hat{K}_i ... Lorentz boosts

Hyperfine Interaction

Low-energy QCD of N_f flavors is characterized by:

- spontaneous breaking of chiral symmetry ($SB\chi S$):

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

- appearance of $(N_f^2 - 1)$ **Goldstone bosons** $\vec{\phi}$
- generation of quasiparticles with dynamical mass,
i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

$$\mathcal{L}_{\text{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^f \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189

E.V. Shuryak: Phys. Rep. 115, 151 (1984)

L.Ya. Glozman and D.O. Riska: Phys. Rep. 268, 263 (1996)

see also:

S. Weinberg: Phys. Rev. Lett. 105, 261601 (2010)

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

RCQM

Mass operator
Dynamics
EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary

$$\Rightarrow H_{free} = \sum_{i=1}^3 \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman, W. Plessas, K. Varga, and R.F. Wagenbrunn: Phys. Rev. D **58**, 094030 (1998)

J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918

J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)

Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C r_{ij}$$

$$\begin{aligned} V_\beta(\vec{r}_{ij}) &= \frac{g_\beta^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi \delta(\vec{r}_{ij}) \right\} \\ &= \frac{g_\beta^2}{4\pi} \frac{1}{12m_i m_j} \left\{ \mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2 \frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}} \right\} \end{aligned}$$

$$B = -402 \text{ MeV}, \quad C = 2.33 \text{ fm}^{-2}$$

$$\beta = 24 : \quad \frac{g_{24}^2}{4\pi} = 0.7, \quad \mu_{24} = \mu_\pi = 139 \text{ MeV}, \quad \Lambda_{24} = 700.5 \text{ MeV}$$

$$\beta = 0 : \quad \left(\frac{g_0}{g_{24}} \right)^2 = 1.5, \quad \mu_0 = \mu_{\eta'} = 958 \text{ MeV}, \quad \Lambda_0 = 1484 \text{ MeV}$$

$$\begin{aligned} m_u &= m_d = 340 \text{ MeV}, & m_s &= 480 \text{ MeV}, \\ m_c &= 1675 \text{ MeV}, & m_b &= 5055 \text{ MeV} \end{aligned}$$

RCQM

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Baryon E.m.

Axial FFs

 $\pi NN, \pi N\Delta$

N Gravitational FF

Summary

Three-Q Mass-Operator Eigenvalue Problem

$$\hat{M} |P, J, \Sigma, F_{abc}\rangle = M |P, J, \Sigma, F_{abc}\rangle \\ = M |M, V, J, \Sigma, F_{abc}\rangle$$

- mass eigenvalues M (i.e. invariant baryon spectra)
- mass-operator eigenstates $|P, J, \Sigma, F_{abc}\rangle$

thus the baryon wave functions (initially in rest frame)

$$\Psi_{PJ\Sigma F_{abc}}(\vec{\xi}, \vec{\eta}) = \langle \vec{\xi}, \vec{\eta} | P, J, \Sigma, F_{abc} \rangle ,$$

where $\vec{\xi}$ and $\vec{\eta}$ are the usual Jacobi coordinates and

- P momentum eigenvalues
- (M, V) mass resp. velocity eigenvalues)
- J intrinsic spin $\hat{=}$ total angular momentum)
- Σ z-component of J
- F_{abc} flavor content

RCQM

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Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

πNN , $\pi N\Delta$

N Gravitational FF

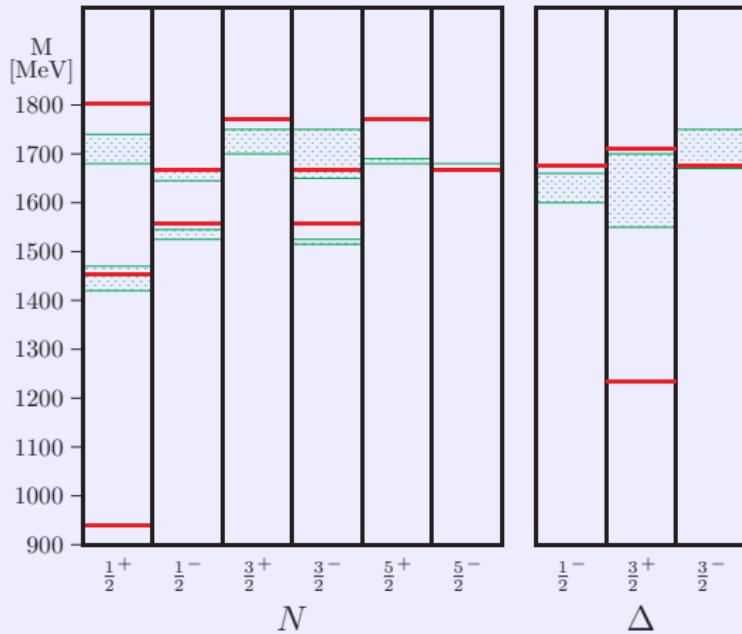
Summary

Mass-Operator **Eigenstates**

—

Baryon **Excitation Spectra**

Light Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

Strange Baryon Spectra

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

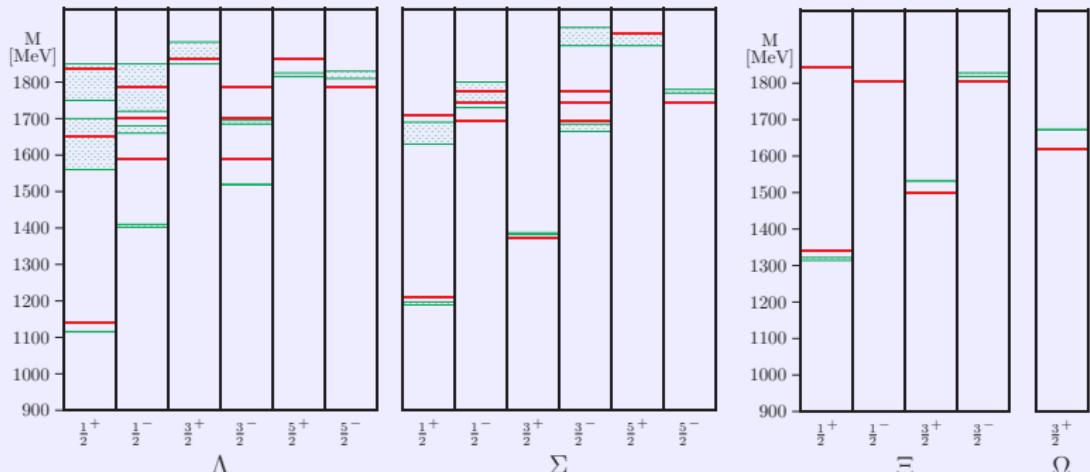
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



red Universal GBE RCQM

green PDG 2013 (experiment)

Charm Baryon Spectra

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

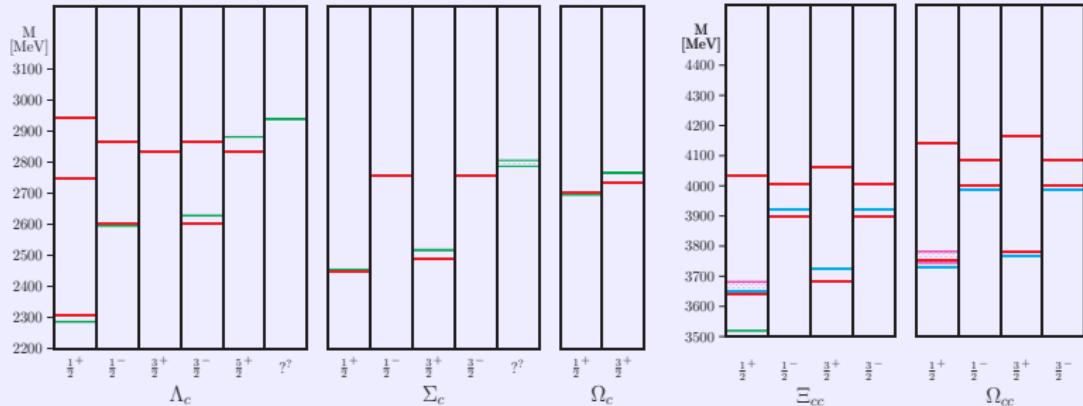
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



Left panel – single charm:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

Right panel – double charm:

green M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Bottom Baryon Spectra

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

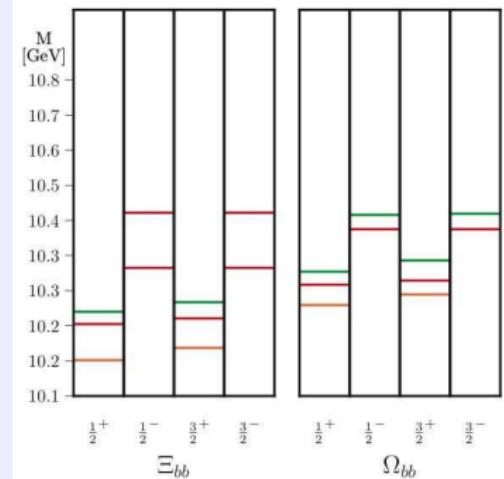
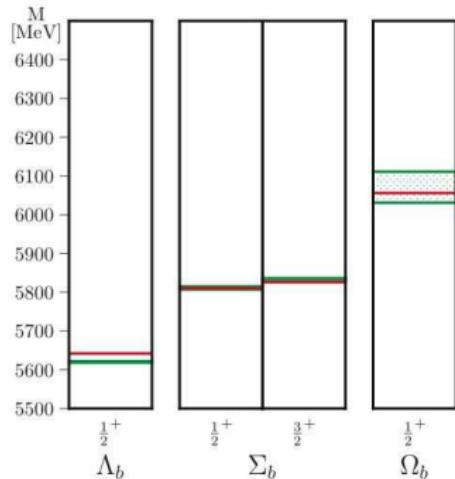
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



Left panel – single bottom:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

Right panel – double bottom:

