Peculiarities of the BFKL approach

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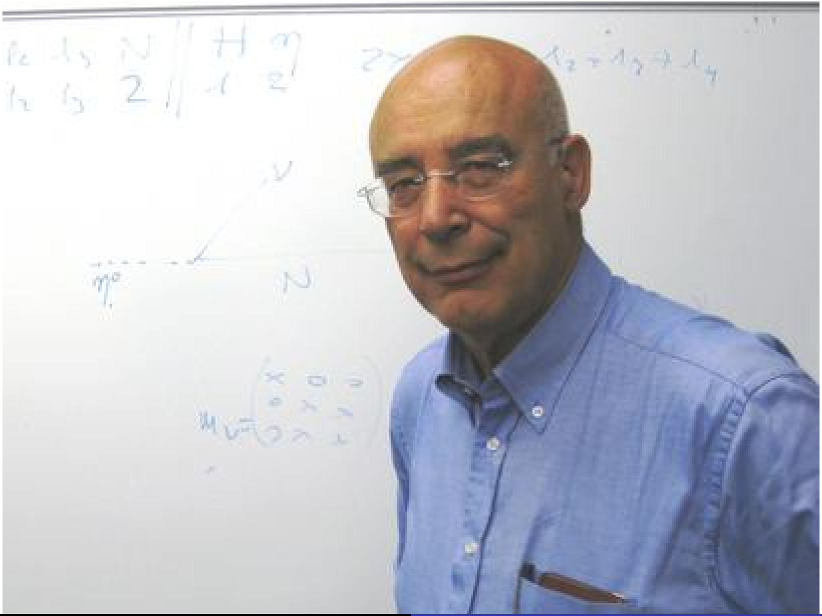
Contents



Asymptotic Freedom in Parton Language

Guido Altarelli (Ecole Normale Superieure) , G. Parisi (IHES, Bures-sur-Yvette)

Nucl.Phys. B126 (1977) 298-318

A novel derivation of the Q^2 dependence of quark and gluon densities as predicted by quantum chromodynamics is presented. The main body of predictions of the theory for deep-inelastic scattering on either unpolarized or polarized targets is re-obtained by a method which only makes use of the simplest tree diagrams and is entirely phrased in parton language with no reference to the conventional operator formalism.

5755 citations in INSPIRES

Introduction

Cross sections of lepton(l)-hadron$A$ collisions with a hard scale Q^2 are given by convolutions of parton distributions $F_A^a(z, Q^2)$ ($a is a parton flavour, x$ is a fraction of longitudinal momenta) and cross sections of lepton-parton interactions $\sigma_{la}(z, Q^2)$

$$\sigma_{lA} = F_A^a \bigotimes \sigma_{la}.$$

The sign $\bigotimes$ means convolution over z and a.
For hadron-hadron collisions

$$\sigma_{AB} = F_A^a \bigotimes \sigma_{ab} \bigotimes F_B^b.$$

$\sigma_{ab}(z_a, z_b; Q^2)$ – cross sections of parton-parton interactions.

Introduction

Evolution of the parton distributions with $\tau = \ln\left(Q^2/\Lambda^2_{QCD}\right)$ is determined by the DGLAP equations

$$\frac{d}{d\tau}F^a_A = \frac{\alpha_s(Q^2)}{2\pi}P^a_b \bigotimes F^b_A \ ,$$

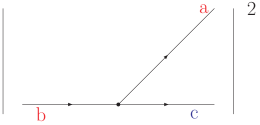
$P^b_a(z)$ are the parton splitting functions,

$$\frac{\alpha_s}{2\pi}\int_0^1 dz z^{j-1} P^b_a(z) = \gamma^{ab}_j(\alpha_s),$$

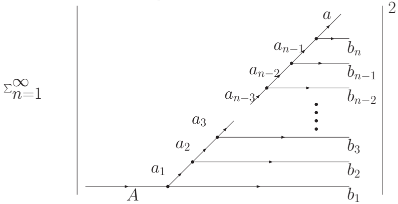
$\gamma^{ab}_j(\alpha_s)$ – anomalous dimensions matrix for twist-2 operators of Lorenz spin j in expansion of products of two currents.

