Properties of the Polyakov loop geometrical clusters and decofinement transition in SU(2) gluodynamics

O. Ivanytskyi

with K. Bugaev, E. Nikonov, E.-M. Ilgenfritz, D. Oliinychenko, V. Sagun, I. Mishustin, V. Petrov, G. Zinovjev

based on **arXiv:1606.04710 [hep-lat]**

ICNFP2016, 6 - 14 July, 2016, Kolymbari, Crete

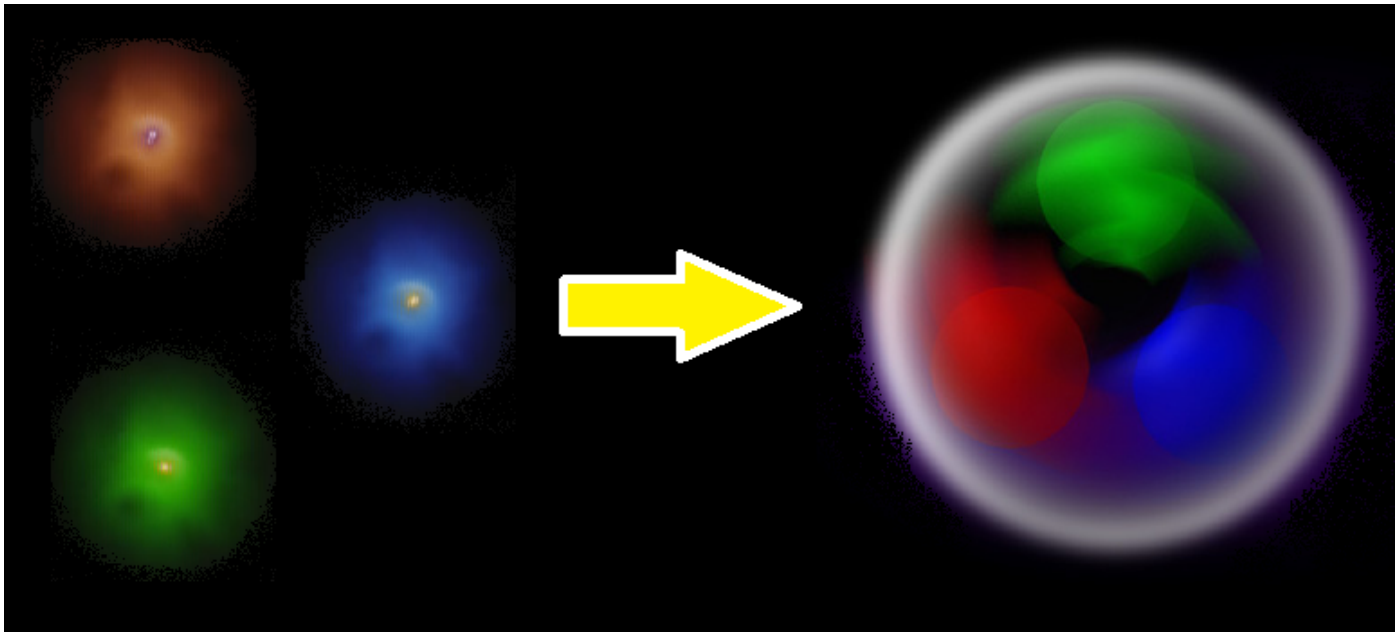
Outline

Outline

Outline

Clusterization in QCD

- Therory of strong interactions (QCD) operates with quarks and gluons
- Only hadrons are directly observed in experiments



Clusterization of partons to hadrons

Svetitsky-Jaffe conjecture

- Deconfinement transitions in (d+1)dimensional SU(N) gluodynamics is equivalent to magnetic transition in the d-dimensional Z(N) spin system
  L. G. Yaffe and B. Svetitsky, PRD, 26, 963, 1982

