Spin Gauge Interactions as Topological Mechanism of Superconductivity

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- I D Choudhury, M. Cristina Diamantini, Guiseppe Guarnaccia, Amitabha Lahiri, Carlo A. Truggenberger.,JHEP 06(2015)081, arXiv:1503.06314
- Chandrasekhar Chatterjee, I D Choudhury, Amitabha Lahiri (Under preperation)

Topological BF Lagrangian

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + MB_{\mu\nu}\epsilon^{\mu\nu\rho\lambda}\partial_\rho A_\sigma + \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} \quad (1)$$

- Photon Propagator

$$iD_{\mu\nu} = -i\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - M^2 + i\epsilon} + i\xi\frac{k_\mu k_\nu}{k^4} \quad (2)$$

[Lahiri et.al.(1991)]

- Vector Gauge invariance

$$B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu \quad (3)$$

- Radiative induction of topological BF term

Spin Current

$$\mathscr{L}^B_{\text{int}} = g B_{\mu\nu} J^{\mu\nu} \tag{4}$$

$$J^{\mu\nu} = \bar{\psi}\sigma^{\mu\nu}\psi \tag{5}$$

or

$$J^{\mu\nu} = \bar{\psi}\gamma^5\sigma^{\mu\nu}\psi \tag{6}$$

These currents are not conserved.

Chosen form

$$J^{\mu\nu} = \frac{-m}{2}\frac{\partial_\alpha}{\Box}\left(\bar{\psi}\gamma^5\{\gamma^\alpha, \sigma^{\mu\nu}\}\psi\right) = \frac{-im}{6}\frac{\partial_\alpha}{\Box}\left(\bar{\psi}\gamma^5\gamma^{[\alpha}\gamma^{\mu}\gamma^{\nu]}\psi\right) \tag{7}$$

Spin Current

$$J^{\mu\nu} = m\ \epsilon^{\alpha\mu\nu\sigma}\frac{\partial_\alpha}{\Box}J_\sigma\ , \tag{8}$$

$$\left[\gamma^{[\alpha}\gamma^{\mu}\gamma^{\nu]} = 6i\ \epsilon^{\alpha\mu\nu\sigma}\gamma^5\gamma_\sigma\right] \tag{9}$$

where

$$J^\mu = \bar{\psi}\gamma^\mu\psi \tag{10}$$

$J^{\mu\nu}$ Contains $\epsilon^{\alpha\mu\nu\sigma}$ $\Rightarrow$ Topologically conserved.
$B_{\mu\nu}J^{\mu\nu}$:

- P and T invariant,
- invariant under vector gauge transformation,

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu\lambda_\nu - \partial_\nu\lambda_\mu \tag{11}$$

physical nature of coupling

We have,

$$J^{\mu\nu} = m\ \epsilon^{\alpha\mu\nu\sigma}\frac{\partial_\alpha}{\Box}J_\sigma\ , \tag{12}$$

For on- shell fermions,

$$J^{\mu\nu} = \frac{1}{m}\epsilon^{\alpha\mu\nu\sigma}\partial_\alpha J_\sigma\ , \tag{13}$$

[Diamantini et al, (2014)]

Therefore,

$$J^{0i} = -\frac{1}{m}\epsilon^{ijk}\partial_j\psi^\dagger\alpha^k\psi \tag{14}$$

- $\psi^\dagger\alpha^k\psi \Rightarrow$ Velocity field of Dirac fermion.
- $J^{0i} \Rightarrow$ Curl of velocity field, i, e. vorticity field.

Spin dependence of current

$$J^{0i} = -m\ \epsilon^{ijk}(\partial_j/\Box)\psi^\dagger\alpha^k\psi \tag{15}$$

Using Gordon decomposition,

$$\psi^\dagger\alpha^k\psi = \frac{i}{2m}\left[\bar{\psi}\partial^k\psi - \partial^k\bar{\psi}\psi\right] + \frac{1}{2m}\partial_\mu\ \bar{\psi}\sigma^{k\mu}\psi \tag{16}$$

In non- relativistic limit,

$$\psi = \begin{bmatrix}\phi \\ \chi\end{bmatrix}, \tag{17}$$

$$\left(\psi^\dagger\alpha^k\psi\right)_{\mathrm{NR}} \to \tfrac{i}{2m}\left[\bar{\phi}\partial^k\phi - \partial^k\bar{\phi}\phi\right] + \tfrac{1}{2m}\epsilon^{ijk}\partial_i\ \phi^\dagger\sigma^j\phi\,, \tag{18}$$

$$\tag{19}$$

In static case,

$$\left(J^{0i}_{\mathrm{spin}}\right)_{\mathrm{NR}} = \frac{1}{2}\left(\phi^\dagger\sigma^i\phi\right)\ , \tag{20}$$

This is the spin magnetic moment density of the particle.