Colour coherence

The splitting functions $P_b^a(z)$ are determined by diagrams



The virtual (proportional to $\delta(z-1)$) contributions to $P_b^a(z)$ can be obtained using flavour conservation and longitudinal momentum conservation. The parton a distribution $F_A^a(z, Q^2)$



with strong ordering of parton transverse momenta.

Another derivation, based on the results of direct summation in perturbation series

Gribov, V. N. and Lipatov, L. N. , Sov. J. Nucl. Phys. 15 (1972) 438 [Yad. Fiz. 15 (1972) 781]
Gribov, V. N. and Lipatov, L. N. , Sov. J. Nucl. Phys. 15 (1972) 675 [Yad. Fiz. 15 (1972) 1218]

was performed in

Lipatov, L. N. , Sov. J. Nucl. Phys. 20 (1975) 94 [Yad. Fiz. 20 (1974) 181]

for several field-theoretical models. For QCD

Dokshitzer, Y. L. , Sov. Phys. JETP 46 (1977) 641 [Zh. Eksp. Teor. Fiz. 73 (1977) 1216]

Introduction

The DGLAP sums terms enhanced by powers of $\ln Q^2$ (collinear logarithms), arising at integration over angles between parton momenta.
There are another logarithms (soft, or Regge logarithms), arising at integration over ratios of parton energies.
These logarithms are present both in parton distributions and in partonic cross sections.

At small $x = Q^2/s$ (s is c.m.s. energy squared), it is necessary to sum the terms of the perturbation series enhanced by powers of $\log(1/x)$.
The resummation of leading $(\alpha_s \ln(1/x))^n$ and next-to-leading $\alpha_s(\alpha_s \ln(1/x))^n$ $\log(1/x)$-terms can be performed using the BFKL equation.

Introduction

The BFKL equation usually is associated with the equation for unintegrated gluon density $\mathcal{F}(x,\vec{k})$ ($\vec{k}$ is transverse momentum), connected with the gluon distribution $F^g(x,Q^2)$ by the equation

$$F^g(x,Q^2) = \int^{Q^2} \mathcal{F}(x,\vec{k})d\vec{k}/(\pi\vec{k}^{\,2}).$$

The equation describes evolution of $\mathcal{F}(x,\vec{k})$ with change of $\ln(1/x)$ and has the structure

$$\frac{\partial\mathcal{F}}{\partial\ln(1/x)} = \mathcal{K}\bigotimes\mathcal{F}\ ,$$

where $\mathcal{K}$ is the BFKL kernel and $\otimes$ means convolution with respect to transverse momenta.

Introduction

The small x resummation:
"A fully small x resummed global PDF analysis is unavoidable in order to quantify the modifications in the PDFs due to small x resummation, and to propagate these modifications into relevant observables at the LHC. Such a programme will be necessary in order to achieve phenomenology at the percent level for many LHC signal, background and standard candle processes. It would be even more important for phenomenology at a future LHeC electron-proton collider, and mandatory for the treatment of extremely high-energy scattering processes, such as those induced by Ultra- High Energy cosmic neutrinos, which are currently under investigation".

Introduction

The small x resummation:

G. Altarelli, R. D. Ball, and S. Forte, Factorization and resummation of small x scaling violations with running coupling, Nucl.Phys. B621 (2002) 359387, [hep-ph/0109178]

G. Altarelli, R. D. Ball, and S. Forte, An Anomalous dimension for small x evolution, Nucl.Phys. B674 (2003) 459483, [hep-ph/0306156]

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G. Altarelli, R. D. Ball, and S. Forte, Small x Resummation with Quarks: Deep-Inelastic Scattering, Nucl.Phys. B799 (2008) 199240, [arXiv:0802.0032]

J. Rojo, G. Altarelli, R. D. Ball, and S. Forte, Towards small x resumed DIS phenomenology, in Proceedings, 17th International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS 2009), 2009. arXiv:0907.0443

By different method

G. Salam, A Resummation of large subleading corrections at small x, JHEP 9807 (1998) 019, [hep-ph/9806482]
M. Ciafaloni, D. Colferai, and G. Salam, Renormalization group improved small x equation, Phys.Rev. D60 (1999) 114036, [hep-ph/9905566]