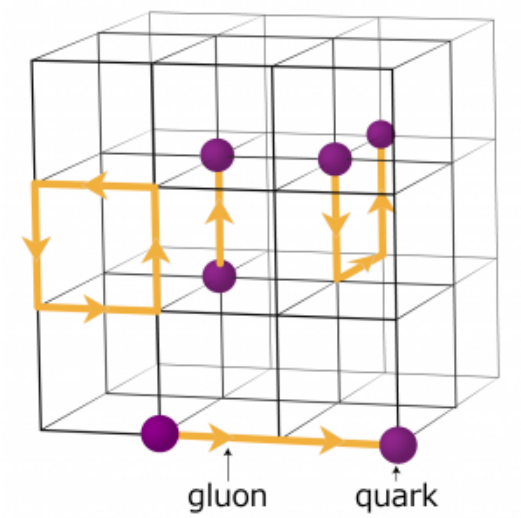
$$\text{SU(2) gluodynamics} \Leftrightarrow \text{Ising spin model}$$

- Local Polyakov loop - gauge invariant analog of continuous spin

$$L(\tilde{x}) = \mathrm{Tr} \prod_{t=0}^{N_\tau - 1} U_4(\tilde{x}, t)$$

$U_4(\tilde{x}, t)$ – temporal gauge link defined by gluon field

$SU(2) \Rightarrow L(\tilde{x}) \in [-1, 1]$, real

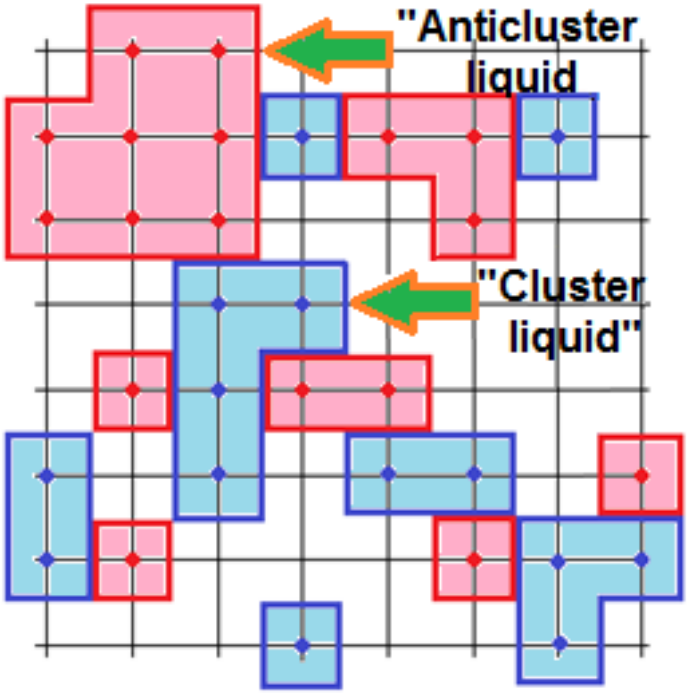


Identification of geometrical clusters

Definition of (anti)clusters

$$|L(\tilde{x})| < L_{cut} \Rightarrow \text{auxiliary vacuum}$$
$$|L(\tilde{x})| \geq L_{cut} \Rightarrow \text{(anti)clusters}$$
$$L_{cut} - \text{vacuum cut} - \text{off parameter}$$

