

Quantum Walks and Deformed Special Relativity

Alessandro Bisio

ICNFP 2016

July 13th 2016
Orthodox Academy of Crete

in collaboration with:

QUIT group in Pavia

Giacomo Mauro D'Ariano

Paolo Perinotti

Alessandro Tosini

Nicola Mosco

Marco Erba

University of Montreal

Alexandre Bibeau-Delisle

supported by

JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE ~ INVESTING IN THE BIG QUESTIONS

Outline

Motivation

QCA (QW) for the free fields evolution

The fate of Lorentz covariance: from QCA to deformed relativity models

Final remarks and (many) open problems

Quantum Theory

Von Neumann, 1932

Each physical system is associated with a Hilbert space

Unit vectors are associated with states of the system

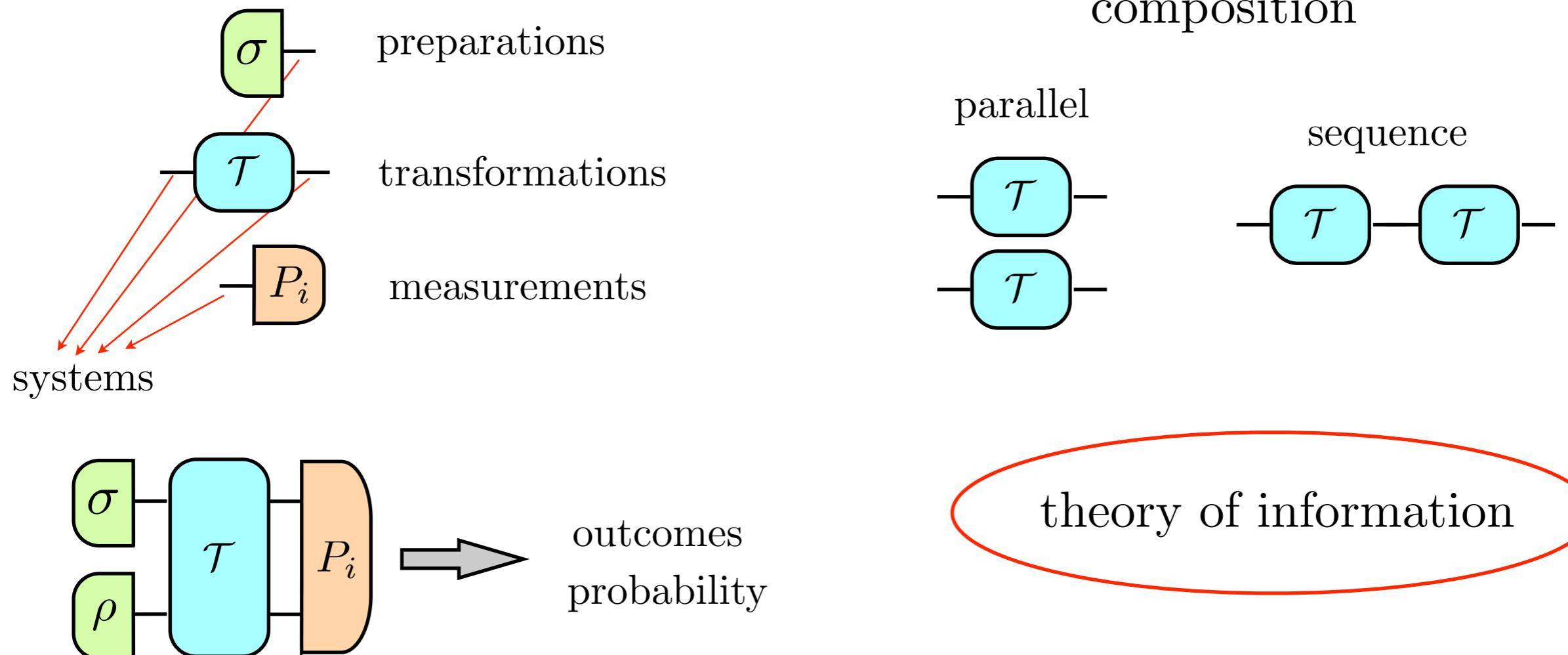
Physical observables are represented by self adjoint operators

The Hilbert space of a composite system is the tensor product of the state spaces associated with the component systems

The probabilities of the outcomes are given by the Born rule

Reconstruction of Quantum Theory

Operational Probabilistic Theory



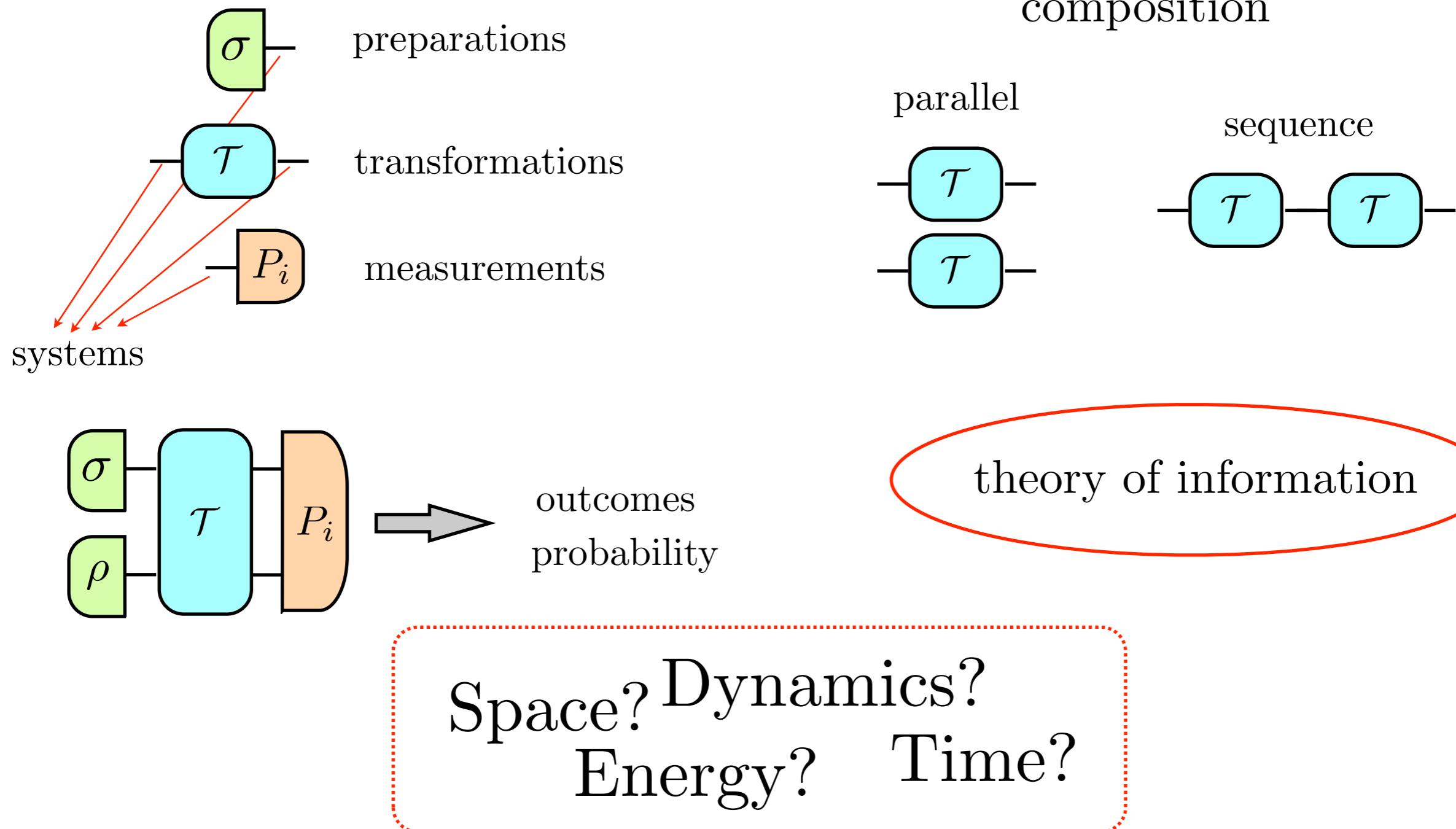
G. Ludwig, *Foundations of Quantum Mechanics* (Springer, New York, 1985).

L. Hardy, e-print arXiv:quant-ph/0101012.

G. Chiribella, G. M. D'Ariano, P. Perinotti, Phys. Rev. A 84, 012311 (2011)

Reconstruction of Quantum Theory

Operational Probabilistic Theory

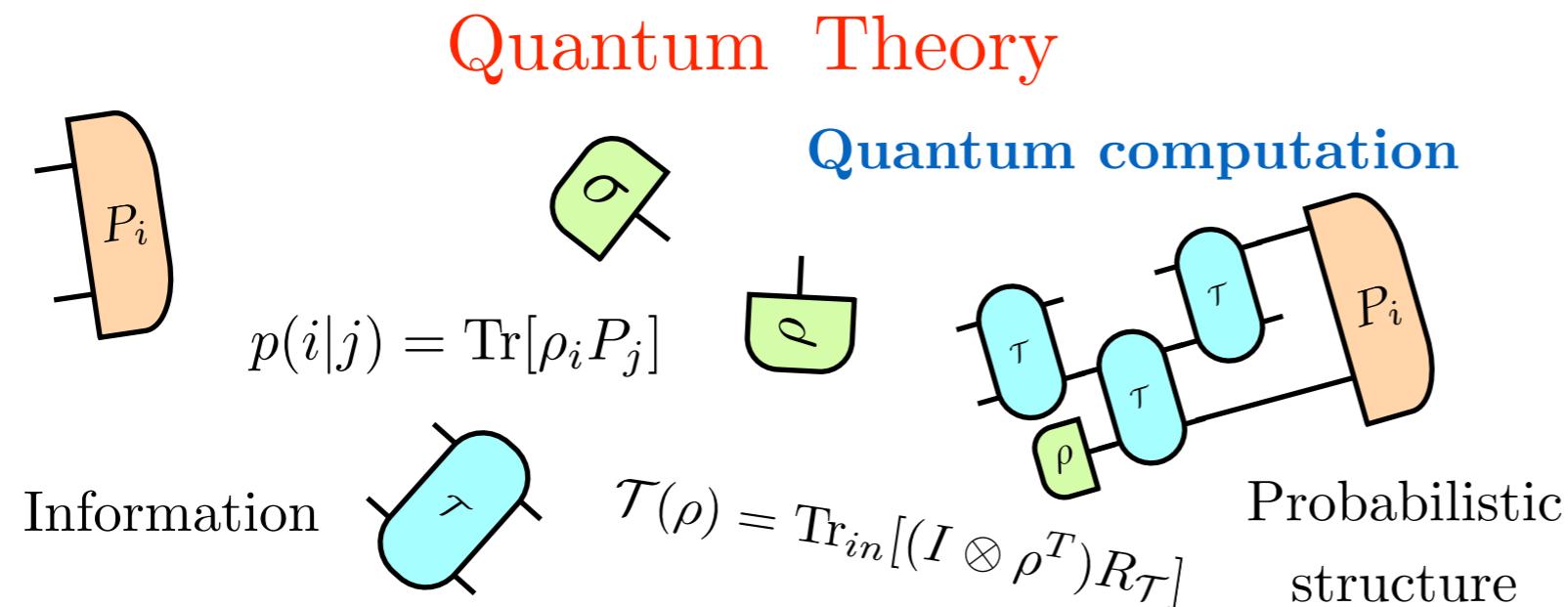


G. Ludwig, *Foundations of Quantum Mechanics* (Springer, New York, 1985).

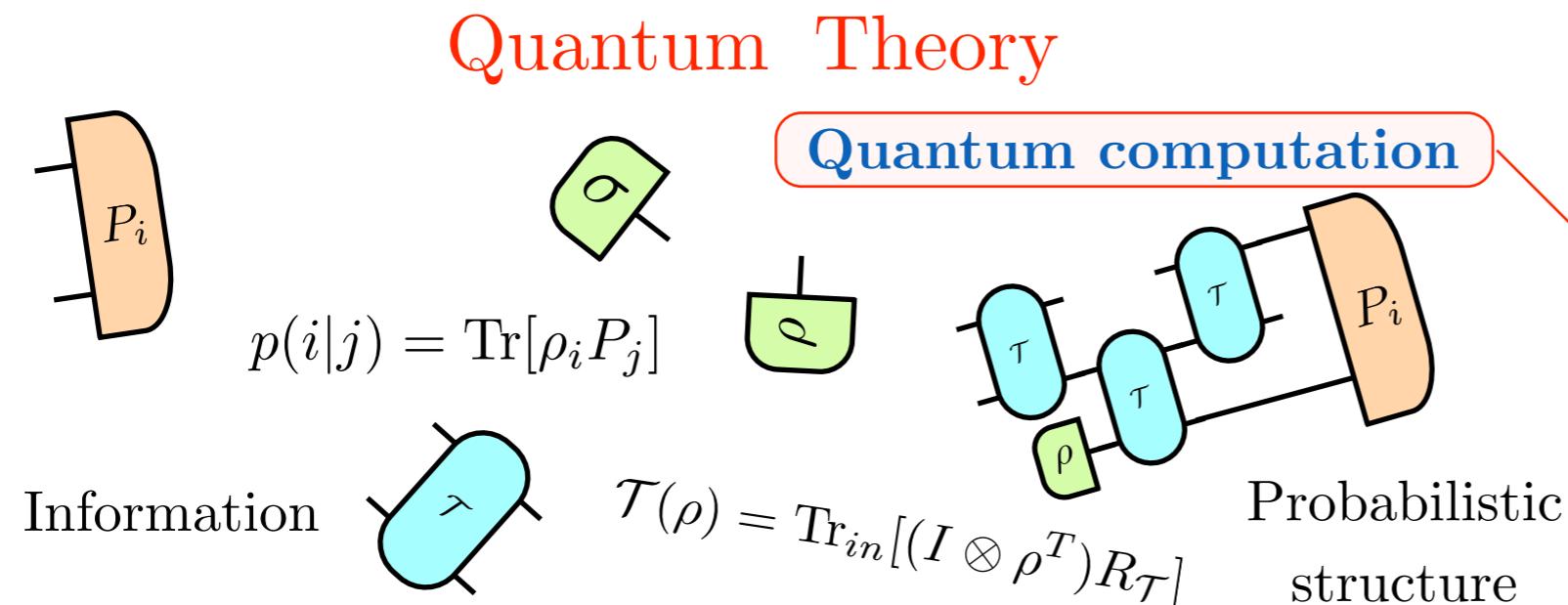
L. Hardy, e-print arXiv:quant-ph/0101012.