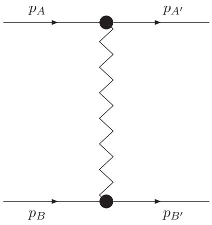
M. Ciafaloni, D. Colferai, G. Salam, and A. Stasto, Renormalization group improved small x Greens function, Phys.Rev. D68 (2003) 114003, [hep-ph/0307188]
M. Ciafaloni, D. Colferai, G. Salam, and A. Stasto, A Matrix formulation for small-x singlet evolution, JHEP 0708 (2007) 046, [arXiv:0707.1453]

Gluon Reggeization

The basis of the BFKL approach
V.S. F., E. A. Kuraev, L. N. Lipatov, Phys. Lett. B60 (1975) 50
E. A. Kuraev, L. N. Lipatov, V.S. F., ZhETF 71 (1976) 840
E. A. Kuraev, L. N. Lipatov, V.S. F., ZhETF 72 (1977) 377
I.I. Balitsky, L.N. Lipatov, Yad. Fiz. 28 (1978) 1597
is the remarkable property of QCD – gluon Reggeization
L. N. Lipatov, Yad. Fiz. 23 (1976) 642.
The Reggeization allows to express an infinite number of amplitudes through several effective vertices and gluon trajectory.
Validity of the Reggeization is proved now in all orders of perturbation theory in the coupling constant g both in the leading logarithmic approximation (LLA), where in each order of the perturbation theory only terms with the highest powers of $\ln s$ are kept, and in the next-to-leading one (NLLA), where terms with one power less are also kept.

Gluon Reggeization

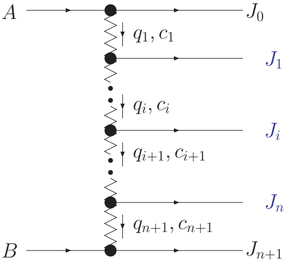
For elastic scattering processes $A+B \to A'+B'$ in the Regge kinematical region: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s) the Reggeization means that scattering amplitudes with the gluon quantum numbers in the t-channel can be presented as



$$\mathcal{A}_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{\omega(t)} - \left(\frac{s}{-t}\right)^{\omega(t)}\right]\Gamma^c_{B'B} \;;$$

Gluon Reggeization

$\Gamma^{c}_{P'P}$–particle-particle-Reggeon (PPR) vertices or scattering vertices ("c" are colour indices); $j(t) = 1 + \omega(t)$ – Reggeon trajectory.
The Reggeization means definite form not only of elastic amplitudes, but of inelastic amplitudes in the multi-Regge kinematics (MRK) as well. It can be presented by the picture



Gluon Reggeization

and written as

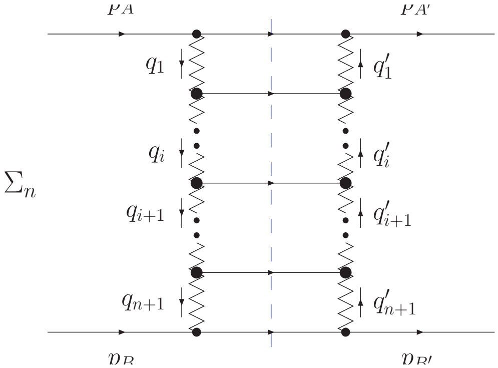
$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{J_i}(q_i,q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

Here $\gamma_{c_i c_{i+1}}^{J_i}(q_i,q_{i+1})$ – the Reggeon-Reggeon-particle (RRP) or production vertices.
MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets.
The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.

Amplitudes of processes with all possible quantum numbers in the t–channel are calculated using s-channel unitarity and analyticity .

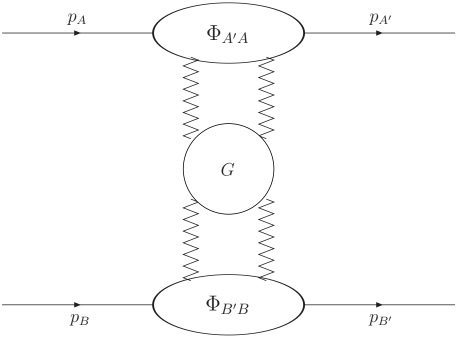
The s-channel discontinuity



BFKL representation of scattering amplitudes

The amplitudes are presented in the form :

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$



BFKL representation of scattering amplitudes

Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$
$B \to B'$,
G – Green's function for two interacting Reggeized gluons,

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.
The kernel is universal (process independent).