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

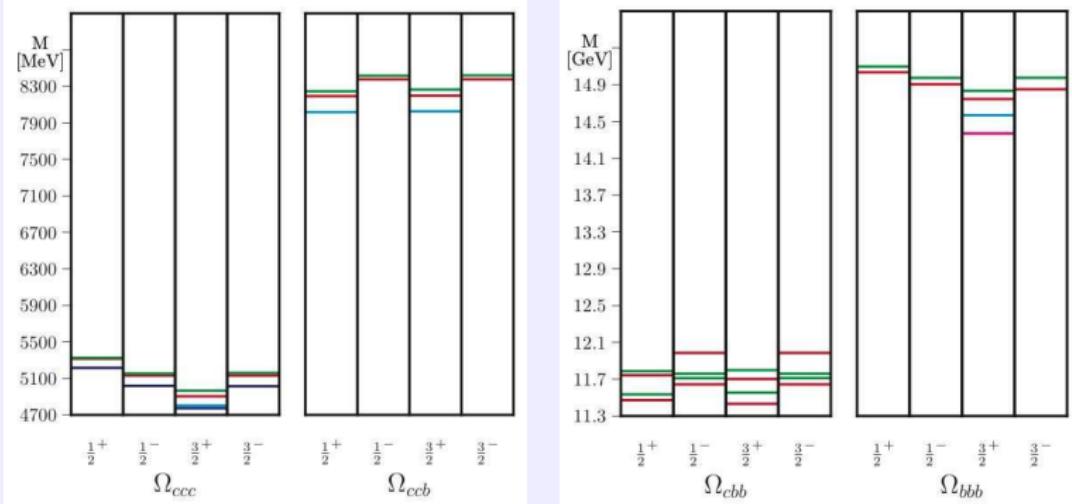
Baryon E.m.

Axial FFs

 πNN , $\pi \Delta$

N Gravitational FF

Summary



red Universal GBE RCQM

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817
(nonrelativistic one-gluon-exchange CQM)

blue S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

cyan A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)

magenta S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Influence of Light-Heavy $Q\bar{Q}$ Interaction

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

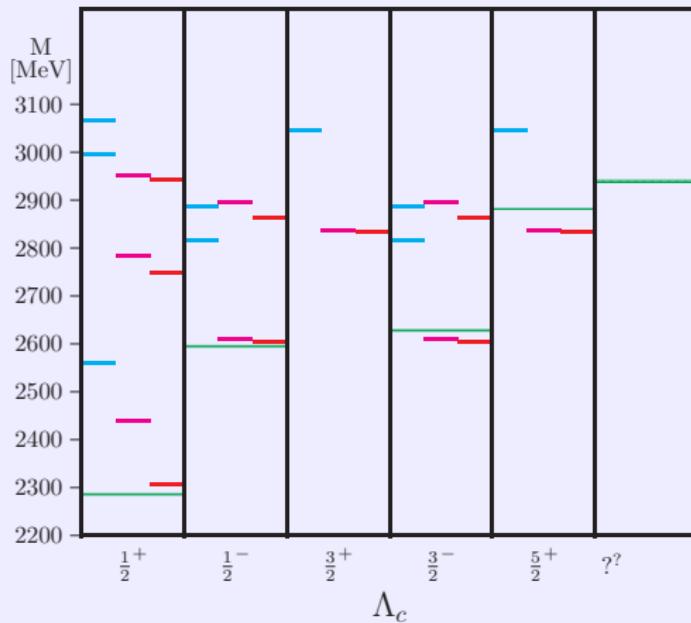
Baryon E.m.

Axial FFs

 $\pi NN, \pi N\Delta$

N Gravitational FF

Summary



leftmost cyan levels

middle magenta levels

rightmost red levels

confinement only

including only light-light GBE

including full GBE RCQM

RCQM

Mass operator

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Light, strange,
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Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary

Mass-Operator **Eigenstates**

—

Baryon **Wave Functions**

Mass operator eigenstates

$$\hat{M} |P, J, \Sigma, T, M_T\rangle = M |P, J, \Sigma, T, M_T\rangle$$

represented in configuration space

$$\langle \vec{\xi}, \vec{\eta} | P, J, \Sigma, T, M_T \rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi}, \vec{\eta})$$

with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

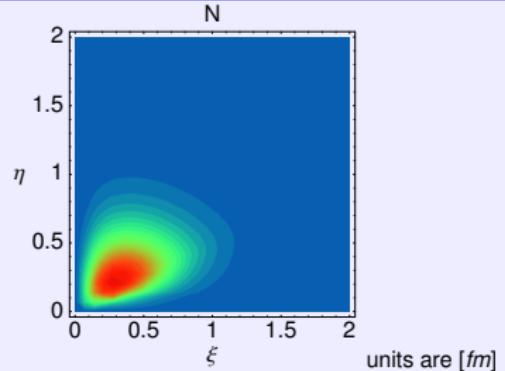
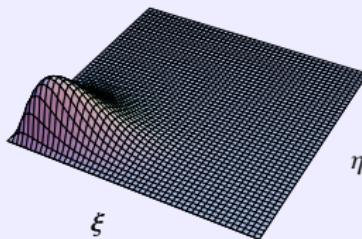
Picture the baryon wave functions through
spatial probability density distributions

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta$$

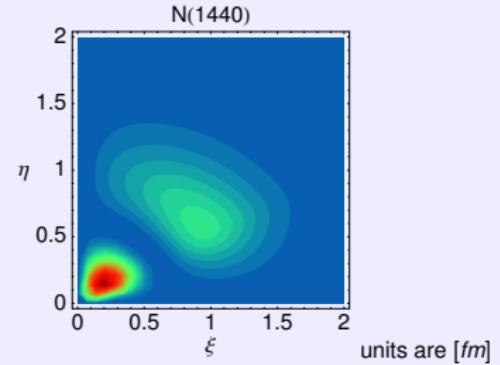
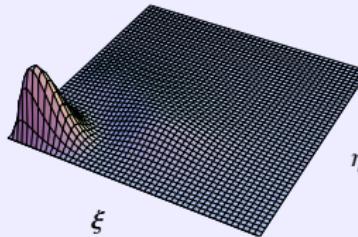
$$\Psi_{PJ\Sigma TM_T}^*(\xi, \Omega_\xi, \eta, \Omega_\eta) \Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$

'Pictures' of Baryons (rest frame)

N GBE CQM

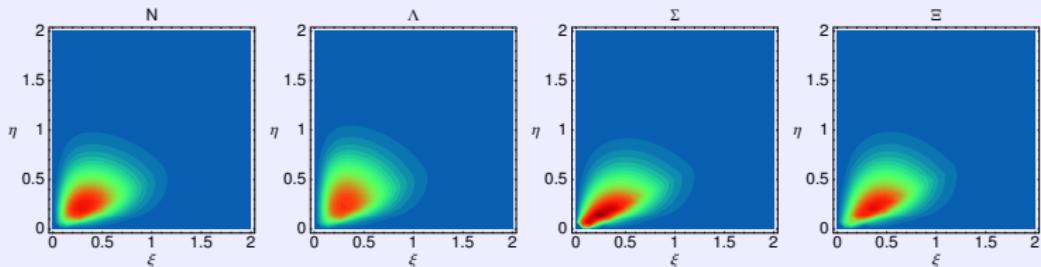


N(1440) GBE CQM

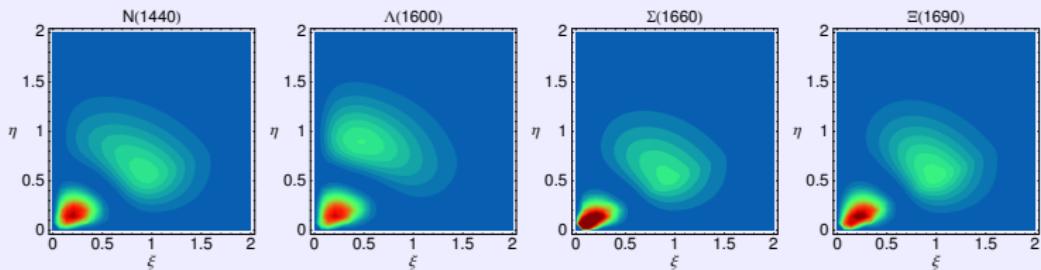


Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ octet baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:

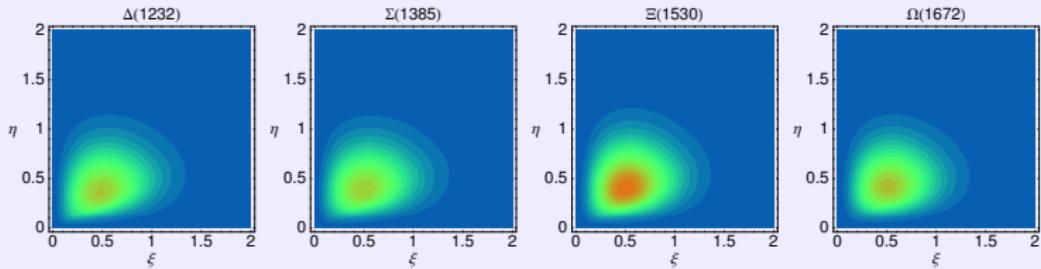


$\rho(\xi, \eta)$ for the $\frac{1}{2}^+$ octet baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:

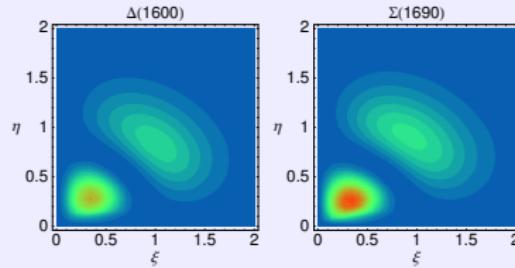


Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ decuplet baryon states $\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$:



$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ decuplet baryon states $\Delta(1600)$, $\Sigma(1690)$:



