# QCD evolution equations at small x:
## pomerons, odderons and more

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*Guido Altarelli memorial session*
$5^{th}$ International Conference on New Frontiers in Physics
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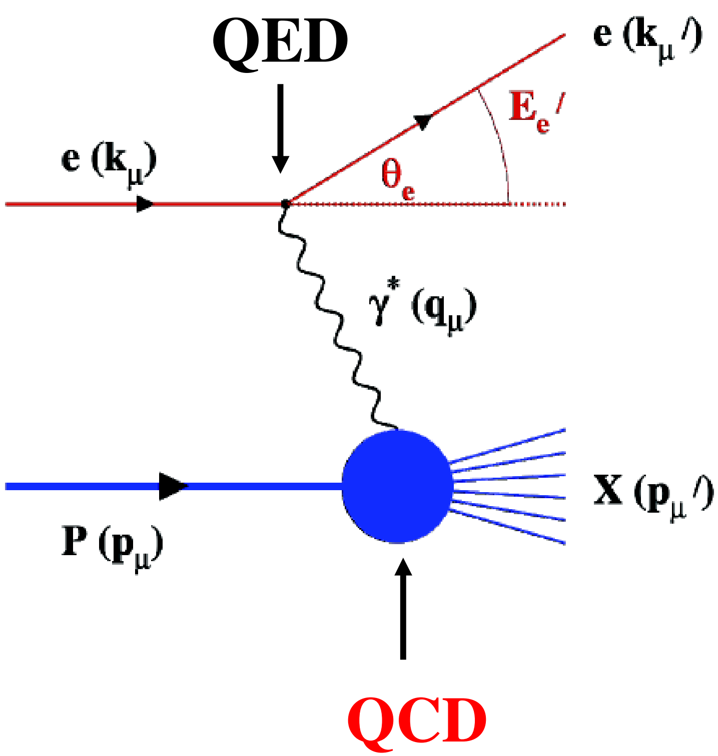
# *Deeply Inelastic Scattering (DIS)*

## probing hadron structure




*Kinematic Invariants*

$$Q^2 = -q^2 = -(k_\mu - k_\mu')^2$$

Measure of resolution power

$$Q^2 = 4E_e E_e' \sin^2\left(\frac{\theta_e'}{2}\right)$$

$$y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\left(\frac{\theta_e'}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

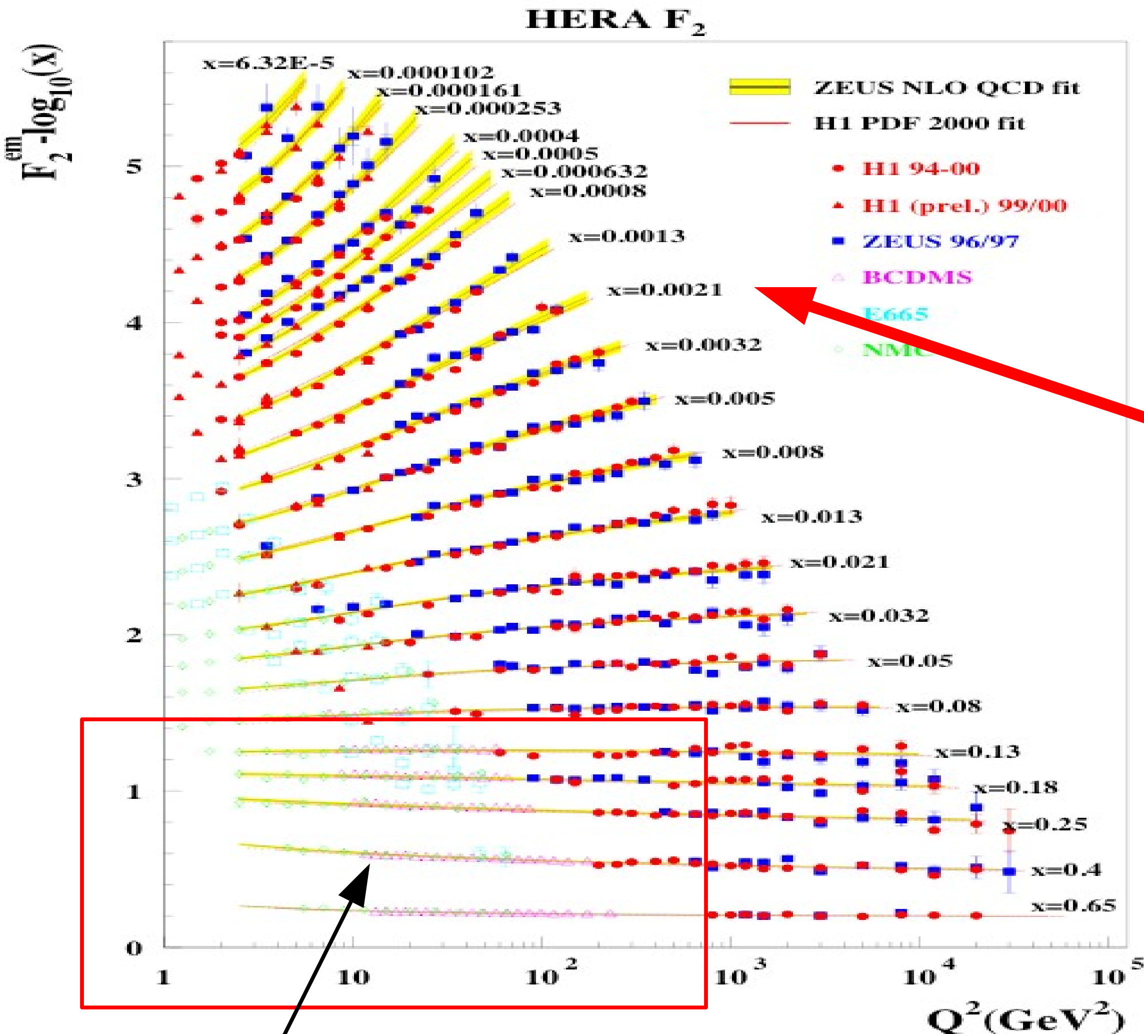
Measure of momentum fraction of struck quark

$$\mathrm{s} \equiv (\mathrm{p} + \mathrm{k})^2$$

*structure functions $F_1$, $F_2$*

$$\mathrm{F_2(x, Q^2)} \equiv \sum^{\mathrm{f}} \mathrm{e_f^2\, x[q_f(x, Q^2) + \bar{q}_f(x, Q^2)]}$$