Violation of the Regge factorization in the NNLLA

The first observation of the violation of the Regge factorization was made by
V.Del Duca, N. Glover, 2001
in the consideration of the the high-energy limit of the two-loop amplitudes for parton-parton scattering.
They consider the interference of the Born amplitude with the two-loop amplitude for gg, gq and qq elastic scattering

$$\mathcal{M}_{ij}\mathcal{M}_{ij}^{(0)} = |\mathcal{M}_{ij}^{(0)}|^2 \left(1 + \tilde{g}_s{}^2\, M_{ij}^{(1)} + \tilde{g}_s{}^4 M_{ij}^{(2)} + \mathcal{O}(\tilde{g}_s{}^6)\right) \,,$$

with $i, j = g, q$,

$$\tilde{g}_s{}^2 = g_s^2 c_\Gamma \left(\frac{\mu^2}{-t}\right)^{\epsilon} \quad c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}}\,\Gamma(1+\epsilon)\,\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,,$$

Violation of the Regge factorization in the NNLLA

and compare the results of using directly calculated amplitudes and the Reggeized amplitudes

$$\mathcal{M}_{ij} = \left[g_s T_i C^i(p_a, p_{a'})\right] \frac{s}{t} \left[\left(\frac{-s}{-t}\right)^{\omega(t)} + \left(\frac{s}{-t}\right)^{\omega(t)}\right] \left[g_s T_j C^j(p_b, p_{b'})\right]$$

$$+\kappa \left[g_s T_i C^i(p_a, p_{a'})\right] \frac{s}{t} \left[\left(\frac{-s}{-t}\right)^{\omega(t)} - \left(\frac{s}{-t}\right)^{\omega(t)}\right] \left[g_s T_j C^j(p_b, p_{b'})\right] ,$$

where κ is zero for *gg* and *gq* scattering and

$$\kappa = \frac{N_c^2 - 4}{N_c^2}$$

for quark-quark scattering.

Violation of the Regge factorization in the NNLLA

They use the perturbative expansion

$$\omega(t) = \tilde{g}_s{}^2 \omega^{(1)}(t) + \tilde{g}_s{}^4 \omega^{(2)}(t)$$

$$C^i = C^{i(0)}(1 + \tilde{g}_s{}^2 C^{i(1)} + \tilde{g}_s{}^4 C^{i(2)})$$

Then the one-loop coefficient

$$M_{ij}^{(1)} = \omega^{(1)}(t) \ln\left(\frac{s}{-t}\right) + C^{i(1)} + C^{j(1)} - i\frac{\pi}{2}\left(1 + \kappa \frac{N^2 - 4}{N^2}\right)\omega^{(1)}(t)\,,$$

and two-loop

Violation of the Regge factorization in the NNLLA

$$M_{ij}^{(2)} = \frac{1}{2}\left(\omega^{(1)}(t)\right)^2 \ln^2\left(\frac{s}{-t}\right) + \ln\left(\frac{s}{-t}\right)$$

$$\times\left[\omega^{(2)}(t) + \left(C^{i(1)} + C^{j(1)}\right)\omega^{(1)}(t) - i\frac{\pi}{2}\left(1 + \kappa\frac{N^2-4}{N^2}\right)\left(\omega^{(1)}(t)\right)^2\right]$$

$$+\left[C^{i(2)} + C^{j(2)} + C^{i(1)}\ C^{j(1)} - \frac{\pi^2}{4}\left(1 + \kappa\frac{N^2-4}{N^2}\right)\left(\alpha^{(1)}(t)\right)^2\right]$$

$$-i\frac{\pi}{2}\left(1 + \kappa\frac{N^2-4}{N^2}\right)\left[\alpha^{(2)}(t) + \left(C^{i(1)} + C^{j(1)}\right)\alpha^{(1)}(t)\right] .$$