C. Gattringer, PLB, 690, 179 (2010)
C. Gattringer, A. Schmidt, JHEP 1101, 051, 2011

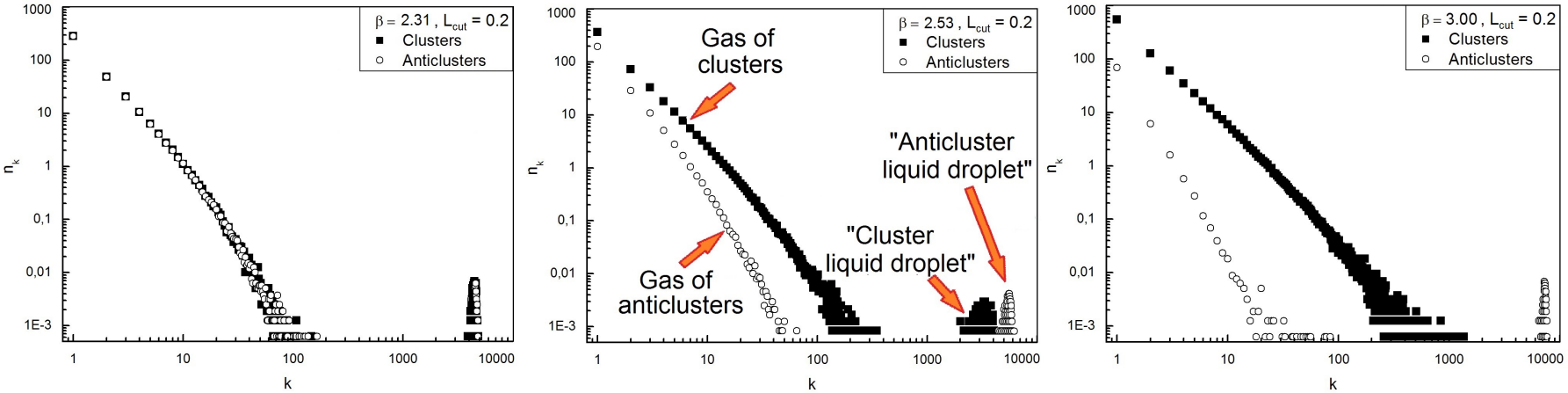


(Anti)clusters can be either “spin up"or “spin down"ones

- Largest fragment - “anticluster liquid droplet"
- Next to the largest fragment - “cluster liquid droplet"
- Gas of (anti)clusters has the same Polykov loop sign as their “liquids"

Size distributions of (anti)clusters

- Numerical simulations at $3+1$ dimensional lattice of size $N_\sigma = 24$, $N_\tau = 8$
- 13 values of inverse coupling $\beta \in [2.31, 3]$ $\Rightarrow$ 13 values of physical temperature
- vacuum cut-off parameter $L_{cut} = 0.1$ and 0.2
- Average over 1600 independent configurations for all β and L_{cut}



Distributions at low $\beta \leq \beta_c \simeq 2.52$ (phase of restored global Z(2) symmetry)

- symmetry between (anti)cluster distributions
- gas and "liquid"domains are well separated

Distributions at high $\beta > \beta_c \simeq 2.52$ (phase of broken global Z(2) symmetry)

- no symmetry between (anti)cluster distributions
- "cluster liquid"evaporates to cluster gas
- anticluster gas condensates to "anticluster liquid"

Liquid droplet approach

- Deconfinement transition in SU(2) gluodynamics is a special kind of the liquid-gas phase transition with two liquids and two gases

Polyakov loop clusters $\Leftrightarrow$ droplets?

- Liquid droplet formula for average number of (anti)clusters of size k

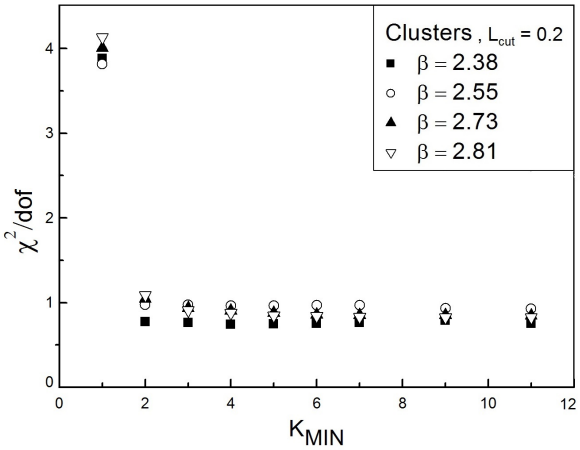
was mentioned in talks of I. Mishustin @ ICNFP21016 and V. Sagun @ ICNFP2016
first introduced in M.E. Fisher, Physics 3, 255 (1967)

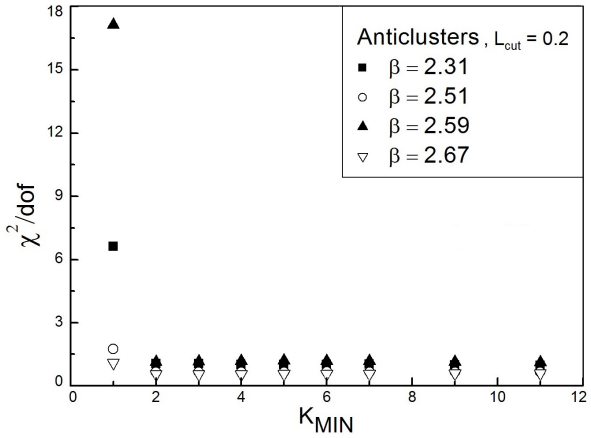
$$n_{k \geq k_{min}} = C \exp\left(\nu k - \sigma k^{2/3} - \tau \ln k\right)$$