# Deeply Inelastic Scattering $\mathbf{e(k)\,p(p) \to e(k')\,X}$




$$\mathbf{F_2} \equiv \frac{\mathbf{Q^2}}{4\,\pi^2\,\alpha_{\mathbf{em}}}\,\sigma^{\gamma^\star \mathbf{p}}$$

***scaling violations***

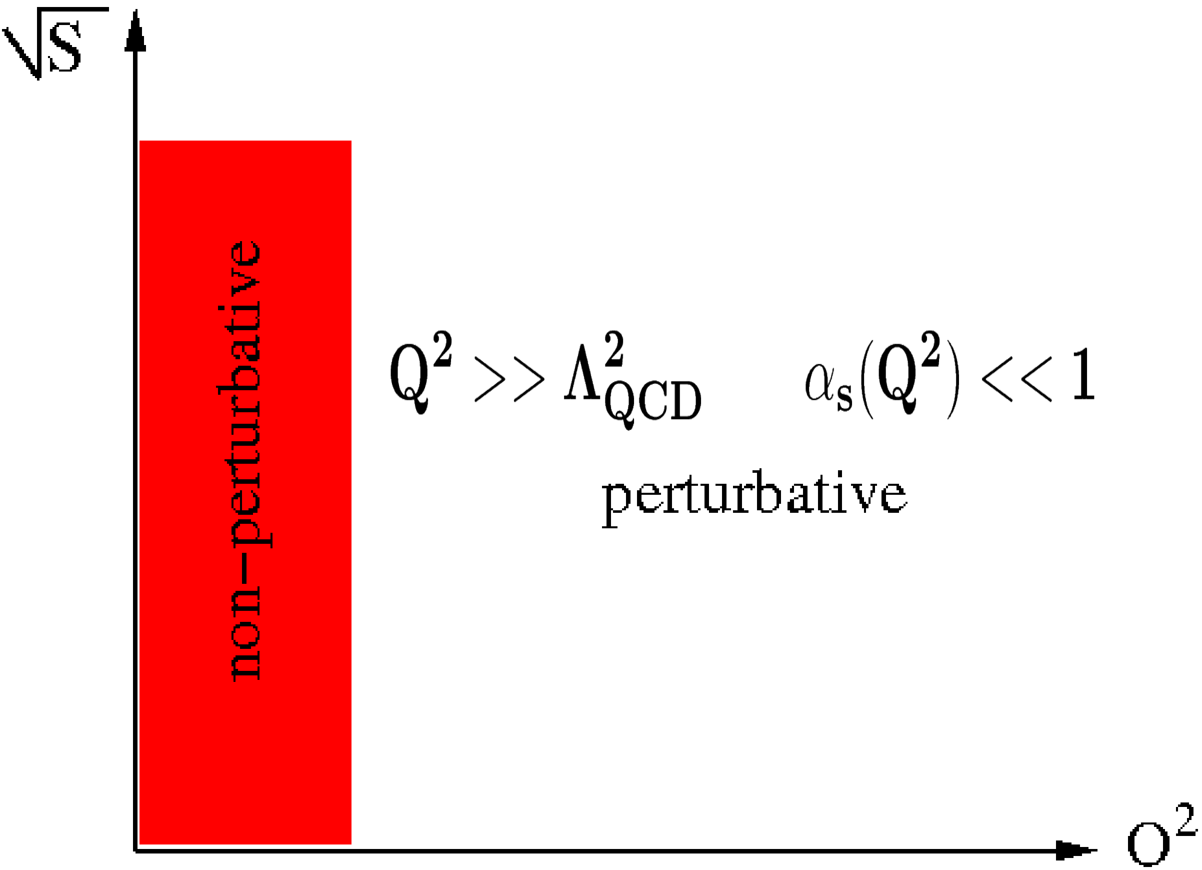
*parton model of a hadron leads to scale invariance*