New Quark-Model Classification

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary

	multiplet	$(LS)J^P$			
	octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$
	octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$
	octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$
	octet	$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$
	octet	$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$
	octet	$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$
	octet	$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$
	octet	$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Xi(1950)^{100}$
	decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$
	decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$	
	decuplet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$	
	decuplet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$		
	singlet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Lambda(1405)^{71}$		
	singlet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Lambda(1520)^{71}$		
	singlet	$(0\frac{1}{2})\frac{1}{2}^+$	$\Lambda(1810)^{92}$		

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D 77, 114002 (2008)

See also the PDG: Chin. Phys. C 38, 090001 (2014)

$SU(3)$ Flavor Multiplets – New

Classification of baryon resonances by the PDG since **2010**
 (results from the GBE relativistic CQM marked by asterisks)

RCQM

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Summary

J^P	$(D, L_N^P) S$	Octet members	Singlets	
$1/2^+$	$(56,0_0^+)$ $1/2 N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$
$1/2^+$	$(56,0_0^+)$ $1/2 N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)^\dagger$
$1/2^-$	$(70,1_1^-)$ $1/2 N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$ $\Lambda(1405)$
				$\Sigma(1560)^\dagger$
$3/2^-$	$(70,1_1^-)$ $1/2 N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$ $\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$ $3/2 N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$
				$\Sigma(1620)^\dagger$
$3/2^-$	$(70,1_1^-)$ $3/2 N(1700)$	$\Lambda(?)$	$\Sigma(1940)^\dagger$	$\Xi(?)$
$5/2^-$	$(70,1_1^-)$ $3/2 N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(1950)^\dagger$
$1/2^+$	$(70,0_2^+)$ $1/2 N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$ $\Lambda(1810)^\dagger$
$3/2^+$	$(56,2_2^+)$ $1/2 N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$
$5/2^+$	$(56,2_2^+)$ $1/2 N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$
$7/2^-$	$(70,3_3^-)$ $1/2 N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$ $\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$ $3/2 N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$
$9/2^+$	$(56,4_4^+)$ $1/2 N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$

PDG: J. Phys. G **37**, 075021 (2010); Phys. Rev. D **86**, 010001 (2012);

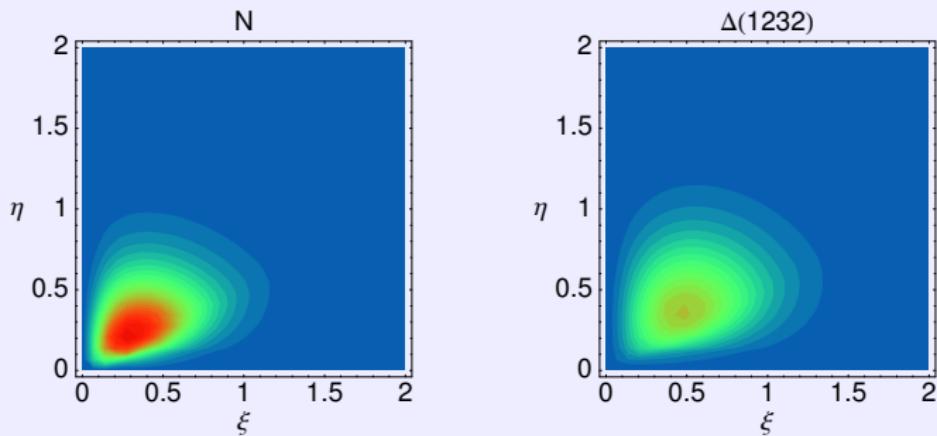
Chin. Phys. C **38**, 090001 (2014)

N and Δ Rest-Frame Wave Functions

Rest-frame **spatial distribution of constituent quarks**
in terms of 3-body Jacobi coordinates $\vec{\xi}$ and $\vec{\eta}$:

$$\rho(\xi, \eta) = \xi^2 \eta^2 \int d\Omega_\xi d\Omega_\eta$$

$$\Psi_{PJ\Sigma TM_T}^*(\xi, \Omega_\xi, \eta, \Omega_\eta) \Psi_{PJ\Sigma TM_T}(\xi, \Omega_\xi, \eta, \Omega_\eta)$$



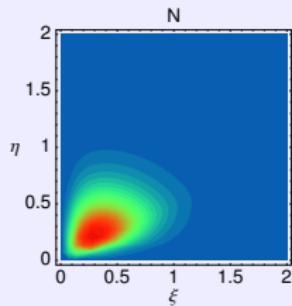
Units on abscissa and ordinates are [fm]

Root-Mean-Square Radii

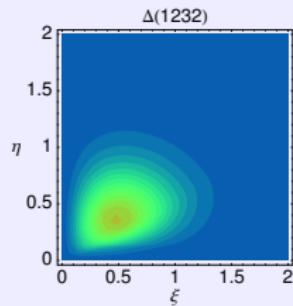
The **root-mean-square radius** (in the rest frame):

$$r_{\text{rms}} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P=0, J, \Sigma \left| \hat{r}_i^2 \right| P=0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable!** Is NOT **relativistically invariant!**
→ Idea about the **spatial distribution** of constituent quarks.



$$r_{\text{rms}}^N = 0.304 \text{ fm}$$



$$r_{\text{rms}}^\Delta = 0.390 \text{ fm}$$

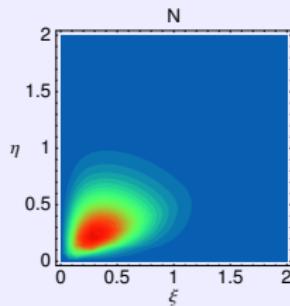
Root-Mean-Square Radii

The **root-mean-square radius** (in the rest frame):

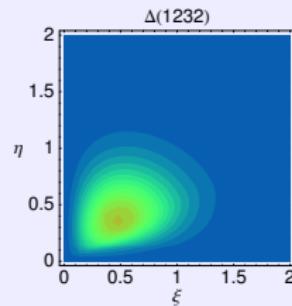
$$r_{\text{rms}} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P=0, J, \Sigma \left| \hat{r}_i^2 \right| P=0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable!** Is NOT **relativistically invariant!**

→ Idea about the **spatial distribution** of constituent quarks.



$$r_{\text{rms}}^N = 0.304 \text{ fm}$$



$$r_{\text{rms}}^\Delta = 0.390 \text{ fm}$$

Exp.: $r_E^p \sim 0.88 \text{ fm}$
 $(r_E^n)^2 \sim -0.12 \text{ fm}^2$

$r_E^{\Delta^{++}} = r_E^{\Delta^+} = r_E^{\Delta^-} = 0.656 \text{ fm}$
 $r_E^{\Delta^0} = 0 \text{ fm}$

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

πNN , $\pi N\Delta$

N Gravitational FF

Summary

Baryon **Reactions**

&

Baryon **Structure**

Various Baryon Reactions

Matrix elements of a transition operator \hat{O} between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T'_3, Y' | \hat{O} | P, J, \Sigma, T, T_3, Y \rangle$$

\hat{O} ... \hat{J}_{em}^μ → electromagnetic FF's

... $\hat{A}_{\text{axial}}^\mu$ → axial FF's

... $\hat{\Theta}^{\mu\nu}$ → gravitational/tensor FF's

... \hat{S} → scalar FF

... \hat{D}_λ^μ → strong FF's and hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3; f_{i'_1}, f_{i'_2}, f_{i'_3} | \hat{O} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3} \rangle$$



boosted 3-body states



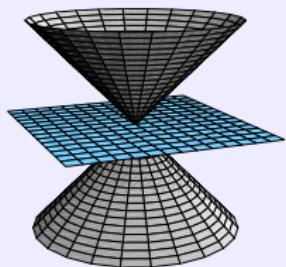
boosted 3-body states

Rel. Dynamics: Poincaré Transf. Generators

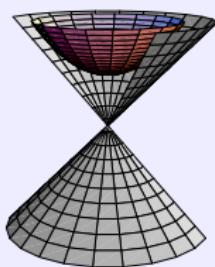
Invariant hypersurfaces in Minkowski space ↠ 3 forms

P.A.M. Dirac: Rev. Mod. Phys. 21 (1949) 392

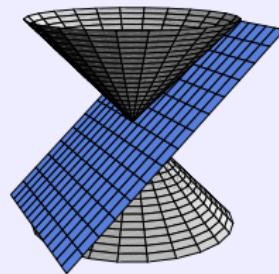
instant



point



front



$$x^0 = 0$$

$$x^2 - a^2 = 0$$

$$x^0 - x^3 = 0$$

J_i, P_i

$P^0 = H, K_1, K_2, K_3$

J_i, K_i

P^0, P^1, P^2, P^3

$P^+, \vec{P}^\perp, E^1, E^2, J_z, K_z$

P^-, F^1, F^2

interaction-free and interaction-dependent generators.

There are 2 and only 2 more forms discussed by Leutwyler & Stern with less, namely only three interaction-free generators !

RCQM

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charm, bottom

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Nucleon E.m.

Baryon E.m.