G. Chiribella, G. M. D'Ariano, P. Perinotti, Phys. Rev. A 84, 012311 (2011)

Reconstruction of Quantum Field Theory



Reconstruction of Quantum Field Theory

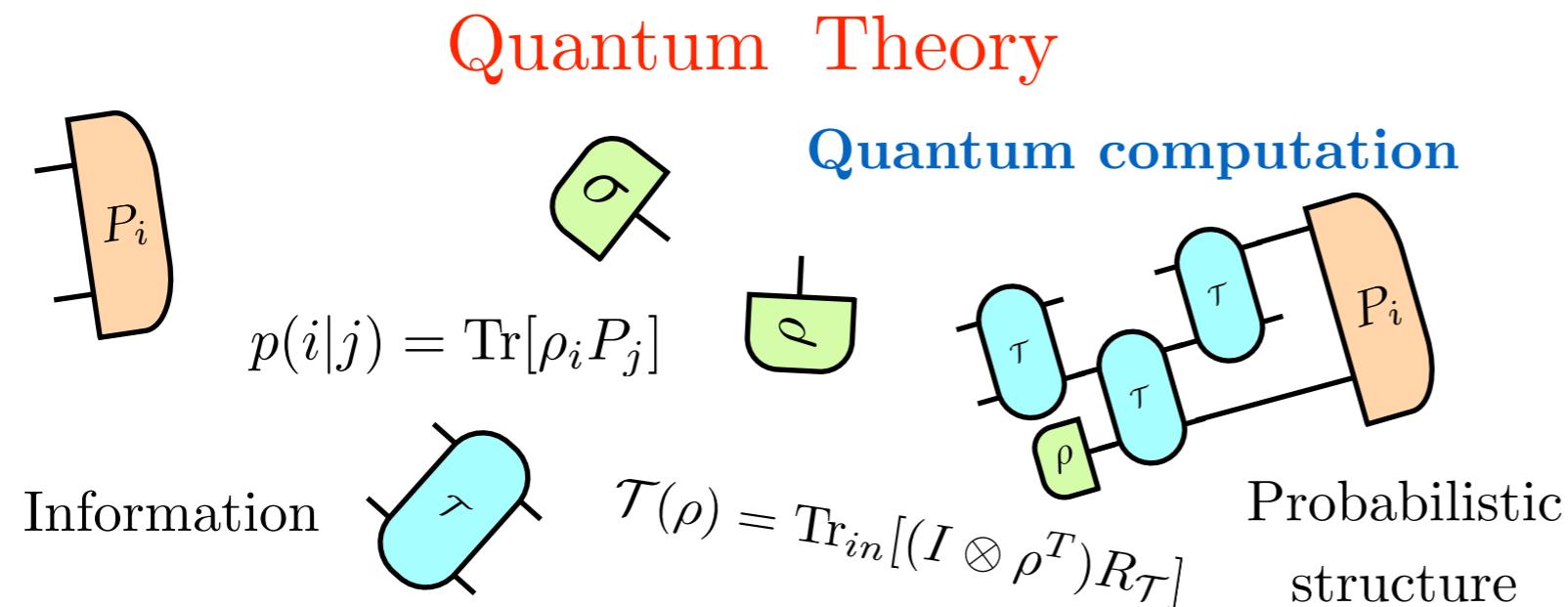


Simulating Physics
with Computers

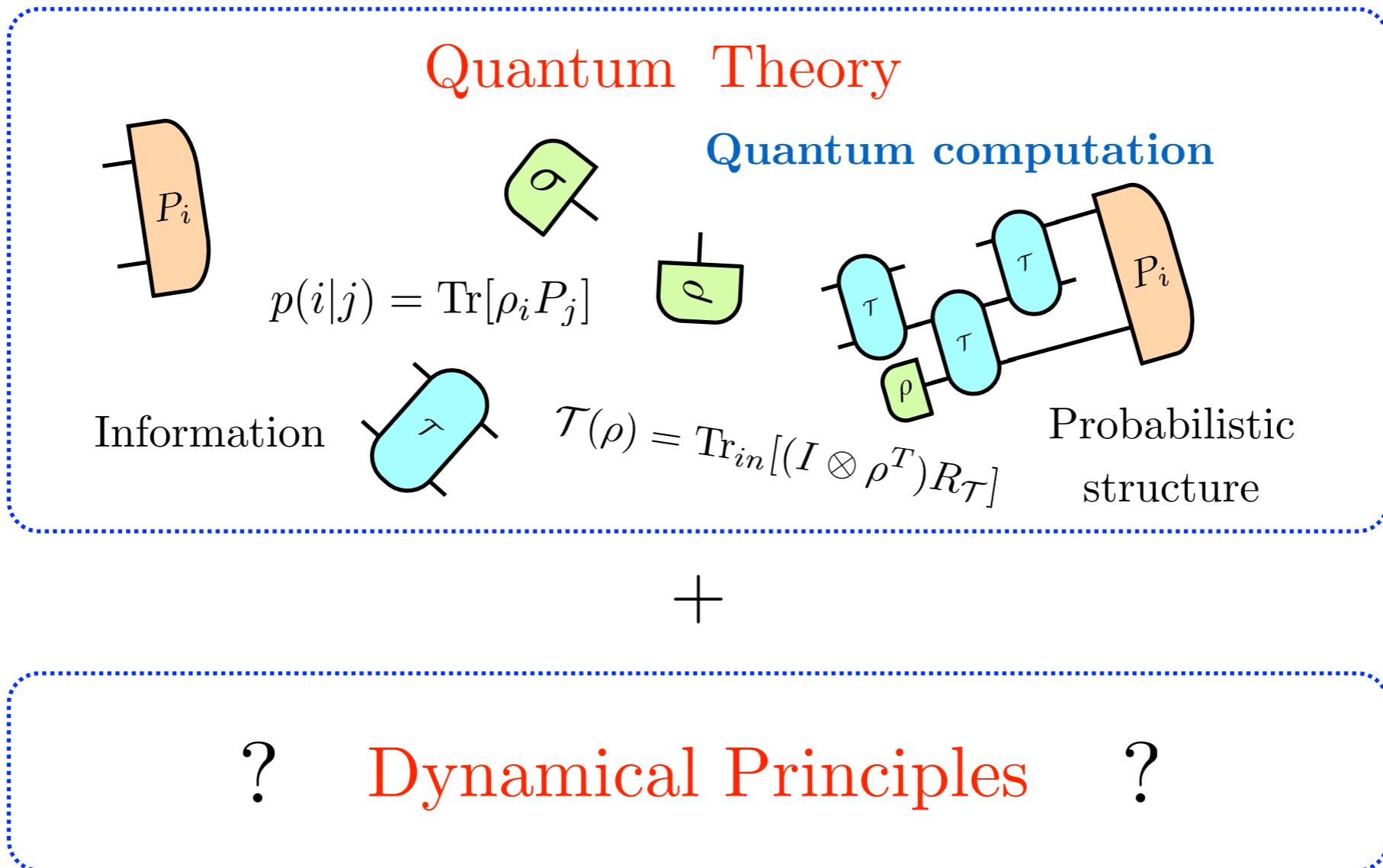
R. P. Feynman, Int. J. Theo. Phys. 21, 467 (1982)

Can a Quantum Computer
exactly simulate physical systems?
Replace physical laws
with quantum algorithms

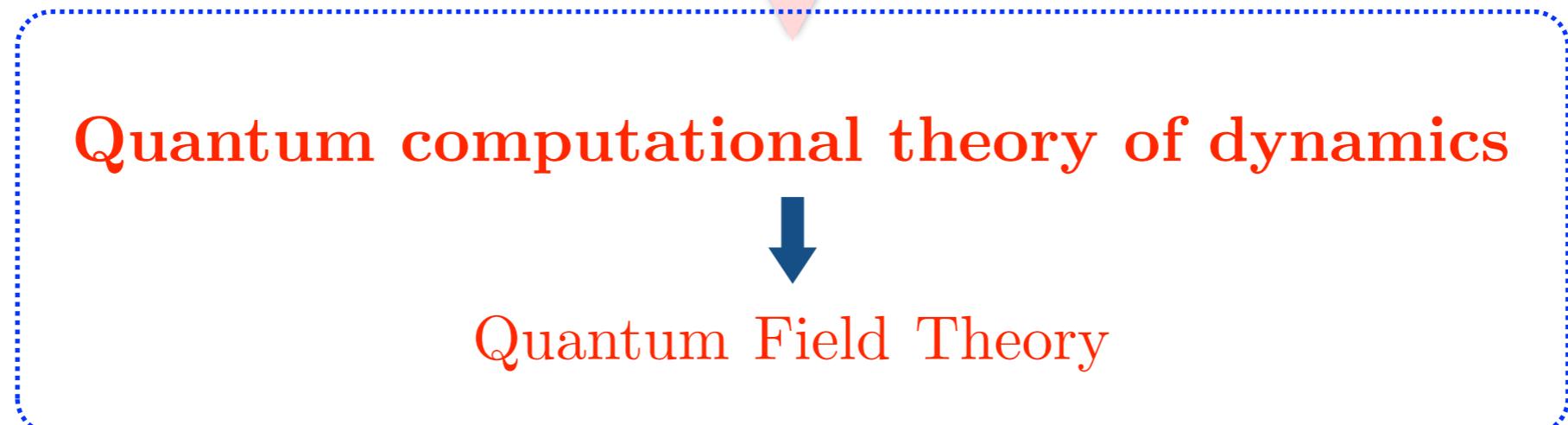
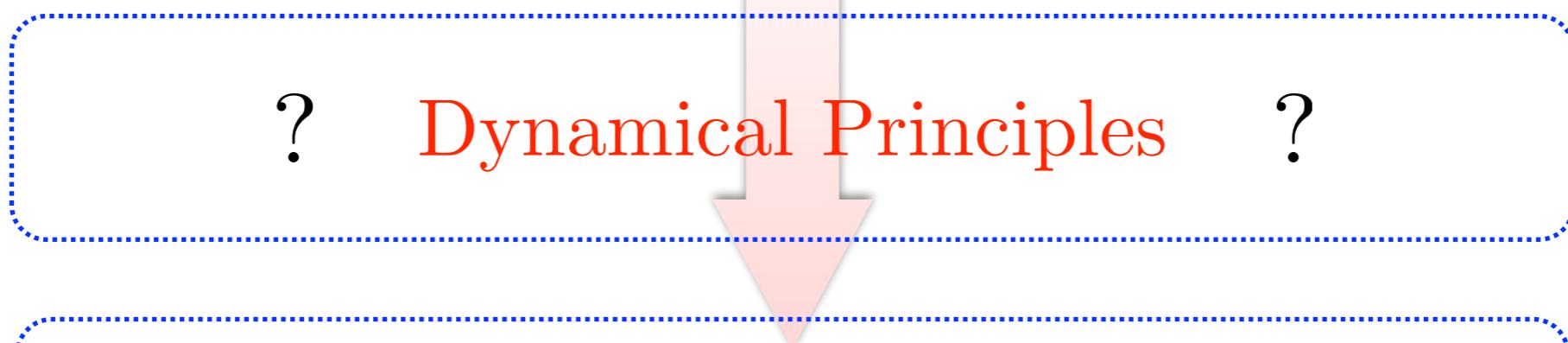
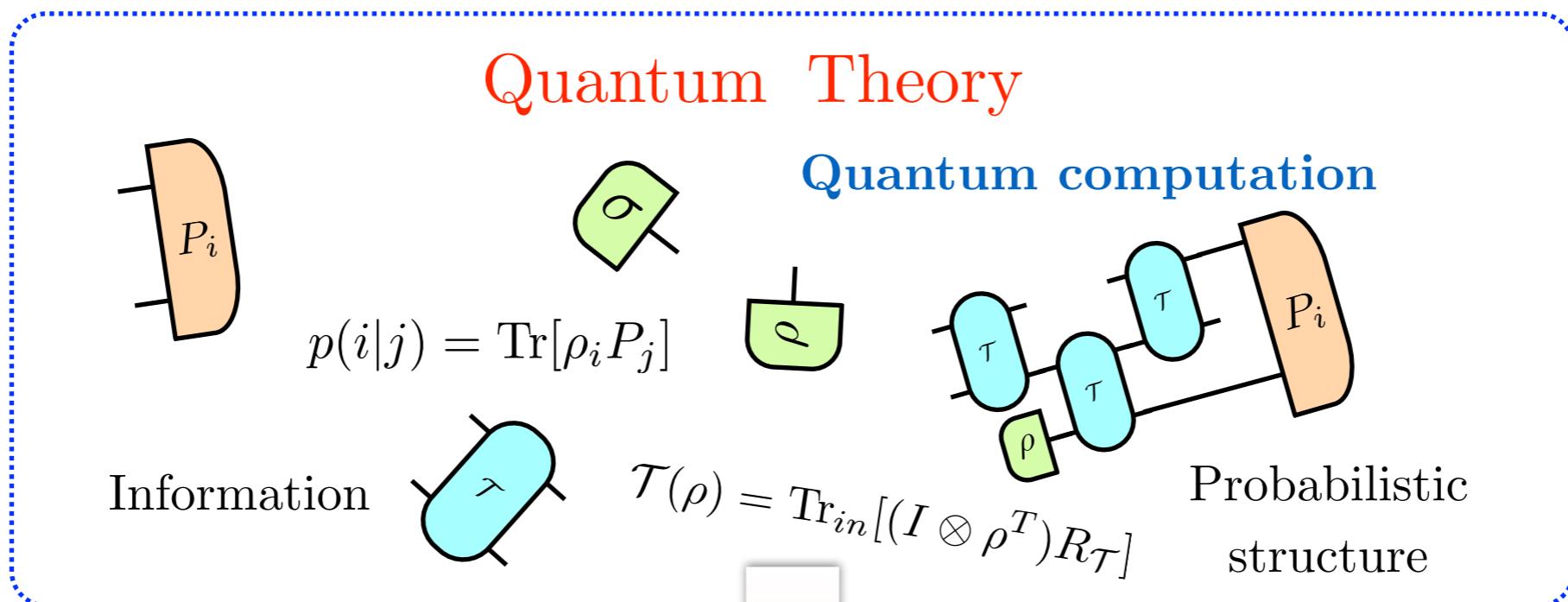
Reconstruction of Quantum Field Theory



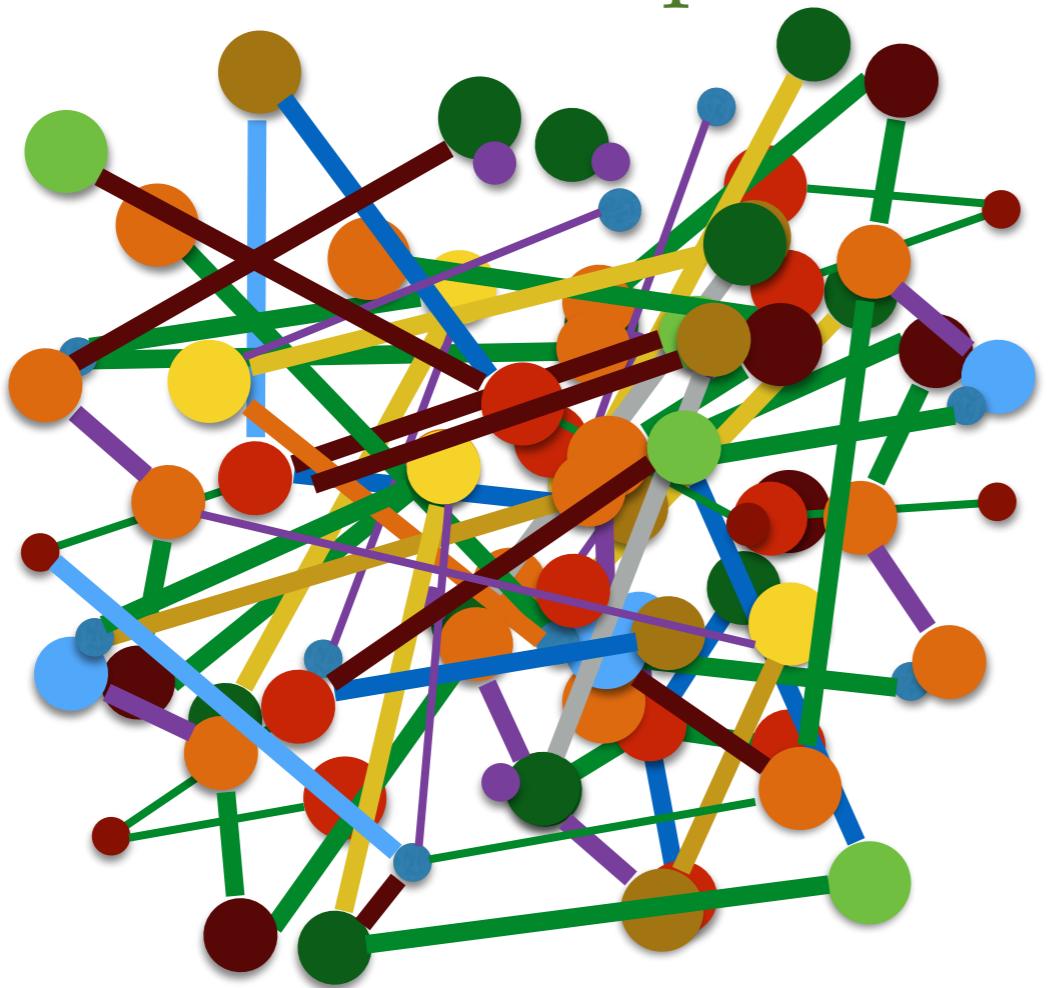
Reconstruction of Quantum Field Theory



Reconstruction of Quantum Field Theory



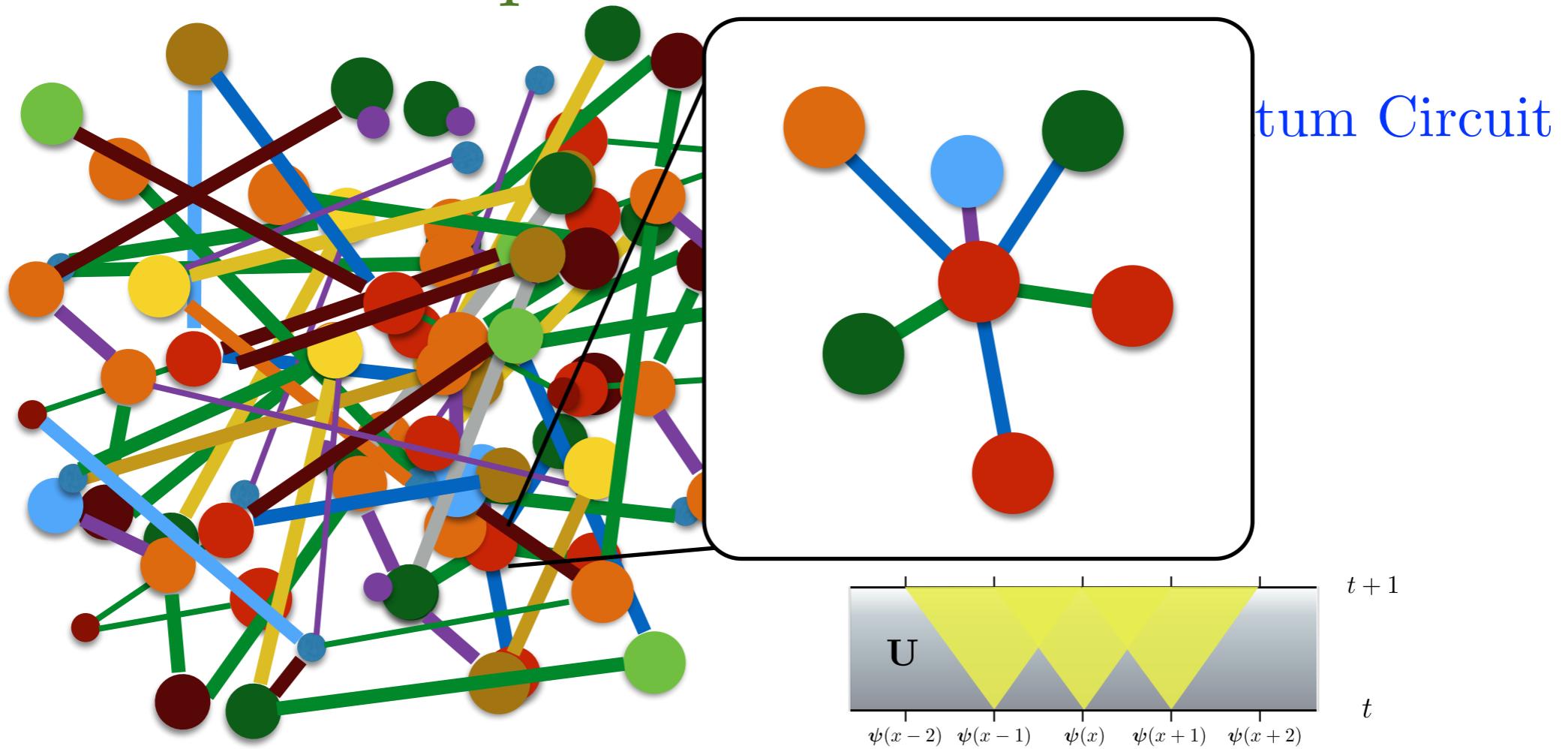
What kind of computer?



Quantum Circuit

Rules of the game \iff axioms

What kind of computer?