Lagrangian :

$$\mathscr{L} = \bar{\psi}\gamma^{\mu}\left(i\partial_{\mu} + eA_{\mu}\right)\psi - m\bar{\psi}\psi + gB_{\mu\nu}J^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} ,$$

We write,

$$\mathscr{L}^{B}_{\text{int}} = gB_{\mu\nu}J^{\mu\nu} = \frac{2mg}{\Box}F_{\mu}J^{\mu} , \tag{21}$$

where $F_{\mu} = (1/2)\epsilon_{\mu\nu\alpha\beta}\partial^{\nu}B^{\alpha\beta} \Rightarrow$ dual Kalb-Ramond field strength.

Interaction term for fermions

$$\begin{aligned} \mathscr{L}_{\text{int}} &= e(A_{\text{eff}})_{\mu}J^{\mu} , \\ (A_{\text{eff}})_{\mu} &= A_{\mu} + \frac{2mg}{e\Box}F_{\mu} . \end{aligned} \tag{22}$$

One loop induced action

Integrating out fermions,

$$\Gamma^{(1-\text{loop})} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}(A_\mu)_{\text{eff}}(-k)(A_\nu)_{\text{eff}}(k)\Pi^{\mu\nu}(k) \tag{23}$$

where $\Pi^{\mu\nu}$ is the usual QED vacuum polarization tensor

$$\begin{aligned} \Pi^{\mu\nu}(k) &= (g^{\mu\nu}k^2 - k^\mu k^\nu)\Pi(k^2) \\ \Pi(k^2) &= \tfrac{e^2}{12\pi^2}\ln\tfrac{\Lambda^2}{m^2}\ , \end{aligned} \tag{24}$$

and Λ is the momentum space ultraviolet cutoff.

One loop induced action

$$\Gamma^{1-\text{loop}} = \frac{e^2}{4}\frac{1}{12\pi^2}\ln\frac{\Lambda^2}{m^2}\int d^4x (F_{\text{eff}})_{\mu\nu}(F_{\text{eff}})^{\mu\nu} \ . \tag{25}$$

where

$$(F_{\text{eff}})_{\mu\nu} = F_{\mu\nu} + \frac{2mg}{e}\left(\frac{\partial_\mu}{\Box}F_\nu - \frac{\partial_\nu}{\Box}F_\mu\right) \ . \tag{26}$$

Expanding the action,

$$\Gamma^{1-\text{loop}} = \Gamma^{1-\text{loop}}_{AA} + \Gamma^{1-\text{loop}}_{AB} + \Gamma^{1-\text{loop}}_{BB} \ , \tag{27}$$

with

$$\Gamma^{1-\text{loop}}_{AA} = \frac{e^2}{4}\frac{1}{12\pi^2}\ln\frac{\Lambda^2}{m^2}\int d^4x \ F_{\mu\nu}F^{\mu\nu} \ , \tag{28}$$

$$\Gamma^{1-\text{loop}}_{AB} = \frac{8mg}{4}\frac{1}{12\pi^2}\ln\frac{\Lambda^2}{m^2}\int d^4x \ A_\mu F^\mu \ , \tag{29}$$

$$\Gamma^{1-\text{loop}}_{BB} = \frac{8m^2g^2}{4}\frac{1}{12\pi^2}\ln\frac{\Lambda^2}{m^2}\int d^4x \ F_\mu\frac{1}{\Box}F^\mu \ . \tag{30}$$

One loop effective Lagrangian

$$\begin{aligned}\mathscr{L} \quad &= -\frac{1}{4e_{\rm ph}^2}F_{\mu\nu}F^{\mu\nu} + \frac{g_{\rm ph}m}{2\pi}B_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\partial_\rho A_\sigma + \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} \\ &+\frac{6g_{\rm ph}^2 m^2}{\ln\frac{\Lambda^2}{m^2}}F_\mu\frac{1}{\Box}F^\mu \ , \end{aligned} \tag{31}$$

where

$$e_{\rm ph}^2 = e^2\left(1+\frac{e^2}{12\pi^2}\ln\frac{\Lambda^2}{m^2}\right) \ , \tag{32}$$

and

$$g_{\rm ph} = \frac{g}{6\pi}\ln\frac{\Lambda^2}{m^2} \ . \tag{33}$$

We may write,

$$
\begin{aligned}
\mathscr{L} \quad &= -\frac{1}{4e_{\rm ph}^2}F_{\mu\nu}F^{\mu\nu} + \frac{g_{\rm ph}m}{2\pi}B_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\partial_\rho A_\sigma + \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} \\
&-\frac{\ln\frac{\Lambda^2}{m^2}}{48\pi^2}G_{\mu\nu}G^{\mu\nu} + \frac{g_{\rm ph}m}{2\pi}B_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\partial_\rho C_\sigma \ , \qquad (34)
\end{aligned}
$$

using a new auxilliary gauge field C_μ.