- C - normalization factor (absolute amount)
- ν - reduced chemical potential (liquid-gas phase transition)
- σ - reduced surface tension coefficient (appearance of critical point)
- τ - Fisher topological exponent (size distribution at critical point)
- k_{min} - size of the minimal (anti)cluster described by the liquid droplet formula

Determination of k_{min} and τ

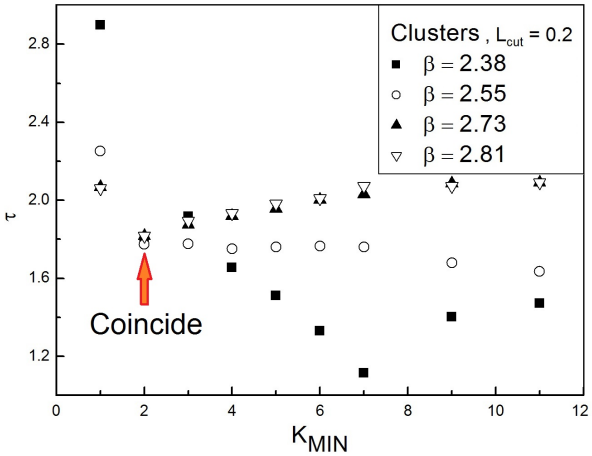
- LDF describes size distributions with almost the same quality for all $k_{min} \geq 2$

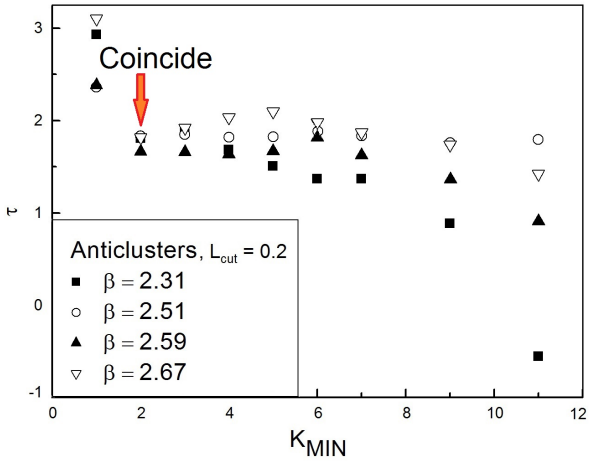




- Fisher topological exponent τ is temperature independent at $k_{min} = 2$

in agreement with Fisher droplet model M.E. Fisher, Physics 3, 255 (1967)

