CGC: new b-dependent model in NLO

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ICNFP'16 , Crete, July 5-15,2016

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Why we are doomed to build a model

- Short answer:

- Because we failed to include in CGC equations the non-perturbative large b behavior : $$\boldsymbol{A \propto e^{-\mu b}}$$

- Perturbative QCD $\Longrightarrow$ $A \propto 1/b^4$ $\Longrightarrow$ $\sigma \propto s^{\lambda} \gg \ln^2 s$

  (Kovner & Wiedemann,2002-2003)

Main building ideas:

1. Non-perturbative large b behavior can be absorbed in $Q_s\left(Y, b\right)$

$$Q_s^2 = Q_0^2\left(Y = Y_0, b\right)\left(\frac{1}{x}\right)^{\lambda_{\text{pert. QCD}}}$$

with

$$Q_0^2\left(Y = Y_0, b\right) = Q_0^2\, S\left(b\right) \xrightarrow{b \gg 1/\mu} Q_0^2\, e^{-\mu b}$$

Theory: $\lambda_{\text{pert. QCD}}$ $\quad e^{-\mu b}$

Fitting parameters: Q_0 and μ

Popular but correct only in semi-classical approach

2. Geometric scaling and matching of two theoretical amplitudes:

- deep inside the saturation domain $(\tau = r^2 Q_S^2(Y,b) \gg 1)$
- in vicinity of the saturation scale $(\tau = r^2 Q_S^2(Y,b) \approx 1)$

(Iancu, Itakura & Munier, 2004)

$$\tau \gg 1 \qquad N = 1 - A\, e^{-\kappa_{\text{pert. QCD}} \ln^2 \tau}$$

(E.L.& Tuchin, 2000)

$$\tau \approx 1 \qquad N = N_0\, (\tau)^{1-\gamma^{cr}_{\text{pert. QCD}}}$$

(Iancu, Itakura & McLerran, Mueller & Triantafyllopoulos)

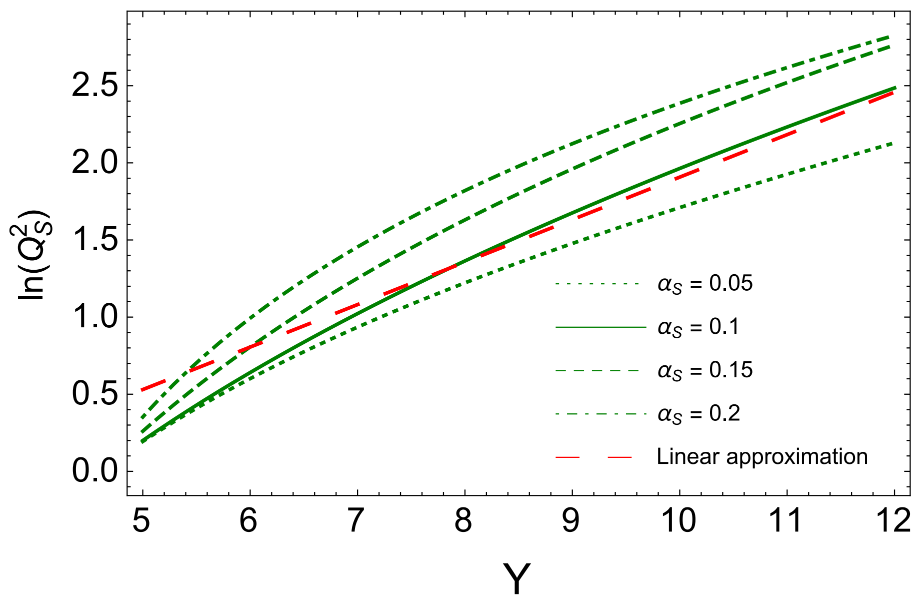
Theory: $\kappa_{\text{pert. QCD}}$ **and** $\gamma^{cr}_{\text{pert. QCD}}$