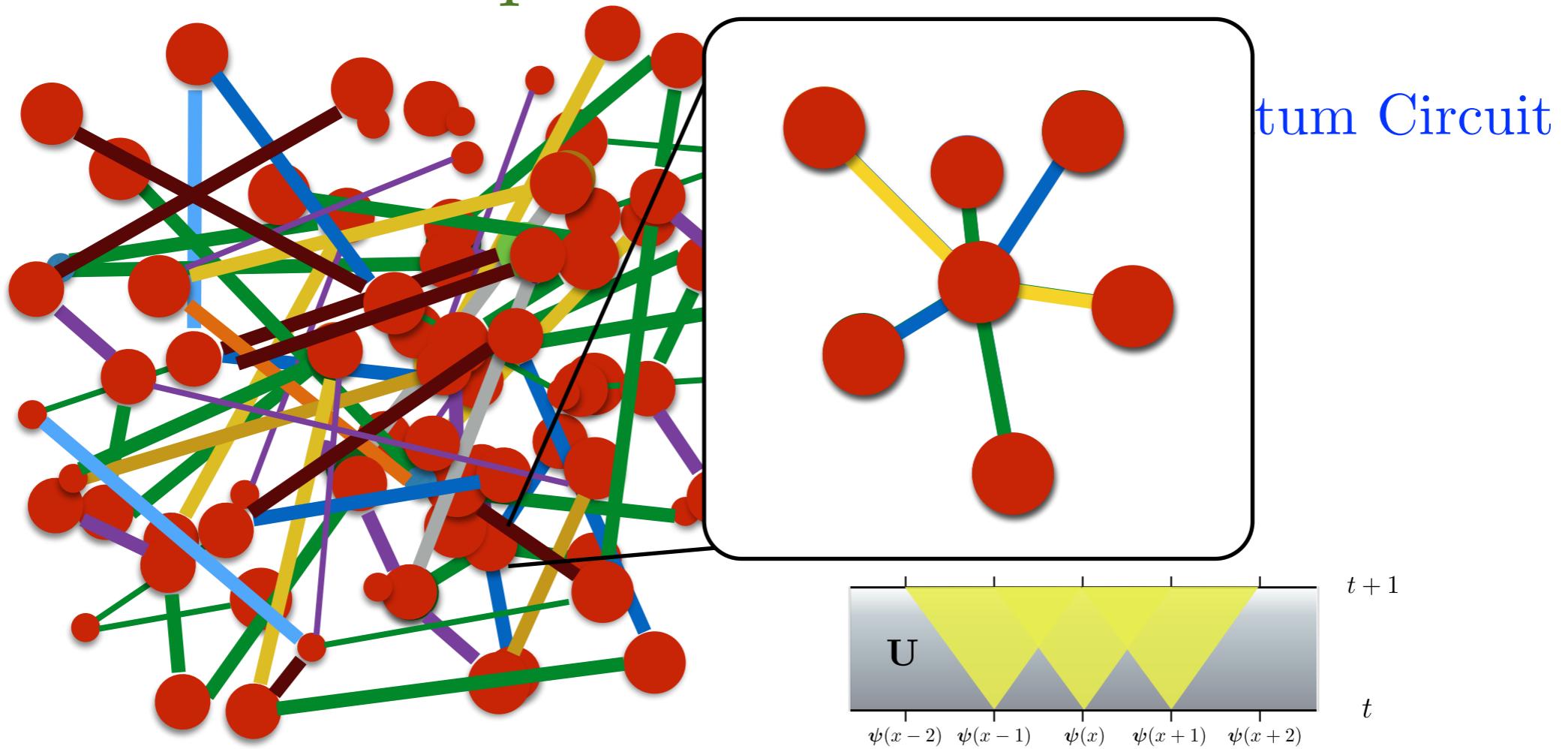
Rules of the game \iff axioms

“[...] everything that happens in a finite volume of space and time would have to be exactly analyzable with a finite numbers of logical operations” R. Feynman

Each system interacts with a finite number of neighbors: **locality**

Reversible Quantum Computation: **unitary evolution**

What kind of computer?



Rules of the game \iff axioms

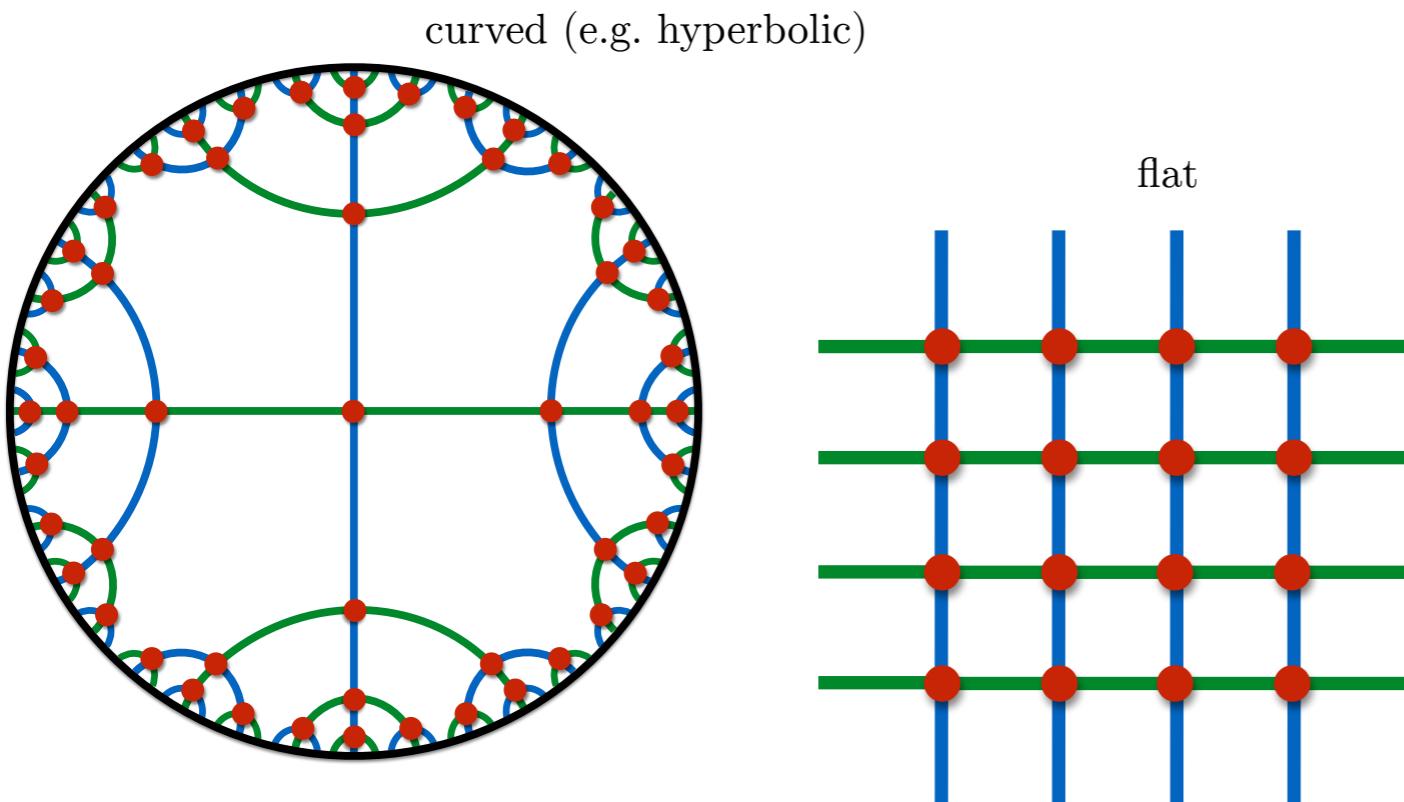
“[...] everything that happens in a finite volume of space and time would have to be exactly analyzable with a finite numbers of logical operations” R. Feynman

Each system interacts with a finite number of neighbors: **locality**

Reversible Quantum Computation: **unitary evolution**

All the nodes are equivalent: **homogeneity**

What kind of computer?



Quantum Circuit



Quantum Cellular
Automaton
on a
Cayley Graph

B. Schumacher, R.F. Werner
e-print arXiv:0405174.

Rules of the game \iff axioms

“[...] everything that happens in a finite volume of space and time would have to be exactly analyzable with a finite numbers of logical operations” R. Feynman

Each system interacts with a finite number of neighbors: **locality**

Reversible Quantum Computation: **unitary evolution**

All the nodes are equivalent: **homogeneity**

G. M. D'Ariano, P. Perinotti, Phys. Rev. A 90, 062106 (2014)

AB, G. M. D'Ariano, P. Perinotti, A. Tosini, Found. Phys. 45, 1137 (2015)

Towards free Quantum Field Theory

Cayley graph **quasi-isometrically** embeddable in flat space



The group G must be **virtually abelian** \longrightarrow We restrict to the abelian case \mathbb{Z}^3

Towards free Quantum Field Theory

Cayley graph **quasi-isometrically** embeddable in flat space



The group G must be **virtually abelian** \rightarrow We restrict to the abelian case \mathbb{Z}^3

Linear evolution $U\psi_i^s U^\dagger = \mathbf{U}_{i,j}^{s,r} \psi_j^r$ \longleftrightarrow Quantum Walk

field operators ψ_i^s ψ_j^r

site label i, j

internal degree of freedom (e.g. spinorial index)

unitary matrix $\mathbf{U}_{i,j}^{s,r}$

```
graph LR; Eq[U psi_i^s U^\dagger = U_{i,j}^{s,r} psi_j^r] --> FOP[psi_i^s]; Eq --> SL[i, j]; Eq --> IDOF["internal degree of freedom (e.g. spinorial index)"]; Eq --> UM[U_{i,j}^{s,r}]; Eq <--> QW[Quantum Walk];
```

Towards free Quantum Field Theory

Cayley graph **quasi-isometrically** embeddable in flat space



The group G must be **virtually abelian** \rightarrow We restrict to the abelian case \mathbb{Z}^3

Linear evolution $U\psi_i^s U^\dagger = \mathbf{U}_{i,j}^{s,r} \psi_j^r$ \longleftrightarrow **Quantum Walk**

Isotropy: the evolution must be covariant under a group of graph automorphisms

Towards free Quantum Field Theory

Cayley graph **quasi-isometrically** embeddable in flat space



The group G must be **virtually abelian** \rightarrow We restrict to the abelian case \mathbb{Z}^3

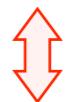
Linear evolution $U\psi_i^s U^\dagger = \mathbf{U}_{i,j}^{s,r} \psi_j^r$ \longleftrightarrow **Quantum Walk**

Isotropy: the evolution must be covariant under a group of graph automorphisms

Minimize the number of degrees of freedom: $s = 1, 2$

Towards free Quantum Field Theory

Cayley graph **quasi-isometrically** embeddable in flat space



The group G must be **virtually abelian** \rightarrow We restrict to the abelian case \mathbb{Z}^3

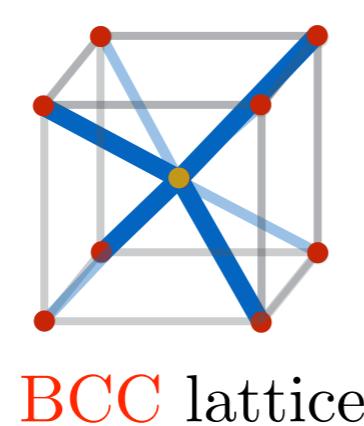
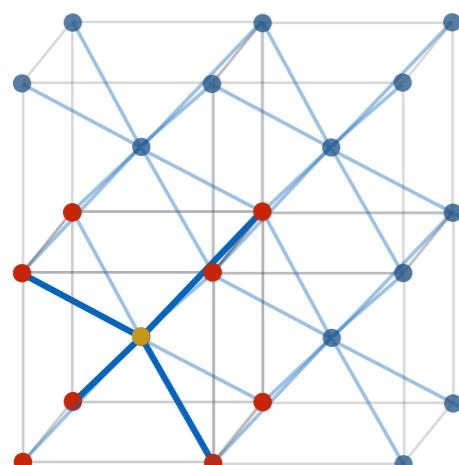
Linear evolution $U\psi_i^s U^\dagger = \mathbf{U}_{i,j}^{s,r} \psi_j^r$ \longleftrightarrow **Quantum Walk**

Isotropy: the evolution must be covariant under a group of graph automorphisms

Minimize the number of degrees of freedom: $s = 1, 2$



Weyl Quantum Walk



$$\mathbf{U} = \sum_{h \in S} T_h \otimes \mathbf{U}_h$$

generators

translations

Fourier

$$\mathbf{U} = \int_B d^3k |\mathbf{k}\rangle \langle \mathbf{k}| \otimes \mathbf{U}(\mathbf{k})$$

Brillouin zone

Momentum cutoff

Towards free Quantum Field Theory

Cayley graph **quasi-isometrically** embeddable in flat space



The group G must be **virtually abelian** \rightarrow We restrict to the abelian case \mathbb{Z}^3

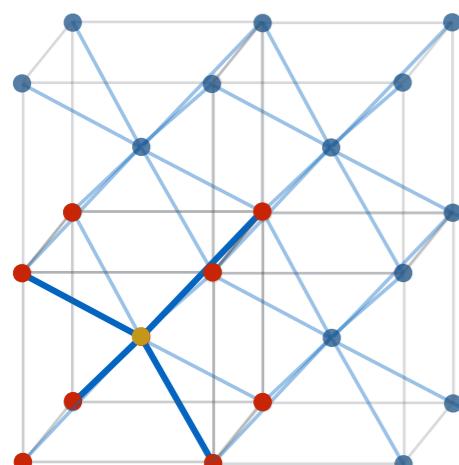
Linear evolution $U\psi_i^s U^\dagger = \mathbf{U}_{i,j}^{s,r} \psi_j^r$ \longleftrightarrow **Quantum Walk**

Isotropy: the evolution must be covariant under a group of graph automorphisms

Minimize the number of degrees of freedom: $s = 1, 2$



Weyl Quantum Walk



BCC lattice

$$\mathbf{U} = \sum_{h \in S} T_h \otimes \mathbf{U}_h$$

Fourier

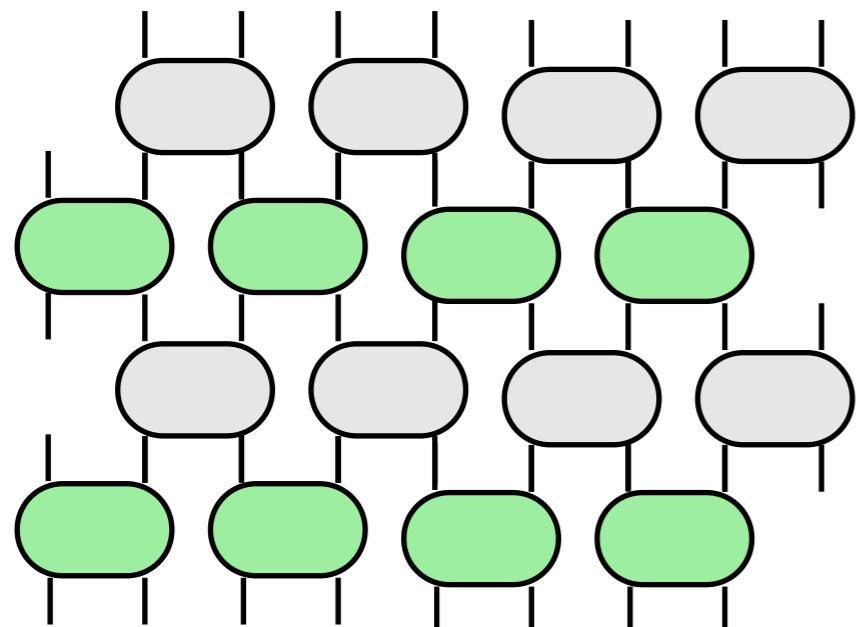
$$\mathbf{U} = \int_B d^3k |\mathbf{k}\rangle \langle \mathbf{k}| \otimes \mathbf{U}(\mathbf{k})$$

$k \ll 1$

$$\sigma^\mu k_\mu \psi = 0$$

Weyl equation

Towards free Quantum Field Theory: 1+1 Dirac QW



(1+1)-dimensional case: 1D Dirac evolution

$$U = \begin{pmatrix} nS & -im \\ -im & nS^\dagger \end{pmatrix} \quad S\psi(x) = \psi(x+1) \quad n^2 + m^2 = 1,$$

$$0 \leq m \leq 1$$

$$\psi(x) = \begin{pmatrix} \psi^R(x) \\ \psi^L(x) \end{pmatrix}$$

bounded rest mass

$$U(k) = \exp(-iH_A(k))$$

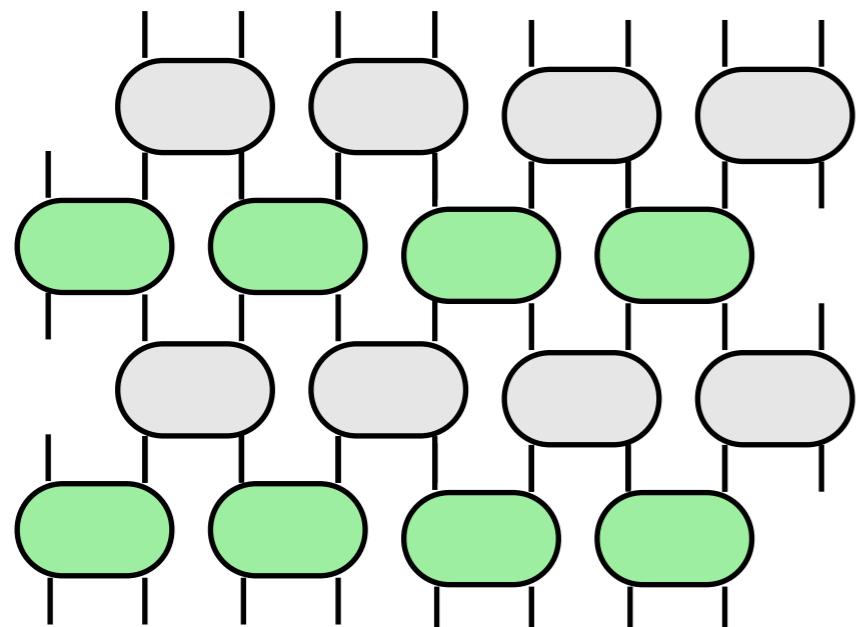
G. M. D'Ariano, P. Perinotti e-print arXiv:1306.1934.