Violation of the Regge factorization in the NNLLA

The interference of the tree- and two-loop amplitudes for each of the parton-parton scattering processes have been explicitly computed
C. Anastasiou, E. W. N. Glover, C. Oleari and
M. E. Tejeda-Yeomans, 2001,
E. W. N. Glover, C. Oleari and M. E. Tejeda-Yeomans, 2001.
After expansion in ϵ and taking the high-energy limit, the interference between the n-loop and the tree amplitudes for ij elastic scattering has the form

$$\mathrm{Re}\left(\mathcal{M}^{(0)*}\mathcal{M}^{(n)}\right)_{ij} = |\mathcal{M}^{(0)}|^2_{ij\to ij}\,\tilde{g}_s{}^{2n}\sum_{m=0}^{n} B^{ij}_{nm}\ln^m\left(-\frac{s}{t}\right),$$

$$B^{ij}_{00} = 1\,.$$

Violation of the Regge factorization in the NNLLA

The discrepancy appears in non-logarithmic two-loop terms. If the Reggeization would be correct in the NNLLA, they should satisfy the equation

$$B_{20}^{qg} - \frac{1}{2}\left(B_{10}^{qg}\right)^2 - \frac{1}{2}\left[B_{20}^{gg} - \frac{1}{2}\left(B_{10}^{gg}\right)^2 \right.$$
$$\left. + B_{20}^{qq} - \frac{1}{2}\left(B_{10}^{qq}\right)^2\right] - \frac{\pi^2}{2\epsilon^2}\left(N^2 - 4\right) = 0\,.$$

Violation of the Regge factorization in the NNLLA

The explicit calculation of the B^{ij} coefficients gives that this relation is violated by terms of $\mathcal{O}(\pi^2/\epsilon^2)$,

$$B_{20}^{qg} - \frac{1}{2}\left(B_{10}^{qg}\right)^2 - \frac{1}{2}\left[B_{20}^{gg} - \frac{1}{2}\left(B_{10}^{gg}\right)^2 + B_{20}^{qq} - \frac{1}{2}\left(B_{10}^{qq}\right)^2\right]$$

$$-\frac{\pi^2}{2\epsilon^2}\left(N^2-4\right) = \frac{3\pi^2}{\epsilon^2}\left(\frac{N^2+1}{N^2}\right) + \mathcal{O}(\epsilon)\,.$$

Violation of the Regge factorization in the NNLLA

Detailed consideration of the terms responsible for the Regge factorization breaking in the case of two-loop and three-loop quark and gluon amplitudes in QCD was performed by
V. Del Duca, G. Falcioni, L, Magnea and L. Vernazza, 2014
In particular, the non-logarithmic double-pole contribution at two-loops was recovered and all non-factorizing single-logarithmic singular contributions at three loops were found using the techniques of infrared factorization.

For comparison of Regge and infrared factorizations the representation of scattering apmlitudes

$$\mathcal{M}_{\mathrm{rs}}^{[8]}\left(\frac{s}{\mu^2}, \frac{t}{\mu^2}, \alpha_s\right) = 2\pi\alpha_s\, H_{\mathrm{rs}}^{(0),[8]}$$

$$\times \left\{ C_{\mathrm{r}}\left(\frac{t}{\mu^2}, \alpha_s\right) \left[A_{+}\left(\frac{s}{t}, \alpha_s\right) + \kappa_{\mathrm{rs}}\, A_{-}\left(\frac{s}{t}, \alpha_s\right) \right] C_{\mathrm{s}}\left(\frac{t}{\mu^2}, \alpha_s\right) \right.$$

$$\left. + \mathcal{R}_{\mathrm{rs}}^{[8]}\left(\frac{s}{\mu^2}, \frac{t}{\mu^2}, \alpha_s\right) \right\}, \quad \kappa_{gg} = \kappa_{qg} = 0, \quad \kappa_{qq} = (4 - N_c^2)/N_c^2\,,$$

was used. $H_{\mathrm{rs}}^{(0)[8]}$ represents the tree-level amplitude. A non-factorizing remainder function $\mathcal{R}_{\mathrm{rs}}$ was introduced.

Violation of the Regge factorization in the NNLLA

The results obtained:

$$R_{qq}^{(2),0,[8]} = \frac{\pi^2}{4\epsilon^2}\left(1-\frac{3}{N_c^2}\right)\left(1-\epsilon^2\zeta(2)\right),$$

$$R_{gg}^{(2),0,[8]} = -\frac{3\pi^2}{2\epsilon^2}\left(1-\epsilon^2\zeta(2)\right),$$

$$R_{qg}^{(2),0,[8]} = -\frac{\pi^2}{4\epsilon^2}\left(1-\epsilon^2\zeta(2)\right).$$

The factor $\left(1-\epsilon^2\zeta(2)\right)$ can be absorbed in the constant c_Γ^2 by performing the expansion in terms of $\tilde{\alpha}_s = \alpha_s c_\Gamma$, instead of using α_s.