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 πNN , $\pi N\Delta$

N Gravitational FF

Summary

Covariant predictions for:

- ▶ **Electromagnetic** nucleon form factors
 $G_E^p(Q^2)$, $G_M^p(Q^2)$; $G_E^n(Q^2)$, $G_M^n(Q^2)$
 - ▶ **Electric radii** and **magnetic moments**
 $r_E^p, \mu^p; r_E^n, \mu^n$
- Comparison to experiment

Electron Scattering and E.m. Form Factors

Elastic electron scattering:

RCQM

- Mass operator
- Dynamics
- EV problem

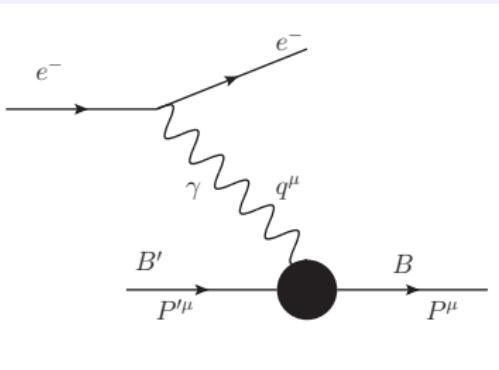
Spectroscopy

- Light, strange,
charm, bottom

Structure

- Nucleon E.m.
- Baryon E.m.
- Axial FFs
- πNN , $\pi N\Delta$
- N Gravitational FF

Summary



Invariant form factors:

$$F_{\Sigma'\Sigma}^\nu(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}_{\text{em}}^\nu | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2; \quad q^\mu = P^\mu - P'^\mu$$

Transition Matrix Elements in Point Form

Incoming baryon state: $|V, M, J, \Sigma\rangle$

$\hat{\equiv} |P, J, \Sigma\rangle$

Outgoing baryon state: $|V', M', J', \Sigma'\rangle$

$\hat{\equiv} |P', J', \Sigma'\rangle$

Transition operator: $\hat{O} = \hat{\mathbf{j}}_{\text{em}}^\mu$

$$\langle V', M', J', \Sigma' | \hat{\mathbf{j}}_{\text{em}}^\mu | V, M, J, \Sigma \rangle =$$

$$= \frac{2}{MM'} \sum_{\sigma_i \sigma'_i} \sum_{\mu_i \mu'_i} \int d^3 \vec{k}_2 d^3 \vec{k}_3 d^3 \vec{k}'_2 d^3 \vec{k}'_3$$

$$\times \sqrt{\frac{(\sum_i \omega'_i)^3}{\prod_i 2\omega'_i}} \prod_{\sigma'_i} D_{\sigma'_i \mu'_i}^{\star \frac{1}{2}} \{ R_W [k'_i; B(V')] \} \Psi_{M' J' \Sigma'}^* (\vec{k}'_1, \vec{k}'_2, \vec{k}'_3; \mu'_1, \mu'_2, \mu'_3)$$

$$\times \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{\mathbf{j}}_{rd}^\mu | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle$$

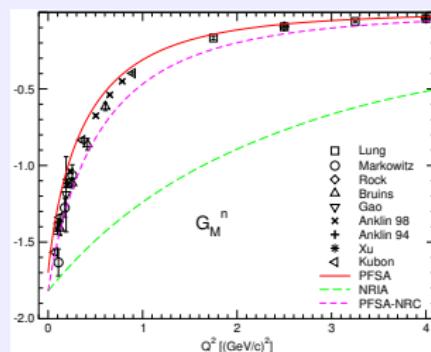
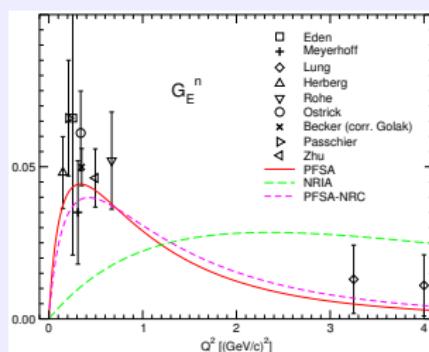
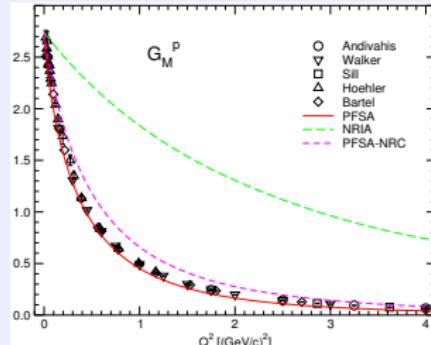
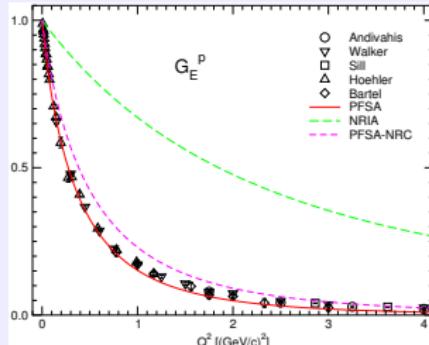
$$\times \sqrt{\frac{(\sum_i \omega_i)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D_{\sigma_i \mu_i}^{\frac{1}{2}} \{ R_W [k_i; B(V)] \} \Psi_{MJ\Sigma} (\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3)$$

$$\times 2MV_0 \delta^3 (M \vec{V} - M' \vec{V}' - \vec{q})$$

where $p_i = B_c(V) k_i$, $p'_i = B_c(V') k'_i$, and $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

Electromagnetic Nucleon Form Factors

Covariant predictions of the GBE CQM:



RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary

Nucleon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm 2]

Baryon	GBE PFSM	Experiment
p	0.82	$0.7692 \pm 0.0123^{1)}$ $0.70870 \pm 0.00113^{2)}$
n	-0.13	-0.1161 ± 0.0022

¹⁾ CODATA value (PDG)

²⁾ Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleon r_E^2 and μ – Nonrelativistic !!!

Electric radii r_E^2 [fm 2]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	0.82	0.10	$0.7692 \pm 0.0123^{1)}$
			$0.70870 \pm 0.00113^{2)}$
n	-0.13	-0.01	-0.1161 ± 0.0022

¹⁾ CODATA value (PDG)

²⁾ Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	GBE NRIA	Experiment
p	2.70	2.74	2.792847356
	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Flavor Analysis of Nucleon E.m. FFs

RCQM

Mass operator
Dynamics
EV problem

Spectroscopy
Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary

Nucleons N

Proton Electric Form Factor

$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

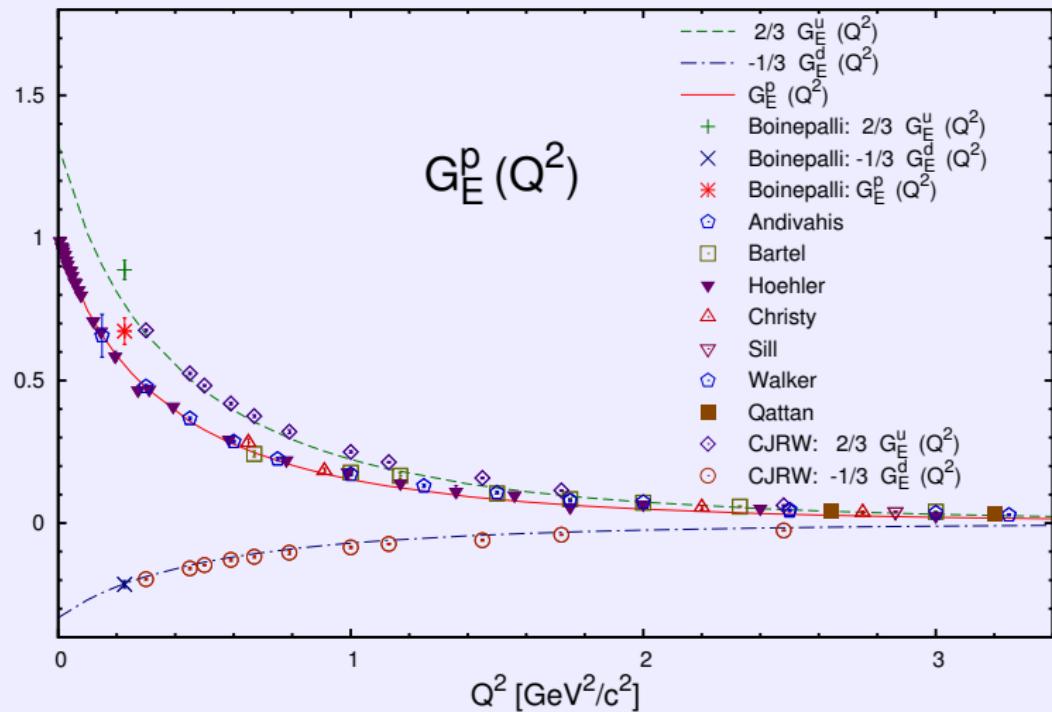
Baryon E.m.

Axial FFs

 $\pi NN, \pi N\Delta$

N Gravitational FF

Summary



Neutron Electric Form Factor

$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$

RCQM

Mass operator
 Dynamics
 EV problem

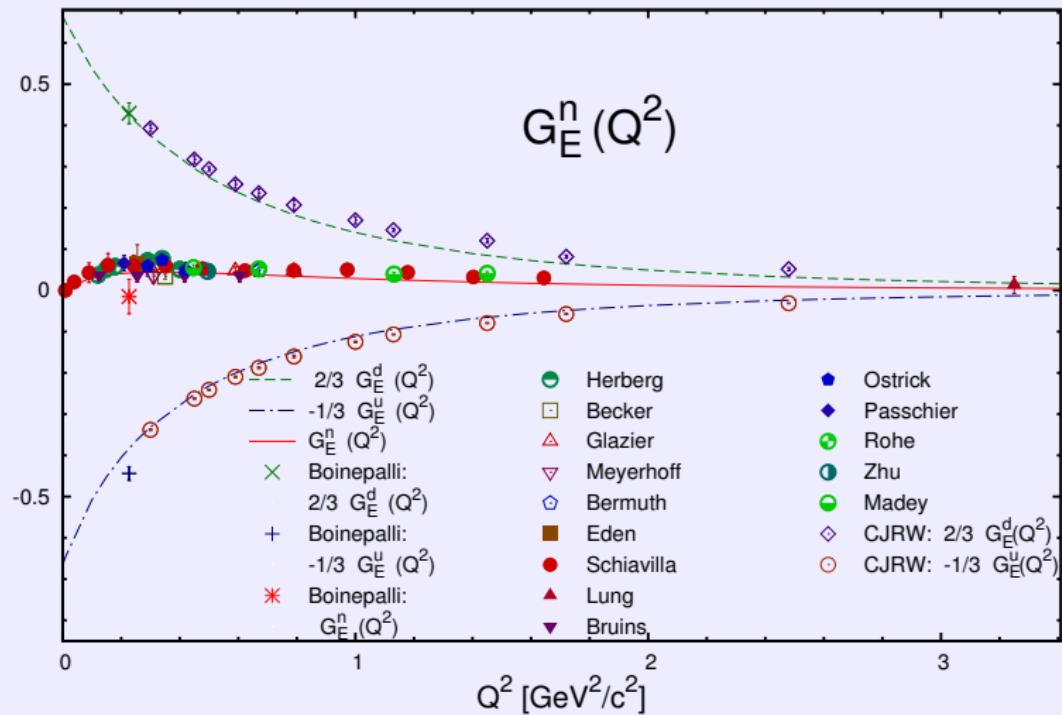
Spectroscopy

Light, strange,
 charm, bottom

Structure

Nucleon E.m.
 Baryon E.m.
 Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary



Proton Magnetic Form Factor

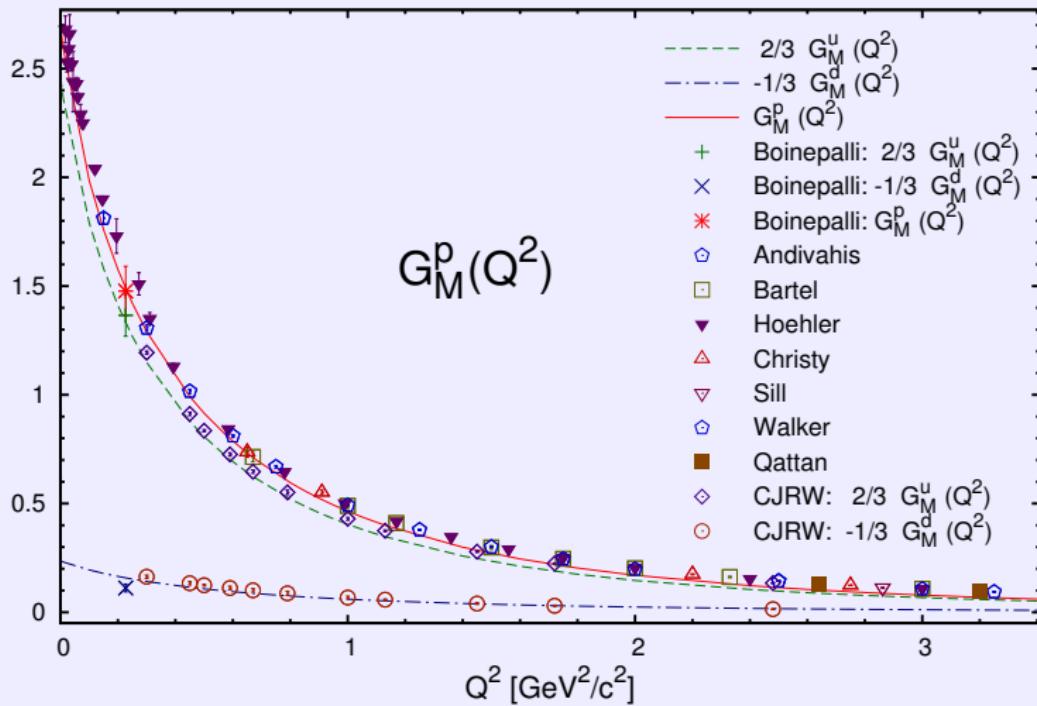
$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$

RCQM
Mass operator
Dynamics
EV problem

Spectroscopy
Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary



Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

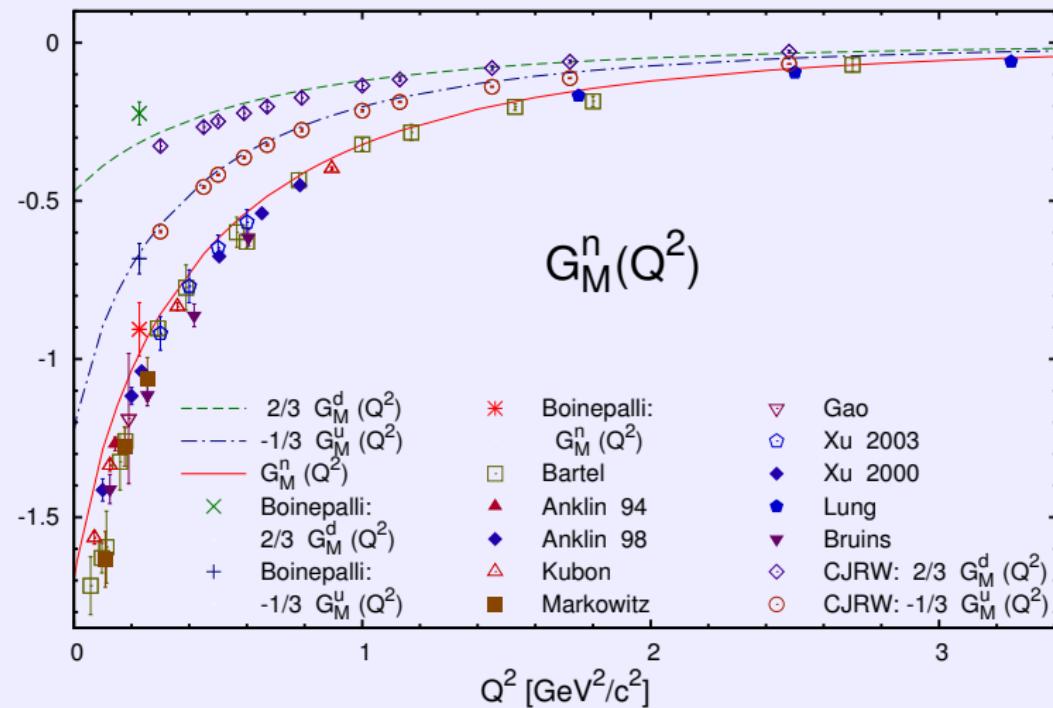
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



Conclusions from Nucleon Flavor Analysis

RCQM

Mass operator
Dynamics
EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary

- ▶ **Flavor analysis of nucleon e.m. form factors** in a relativistically invariant framework (point form).
- ▶ The **GBE RCQM** predicts flavor contributions in reasonable agreement with **experimental data**.
- ▶ The GBE RCQM relies on $\{QQQ\}$ degrees of freedom only; no explicit $\{QQQQ\bar{Q}\}$ etc.
- ▶ No explicit **meson-cloud effects** are included.
- ▶ No **strangeness content** in the nucleon for the low momentum transfers considered here.
- ▶ With respect to F_2^d/F_2^u three different phenomenological analyses give **distinct answers**.
- ▶ Details:
 - M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665
 - W. Plessas: Mod. Phys. Lett. A **28**, 136022 (2013)

Δ and Hyperon E.m. Form Factors

RCQM

Mass operator
Dynamics
EV problem

Spectroscopy
Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary

Δ

Λ , Σ , Ξ

Σ^* , Ξ^* , Ω

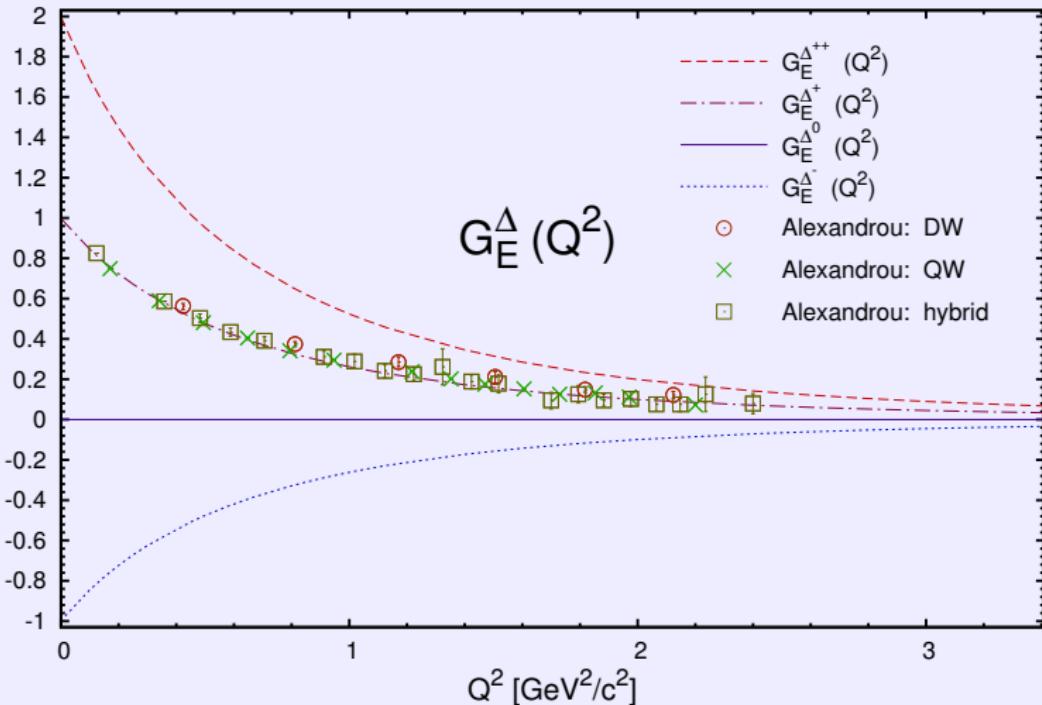
Electric Δ Form Factors

RCQM
Mass operator
Dynamics
EV problem

Spectroscopy
Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D 79 (2009) 014507