AB, G. M. D'Ariano, P. Perinotti, e-print arXiv:1407.6928.

AB, G. M. D'Ariano, A. Tosini, e-print arXiv:1212.2839.

AB, G. M. D'Ariano, A. Tosini, Phys. Rev. A 88, 032301 (2013).

Towards free Quantum Field Theory: 1+1 Dirac QW



(1+1)-dimensional case: 1D Dirac evolution

$$U = \begin{pmatrix} nS & -im \\ -im & nS^\dagger \end{pmatrix} \quad S\psi(x) = \psi(x+1) \quad n^2 + m^2 = 1,$$

$$0 \leq m \leq 1$$

$$\psi(x) = \begin{pmatrix} \psi^R(x) \\ \psi^L(x) \end{pmatrix}$$

bounded rest mass

$$U(k) = \exp(-i\mathbf{H}_A(k))$$

$$\mathbf{H}_A(k) \xrightarrow{m, k \rightarrow 0} \boxed{\mathbf{H}_D(k)} + O(m^2k)$$

$$\mathbf{H}_D(k) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix}$$

Dispersion relation

$$\cos^2(\omega_A) = (1 - m^2) \cos^2(k) \xrightarrow{m, k \rightarrow 0} \omega_A^2 - k^2 = m^2$$

For small k and m we recover the usual physics

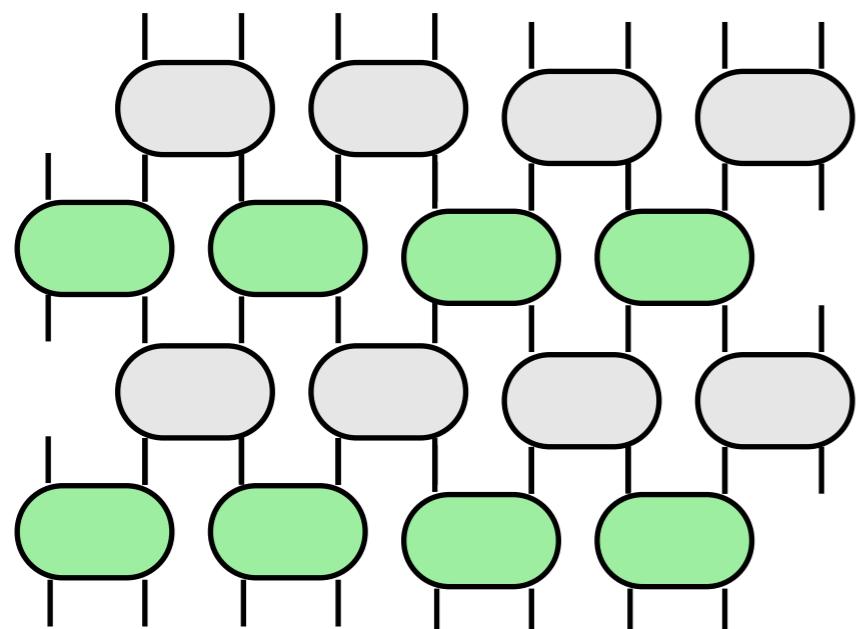
G. M. D'Ariano, P. Perinotti e-print arXiv:1306.1934.

AB, G. M. D'Ariano, P. Perinotti, e-print arXiv:1407.6928.

AB, G. M. D'Ariano, A. Tosini, e-print arXiv:1212.2839.

AB, G. M. D'Ariano, A. Tosini, Phys. Rev. A 88, 032301 (2013).

Towards free Quantum Field Theory: 1+1 Dirac QW



(1+1)-dimensional case: 1D Dirac evolution

$$U = \begin{pmatrix} nS & -im \\ -im & nS^\dagger \end{pmatrix} \quad S\psi(x) = \psi(x+1) \quad n^2 + m^2 = 1,$$

$$0 \leq m \leq 1$$

$$\psi(x) = \begin{pmatrix} \psi^R(x) \\ \psi^L(x) \end{pmatrix}$$

bounded rest mass

$$U(k) = \exp(-i\mathbf{H}_A(k))$$

$$\mathbf{H}_A(k) \xrightarrow{m, k \rightarrow 0} \boxed{\mathbf{H}_D(k)} + O(m^2k)$$

$$\mathbf{H}_D(k) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix}$$

Dispersion relation

$$\cos^2(\omega_A) = (1 - m^2) \cos^2(k) \xrightarrow{m, k \rightarrow 0} \omega_A^2 - k^2 = m^2$$

For small k and m we recover the usual physics

Generalization to 3+1 dimensions and
to free Maxwell's equations is possible

G. M. D'Ariano, P. Perinotti e-print arXiv:1306.1934.

AB, G. M. D'Ariano, P. Perinotti, e-print arXiv:1407.6928.

AB, G. M. D'Ariano, A. Tosini, e-print arXiv:1212.2839.

AB, G. M. D'Ariano, A. Tosini, Phys. Rev. A 88, 032301 (2013).

QCA and Lorentz transformations

Quantum Cellular Automata

$$m, k \rightarrow 0$$

Quantum Field Theory
Lorentz invariant equations

QCA and Lorentz transformations

Quantum Cellular Automata

$m, k \rightarrow 0$

Quantum Field Theory
Lorentz invariant equations

The observer is the same!
Boosted observer?

Relativity 

QCA and Lorentz transformations

Quantum Cellular Automata

$m, k \rightarrow 0$

Quantum Field Theory
Lorentz invariant equations

The observer is the same!
Boosted observer?



Relativity

1D Dirac QW dispersion relation

$$\cos^2(\omega) = (1 - m^2) \cos^2(k)$$

classical kinematics
emergent from the automaton

non Lorentz invariant

Lorentz transformation

$$\begin{pmatrix} \omega' \\ k' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} \omega \\ k \end{pmatrix}$$

$$\gamma := \frac{1}{\sqrt{1 - \beta^2}}$$

Violations of Lorentz invariance
at ultra-relativistic scales

different
transformation

or

privileged
reference frame

Deformed relativity

A simple speculation
from Quantum Gravity



In whose reference frame is the Planck energy the threshold for new phenomena?

Preserve relativity principle
Lorentz group

AND

invariant energy scale

Deformed relativity

A simple speculation
from Quantum Gravity



In whose reference frame is the Planck
energy the threshold for new phenomena?

Preserve relativity principle
Lorentz group

AND

invariant energy scale

Modify the action of Lorentz group

non-linear action in **momentum** space

$$L_\beta^D := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D},$$

$$L_\beta = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}$$

momentum space
is more fundamental

\mathcal{D} is a non-linear map

- $J_{\mathcal{D}}(0, 0) = I$

- singular point invariant energy

- invertible

Deformed relativity

A simple speculation
from Quantum Gravity



In whose reference frame is the Planck energy the threshold for new phenomena?

Preserve relativity principle
Lorentz group

AND

invariant energy scale

Modify the action of Lorentz group

non-linear action in **momentum** space

$$L_\beta^D := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D},$$

$$L_\beta = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}$$

momentum space
is more fundamental

which \mathcal{D} ?

\mathcal{D} is a non-linear map

- $J_{\mathcal{D}}(0, 0) = I$

- singular point



invariant
energy

- invertible

G. Amelino-Camelia, Physics Letters B 510, 255 (2001).

J. Magueijo, L. Smolin, Phys. Rev. Lett. 88, 190403 (2002).

Deformed relativity and QW

Dirac QW dispersion relation

$$\cos^2(\omega) = (1 - m^2) \cos^2(k) \quad \xrightarrow{\text{blue arrow}} \quad \frac{\sin^2(\omega)}{\cos^2(k)} - \tan^2(k) = m^2$$

$$\tilde{\omega}^2 - \tilde{k}^2 = m^2$$

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

Deformed relativity and QW

Dirac QW dispersion relation

$$\cos^2(\omega) = (1 - m^2) \cos^2(k) \quad \xrightarrow{\text{blue arrow}} \quad \frac{\sin^2(\omega)}{\cos^2(k)} - \tan^2(k) = m^2$$

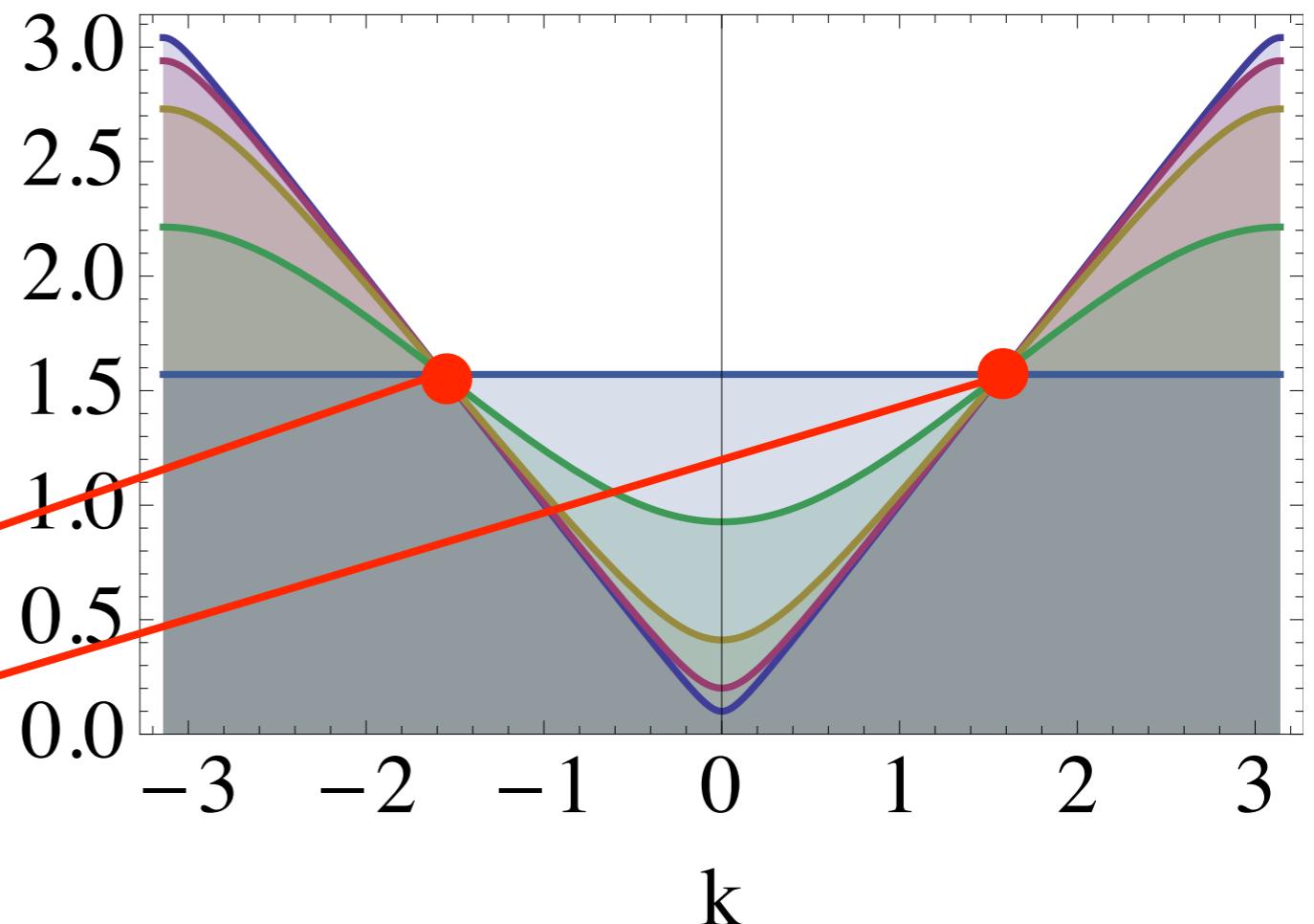
$$\tilde{\omega}^2 - \tilde{k}^2 = m^2$$

$$\mathcal{D} \begin{pmatrix} \omega \\ k \end{pmatrix} = \begin{pmatrix} \frac{\sin(\omega)}{\cos(k)} \\ \tan(k) \end{pmatrix}$$

$$-\frac{\pi}{2} \leq k \leq \frac{\pi}{2}$$

$$\omega(k)$$

$$\omega_{inv} = \frac{\pi}{2}$$



A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

Deformed relativity in position space

The model is defined in
the momentum space



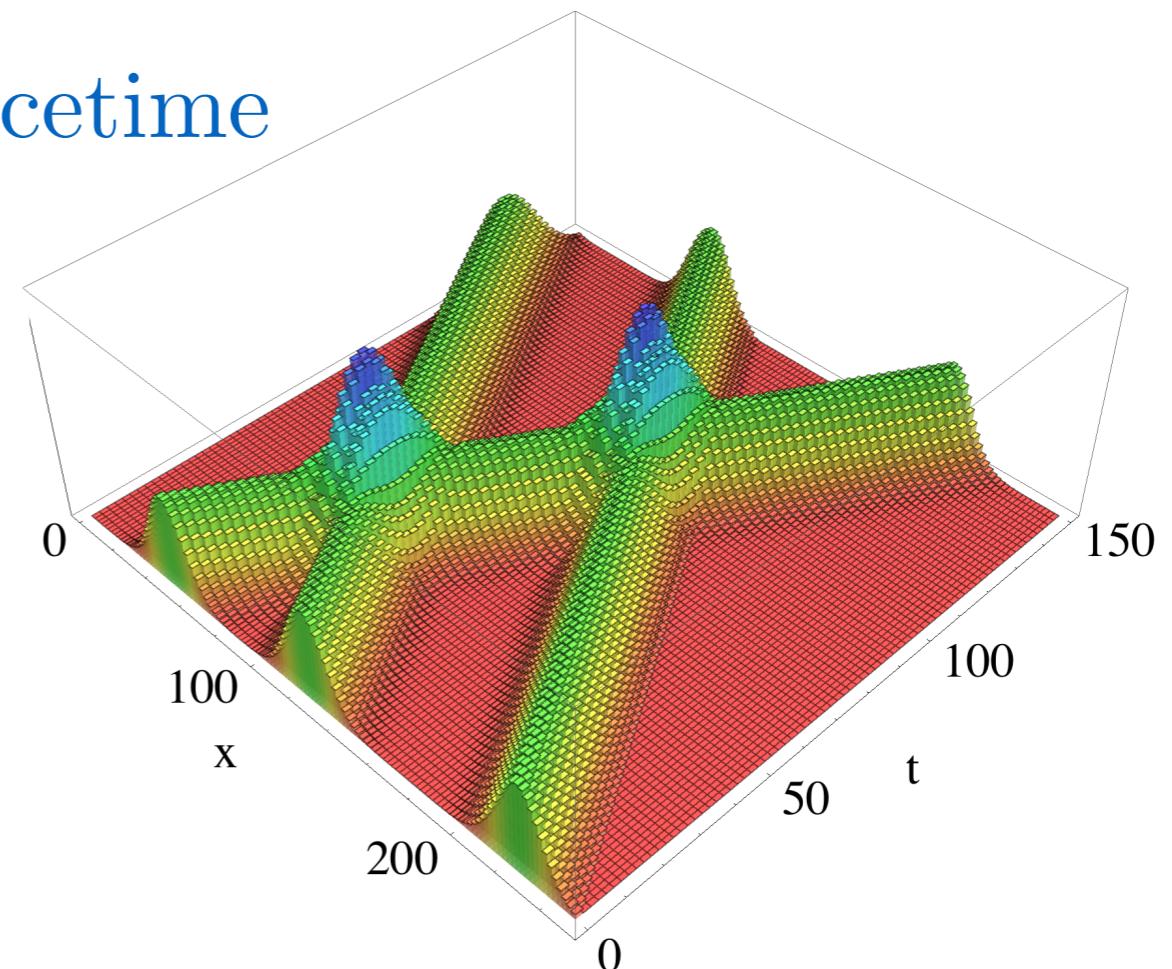
Transformations in the position space?

Operational toy model of spacetime

coincidence of wavepackets



point in spacetime



A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

Deformed relativity in position space

The model is defined in
the momentum space



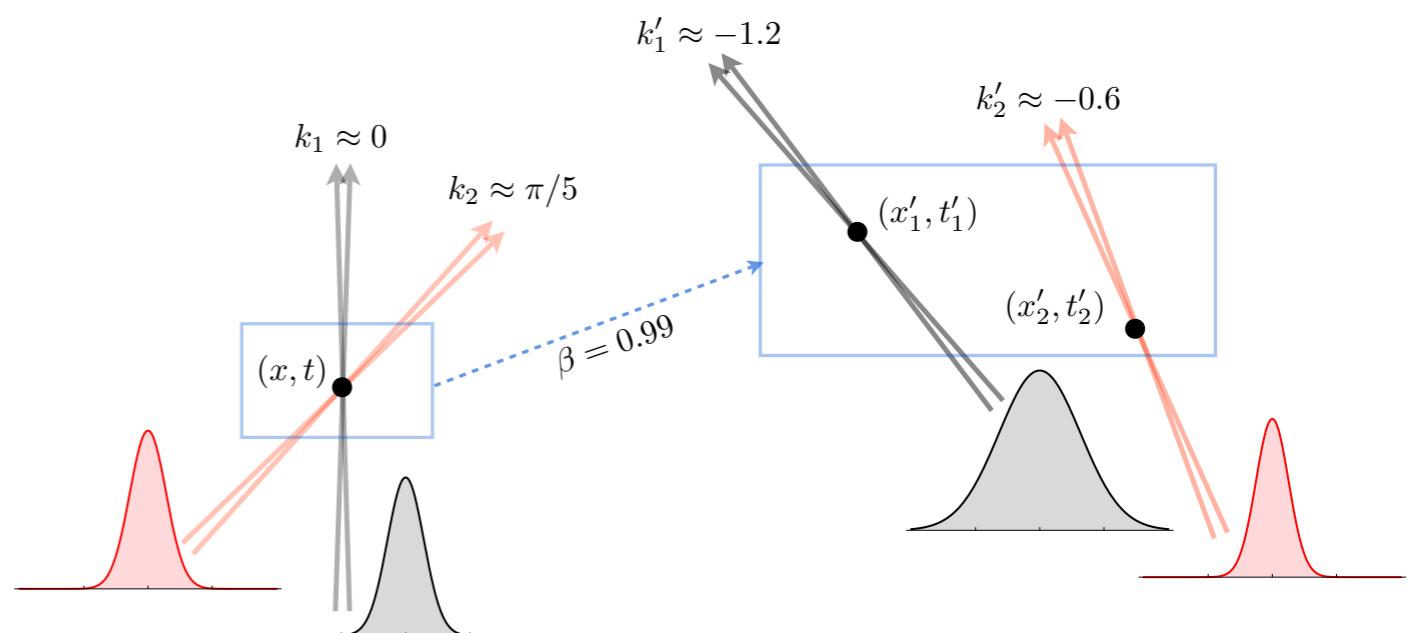
Transformations in the position space?