Fitting parameters: A **and** N_0

3. Saturation momentum $Q_s\,(Y, b)$:

- $$\ln\left(Q_s^2\,(Y, b)\Big/Q_s^2\,(Y = Y_0, b)\right) =$$

$$d_1^{\rm pert.QCD}\,(\bar\alpha_S)\,(Y - Y_0) - d_2^{\rm pert.QCD}\,(\bar\alpha_S)\ln\left(Y/Y_0\right)$$
$$+ d_3^{\rm pert.QCD}\,(\bar\alpha_S)\left(\frac{1}{\sqrt{Y_0}} - \frac{1}{\sqrt{Y}}\right)$$



Linear approximation $\longrightarrow$ **0.7** $d_1^{\rm pert.QCD}\,(\bar\alpha_S)\,(Y - Y_0)$ **at** $\bar\alpha_S = 0.1$.

4. Phenomenological input: impact parameter dependence

- **Theoretical restrictions:**
- **At large b** $\quad A \propto e^{-\mu b}$ **(Froissart ,1961);**
- **At large Q_T (conjugated variable to b)** $\quad A \propto 1/Q_T^4$ **(Brodsky & Lepage, ,1979);**

♠ **Our choice:**

- $$A\left(Q_T\right) = \frac{1}{\left(1 + Q_T^2/m^2\right)^2} \quad \Longrightarrow \quad A(b) = mb\,K_1\left(mb\right)$$

- $$S\left(b\right) = \Bigg(mb\,K_1\left(mb\right)\Bigg)^{1/(1-\gamma^{cr})}$$

Theoretical results for $\tau \gg 1$

Non-linear equation: **(Balisky & Chirilli, 2008)**

$$\frac{\partial S_{12}}{\partial Y} = \frac{\bar{\alpha}_S}{2\pi}\int d^2 x_3 \frac{x_{12}^2}{x_{13}^2 x_{23}^2}\left\{1+\bar{\alpha}_S b\left(\ln x_{12}^2\mu^2 - \frac{x_{13}^2 - x_{23}^2}{x_{12}^2}\ln\frac{x_{13}^2}{x_{23}^2}\right)\right.$$

$$+ \bar{\alpha}_S\left(\frac{67}{36} - \frac{\pi^2}{12} - \frac{5}{18}\frac{N_f}{N_c} - \frac{1}{2}\ln\frac{x_{13}^2}{x_{12}^2}\ln\frac{x_{23}^2}{x_{12}^2}\right)\left.\right\}(S_{13}S_{32} - S_{12})$$

$$+ \frac{\bar{\alpha}_S^2}{8\pi^2}\int\frac{d^2x_3\, d^2x_4}{x_{34}^4}\left\{-2 + \frac{x_{13}^2x_{24}^2 + x_{14}^2x_{23}^2 - 4x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2 - x_{14}^2x_{23}^2}\right.$$

$$\ln\frac{x_{13}^2x_{24}^2}{x_{14}^2x_{23}^2} + \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}\left(1 + \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2 - x_{14}^2x_{23}^2}\right)\ln\frac{x_{13}^2x_{24}^2}{x_{14}^2x_{23}^2}\left.\right\}$$

$$\times \left(S_{13}S_{34}S_{42} - S_{13}S_{32}\right)$$

For $S_{ij} \ll 1$ the equation degenerates to **(E.L. & Tuchin , 2000)**

- $$\frac{\partial S_{12}}{\partial Y} = -\frac{\bar{\alpha}_S}{2\pi}\int d^2 x_3 \frac{x_{12}^2}{x_{13}^2 x_{23}^2}\left\{\dots\right\}S_{12}$$

$$\bullet\quad \frac{\mathrm{d}\ln S_{12}}{\mathrm{d}Y} = -\bar{\alpha}_S\left[1+\bar{\alpha}_S\, b\ln(\mu^2 x_{12}^2)+\bar{\alpha}_S\left(\frac{67}{36}-\frac{\pi^2}{12}-5\frac{N_f}{N_c}\right)\right]\ln(Q_s^2 x_{12}^2)$$