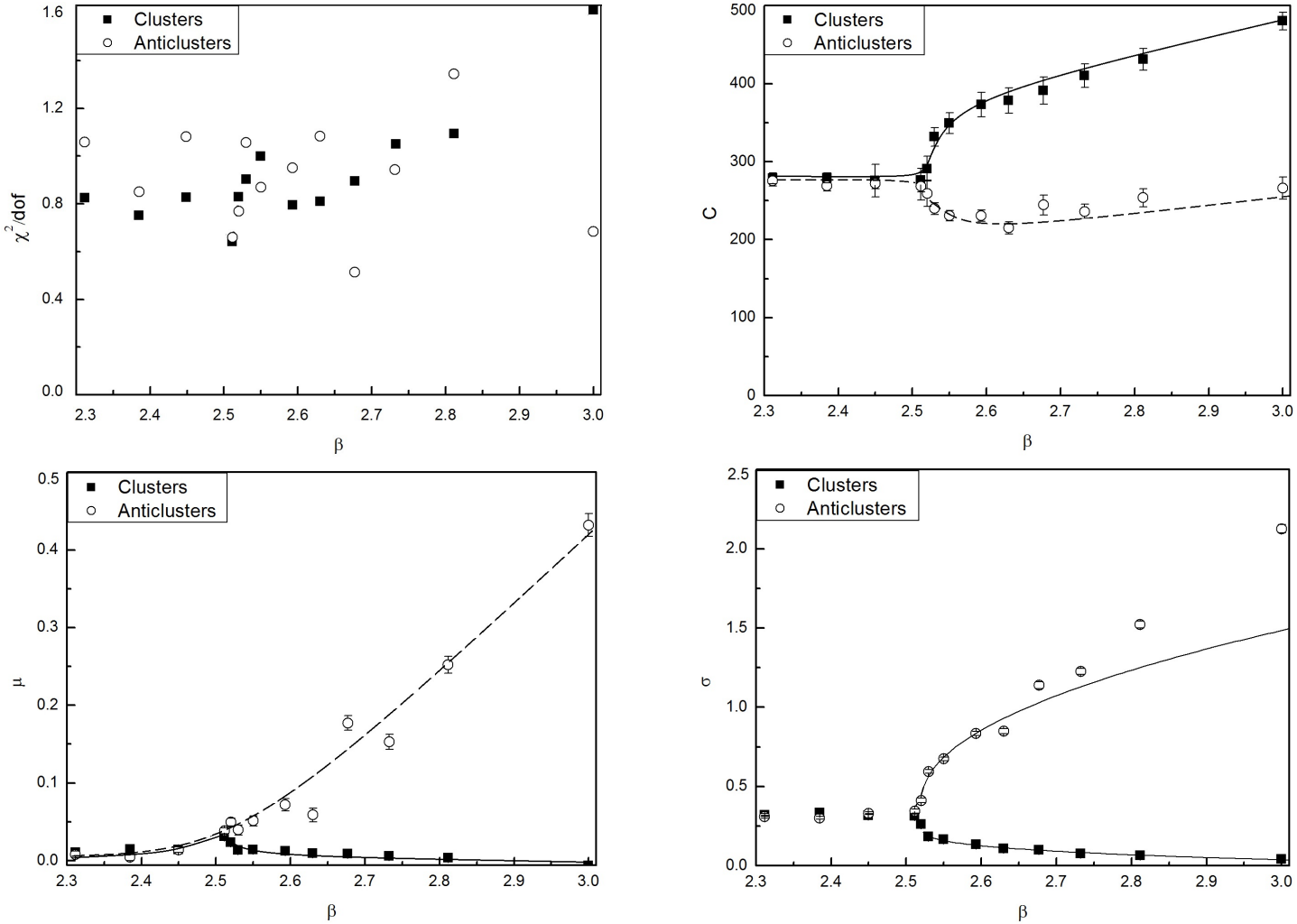




$$k_{min} = 2, \quad \tau = 1.806 \pm 0.008$$

value of τ agrees with NPA 924, 24 (2014)

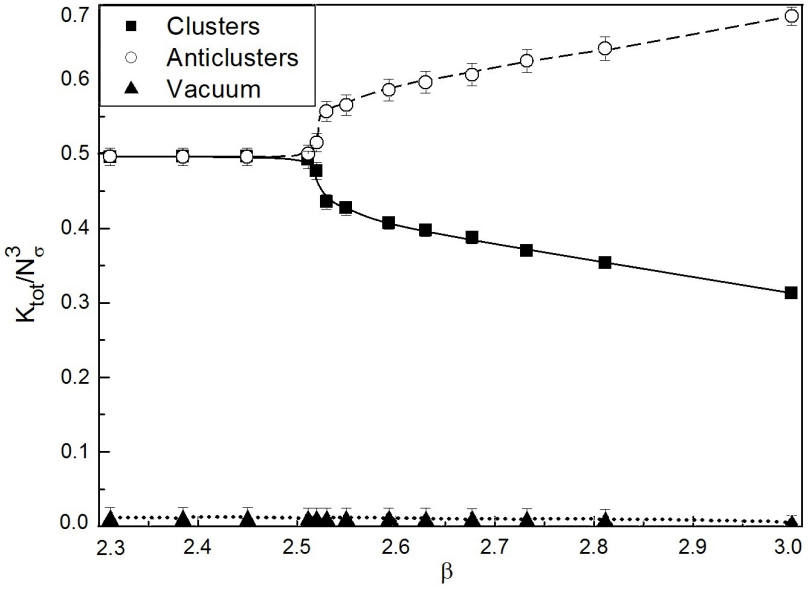
Reduced chemical potential and surface tension