Operational toy model of spacetime

coincidence of wavepackets



point in spacetime



transformation of the coincidence points:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} \approx \begin{pmatrix} -\partial_{\omega'} k & \partial_{k'} k \\ \partial_{\omega'} \omega & -\partial_{k'} \omega \end{pmatrix}_{k'=k'_0} \begin{pmatrix} t \\ x \end{pmatrix}$$

momentum-dependent spacetime

Relative Locality

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

Deformed relativity in position space

Relative locality



Observer-dependent spacetime

“before”

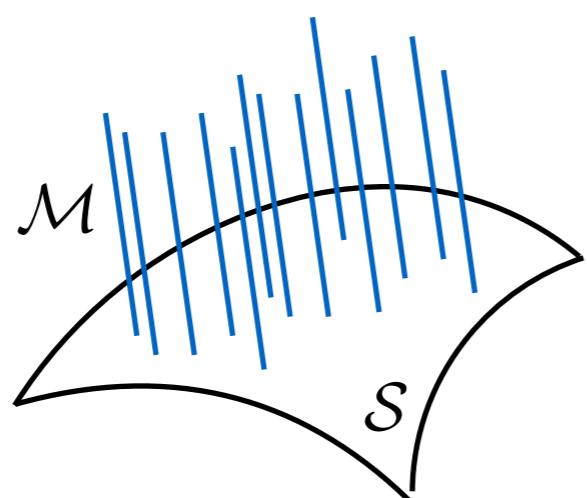
spacetime \mathcal{S}

“objective arena”

flat momentum space \mathcal{M}

phase space

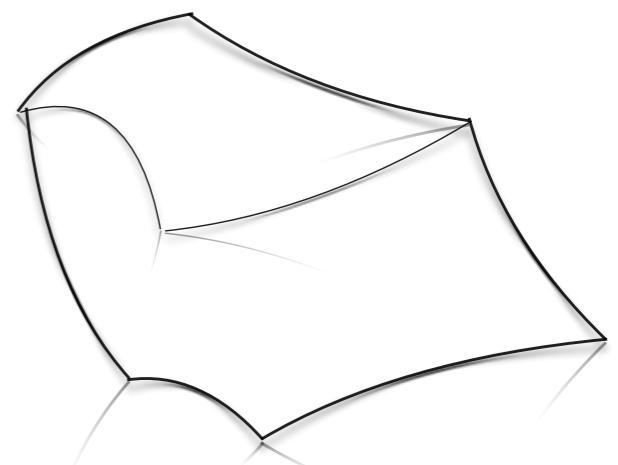
$$\mathcal{P} = \mathcal{T}^* \mathcal{S}$$



“after”

phase space

$$\mathcal{P} \neq \mathcal{T}^* \mathcal{S}$$



no canonical projection that gives a description of processes in spacetime

R. Schutzhold, W. G. Unruh, JETP Lett. 78, 431 (2003).

G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, Phys. Rev. D 84, 084010 (2011).

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

AB, G. M. D'Ariano, P. Perinotti, arXiv 1503.01017

A final overlook

Main idea

Quantum
theory



“Quantum computational
field theory”

Quantum “ab initio” theory of dynamics

A final overlook

Main idea

Quantum theory



“Quantum computational field theory”

Quantum “ab initio” theory of dynamics

QCA model

Free Dirac Field

Free QED
AB, G. M. D'Ariano, P. Perinotti
Annals Phys. 368 177-190 (2016)

Interactions
Energy?
Momentum?

A final overlook

Main idea

Quantum theory



“Quantum computational field theory”

Quantum “ab initio” theory of dynamics

QCA model

Free Dirac Field

Free QED
AB, G. M. D'Ariano, P. Perinotti
Annals Phys. 368 177-190 (2016)

Boost?

Interactions
Energy?
Momentum?

momentum space DSR

Deformed relativity

operational toy-model
of spacetime

emergent
spacetime

Thank you!

A final overlook

Not only foundations... QCA as a simulation tool

Discrete QCA model

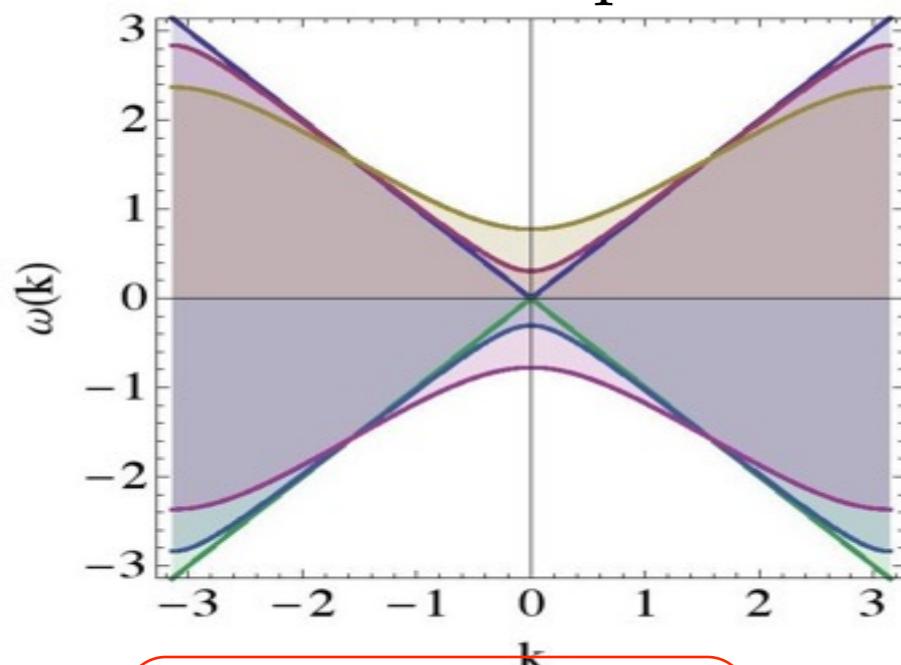
- discrete unitary evolution
- “strictly local”

\neq

Lattice Gauge theory

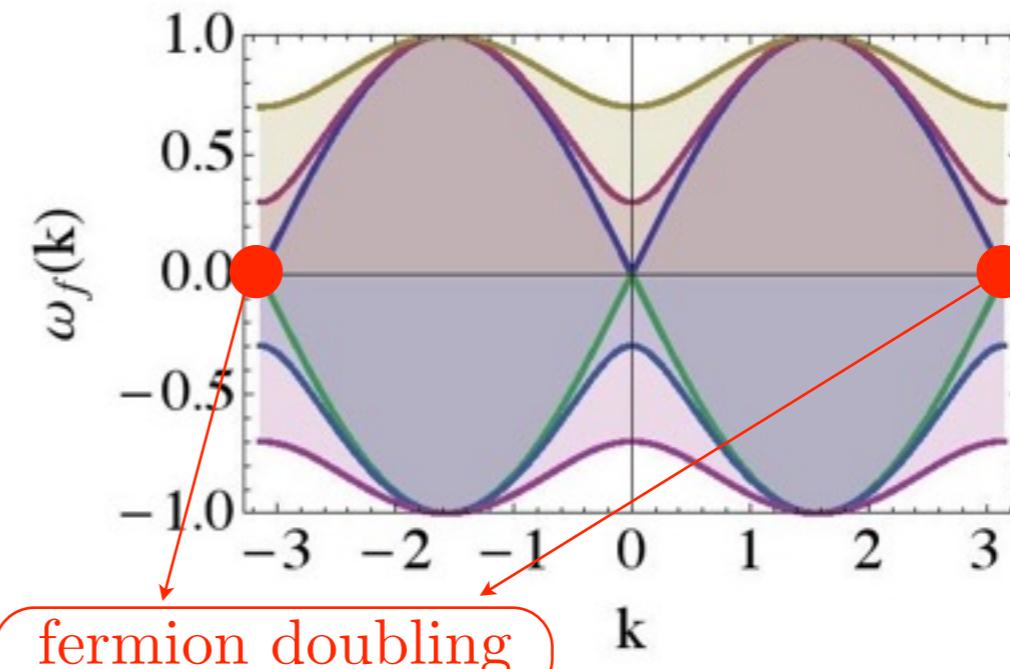
- discrete Lagrangian
- finite difference equations

automaton disp. relation



no fermion doubling

finite difference Dirac



I. Bialynicki-Birula, Phys.Rev. D 49, 6920 (1994).

Open problems

Interacting theory

relax linearity: a “true” QCA

Real space formulation

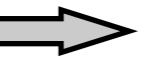
operational characterization of boosts

k-Poincare algebra and k-Minkowski

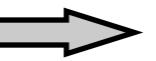
More general symmetries

Quantum Walks on non abelian graphs

New phenomenology

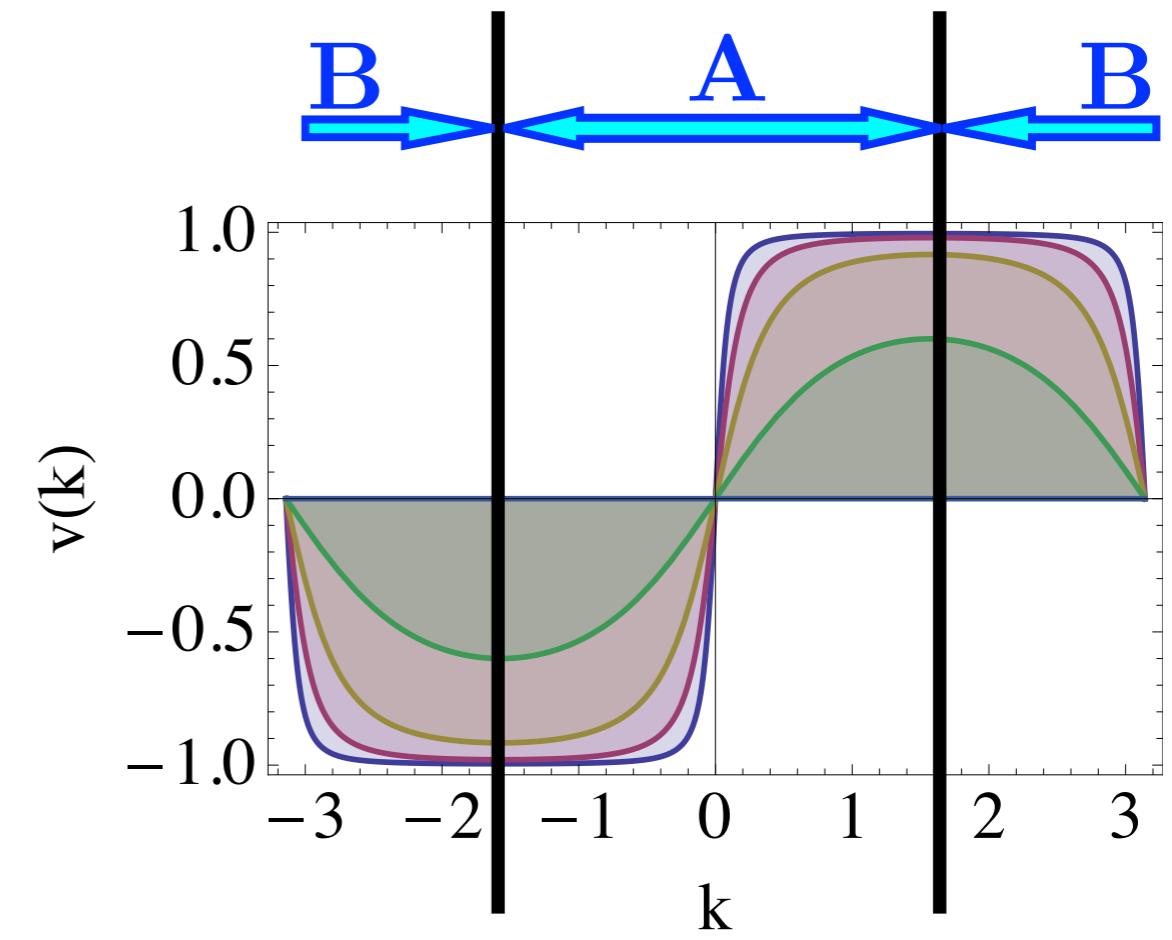
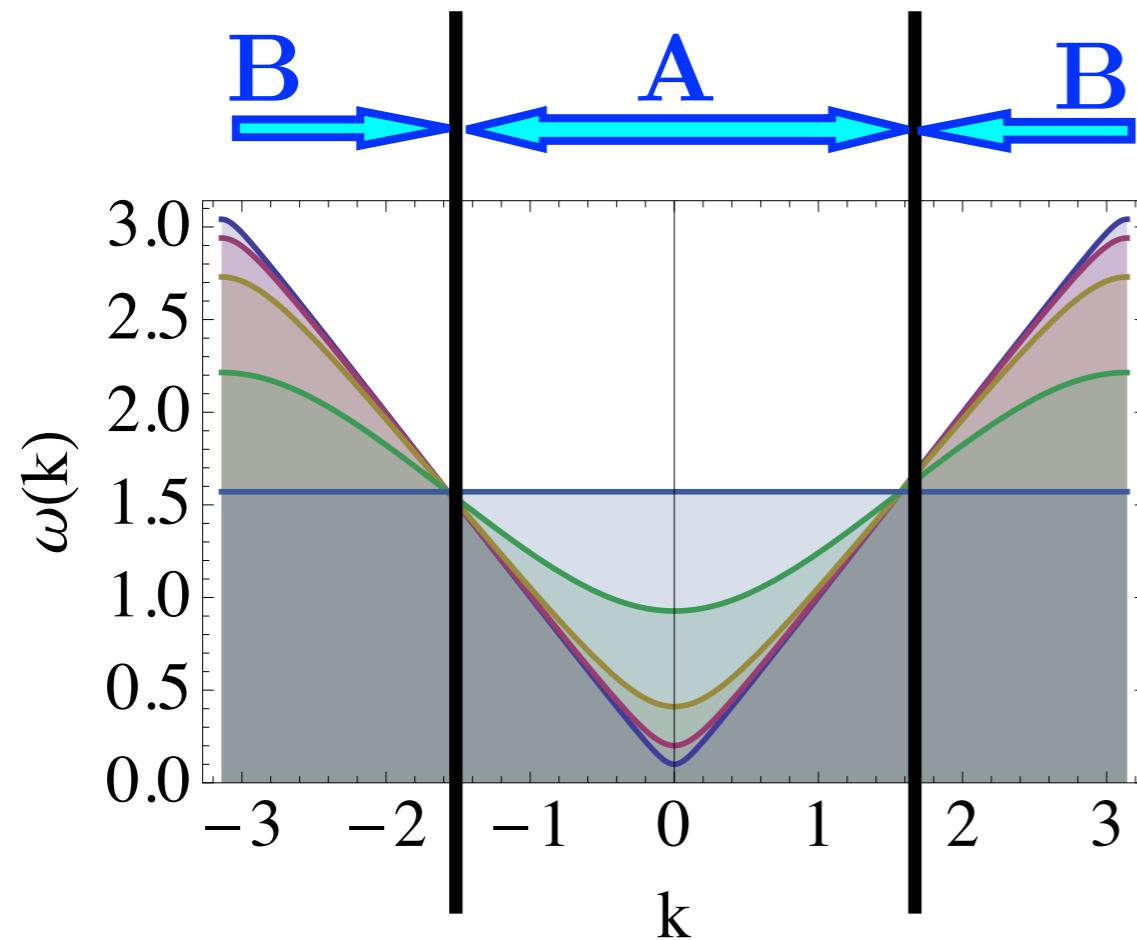
Modified Dispersion relation  light from distant astrophysical objects