$$+\quad \frac{\bar{\alpha}_S^2 b}{2}\ln^2 Q_s^2 x_{12}^2+\frac{\bar{\alpha}_S\zeta(3)}{16}$$

$$\mu = Q_s \quad = \quad -\bar{\alpha}_S\left(Q_s\right)\left[1+\frac{3}{2}\bar{\alpha}_S\left(Q_s\right)\, b\ln(Q_s^2 x_{12}^2)\right.$$

$$\left.+\quad \bar{\alpha}_S(Q_s)\left(\frac{67}{36}-\frac{\pi^2}{12}-5\frac{N_f}{N_c}\right)\right]\ln(Q_s^2 x_{12}^2)+\frac{\bar{\alpha}_S(Q_s)\zeta(3)}{16}$$

$$\bullet\quad \ln S_{12} = -\ \mathcal{Z} =$$

$$-\frac{1}{2\,\varrho}\left(\left[z+\bar{\alpha}_S\left(Q_s\right) b z^2+\bar{\alpha}_S(Q_s)\, z\left(\frac{67}{36}-\frac{\pi^2}{12}-5\frac{N_f}{N_c}\right)\right] z+\frac{\zeta(3)}{8}\, z\right)$$

$$z = \bar{\alpha}_S\left(Q_s\right)\varrho\left(Y-Y_0\right)+\ln\left(Q_0^2\left(Y=Y_0\right)x_{12}^2\right)$$

- **Results:**
- Geometric scaling behavior with $\bar{\alpha}_S\,(Q_s)$;
- Particular form $\tau = x^2_{12}\,Q^2_s$ dependence for $\tau \gg 1$;

$$N\,(z)\;=\;1\;-\;A\exp\left(-\mathcal{Z}\right)$$

(Contreras, E.L. & Meneses, 2014)

- $N^{z\,\gg\,1}\,(z) \;=\; 1 - 2\,Ae^{-\mathcal{Z}} \;-\; A^2\,\frac{1}{\mathcal{Z}}\,e^{-2\mathcal{Z}} \;+\; \mathcal{O}\left(e^{-3\mathcal{Z}}\right)$
- $\mathcal{Z} \;=\; \mathcal{Z}\left(z \to z - \frac{1}{2}A\sqrt{\varrho\,\pi/2} - 2\psi(1)\right)$