Magnetic Δ Form Factors

RCQM

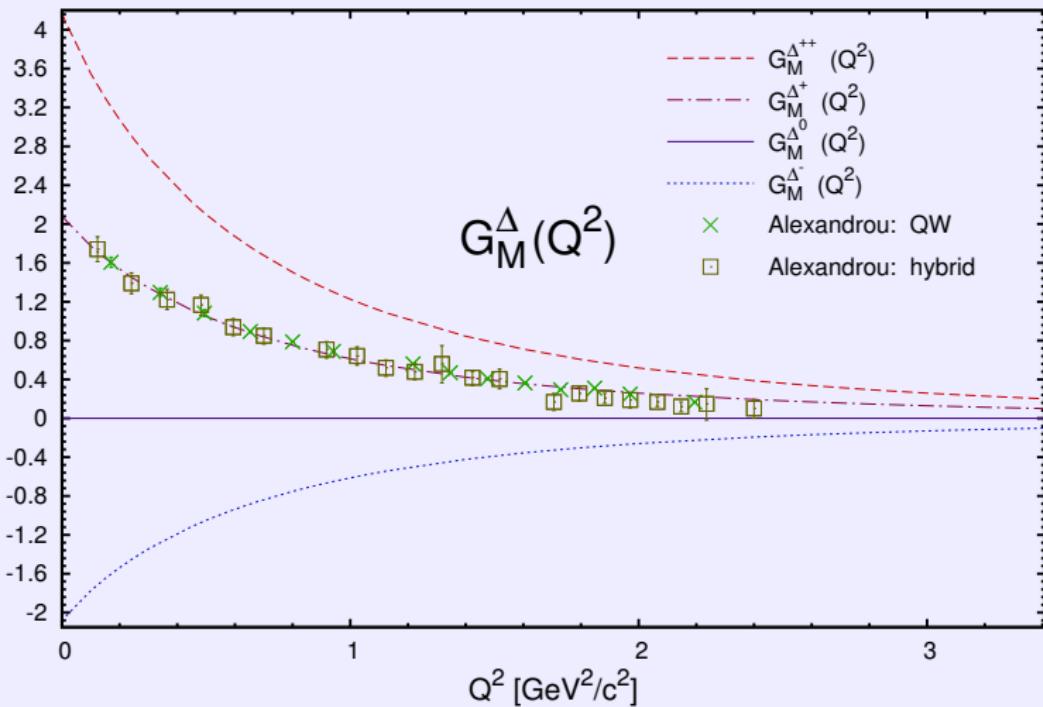
Mass operator
Dynamics
EV problemSpectroscopy
Light, strange,
charm, bottom

Structure

Nucleon E.m.
Baryon E.m.
Axial FFs πNN , $\pi N\Delta$

N Gravitational FF

Summary



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

Octet $\Lambda(uds)$ Electric Form Factor

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

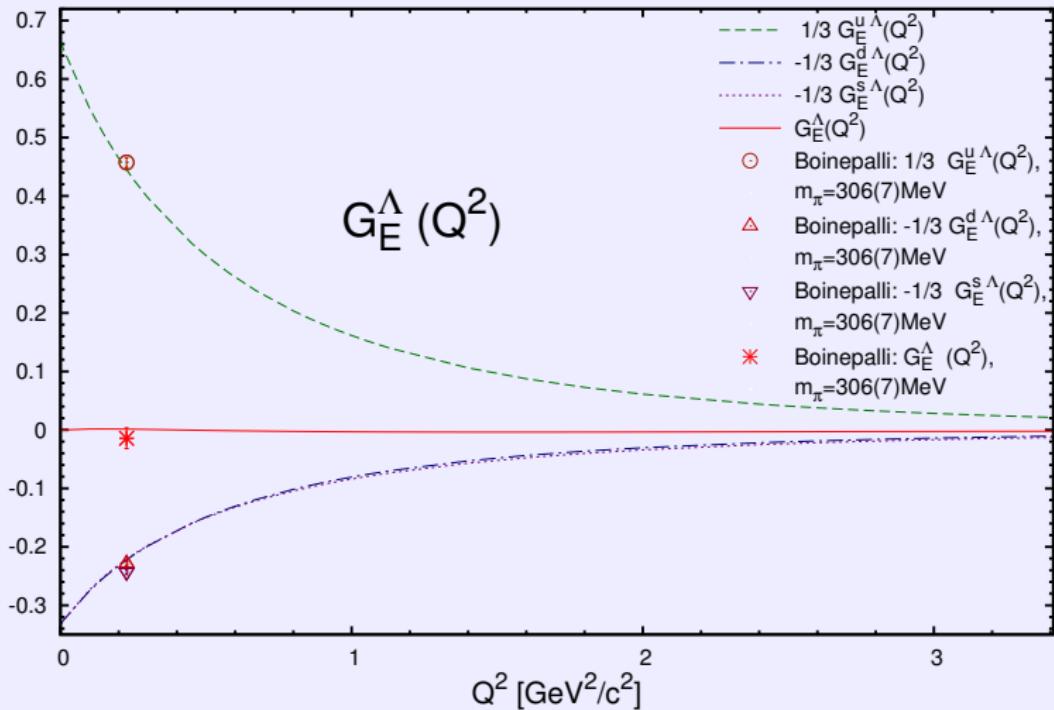
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



Octet $\Lambda(uds)$ Magnetic Form Factor

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

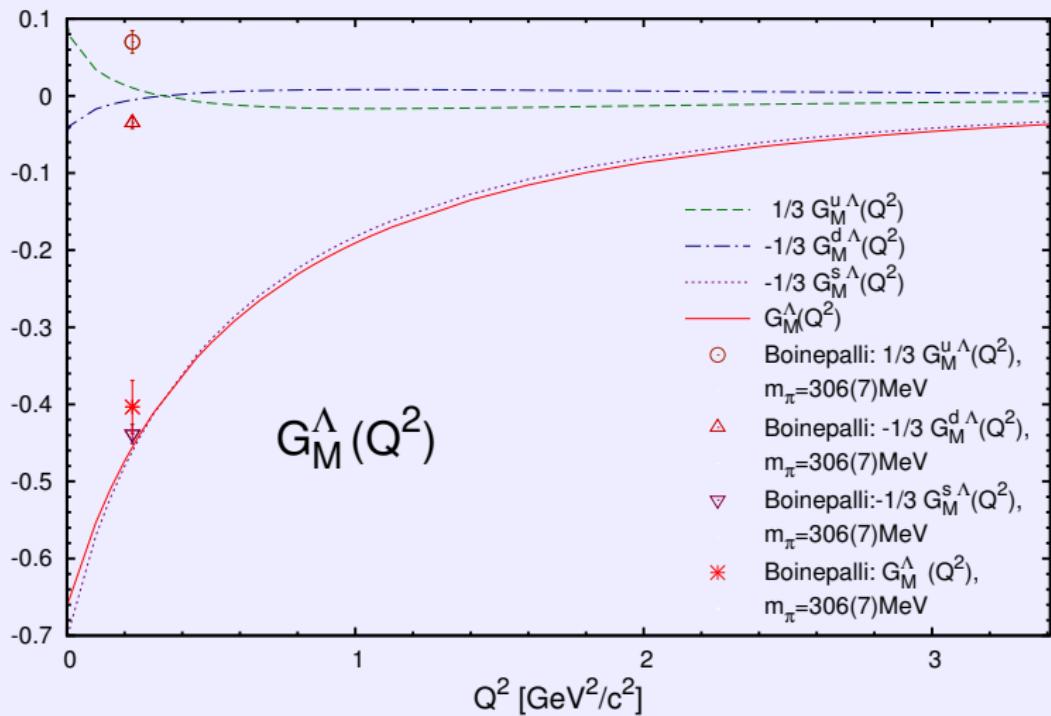
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



Decuplet Ω^- (sss) Electric Form Factor

RCQM

Mass operator

Dynamics

EV problem

Spectroscopy

Light, strange,
charm, bottom

Structure

Nucleon E.m.

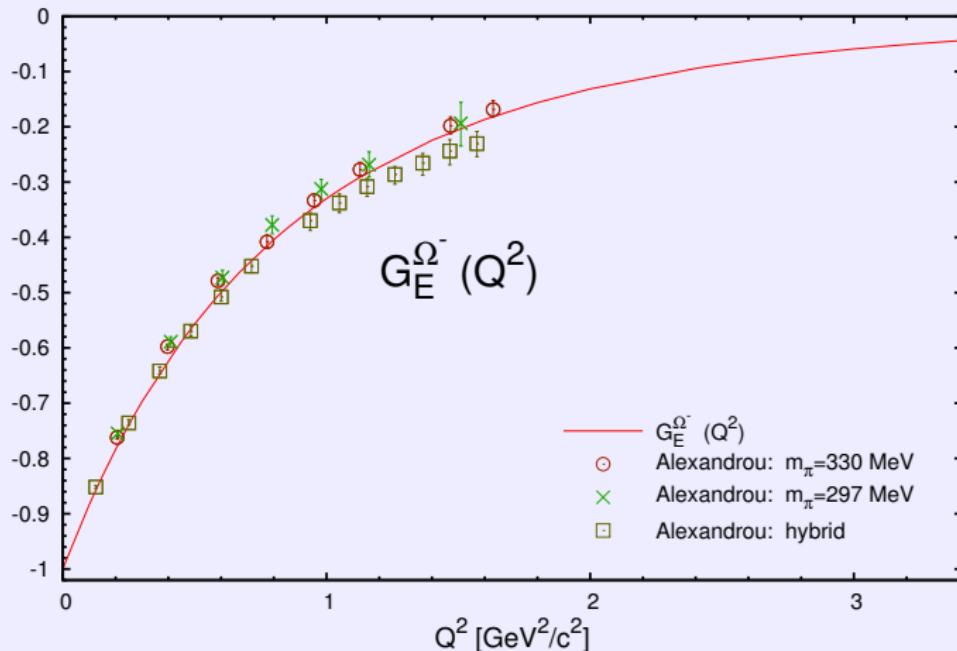
Baryon E.m.

Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary



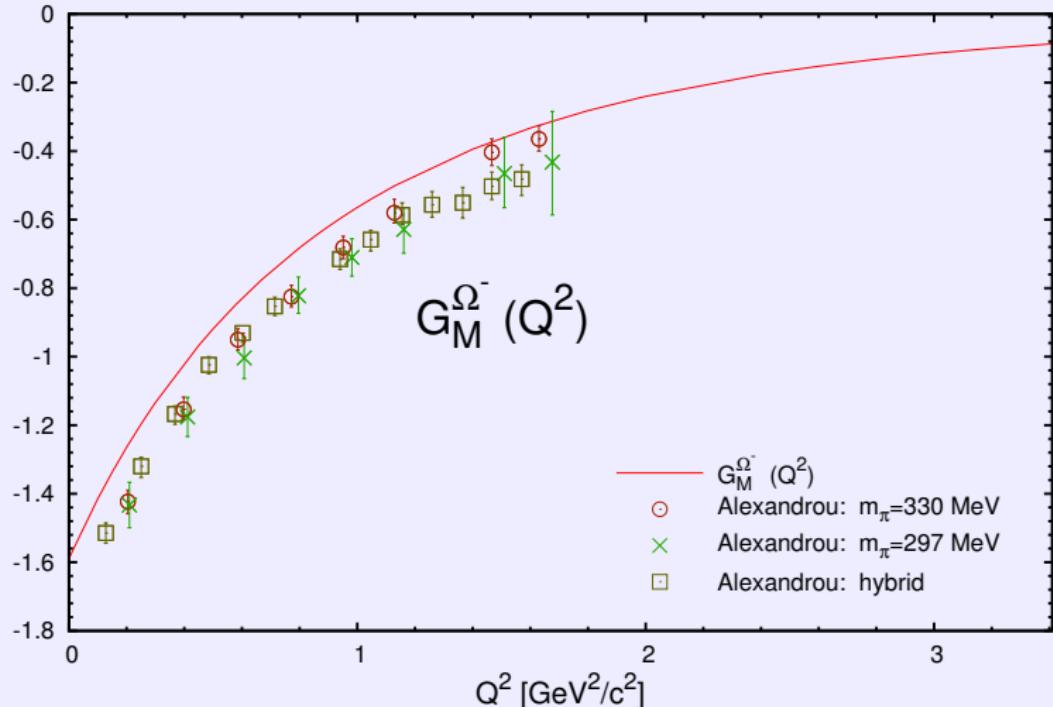
Decuplet Ω^- (sss) Magnetic Form Factor

RCQM
Mass operator
Dynamics
EV problem

Spectroscopy
Light, strange,
charm, bottom

Structure
Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary



Octet $\Sigma^0(uds)$ vs. Decuplet $\Sigma^{*0}(uds)$

RCQM

Mass operator
 Dynamics
 EV problem

Spectroscopy

Light, strange,
 charm, bottom

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

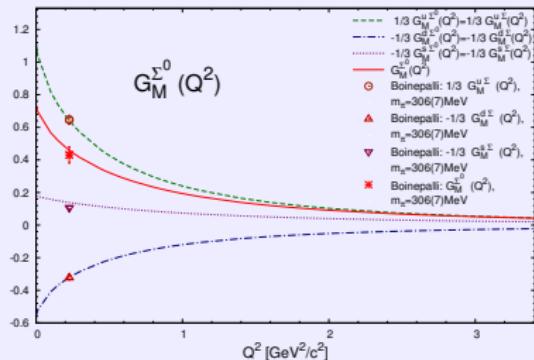
 πNN , $\pi N\Delta$

N Gravitational FF

Summary

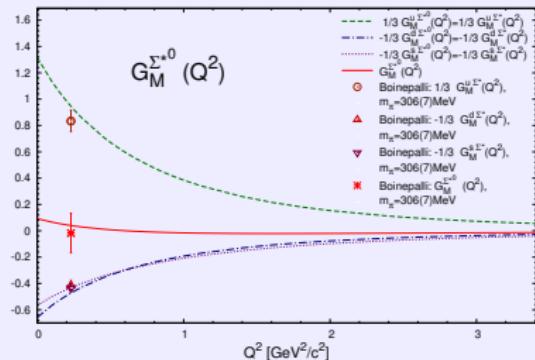
Octet

$$G_M^{\Sigma^0} = \frac{1}{3} G_M^{u,\Sigma} - \frac{1}{3} G_M^{d,\Sigma} - \frac{1}{3} G_M^{s,\Sigma}$$



Decuplet

$$G_M^{\Sigma^{*0}} = \frac{1}{3} G_M^{u,\Sigma^*} - \frac{1}{3} G_M^{d,\Sigma^*} - \frac{1}{3} G_M^{s,\Sigma^*}$$



Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Octet Ξ^- (dss) vs. Decuplet Octet Ξ^{*-} (dss)

RCQM

Mass operator
 Dynamics
 EV problem

Spectroscopy

Light, strange,
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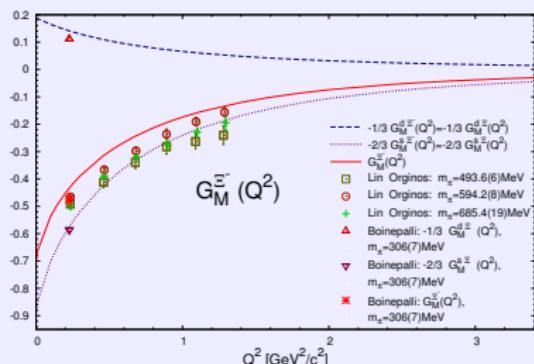
Structure

Nucleon E.m.
 Baryon E.m.
 Axial FFs
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Summary

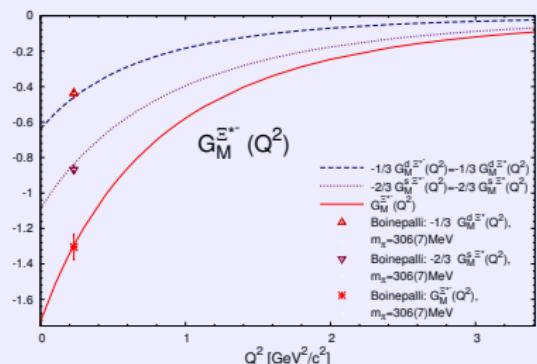
Octet

$$G_M^{\Xi^-} = -\frac{1}{3} G_M^{d,\Xi} - \frac{2}{3} G_M^{s,\Xi}$$



Decuplet

$$G_M^{\Xi^{*-}} = -\frac{1}{3} G_M^{d,\Xi^*} - \frac{2}{3} G_M^{s,\Xi^*}$$



Lattice-QCD: S. Boinepalli et al.: Phys. Rev. D **74**, 093005 (2006)

S. Boinepalli et al.: Phys. Rev. D **80**, 054505 (2009)

Baryon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm 2]

Baryon	GBE PFSM	Experiment
p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

Magnetic moments μ [n.m.]

Baryon	GBE PFSM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
Δ^{++}	4.17	$3.7 - 7.5$
Ω^-	-1.59	-2.020 ± 0.05

RCQM

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Nucleon E.m.
Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
 N Gravitational FF

Summary

Axial **Charges** and Axial **Form Factors**

of

N Ground State and **N^*** Resonances

as well as

$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Axial Nucleon Form Factors

RCQM

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Nucleon E.m.

Baryon E.m.

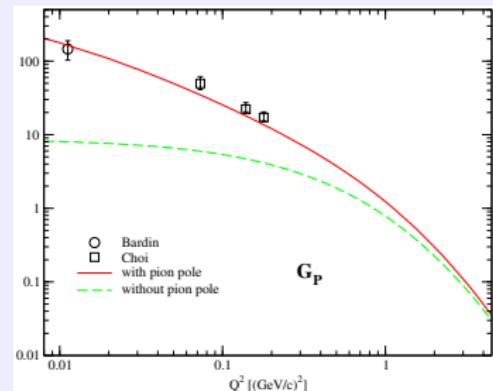
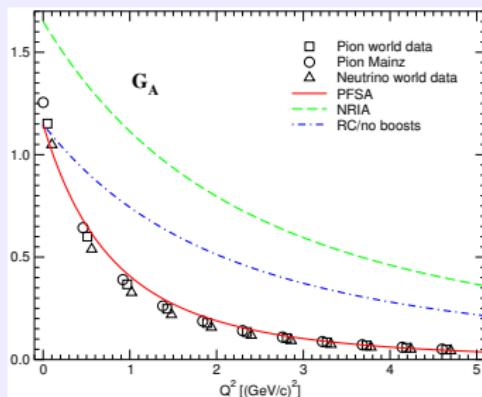
Axial FFs

 πNN , $\pi N\Delta$

N Gravitational FF

Summary

Covariant predictions of the GBE RCQM:



$$g_A^{GBE} = 1.15 \quad \text{vs.}$$

$$g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Axial Charges of N and N^* Resonances

RCQM
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 Baryon E.m.
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Summary

State	J^P	EGBE	Lattice QCD	GN	NR
$N(939)$	$\frac{1}{2}^+$	1.15	1.18~1.31	1.66	1.65
$N(1440)$	$\frac{1}{2}^+$	1.16	?	1.66	1.61
$N(1535)$	$\frac{1}{2}^-$	0.02	~ 0.00	-0.11	-0.20
$N(1710)$	$\frac{1}{2}^+$	0.35	?	0.33	0.42
$N(1650)$	$\frac{1}{2}^-$	0.51	~ 0.55	0.55	0.64

- EGBE Extended **GBE** RCQM covariant result
 Lattice **Lattice QCD** calculations by LHPC Collaboration and
 Takahashi-Kunihiro (Kyoto)
 GN **Glozman-Nefediev** $SU(6) \times O(3)$ nonrelativistic QM
 NR **Non-Relativistic** EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Axial Form Factor of the Δ

Covariant predictions of the GBE and OGE RCQMs:

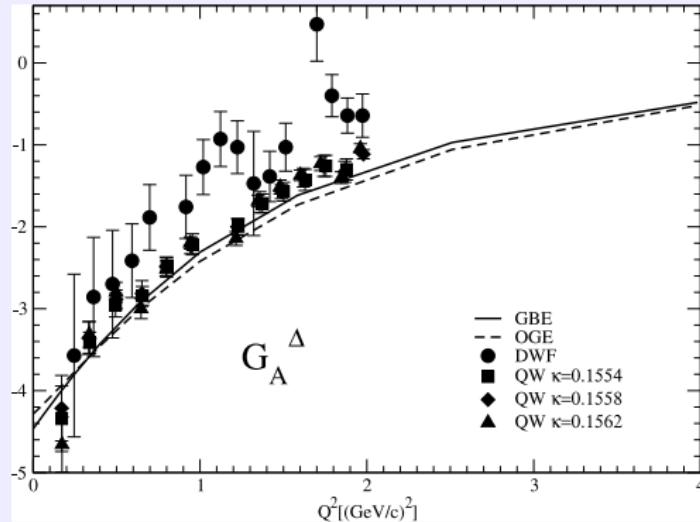
RCQM

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Baryon E.m.
Axial FFs
 πNN , $\pi N\Delta$
N Gravitational FF