G. Amelino-Camelia and L. Smolin
Phys. Rev. D 80, 084017 (2009)

Non-commutative spacetime  holographic noise

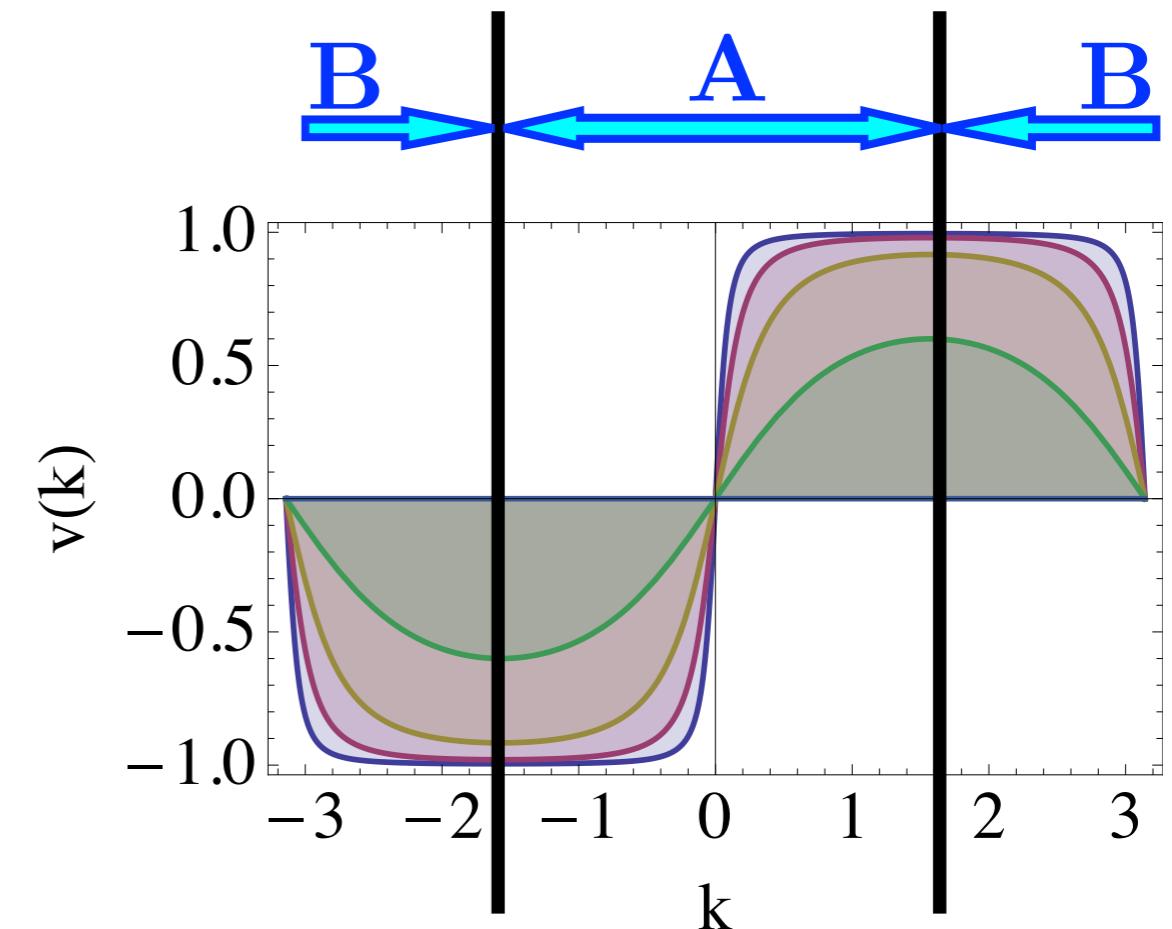
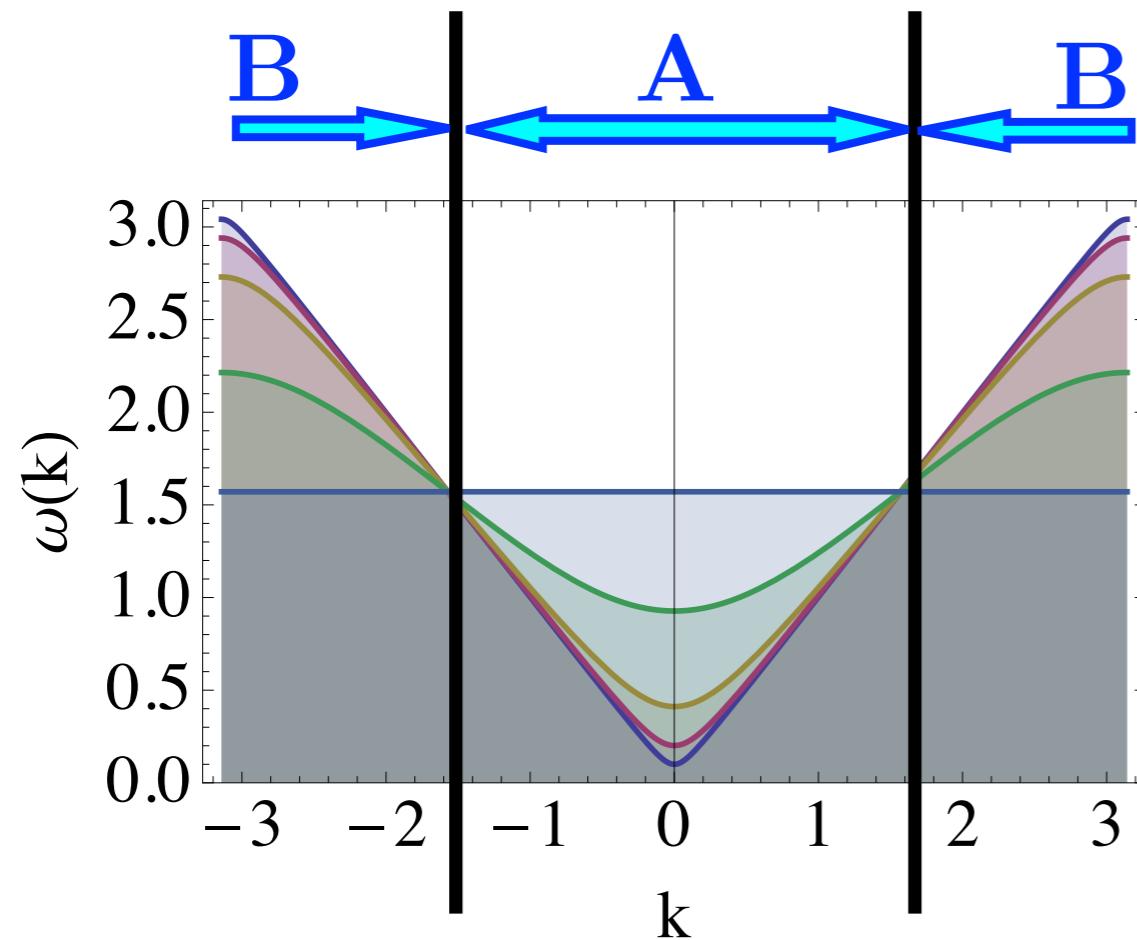
C. J. Hogan Phys. Rev. D 85, 064007 (2012)

Deformed relativity and QCA



A and **B** exhibits the same kinematics

Deformed relativity and QCA



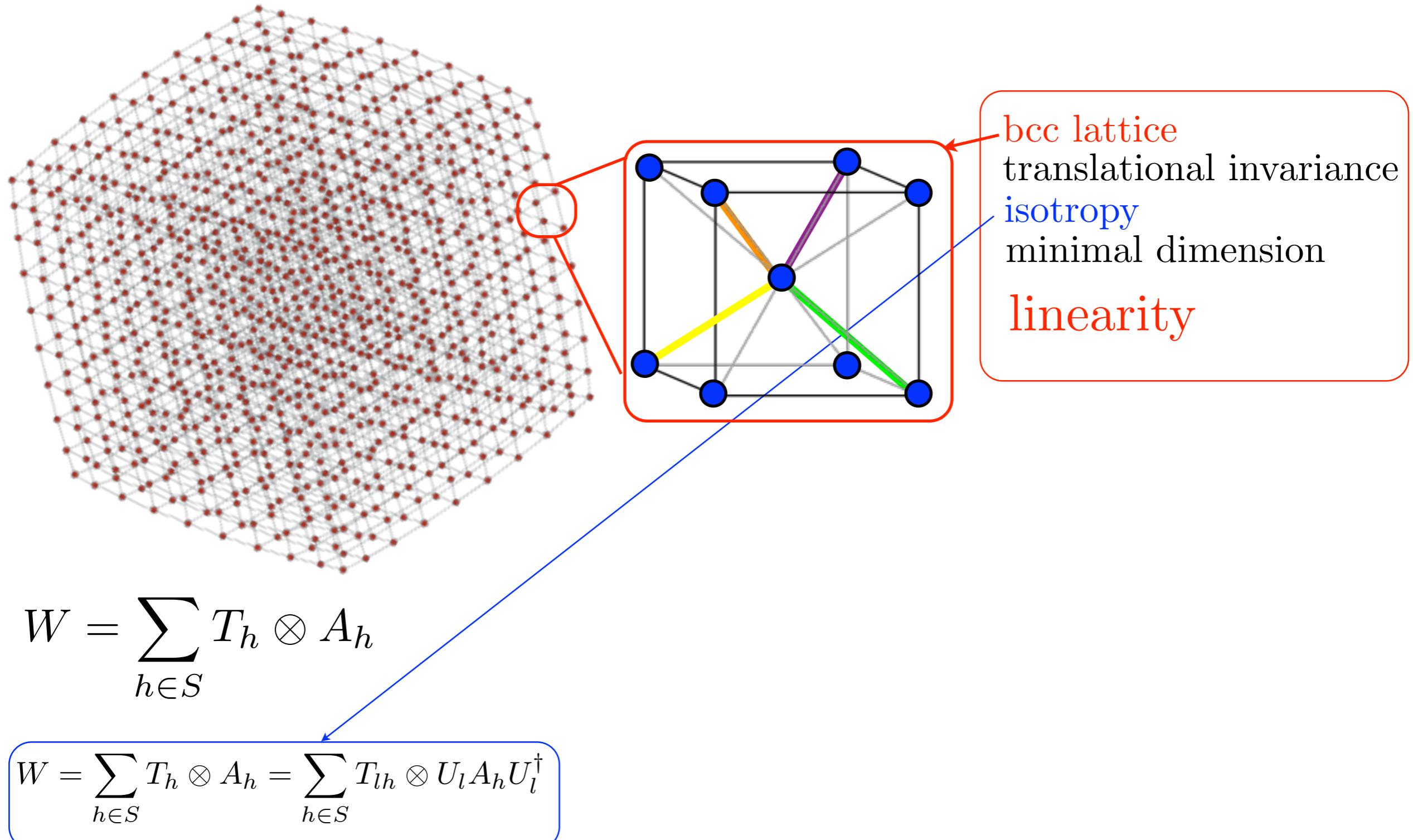
A and B exhibits the same kinematics

State transformation

$$|\psi\rangle = \int d\mu_k \hat{g}(k)|k\rangle \xrightarrow{L_\beta^D} \int d\mu_k \hat{g}(k)|k'\rangle = \int d\mu_{k'} \hat{g}(k(k'))|k'\rangle$$

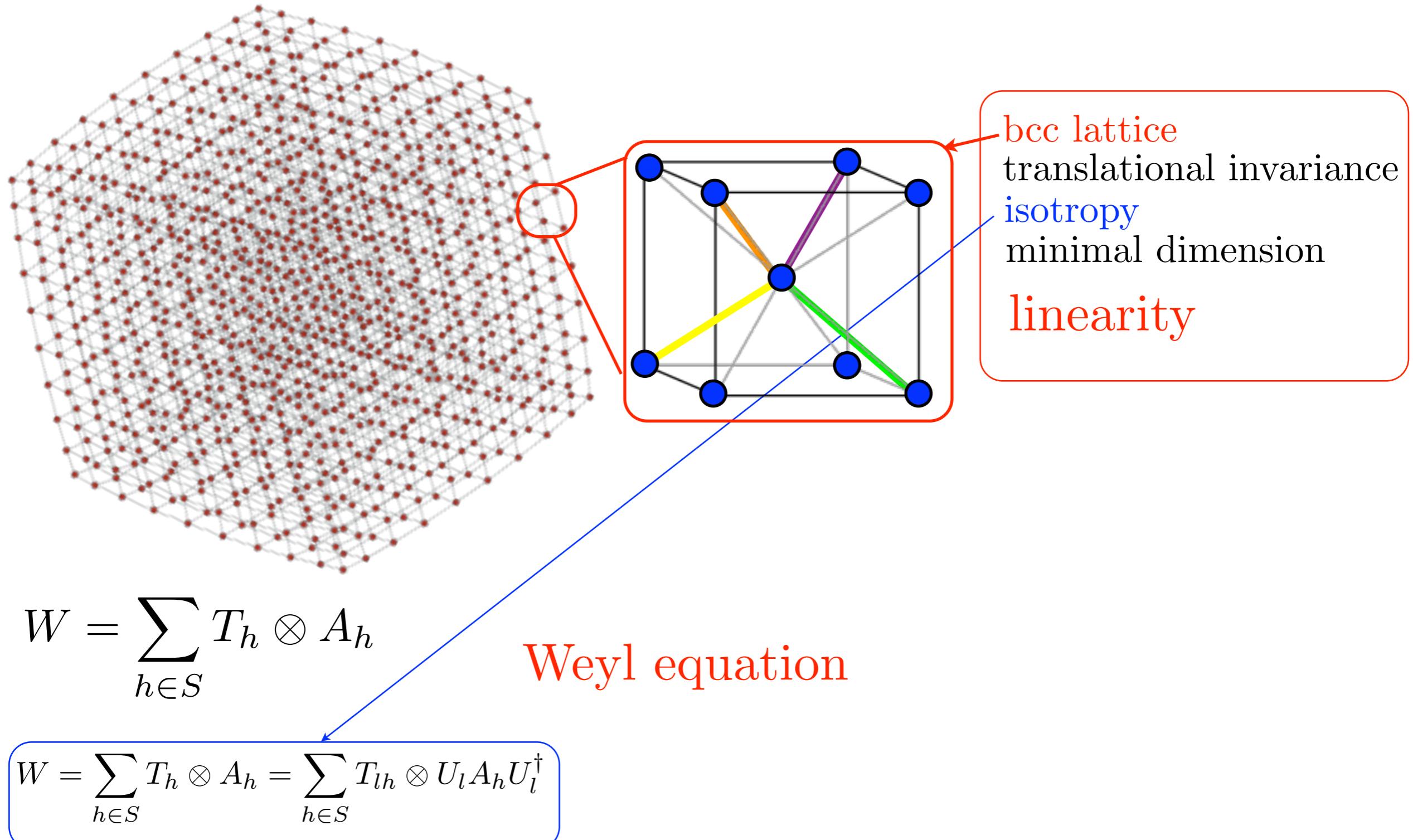
QCA for the Dirac field

(3+1)-dimensional case



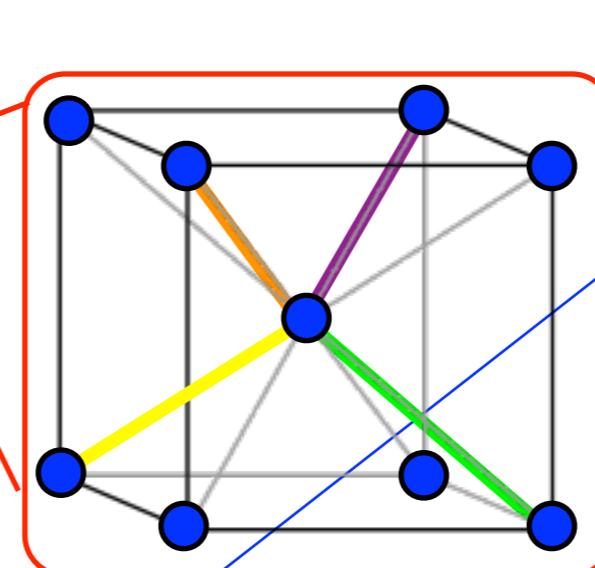
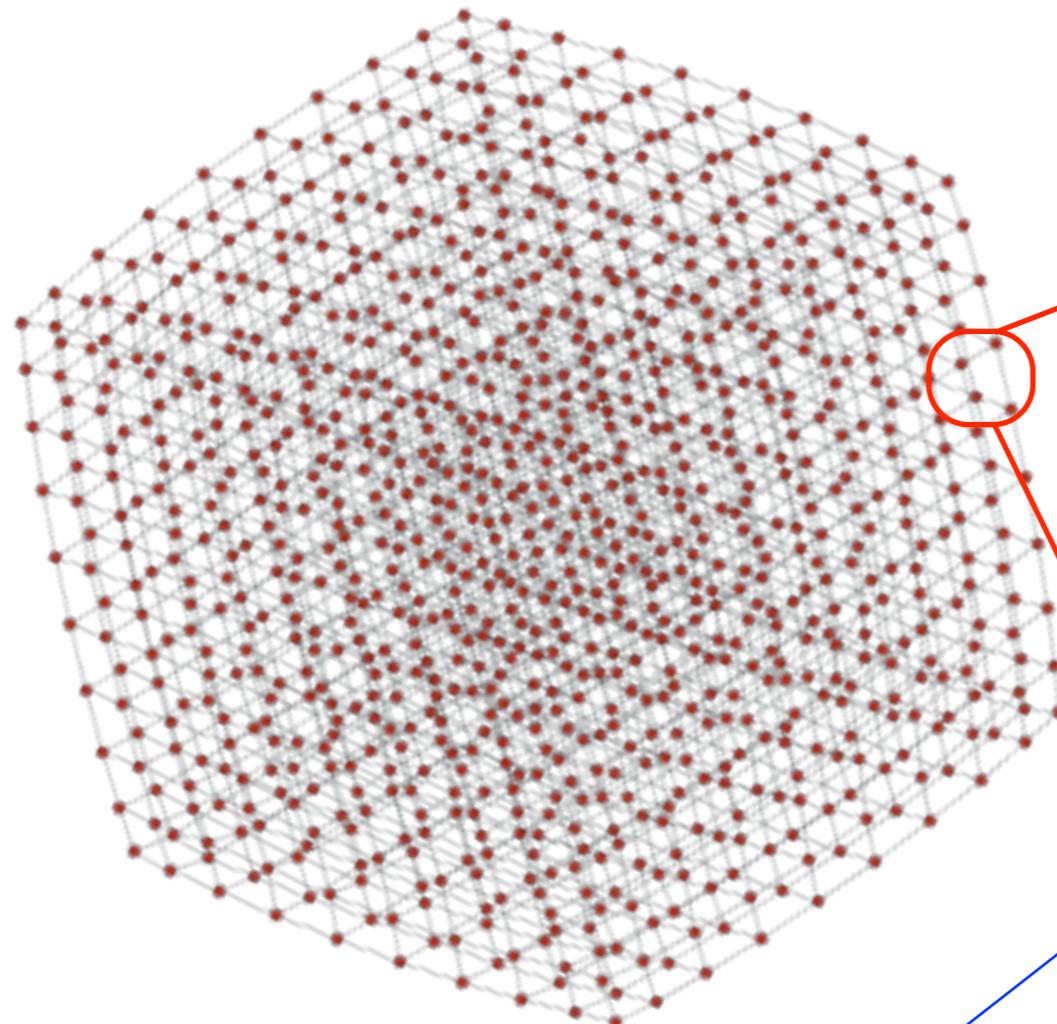
QCA for the Dirac field

(3+1)-dimensional case



QCA for the Dirac field

(3+1)-dimensional case



bcc lattice
translational invariance
isotropy
minimal dimension
linearity

$$W = \sum_{h \in S} T_h \otimes A_h$$

Weyl equation

$$W = \sum_{h \in S} T_h \otimes A_h = \sum_{h \in S} T_{lh} \otimes U_l A_h U_l^\dagger$$

2 Weyl + mass



Dirac (with spin)

A simple paradox from Quantum Gravity

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad E_P = \sqrt{\frac{\hbar c^5}{G}}$$

Threshold for quantum spacetime

A simple paradox from Quantum Gravity

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad E_P = \sqrt{\frac{\hbar c^5}{G}}$$

Threshold for quantum spacetime

BUT

length and energy are not Lorentz invariant

A simple paradox from Quantum Gravity

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \quad E_P = \sqrt{\frac{\hbar c^5}{G}}$$

Threshold for quantum spacetime

BUT

length and energy are not Lorentz invariant



In whose reference frame

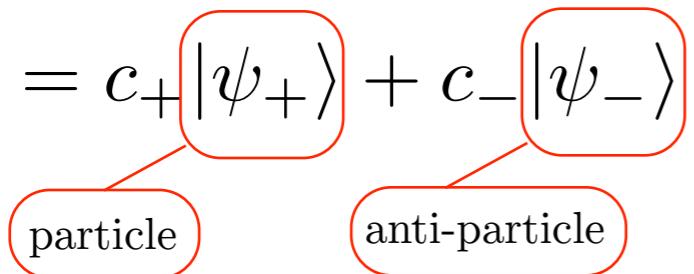
E_P is a threshold for new phenomena?