Diagonalization of the Lagrangian:

$$
\begin{aligned}
A_\mu \quad &\to A_\mu + C_\mu \ , \\
C_\mu \quad &\to A_\mu - \frac{4e_{\rm ph}^2}{12\pi^2}\ln\frac{\Lambda^2}{m^2}\ C_\mu \ , \qquad (35)
\end{aligned}
$$

Induced BF Lagrangian and photon mass generation

$$\begin{aligned}
\mathscr{L} &= \mathscr{L}_1 + \mathscr{L}_2 \ , \\
\mathscr{L}_1 &= -\frac{1}{4e_1^2} F_{\mu\nu} F^{\mu\nu} + \frac{g_{\rm ph} m}{2\pi} B_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} \partial_\rho A_\sigma + \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \ , \\
\mathscr{L}_2 &= -\frac{1}{4e_2^2} G_{\mu\nu} G^{\mu\nu} \ ,
\end{aligned} \tag{36}$$

where

$$\begin{aligned}
e_1^2 &= e_{\rm ph}^2 \frac{1 + \frac{4e_{\rm ph}^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}}{\frac{4e_{\rm ph}^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}} \ , \\
e_2^2 &= e_{\rm ph}^2 \left(1 + \frac{4e_{\rm ph}^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2} \right) \ .
\end{aligned} \tag{37}$$

Equations of motion

From the topological sector we get

$$\partial_\mu F^{\mu\nu} = -\frac{e_1^2 g_{\rm ph} m}{6\pi}\epsilon^{\nu\mu\alpha\beta} H_{\mu\alpha\beta} = J^\nu \ ,$$
$$\partial_\mu H^{\mu\alpha\beta} = \frac{g_{\rm ph} m}{2\pi}\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu} \ , \tag{38}$$

Klein-Gordon Equation for photon

$$\left[\Box + \left(\frac{e_1 g_{\rm ph} m}{2\pi}\right)^2\right] F_{\mu\nu} = 0 \ . \tag{39}$$

London equations

$$\epsilon^{\mu\nu\alpha\beta}\partial_\alpha J_\beta = -\left(\frac{e_1 g_{\rm ph} m}{2\pi}\right)^2 \tilde{F}_{\mu\nu} \ ,$$
$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \tag{40}$$

Lagrangian

$$\mathscr{L} = \frac{1}{12}H_{\mu\nu\alpha}H^{\mu\nu\alpha} + gB_{\mu\nu}J^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + eA_\mu J^\mu + \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi \quad (41)$$

where

$$J^{\mu\nu} = m\epsilon^{\alpha\mu\nu\sigma}\frac{\partial_\alpha}{\Box}J_\sigma \quad (42)$$

$$J_\mu = \bar{\psi}\gamma_\mu\psi\,. \quad (43)$$

Action after integrating out the gauge fields

$$S[\psi,\bar{\psi}] = \int \frac{d^4k}{(2\pi)^4}J^\sigma(-k)\left\{\frac{e^2}{k^2} + \frac{g^2m^2}{k^4}\right\}J_\sigma(k) \quad (44)$$

Static Potential

In non-relativistic limit, the contribution from the leading term is

$$\int \cdots \sum_{s,s',r,r'} \delta_{rr'}\delta_{ss'} a^{\dagger r'}(\mathbf{q}')a^{r}(\mathbf{q}) \left\{ \frac{e^2}{(p-p')^2} + \frac{g^2 m^2}{(p-p')^4} \right\} a^{\dagger s'}(\mathbf{p}')a^{s}(\mathbf{p})\,. \tag{45}$$

We may write,

$$\begin{aligned} p = (m,\mathbf{p})\,, p' = (m,\mathbf{p}')\,, \\ k = p - p'\,, \end{aligned} \tag{46}$$

so that

$$(p-p')^2 \approx -|\mathbf{p}-\mathbf{p}'|^2 \tag{47}$$

There should be an additional sign in the potential.

Linear Potential

In three dimensional momentum space the 'static potential' is,

$$V(\mathbf{k}) = \frac{e^2}{\mathbf{k}^2} - \frac{g^2 m^2}{\mathbf{k}^4}\,, \tag{48}$$

Fourier transform gives,

$$V(r) = \frac{e^2}{4\pi r} + \frac{g^2 m^2 r}{4\pi} \tag{49}$$

Effective force

$$-\overrightarrow{\nabla} V(r) = \frac{e^2 \mathbf{r}}{4\pi r^3} - \frac{g^2 m^2 \mathbf{r}}{4\pi r} \tag{50}$$

There is a length scale R where the effective force vanishes and

$$\frac{1}{R^2} = \frac{g^2 m^2}{e^2}\,. \tag{51}$$

Summary

- We propose a low energy effective theory of superconductivity valid for the energy scales well below an ultraviolet cut-off.
- The theory consists of a non-local interaction between fermions and the anti-symmetric tensor field $B_{\mu\nu}$.
- Using this non- local fermion current we radiatively induce topological BF term at one loop.
- This current shows a vortex formation for on- shell fermions.
- We have to consider the non local form of the current in order to get the desired result.
- Photon obeys a massive Klein- Gordon equation obtained from the equation of motions coming from the effective Lagrangian.

Summary

- The Static potential of the theory has two terms.
- One terms is the Coulomb potential repulsive for $e^- - e^-$ interaction.
- The other term is linear and always attractive in nature.
- There is a length scale at which the effective force vanishes.
- The system can be realised as two dipoles connected by a string may be formed by spin alignment.

Thank you