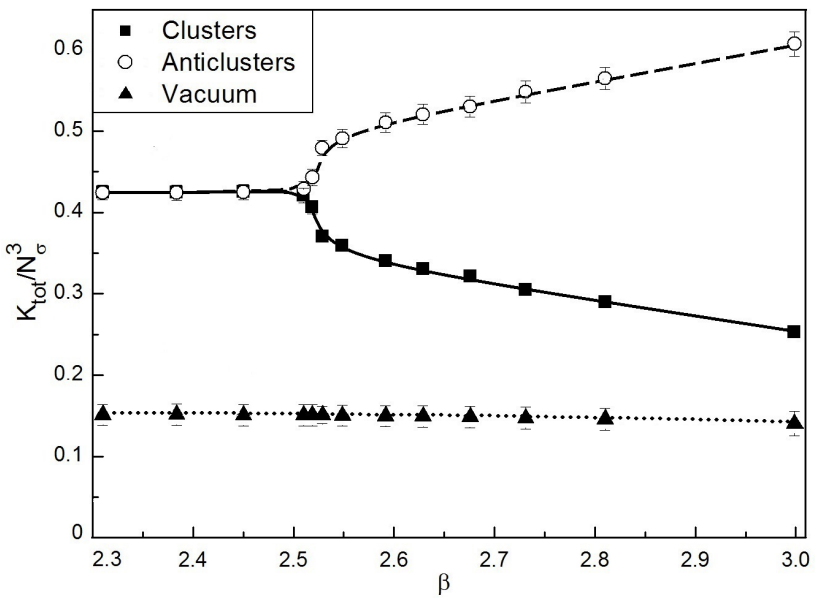


At $\beta = 2.52$ global Z(2) symmetry breaks down $\Rightarrow$
chemical nonequilibrium between (anti)clusters ($\nu_{\mathrm{cl}} \neq \nu_{\mathrm{acl}}$)

Volume fraction

$$K_{tot} = \begin{cases} \sum_k k n_k^{(a)Cl} \Big/ N_\sigma^3, & \text{(anti)clusters} \\ 1 - K_{tot}^{aCl} - K_{tot}^{Cl}, & \text{auxiliary vacuum} \end{cases}$$



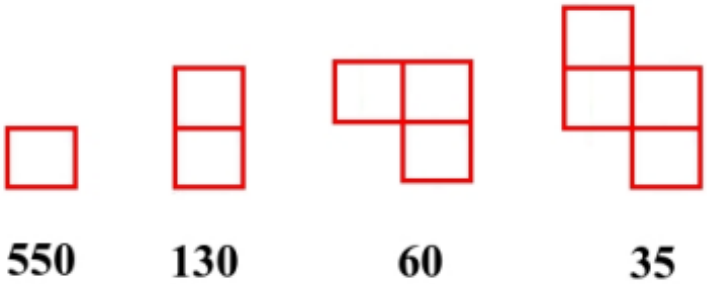


Volume fraction of vacuum is independent on β and/ore temperature

Incompressible auxiliary vacuum?