Give up relativity principle?

The 1D Dirac QCA 1-particle sector

Zitterbewegung

$$|\psi\rangle = c_+ |\psi_+\rangle + c_- |\psi_-\rangle$$

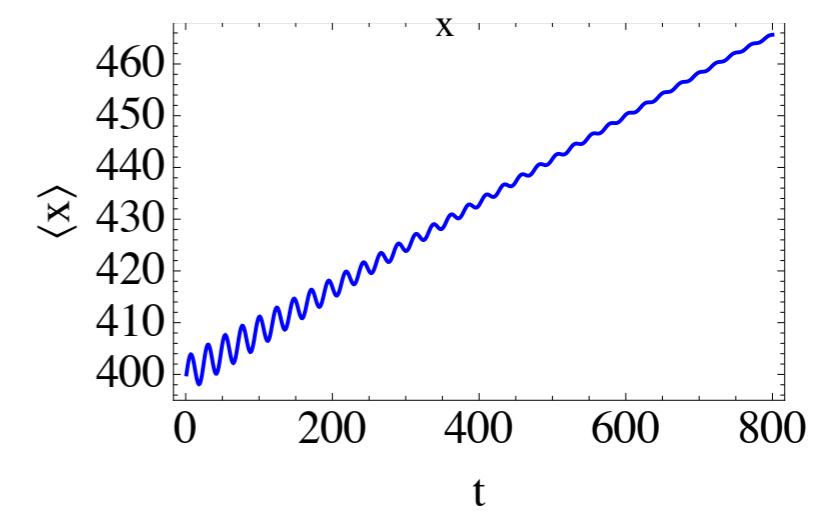
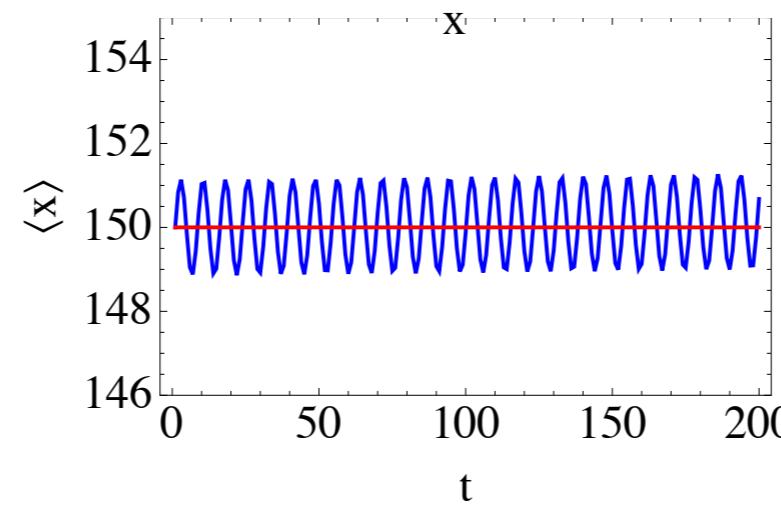
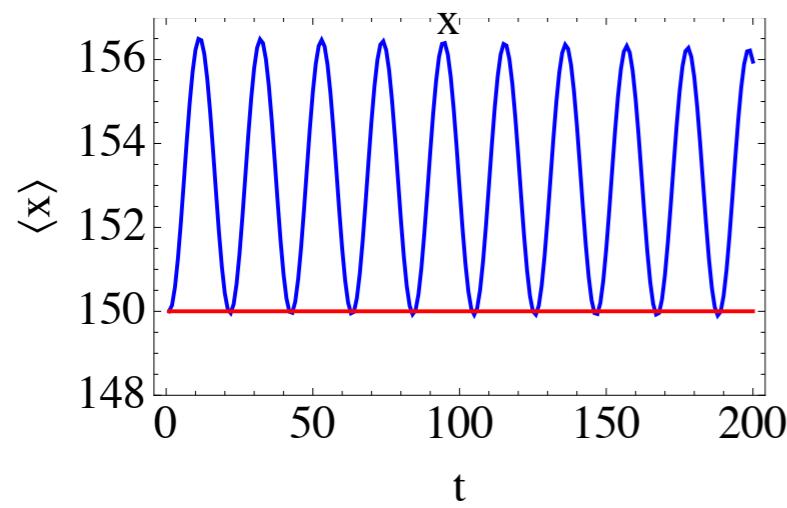
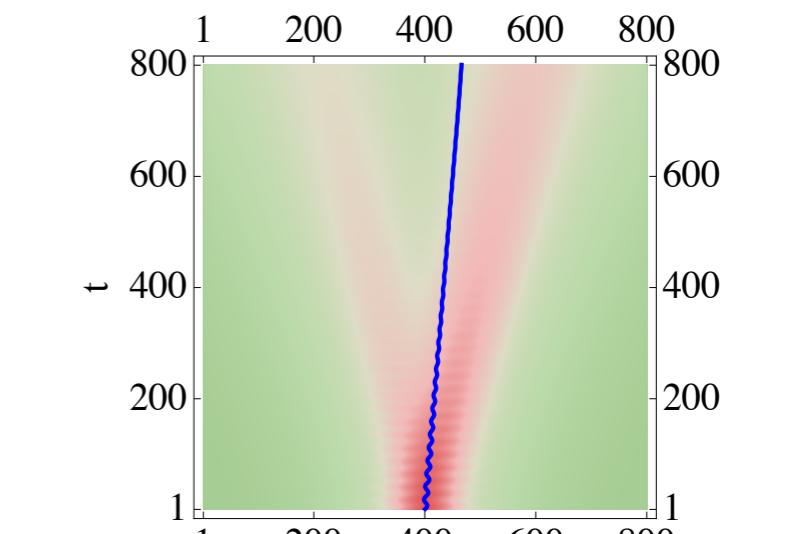
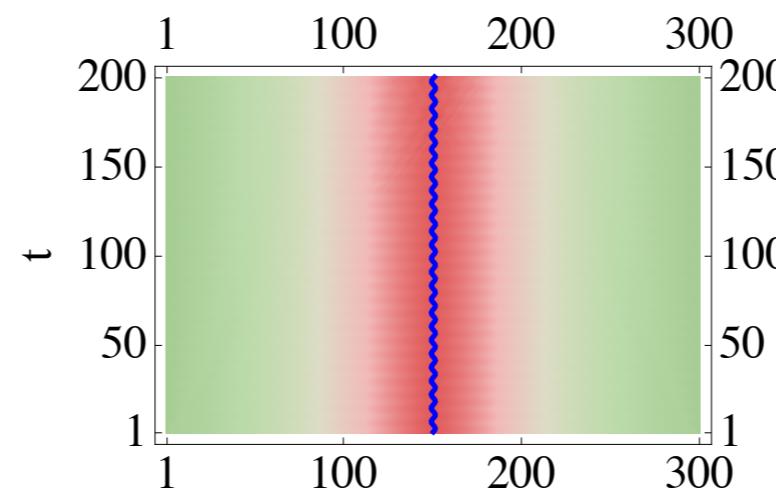
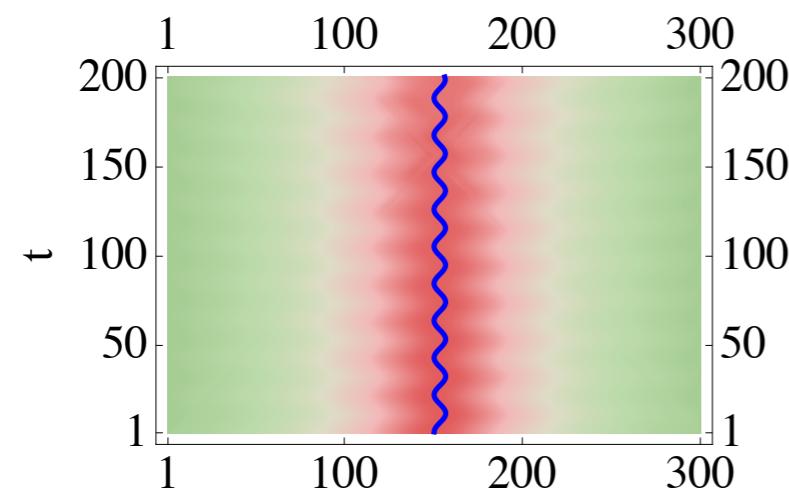


The 1D Dirac QCA 1-particle sector

Zitterbewegung

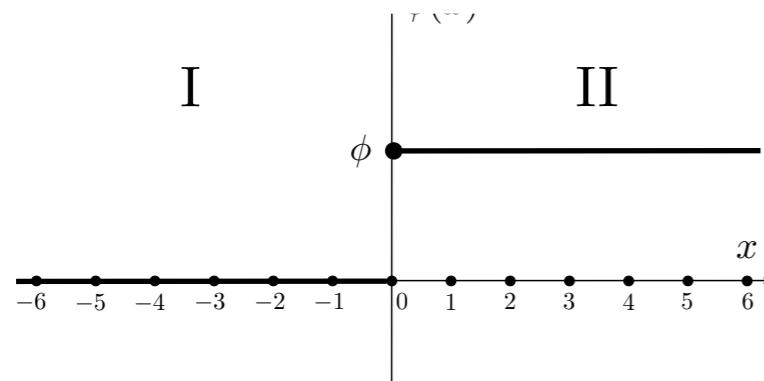
$$|\psi\rangle = c_+ |\psi_+\rangle + c_- |\psi_-\rangle$$

particle anti-particle

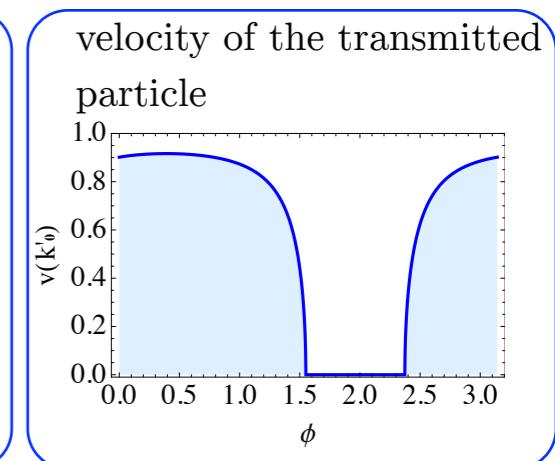
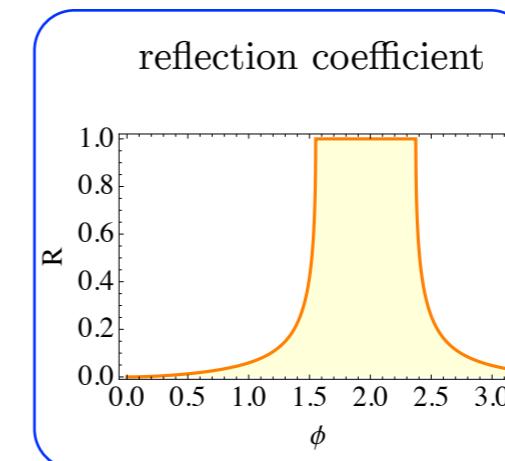
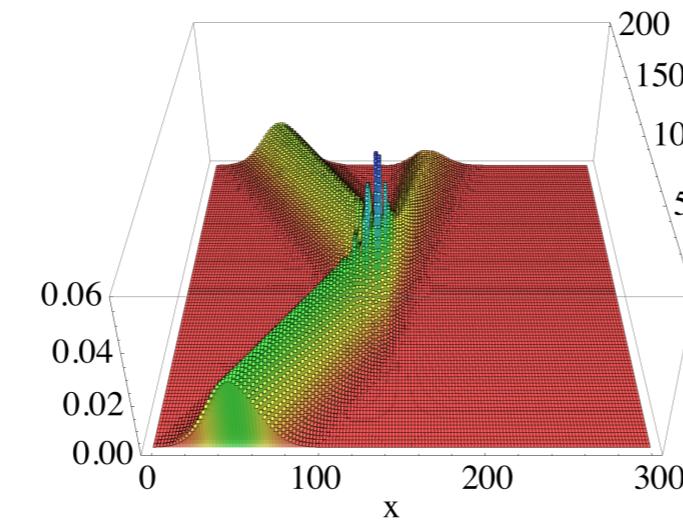
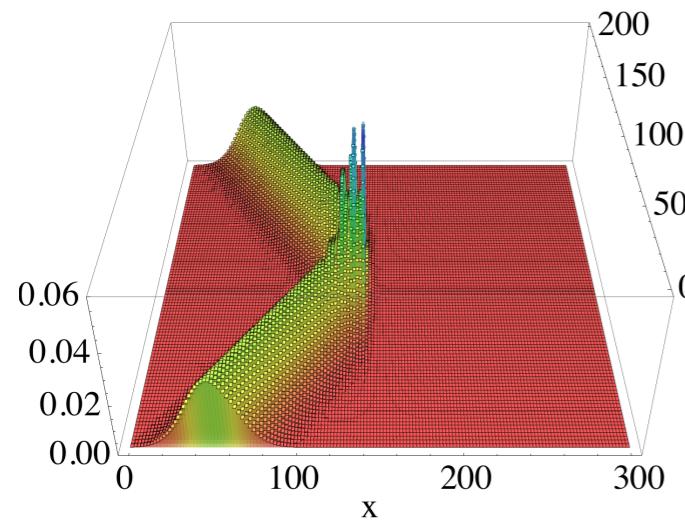
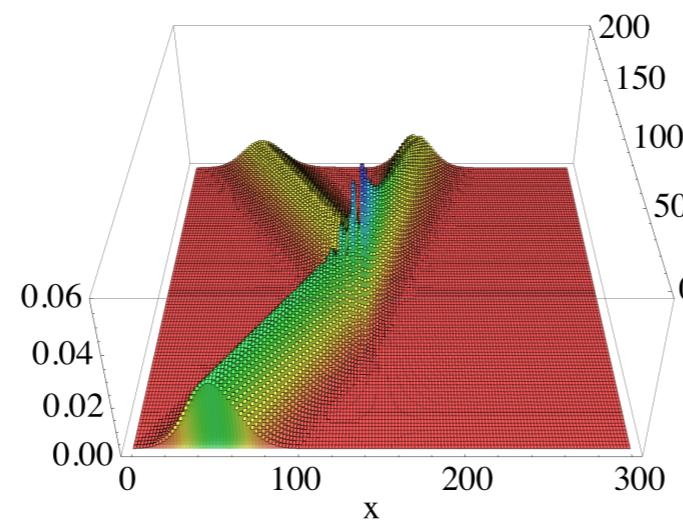
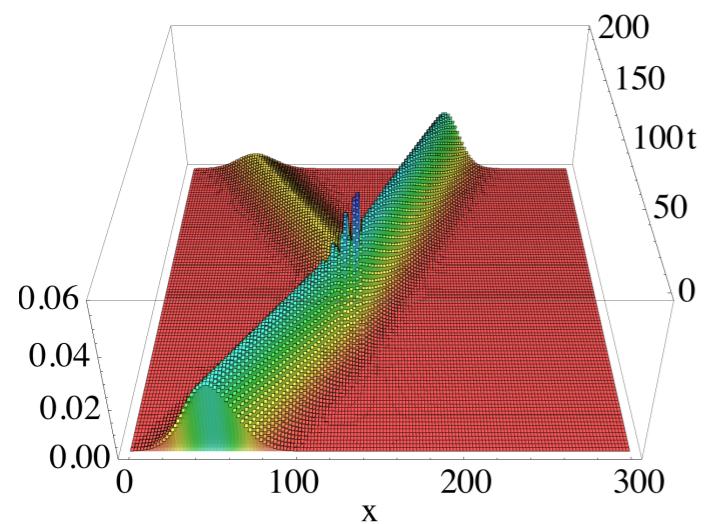


The 1D Dirac QCA 1-particle sector

Scattering against a potential

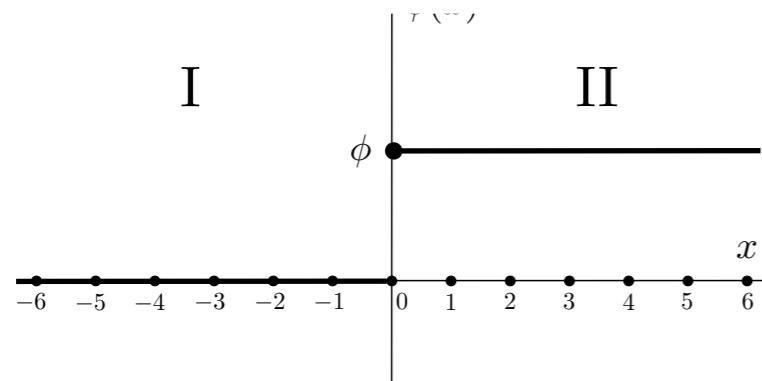


$$U_\phi := \sum_x e^{-i\phi(x)} \begin{pmatrix} n|x-1\rangle\langle x| & -im|x\rangle\langle x| \\ -im|x\rangle\langle x| & n|x+1\rangle\langle x| \end{pmatrix}$$