Theoretical results for $\tau \approx 1$ in NLO

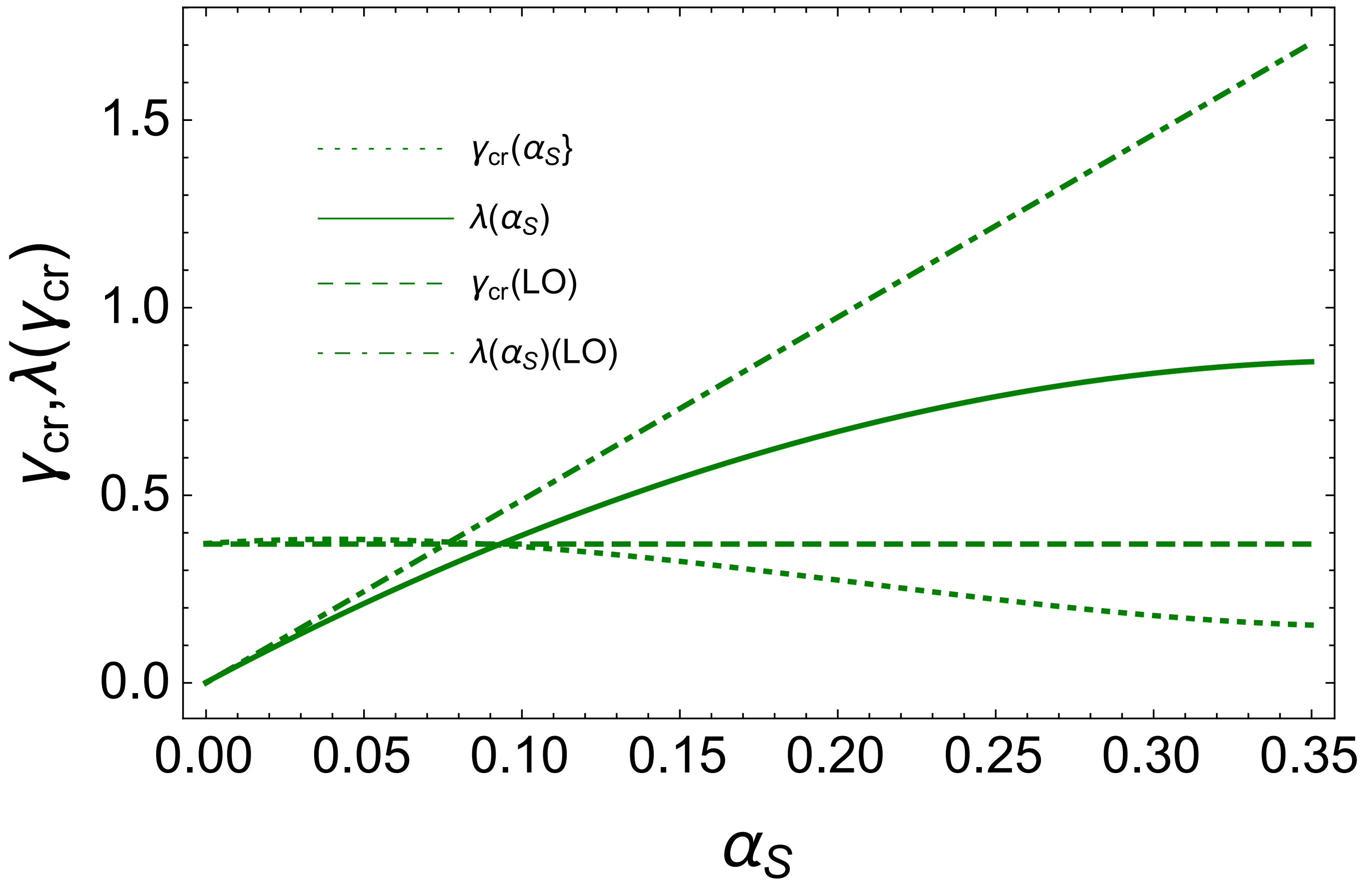
We follow the well known procedure:

- **(Lipatov & Fadin, 1998)** $\omega = \bar{\alpha}_S \left(\chi_{\mathrm{LO}}(\gamma) + \bar{\alpha}_S \chi_{\mathrm{NLO}}(\gamma) \right)$

$$\xrightarrow{\text{Salam, 1998}} \omega = \bar{\alpha}_S \left(\chi_0(\omega, \gamma) + \omega \frac{\chi_1(\omega, \gamma)}{\chi_0(\omega, \gamma)} \right)$$

- $$\frac{\omega^{NLO}(\gamma_{cr})}{1 - \gamma_{cr}} = \left| \frac{d\omega^{NLO}(\gamma_{cr})}{d\gamma_{cr}} \right|$$