***DGLAP (Q²)***

*evolution of quark and gluon distribution functions in pQCD*

*early experiments (SLAC,...): scale invariance of hadron structure*

# QCD: the standard paradigm




pQCD tools:
twist expansion,
collinear factorization

$$\sqrt{\mathbf{S}} \rightarrow \infty$$

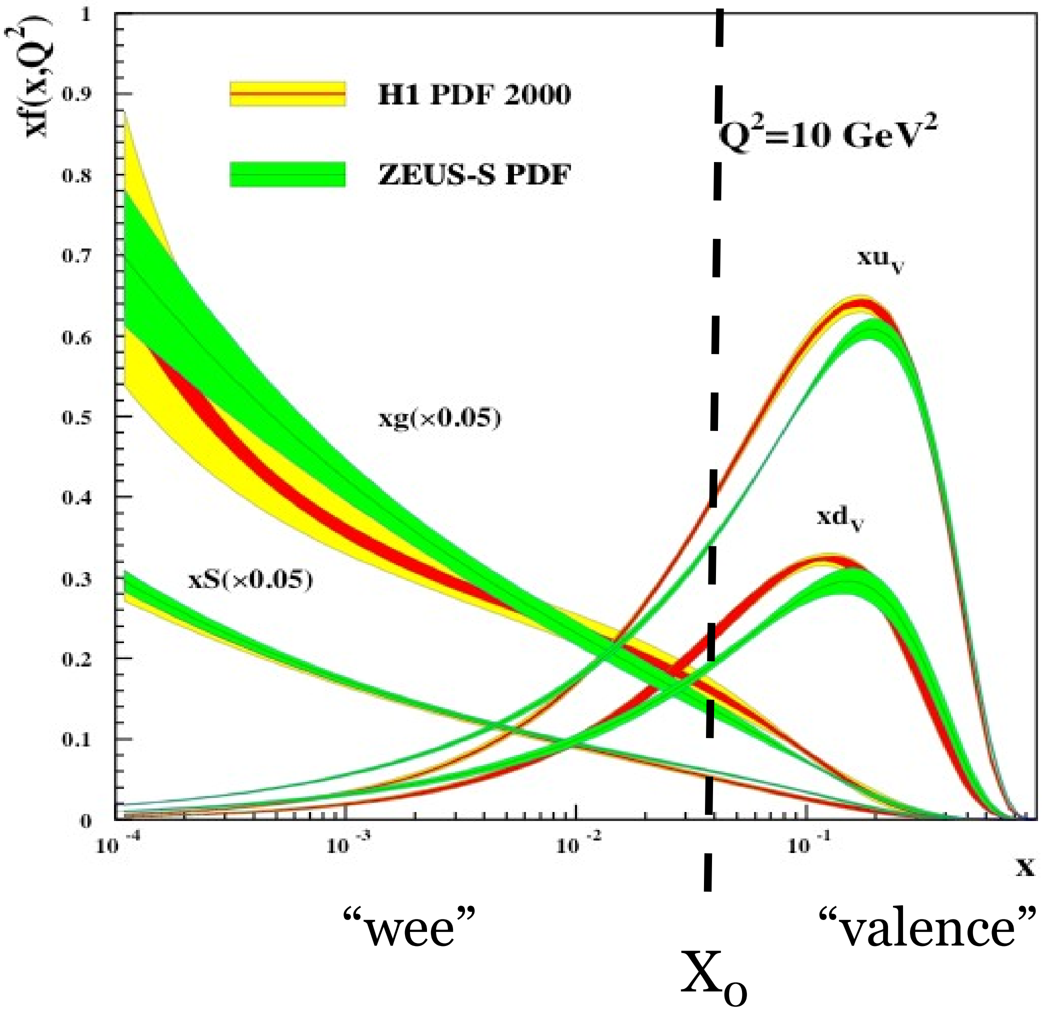
$$\mathbf{Q^2} \rightarrow \infty$$

***but bulk of QCD phenomena happens at low $p_t$***

$$\mathbf{x} \sim \frac{\mathbf{p_t}}{\sqrt{\mathbf{S}}} \mathbf{e^{-y}} \rightarrow \mathbf{0}$$

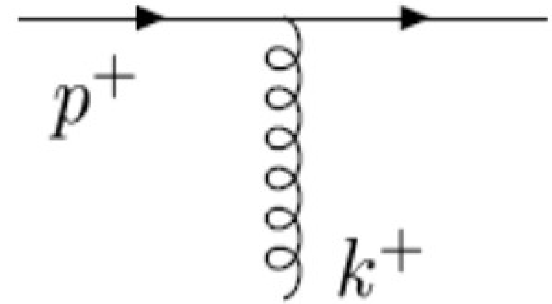
***QCD cross sections at small x***

# A hadron at small x




recall for any 4-momentum

$$\mathbf{p^2 = 2p^+p^- - p_t^2}$$




**small x:** $\mathbf{k^+ << p^+}$

$$\tau_{\mathbf{val}} \sim \frac{\mathbf{1}}{\mathbf{p^-}} = \frac{\mathbf{2p^+}}{\mathbf{k_t^2}} \gg \tau_{\mathbf{wee}} \sim \frac{\mathbf{1}}{\mathbf{k^-}} = \frac{\mathbf{2k^+}}{\mathbf{k_t^2}}$$

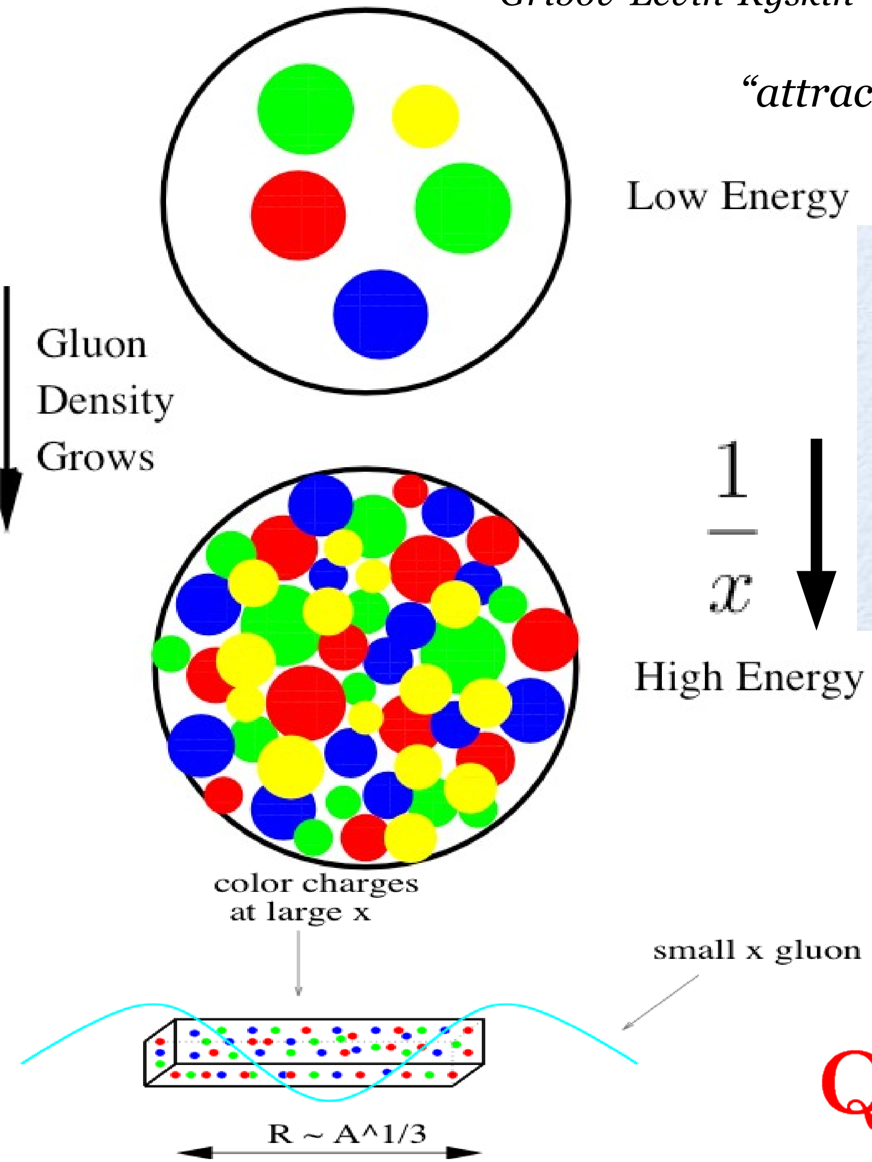
*high x partons as* *__static__* *color charges* $\rho$