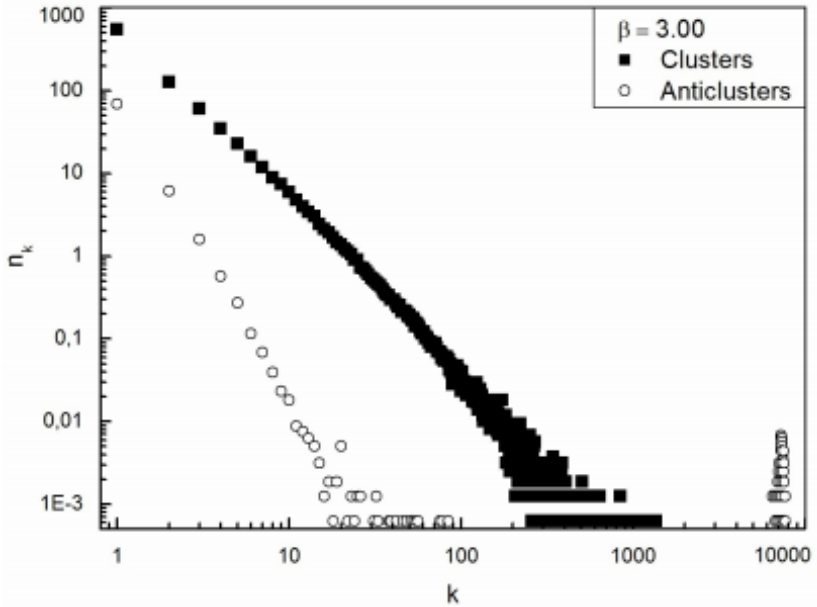
Space inhomogeneity

$^{max}K_{acl}$ = 7300 - the volume of largest anticluster;
$^{max}K_{cl}$ = 223 - the volume of largest cluster;
V^{gas}_{acl} = 89 - the total volume of the gas of anticlusters;
V^{gas}_{cl} = 2848 - the total volume of the gas of clusters;
V_{vac} = 1707 - the volume of auxiliary vacuum;



$$N^{near}_{cl} \simeq 6\, n_{cl}(1) + 10\,(n_{cl}(2) + n_{cl}(3) + n_{cl}(4)) \simeq 5600.$$

$\beta = 3.0,\ L_{cut} = 0.2$



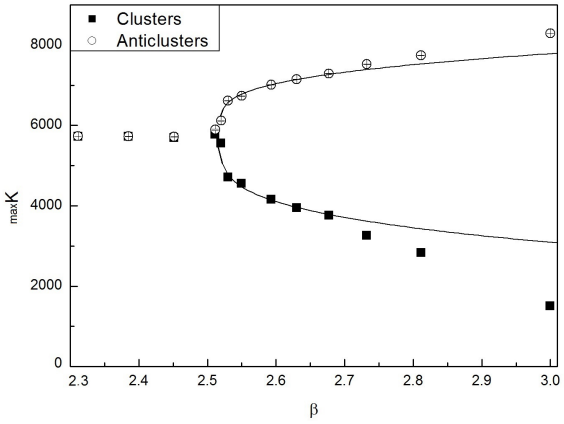
Visually these liquids resemble two pieces of different Swiss cheeses!

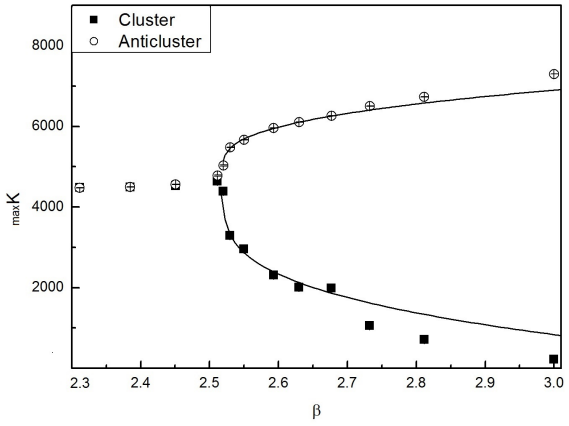
Average maximal (anti)cluster

- **Average Polyakov loop** is SU(2) gluodynamics order parameter, not observable
- **Largest (anti)cluster** occupies almost all lattice $\Rightarrow |\mathrm{L}| \sim \max \mathrm{K}_{\mathrm{aCl}} - \max \mathrm{K}_{\mathrm{Cl}}$

$$\max \mathrm{K} = \sum_{\tilde{x}} k^{1+\tau} n_k \Big/ \sum_{\tilde{x}} k^{\tau} n_k$$

$$\beta > \beta_c : \ \max \mathrm{K}(\beta) - \max \mathrm{K}(\beta_c) = a \cdot (\beta_c - \beta)^b$$