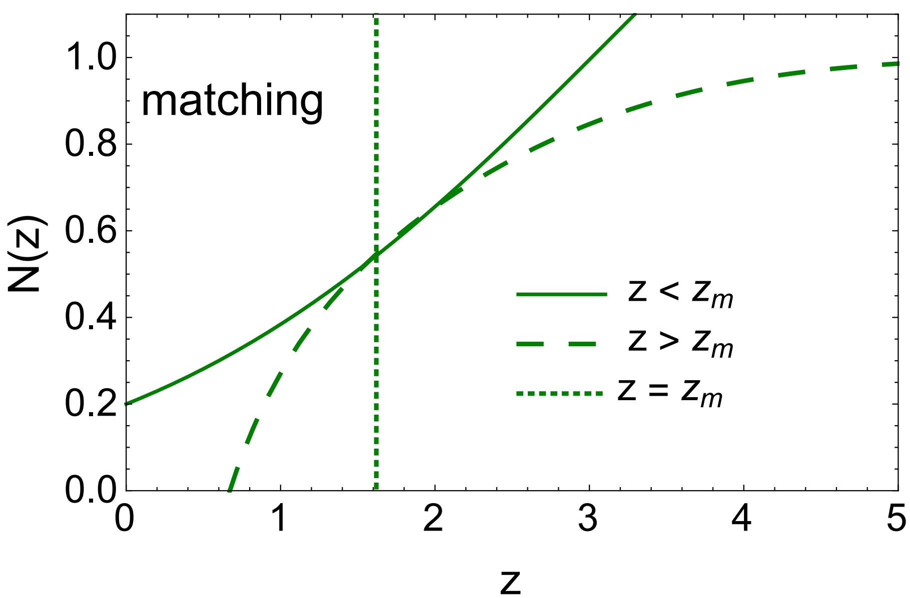
(Triantafyllopoulos, 2003; Khoze, Martin, Ryskin & Stirling,2004)



- $$\ln\left(Q_s^2\left(Y\right)/Q_s^2\left(Y_0\right)\right) = \lambda\left(\gamma_{cr}\right)\left(Y - Y_0\right)$$

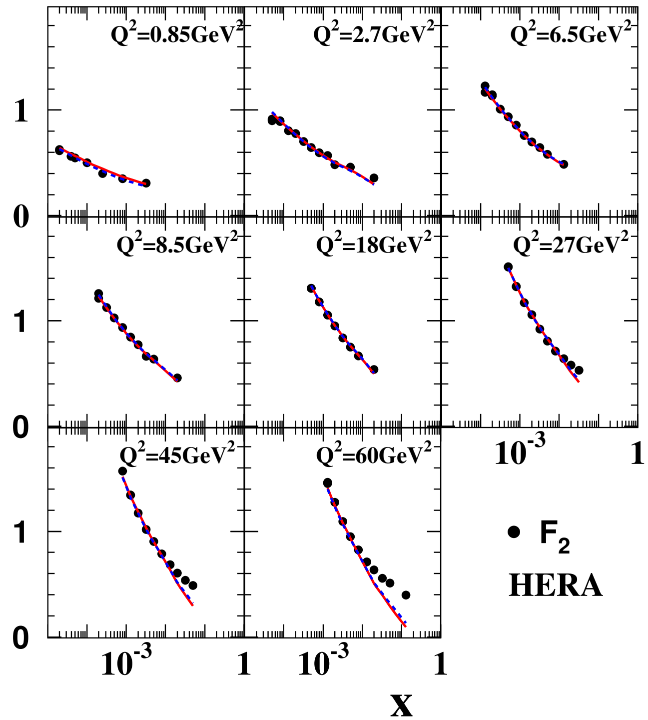
Matching

- $$N^{0<z\ll 1}\left(z = z_m\right) = N^{z\gg 1}\left(z = z_m\right);$$
- $$\frac{dN^{0<z\ll 1}\left(z = z_m\right)}{dz_m} = \frac{dN^{z\gg 1}\left(z = z_m\right)}{dz_m};$$



Descriptions of HERA data

$\bar{\alpha}_S$	N_0	Y_0	m (GeV)	Q_0^2 (GeV^2)	m_u(MeV)	m_d(MeV)	m_s(MeV)	m_c(GeV)
0.133	0.1075	3.77	0.83	3.0	2.3	4.8	95	1.4



$$\chi^2/d.o.f. = 183/153 = 1.2$$

Difficulties of the model

- $\bar{\alpha}_S\,(Q_0)\;=\;0.133$ **is too small for** $\Lambda_{\rm QCD}\approx$ **200** MeV;
- $Q_s^2\,(Y\,=\,Y_0)\;=\;3\,GeV^2$ **is too large ;**
- $Q_s^2\,(Y)$ **tends to be extremely large at small** x **;**