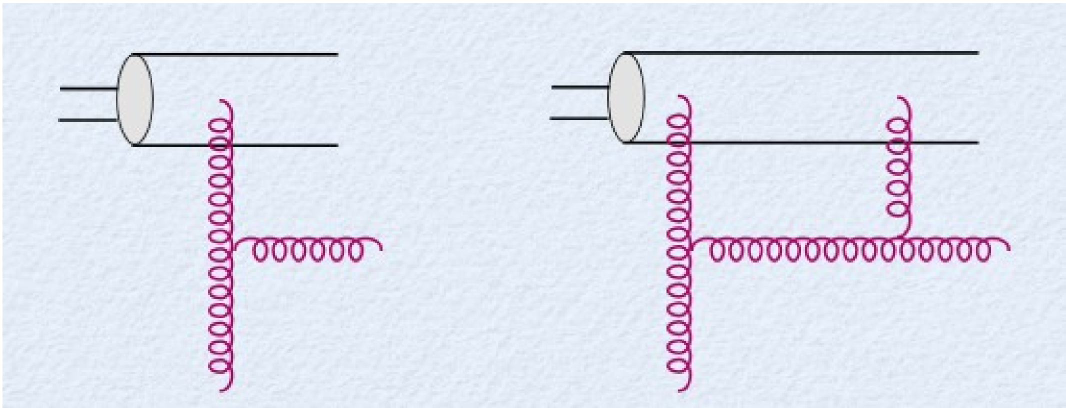
# Gluon saturation/CGC

$$\mathbf{S \to \infty, \quad Q^2 \ \ fixed, \quad x_{Bj} \equiv \frac{Q^2}{S} \to 0}$$

*Gribov-Levin-Ryskin*

*"attractive" bremsstrahlung vs. "repulsive" recombination*



$$\frac{\alpha_s \, xG(x, b_t, Q^2)}{S_\perp Q^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

# *MV effective Action + RGE*

$$\mathbf{S[A,\rho] = -\frac{1}{4}\int d^4x\, F^2_{\mu\nu} + \frac{i}{N_c}\int d^2x_t dx^-\ \delta(x^-) Tr[\rho(x_t) U(A^-)]}$$

Large x: color source $\rho$ small x: gluon field $\mathbf{A}^{\mu}$

$$\mathbf{U(A^-) = \hat{P}\,Exp\left[ig\int dx^+\, A^-_a\, T_a\right]}$$

$$\mathbf{Z[j] = \int [D\rho]\, W_{\Lambda^+}[\rho]\left[\frac{\int^{\Lambda^+}[DA]\delta(A^+)e^{iS[A,\rho]-\int j\cdot A}}{\int^{\Lambda^+}[DA]\delta(A^+)e^{iS[A,\rho]}}\right]}$$

$\mathbf{W}_{\mathbf{\Lambda}^+}[\rho]$ weight functional:
probability distribution of color sources $\rho$
at longitudinal scale $\mathbf{\Lambda}^+$

# Wilsonian RG: JIMWLK evolution eq

invariance under change of $\Lambda^+$ $\longrightarrow$ RGE for $W_{\Lambda^+}[\rho]$

**Field** $A_\mu^a$

**source** $\rho^a$

**weight functional** $W_x[\rho]$

**0** **x** **1**

$$x \equiv \frac{\Lambda^+}{P^+}$$

# Wilsonian RG: JIMWLK evolution eq

$$\mathbf{resum}\ \ \alpha_s \log\frac{1}{x}$$

**Field** $\mathbf{A}^a_\mu$

**source** $\rho^a$

**weight functional** $\mathbf{W}_x[\rho]$


$$\mathbf{A}^\mu = \mathbf{A}^\mu_{\mathbf{class}} + \delta\,\mathbf{A}^\mu$$

integrate out field fluctuations quadratically

$$\rho \to \rho' = \rho + \delta\rho \qquad \mathbf{W}[\rho] \to \mathbf{W}[\rho']$$

# Wilsonian RG: JIMWLK evolution eq

**resum** $\alpha_s \log\frac{1}{x}$

**Field** $A_\mu^a$

**source** $\rho^a$

**weight functional** $W_x[\rho]$

**0** **x** **1**

$$\frac{\partial}{\partial\, ln 1/x}\langle O\rangle = \left\langle \frac{1}{2}\int d^2x_t d^2y_t \, \frac{\delta}{\alpha_a(x_t)}\eta^{ab}(x_t,y_t)\frac{\delta}{\delta\alpha_b(y_t)}O\right\rangle$$

$$\eta^{ab}(x_t,y_t) = K\,\left[1 + U^\dagger(x_t)U(y_t) - U^\dagger(x_t)U(z_t) - U^\dagger(z_t)U(y_t)\right]^{ab}$$

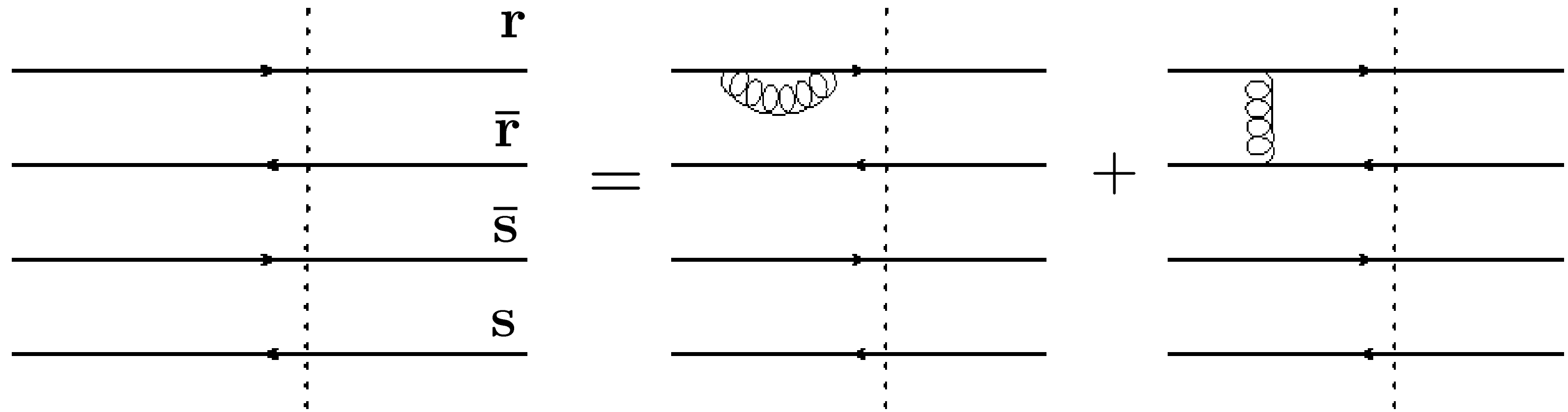
$$K = \frac{1}{\pi}\int \frac{d^2z_t}{(2\pi)^2}\frac{(x_t - z_t)\cdot(y_t - z_t)}{(x_t - z_t)^2\,(y_t - z_t)^2}$$