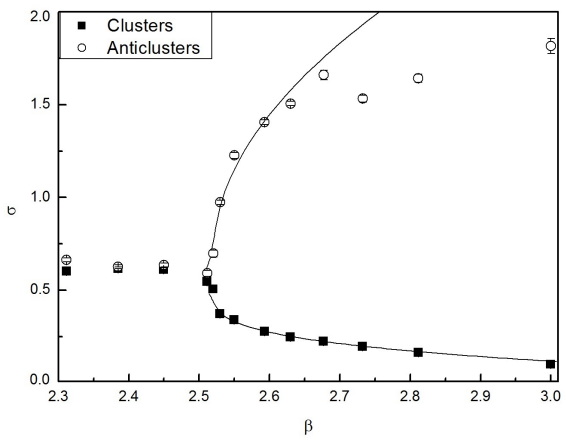


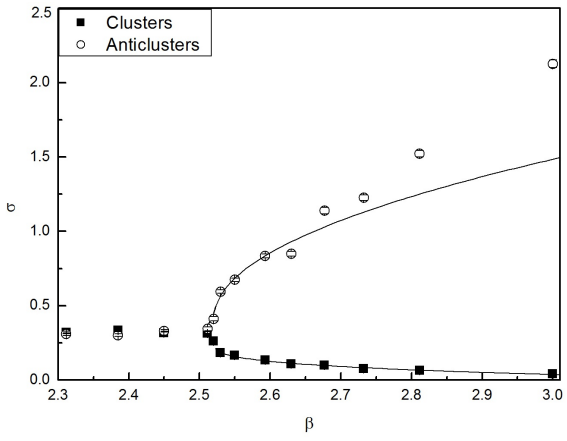
L_{cut}	type	a	b	χ^2/dof
0.1	Cl	-3056 ± 246	0.2964 ± 0.0284	$16.32/4 \simeq 4.08$
0.1	aCl	2129 ± 160	0.3315 ± 0.0269	$8.94/4 \simeq 2.235$
0.2	Cl	-4953 ± 443	0.3359 ± 0.0289	$12.3/3 \simeq 4.01$
0.2	aCl	2462 ± 87.7	0.3750 ± 0.0129	$2.068/4 \simeq 0.517$

Exponent b coinside with b_{Ising} of the Ising model - Svetitsky-Jaffe conjecture

Reduced surface tension coefficient

$$\beta > \beta_c : \ \sigma(\beta) - \sigma(\beta_c) = \mathrm{d} \cdot (\beta_c - \beta)^{\mathrm{B}}$$





L_{cut}	type	d	B	χ^2/dof
0.1	Cl	-0.485 ± 0.014	0.2920 ± 0.0012	$1.43/4 \simeq 0.36$
0.1	aCl	2.059 ± 0.028	0.4129 ± 0.0077	$1.68/4 \simeq 0.48$
0.2	Cl	-0.2796 ± 0.0118	0.2891 ± 0.0016	$1.11/4 \simeq 0.28$
0.2	aCl	1.344 ± 0.033	0.4483 ± 0.0021	$0.66/2 \simeq 0.33$

$$|\mathrm{L}| \sim \max \mathrm{K}_{\mathrm{aCl}} - \max \mathrm{K}_{\mathrm{Cl}} \sim (\sigma_{\mathrm{aCl}} - \sigma_{\mathrm{Cl}})^{\mathrm{b}/\mathrm{B}}$$

Reduced surface tension coefficient - order parameter

Conclusion

- The approach to study the properties of the Polyakov loop geometrical (anti)clusters is developed
- It is shown that the deconfinement phase transition can be explained by the condensation/evaporation of large anticluster/cluster "liquid droplet which corresponds to Z(2) global symmetry breaking
- The size distributions of the gas of (anti)clusters are analyzed on the basis of the Liquid Droplet Model It is shown that even dimers are described within this approach with high accuracy
- The Fisher topological constant τ is found to be 1.806 ± 0.008
- It is shown that the reduced surface tension of (anti)clusters can serve as an order parameter which is able to distinguish the phases of restored and broken Z(2) global symmetry
- Need model: Svetitsky-Jaffe inspired statistical models, instanton-dyon model for gluodynamics ...


THANK YOU
FOR YOUR ATTENTION