Violation of the Regge factorization in the NNLLA

$$R_{qq}^{(3),1,[8]} = \left(\frac{\alpha_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{2N_c^2-5}{12N_c} \left(1 - \frac{3}{2}\epsilon^2\zeta(2)\right) + \mathcal{O}\left(\epsilon^0\right),$$

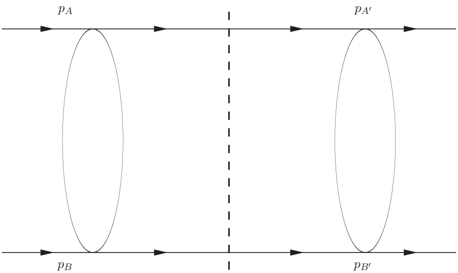
$$R_{gg}^{(3),1,[8]} = -\left(\frac{\alpha_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{2}{3} N_c \left(1 - \frac{3}{2}\epsilon^2\zeta(2)\right) + \mathcal{O}\left(\epsilon^0\right),$$

$$R_{qg}^{(3),1,[8]} = -\left(\frac{\alpha_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{N_c}{24} \left(1 - \frac{3}{2}\epsilon^2\zeta(2)\right) + \mathcal{O}\left(\epsilon^0\right),$$

Account of imaginary parts of amplitudes

Another difficulty in development of the BFKL approach in the NNLLA is necessity to account imaginary parts of amplitudes in the unitarity relations.

In the simplest two-particle intermediate state:



In imaginary parts, one $\ln s$ is lost. Products of imaginary and real parts in the unitarity relations cancel due to summation of contributions complex conjugated to each other. Due to this, imaginary parts don't play any role in the NLLA.

But they become important in the NNLLA.

Account of imaginary parts of amplitudes

Attenuating circumstance:
For the amplitudes in the unitarity relation, LLA is sufficient.

Nevertheless, account of imaginary parts leads to great changes.
Remind that for two Reggeized gluons in the t-channel in QCD, that is for tree colours, there are 6 irreducible representations:

$$\underline{1}, \underline{8_a}, \underline{8_s}, \underline{10}, \underline{\overline{10}}, \underline{27}.$$

For $N_c > 3$ there are 7 possible representations.
The representations $\underline{8_a}, \underline{10}, \underline{\overline{10}}$ are anti-symmetric
the representations $\underline{1}, \underline{8_s}, \underline{27}$ and the extra one are symmetric.

Account of imaginary parts of amplitudes

In real parts, with the NLLA accuracy, only the Reggeon channel, $\underline{8}_a$, is important.

It provides universality of the NLLA:

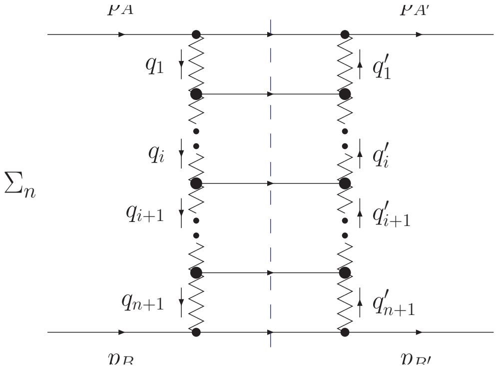
gg, qg and qq scattering can be considered in an unique way.

But account imaginary parts violate the universality.

gg scattering amplitudes can contain all the representations, while qg and qq scattering amplitudes only $\underline{1}, \underline{8}_a, \underline{8}_s$.

Account of imaginary parts of amplitudes

Consideration of many-particle states in the unitarity condition



is even more complicated problem.

Summary

- The basis of the BFKL approach is the remarkable property of QCD – gluon Reggeization
- The Reggeization hypothesis is extremely powerful: all scattering amplitudes are expressed in terms of the gluon trajectory and several Reggeon vertices
- In the LLA and NLLA the Reggeization means a simple factorized form for real parts of amplitudes with gluon exchanges, which is proved to be true
- This simple form is broken in the NNLLA
- In the NNLLA one has to account also imaginary parts of multiparticle amplitudes in the unitarity relations