# Evolution of quadrupole from JIMWLK

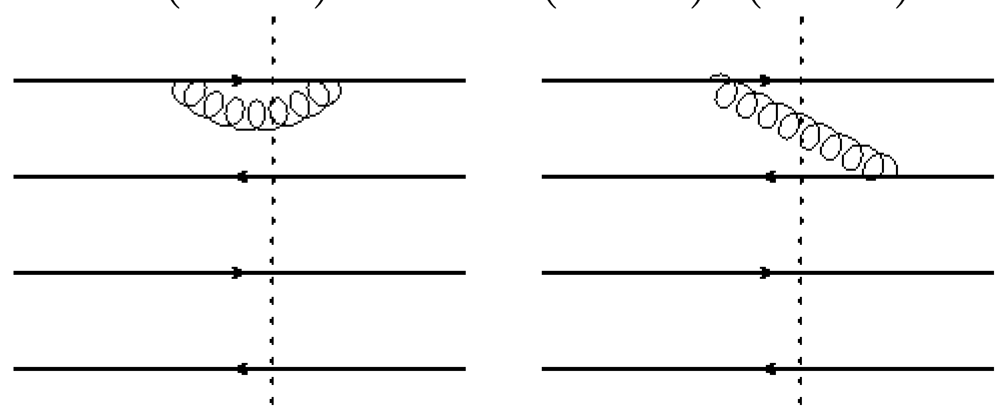
$$Q(\mathbf{r}, \bar{\mathbf{r}}, \bar{\mathbf{s}}, \mathbf{s}) \equiv \frac{1}{N_c} < \mathrm{Tr}\, V(\mathbf{r})\, V^\dagger(\bar{\mathbf{r}})\, V(\bar{\mathbf{s}})\, V^\dagger(\mathbf{s}) >$$

radiation kernels
as in dipole

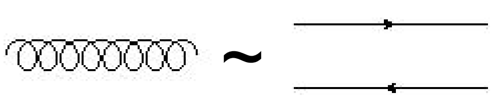
$$\frac{d}{dy}$$




$$\frac{1}{(\mathbf{r}-\mathbf{z})^2} \qquad \frac{(\mathbf{r}-\mathbf{z})\cdot(\bar{\mathbf{r}}-\mathbf{z})}{(\mathbf{r}-\mathbf{z})^2(\bar{\mathbf{r}}-\mathbf{z})^2}$$