Summary



K.-S. Choi and W. Plessas: Few-Body Syst. **54**, 1055 (2013)

(Lattice QCD data from C. Alexandrou et al., PoS LATTICE2010, 141 (2010))

Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

	J^P	Exp	EGBE	LO	EOT	JT	NR
RCQM	$N \frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
Mass operator	$\Sigma \frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
Dynamics	$\Xi \frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
EV problem	$\Delta \frac{3}{2}^+$	-	-4.48	-	-	~ -4.5	-6.00
Spectroscopy	$\Sigma^* \frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
Light, strange, charm, bottom	$\Xi^* \frac{3}{2}^+$	-	-0.75	-	-	-	-1.00
Structure							
Nucleon E.m.							
Baryon E.m.							
Axial FFs							
$\pi NN, \pi N\Delta$							
N Gravitational FF							
Summary							

- EGBE Extended GBE RCQM covariant result
 LO Lin and Orginos lattice-QCD calculation
 EOT Erkol, Oka, and Takahashi lattice-QCD calculation
 JT Jiang and Tiburzi χ PT calculation
 NR Non-Relativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

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Microscopic Description

of

Meson-Baryon Interaction Vertices

Meson-Baryon Interaction Vertices

Interaction vertices

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Nucleon E.m.

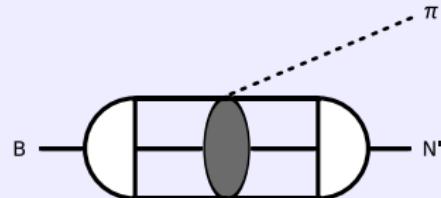
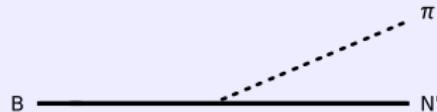
Baryon E.m.

Axial FFs

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N Gravitational FF

Summary



$$F_{i \rightarrow f} = (2\pi)^4 \langle f | \mathcal{L}_I(0) | i \rangle \equiv \langle V', M', J', \Sigma' | \hat{D}_{rd}^\pi | V, M, J, \Sigma \rangle$$

where

$$\left\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \left| \hat{D}_{rd}^\pi \right| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle =$$

$$3\mathcal{N}_S \frac{ig_{qqm}}{2m_1(2\pi)^{\frac{3}{2}}} \bar{u}(p'_1, \sigma'_1) \gamma_5 \gamma_\mu \lambda_m u(p_1, \sigma_1) \tilde{q}^\mu 2p_{20} \delta(\vec{p}_2 - \vec{p}'_2) 2p_{30} \delta(\vec{p}_3 - \vec{p}'_3) \delta_{\sigma_2 \sigma'_2} \delta_{\sigma_3 \sigma'_3}$$

and

$$G_{\pi NN}(Q^2) = \frac{1}{f_{\pi NN}} \frac{m_\pi \sqrt{2\pi}}{\sqrt{2M_N}} \frac{\sqrt{E'_N + M'_N}}{E'_N + M'_N + \omega} \frac{F_{i \rightarrow f}}{Q_z}$$

$$G_{\pi N\Delta}(Q^2) = -\frac{1}{f_{\pi N\Delta}} \frac{3\sqrt{2\pi}}{2} \frac{m_\pi}{\sqrt{E'_N + M'_N} \sqrt{2M_\Delta}} \frac{F_{i \rightarrow f}}{Q_z}$$

πNN and $\pi N\Delta$ Interaction Vertices

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Nucleon E.m.

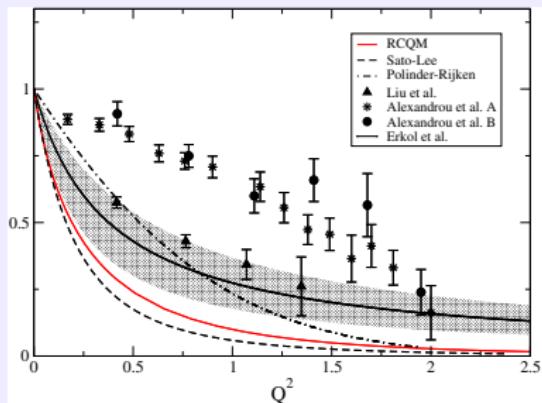
Baryon E.m.

Axial FFs

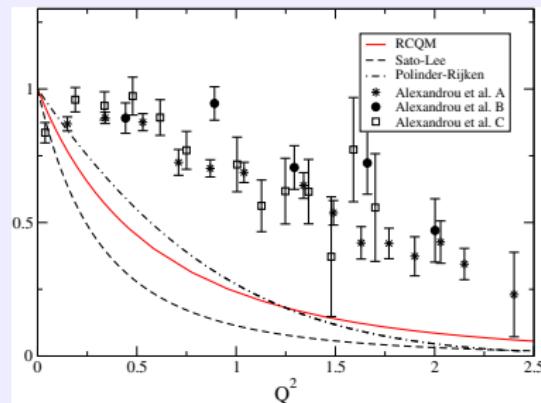
 πNN , $\pi N\Delta$

N Gravitational FF

Summary



$$G_{\pi NN}$$



$$G_{\pi N\Delta}$$

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Form-Factor Parametrizations

$$G(\vec{q}^2) = \frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_1}\right)^2 + \left(\frac{\vec{q}}{\Lambda_2}\right)^4}$$

$$G(Q^2) = \frac{1}{1 + \left(\frac{Q}{\Lambda}\right)^2}$$

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	RCQM	SL	PR	LIU	ERK	ALX
N	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075	0.0649	0.0481
	Λ_1	0.451	0.453	0.940	0.747	0.614
	Λ_2	0.931	0.641	1.102	-	-
Δ	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478		
	Λ_1	0.594	0.458	0.853		
	Λ_2	0.998	0.648	1.014		

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

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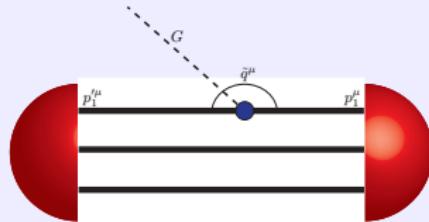
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Summary

Gravitational Form Factors of the Nucleon

Gravitational Form Factors



Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P' J \Sigma' | \hat{\Theta}^{\mu\nu} | P J \Sigma \rangle = \bar{U}(P') \left[\gamma^{(\mu} \bar{P}^{\nu)} A(Q^2) + \frac{i}{2M} \bar{P}^{(\mu} \sigma^{\nu)} B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} C(Q^2) \right] U(P)$$

$$A(Q^2) \sim \langle P' J \Sigma' | \Theta^{00} | P J \Sigma \rangle$$

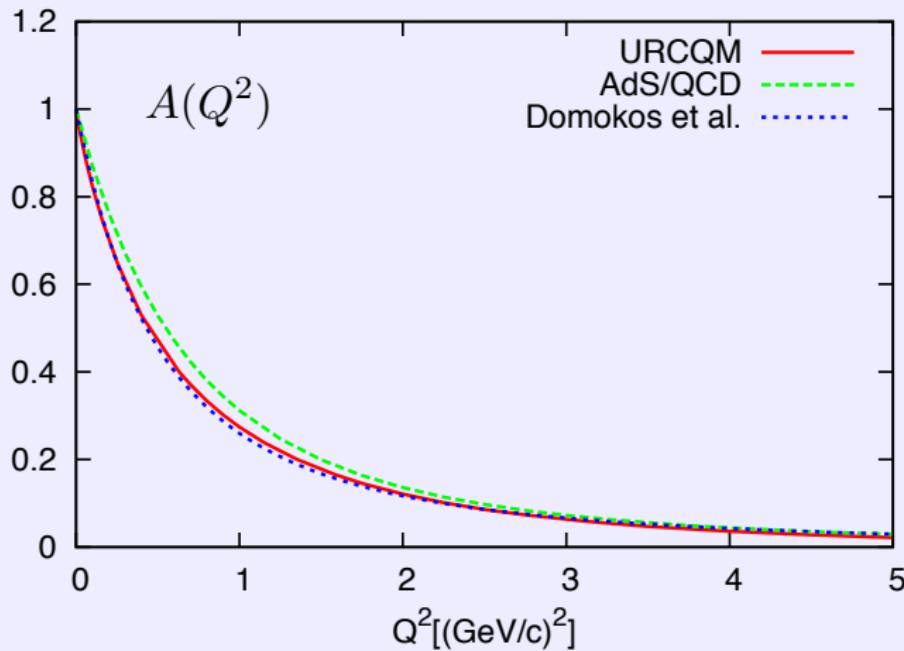
Nucleon Gravitational Form Factor $A(Q^2)$

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Summary and Conclusions

- ▶ Surprisingly **good agreement** of predictions by the $\{QQQ\}$ GBE RCQM with experimental data (wherever such data are available)
- ▶ **Small deviations** left in some observables, such as electric radii and magnetic moments
- ▶ Surprisingly **good agreement** of predictions by the GBE RCQM with lattice-QCD results
- ▶ Most important symmetries of the GBE RCQM:
 - ▶ **$S\bar{B}X\bar{S}$**
 - ▶ **Lorentz invariance**
 - ▶ **time-reversal invariance**
 - ▶ **current conservation**
- ▶ The **non-relativistic** quark model **does not work** in any instance

Constituent-Quark-Model Review

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W. Plessas:

Int. J. Mod. Phys. A30 (2015) 02, 1530013

also in the book:

"50 Years of Quarks"

ed. by M. Gell-Mann and H. Fritzsch

(World Scientific, Singapore, 2015)

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Thank you very much
for
your attention!