large $N_c$



$+ \cdots$

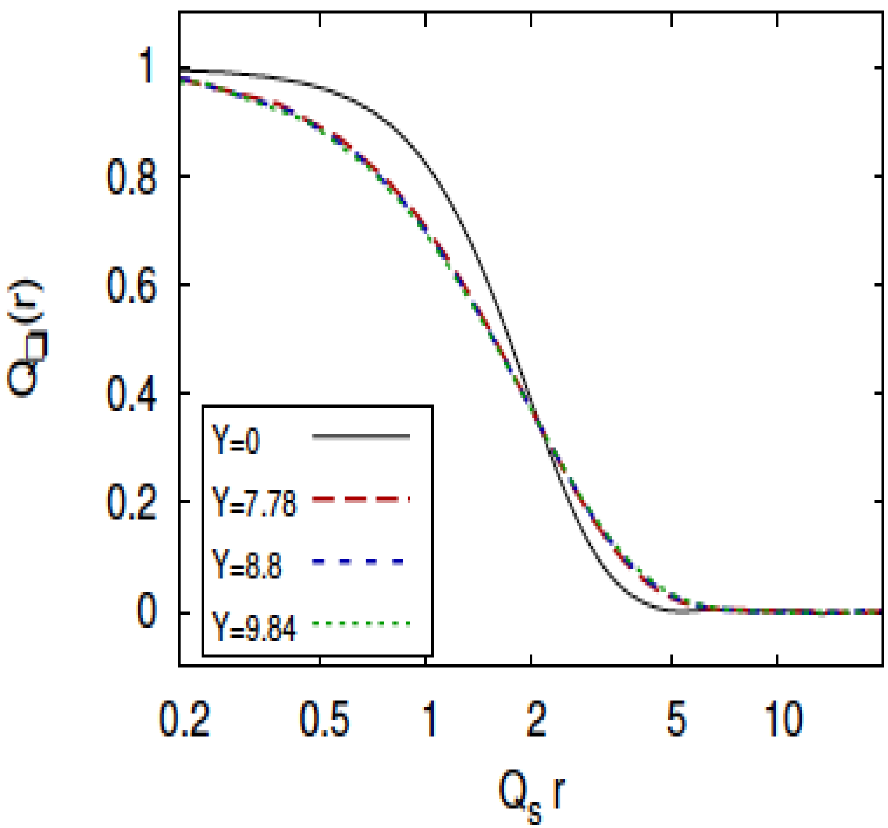
# Evolution of quadrupole from JIMWLK

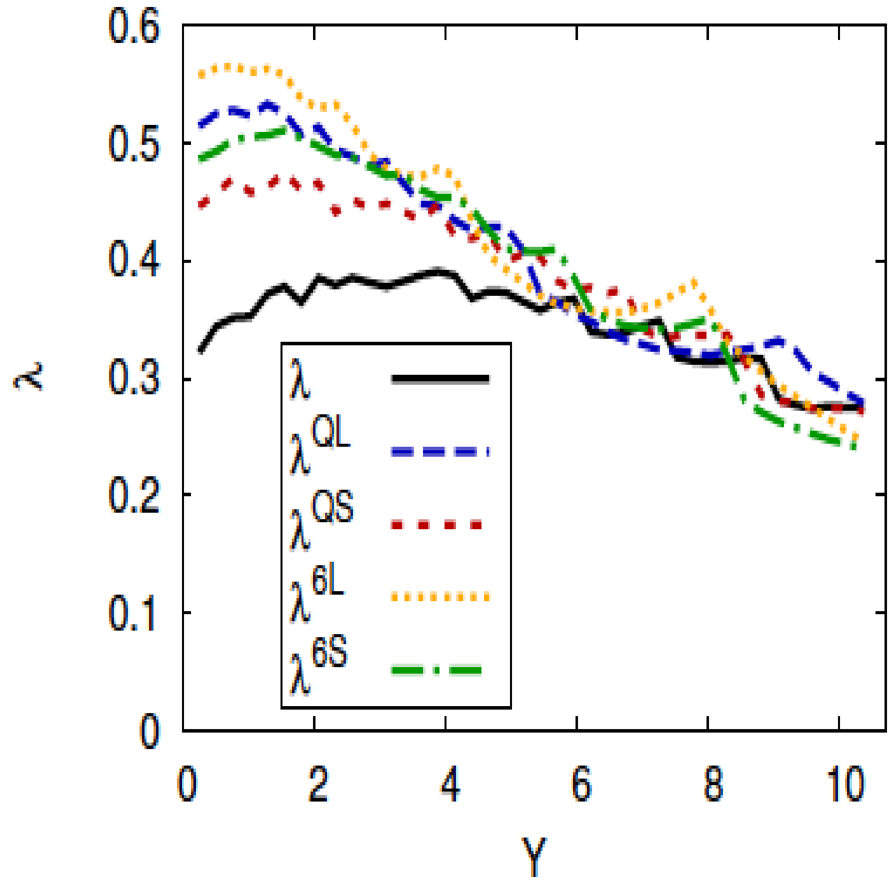
$$
\begin{aligned}
&\frac{d}{dy}\langle Q(r,\bar r,\bar s,s)\rangle \\
= \;& \frac{N_c\,\alpha_s}{(2\pi)^2}\int d^2z \Bigg\{\Bigg\langle \left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right] Q(z,\bar r,\bar s,s)\,S(r,z) \\
+\;& \left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right] Q(r,z,\bar s,s)\,S(z,\bar r) \\
+\;& \left[\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(s-z)^2(\bar r-z)^2}\right] Q(r,\bar r,z,s)\,S(\bar s,z) \\
+\;& \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right] Q(r,\bar r,\bar s,z)\,S(z,s) \\
-\;& \left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}+\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}\right] Q(r,\bar r,\bar s,s) \\
-\;& \left[\frac{(r-s)^2}{(r-z)^2(s-z)^2}+\frac{(\bar r-\bar s)^2}{(\bar r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}\right] S(r,s)\,S(\bar r,\bar s) \\
-\;& \left[\frac{(r-\bar r)^2}{(r-z)^2(\bar r-z)^2}+\frac{(s-\bar s)^2}{(s-z)^2(\bar s-z)^2}-\frac{(r-\bar s)^2}{(r-z)^2(\bar s-z)^2}-\frac{(\bar r-s)^2}{(\bar r-z)^2(s-z)^2}\right] S(r,\bar r)\,S(\bar s,s)\Bigg\rangle\Bigg\}
\end{aligned}
$$

$$\frac{\mathbf{d}}{\mathbf{d\,y}}\mathbf{Q} = \int \mathbf{P_1}\,[\mathbf{Q\,S}] - \mathbf{P_2}\,[\mathbf{Q}] + \mathbf{P_3}[\mathbf{S\,S}] \qquad \text{with} \qquad P_1 - P_2 + P_3 = 0$$

J. Jalilian-Marian, Y. Kovchegov: PRD70 (2004) 114017, ............

# Quadrupole evolution: JIMWLK

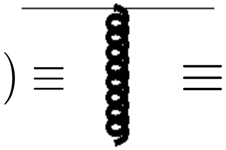
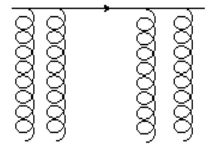



*geometric scaling also present in quadrupoles* *energy dependence of saturation scale*

*Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219*

# *JIMWLK evolution in the linear regime*

$V(x_t) \equiv$  $\equiv$  $\sim 1 + O(g\,A) + O(g^2\,A^2)$

*BJKP: evolution of n-Reggeized gluons in a singlet state*

$\mathbf{O(A^2)}$ 2-gluon exchange (BFKL pomeron)

$\mathbf{O(A^3)}$ 3-gluon exchange (odderon)

i) a dipole

ii) 3-quarks

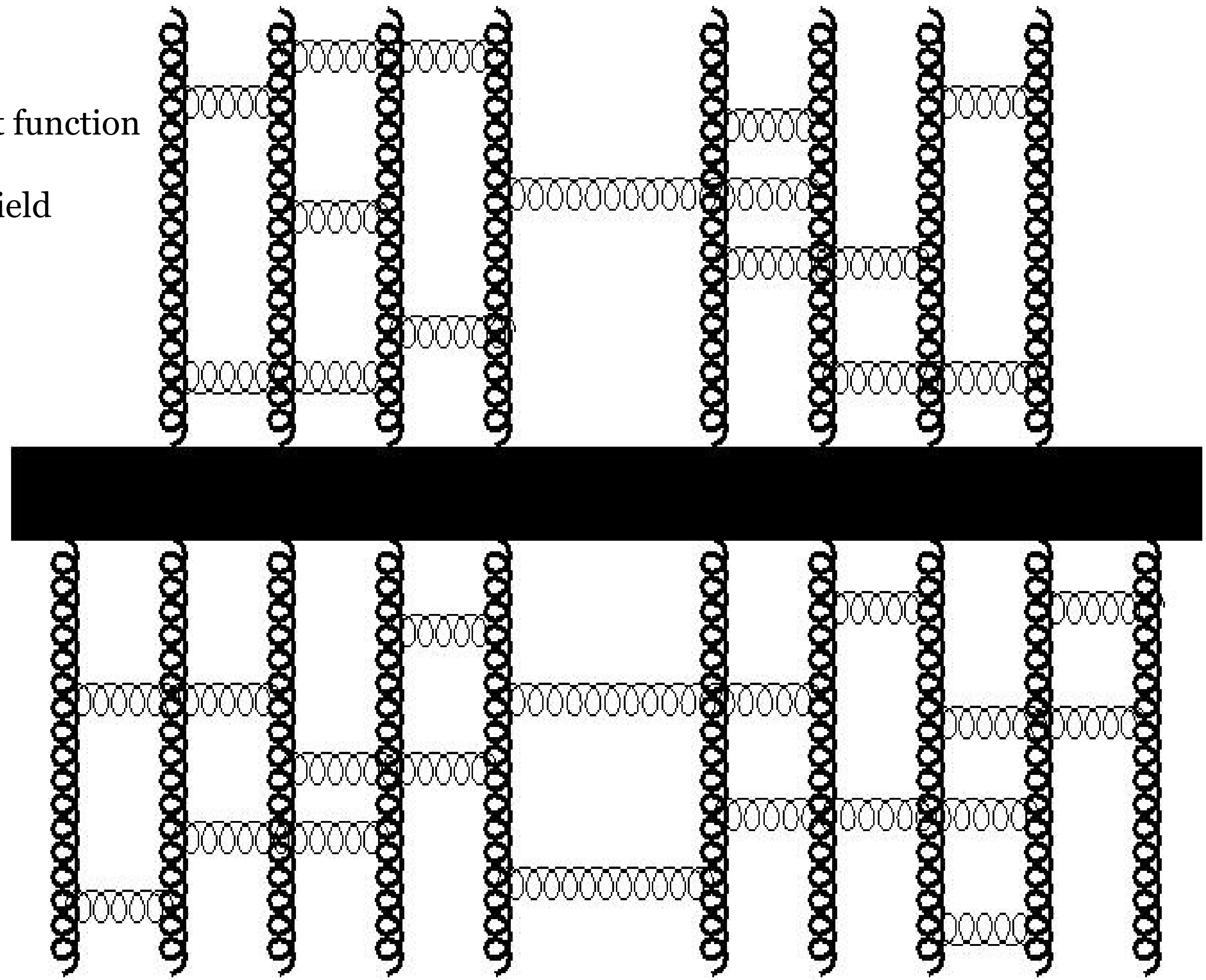
$\mathbf{O(A^4)}$ 4-gluon exchange

**JIMWLK (linear) and BJKP eqs. agree for n=2,3,4,...,n**

# JIMWLK in non-linear regime: n ----> n+1 “pomeron” vertices

Follow the same strategy:

- Evolution equation for 2n-pt function
- Keep the non-linear terms
- Expand in powers of gluon field
- Extract the vertex



**1 ---> 2 vertex**
**(triple pomeron)**

**2 ---> 3 vertex**

**n ---> n + 1 vertex**

# ***CGC signatures?***

***two main effects:***

*multiple scatterings*
*evolution with x (rapidity)*

***dense-dense (AA, pA, pp) collisions***

*initial conditions*

***dilute-dense (pA, forward pp ) collisions***

*multiplicities*
*$p_t$ spectra*
*angular correlations*

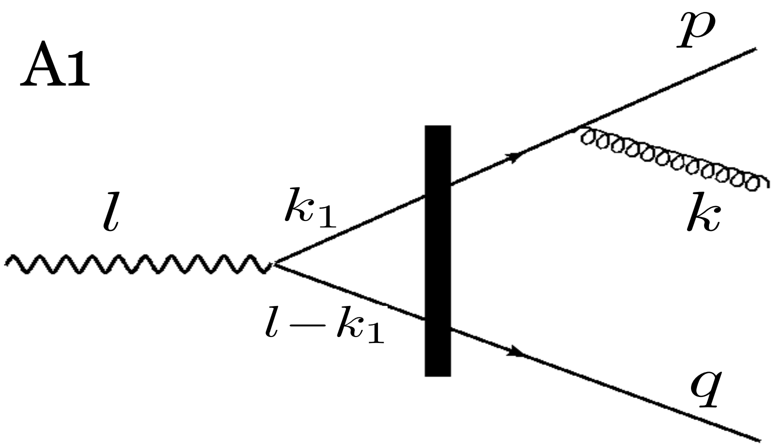
***spin asymmetries***

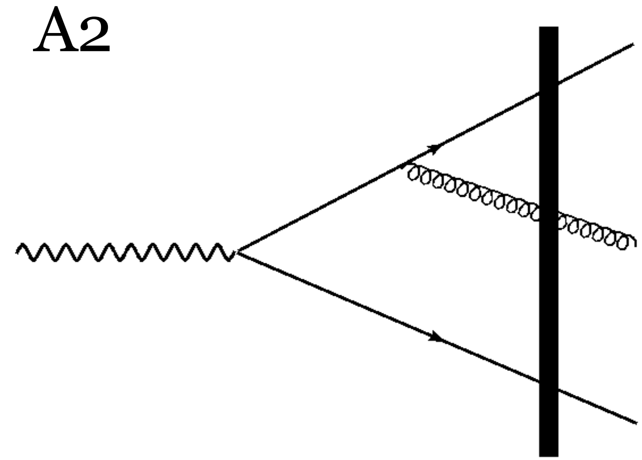
***DIS***

*structure functions (diffraction)*
**NLO** di-hadron correlations

***3-hadron/jet correlations***

# 3-jet production in DIS: azimuthal correlations




$$A^L_{1,hg} = \sqrt{2Q^2}\, e^{ix_t(k_t+p_t)+iq_ty_t}\, K_0\left[\sqrt{Q^2x_{12}^2z_2\,(z_1+z_3)}\right]\cdot a^L_{1,hg}$$

with $\quad x_{12}^2 \equiv (x_t - y_t)^2$

$$\begin{aligned}
a^L_{1,++} &= \frac{z_1z_2\sqrt{z_1z_2}\,(z_1+z_3)}{z_3e^{-i\theta_p}|p_t| - z_1e^{-i\theta_k}|k_t|} \\
a^L_{1,-+} &= \frac{\sqrt{z_1z_2}z_2\,(z_1+z_3)^{\,2}}{z_3e^{-i\theta_p}|p_t| - z_1e^{-i\theta_k}|k_t|} \\
a^L_{1,--} &= (a^L_{1,++})^\star \\
a^L_{1,+-} &= (a^L_{1,+--+})^\star
\end{aligned}$$

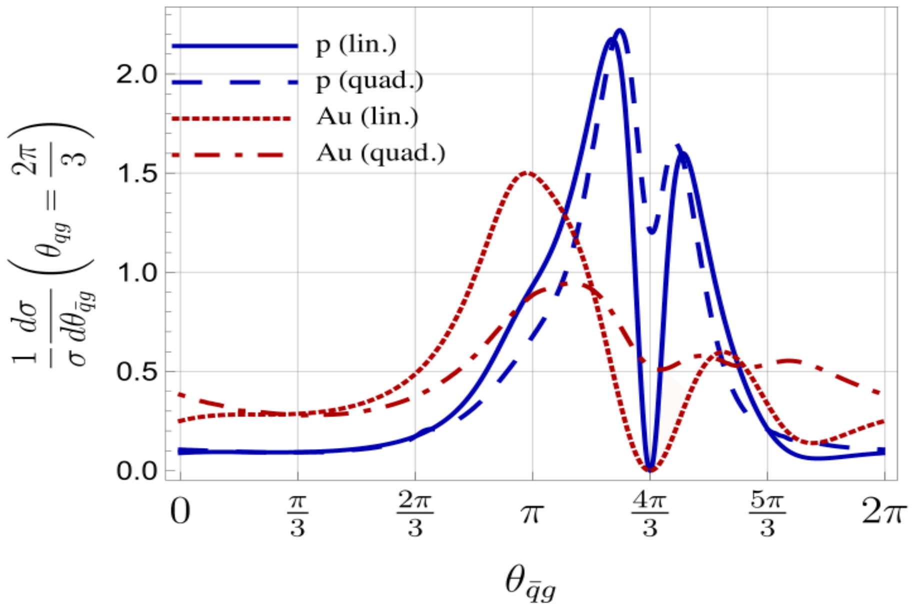
<u>*spinor helicity methods*</u>

longitudinal photons

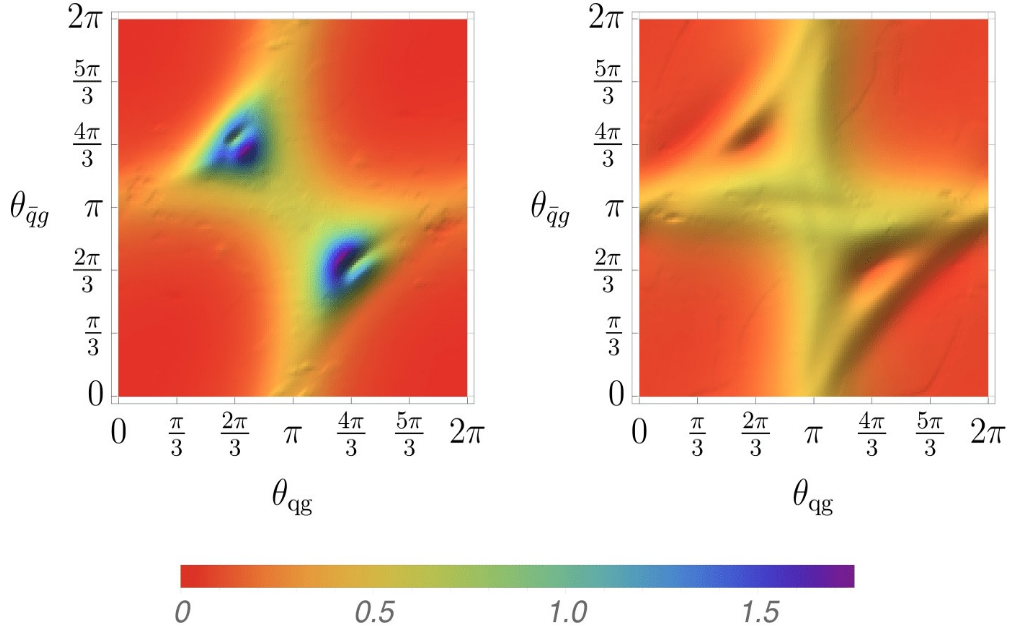
gluon, quark helicity

$g, h = \pm$

# *3-parton azimuthal angular correlations in DIS*




*disappearance of the away side peaks*




*Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans, arXiv:1604:08526*

# Possible extensions to other processes

real photons: $Q^2 \to 0$

**ultra-peripheral nucleus-nucleus collisions**

crossing symmetry:

$$\gamma^{(\star)}\, T \longrightarrow q\, \bar{q}\, g\, X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\, T \longrightarrow q\, g\, \gamma^{(\star)}\, X \\ \bar{q}\, T \longrightarrow \bar{q}\, g\, \gamma^{(\star)}\, X \\ g\, T \longrightarrow q\, \bar{q}\, \gamma^{(\star)}\, X \end{array} \right\}$$

**proton-nucleus collisions** (collinear factorization in proton?)

$$pA \longrightarrow h_1\, h_2\, \gamma^{(\star)}\, X$$

**MPI** (double/triple parton interactions)

$$\gamma^{(\star)}\, T \longrightarrow q\, \bar{q}\, g\, X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\, \bar{q}\, T \longrightarrow g\, \gamma^{(\star)}\, X \\ g\, \bar{q}\, T \longrightarrow \bar{q}\, \gamma^{(\star)}\, X \\ g\, q\, T \longrightarrow q\, \gamma^{(\star)}\, X \end{array} \right\}$$

# ***SUMMARY***

***CGC is a systematic approach to high energy collisions***

***JIMWLK eq reduces to BJKP eq  in the linear regime:***

***Pomerons, odderons,....***

***n ---> n+1 "pomeron" vertices from non-linear terms in JIMWLK***

***CGC hes been used to fit a wealth of data:***

***ep, eA, pp, pA, AA***

***Azimuthal angular correlations offer a unique probe of CGC:***

***3-hadron/jet correlations should be even more discriminatory***

***Must generalize CGC to include high $p_t$ (DGLAP) physics***

***signs of trouble at $p_t > Q_s$ ?***