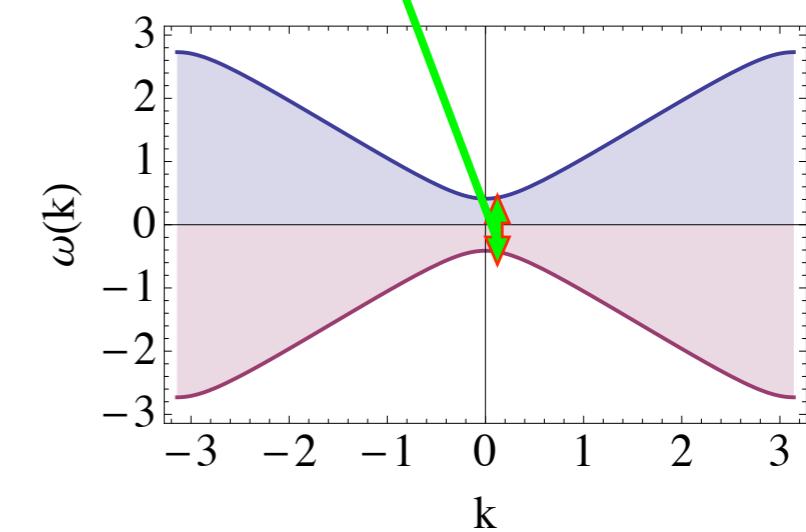
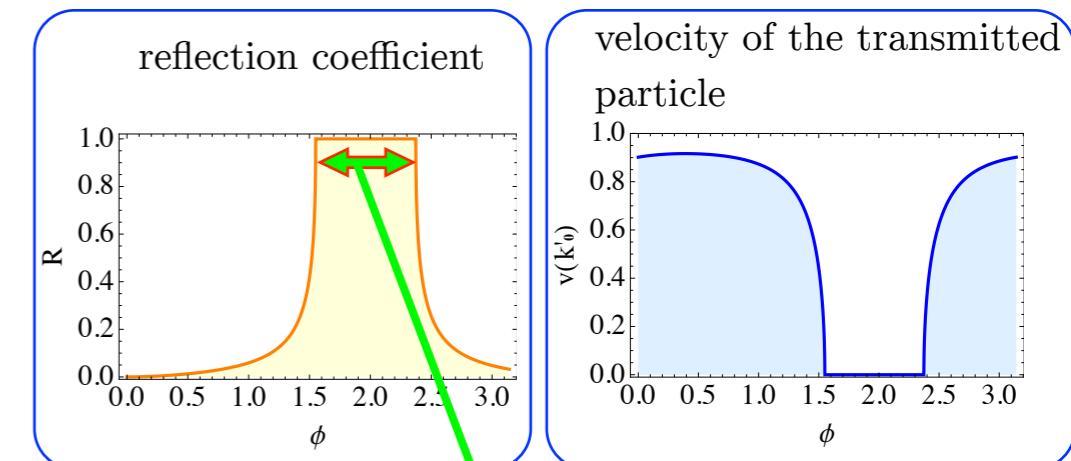
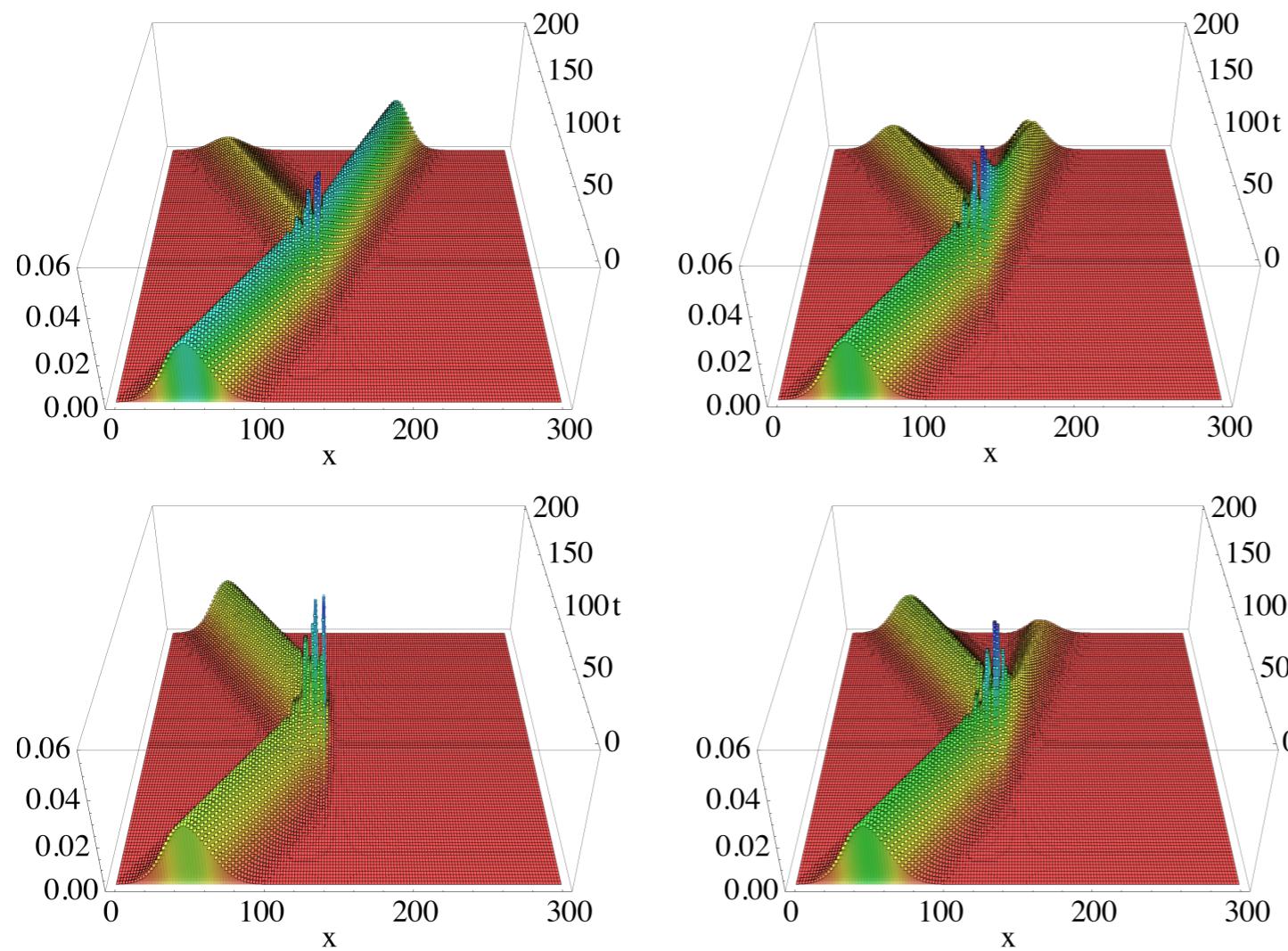


The 1D Dirac QCA 1-particle sector

Scattering against a potential

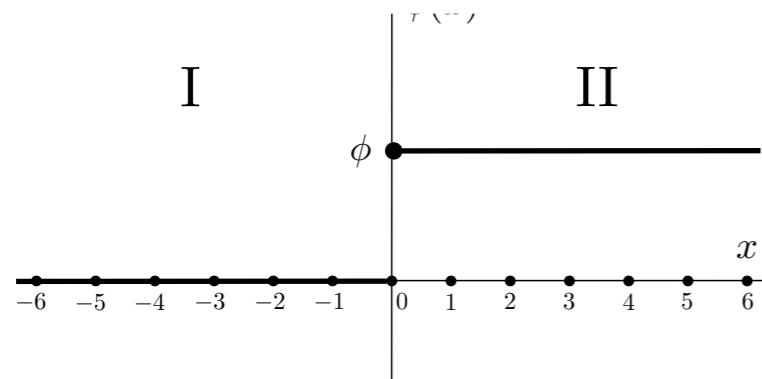


$$U_\phi := \sum_x e^{-i\phi(x)} \begin{pmatrix} n|x-1\rangle\langle x| & -im|x\rangle\langle x| \\ -im|x\rangle\langle x| & n|x+1\rangle\langle x| \end{pmatrix}$$

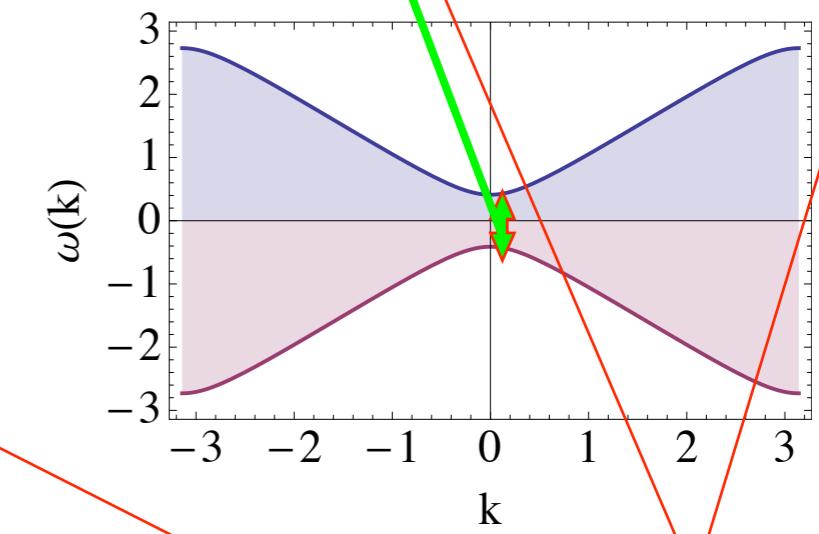
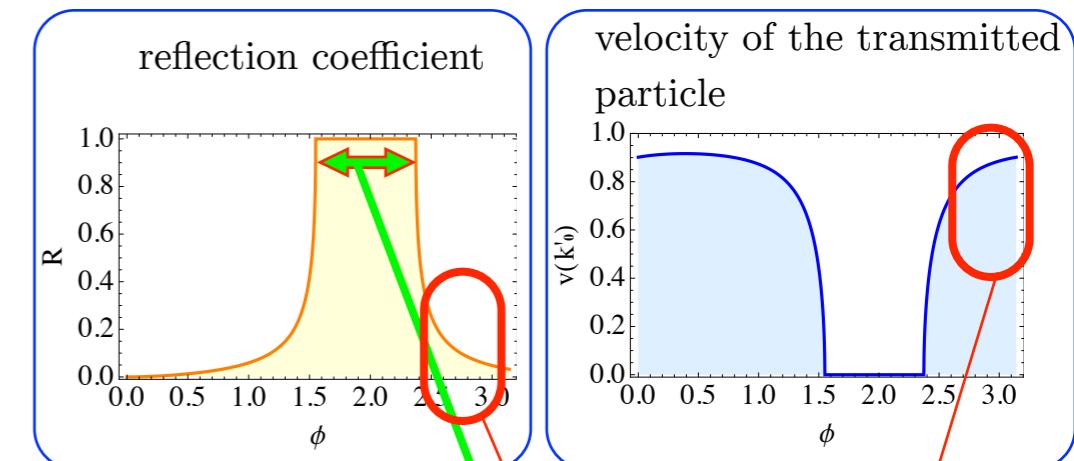
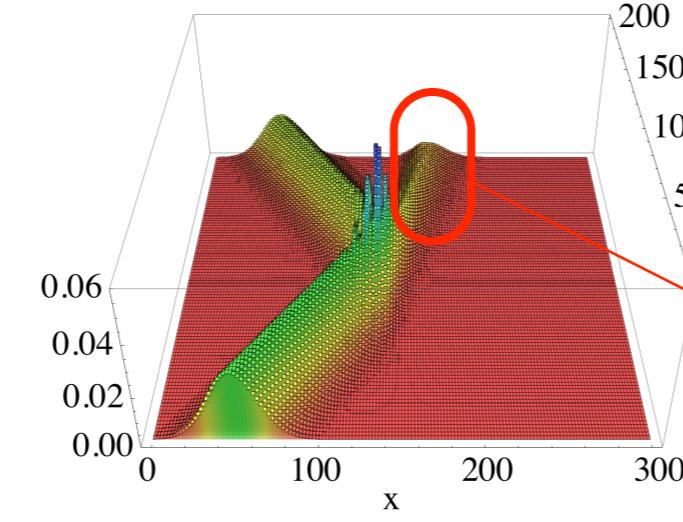
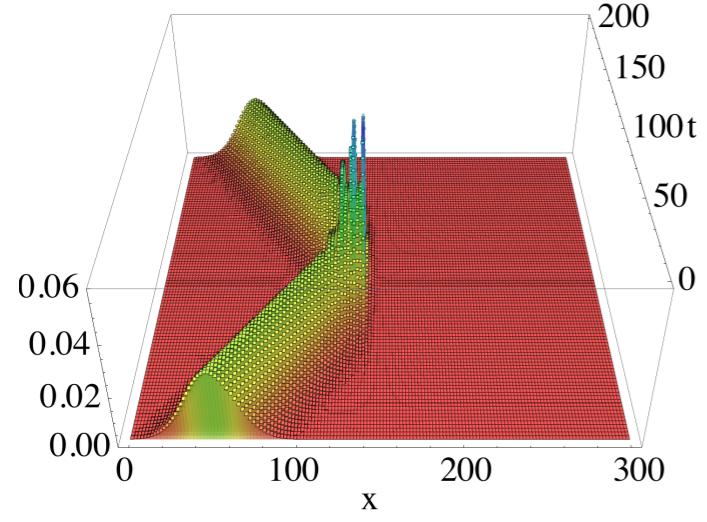
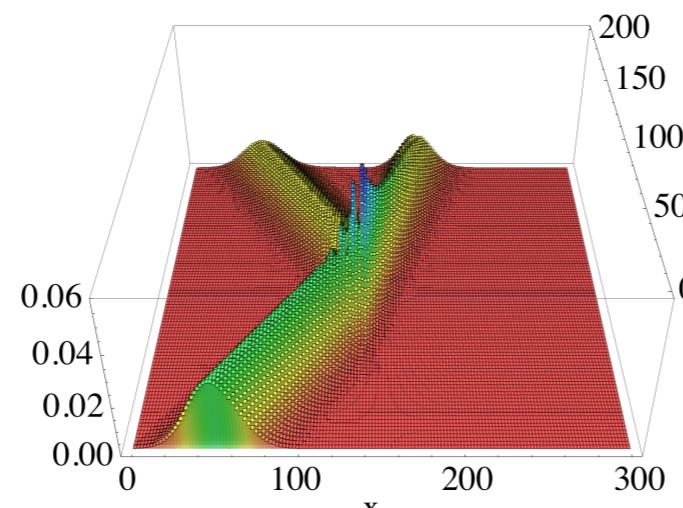
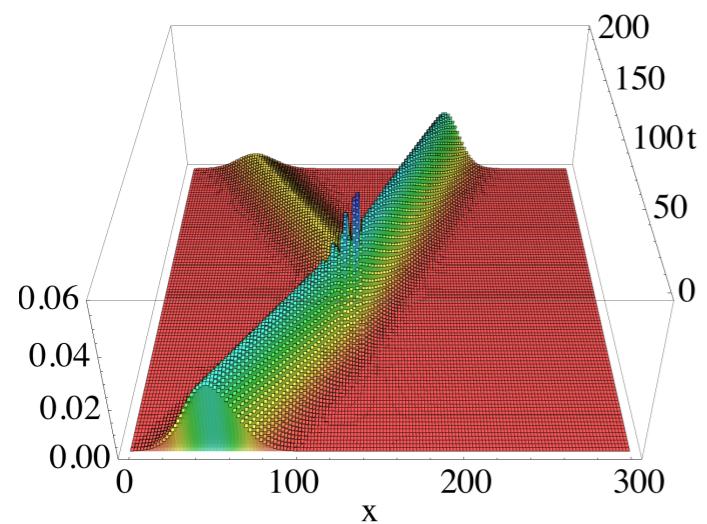


The 1D Dirac QCA 1-particle sector

Scattering against a potential

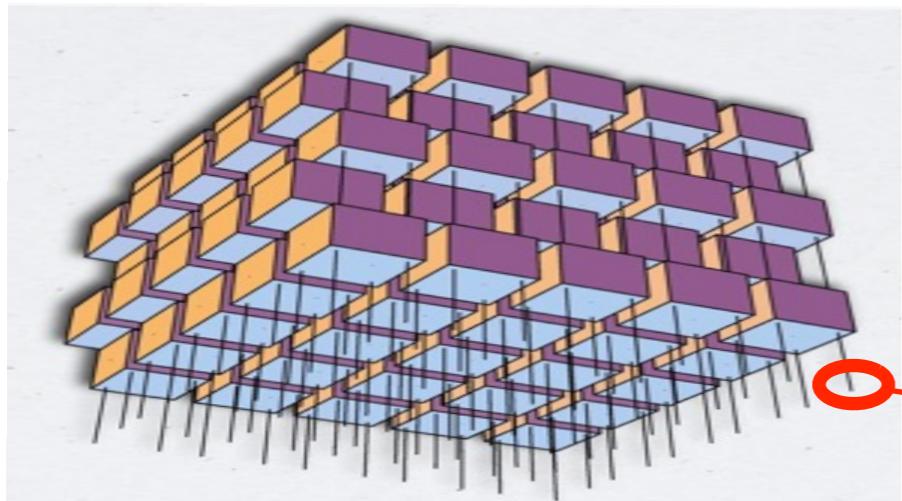


$$U_\phi := \sum_x e^{-i\phi(x)} \begin{pmatrix} n|x-1\rangle\langle x| & -im|x\rangle\langle x| \\ -im|x\rangle\langle x| & n|x+1\rangle\langle x| \end{pmatrix}$$



Klein paradox

Spin and statistics



elementary system
field local algebra
 $\{\psi^{(i)}(x), \psi^{(i)\dagger}(x)\}_{i=1,\dots,N}$

fermionic?
qubit?
bosonic?

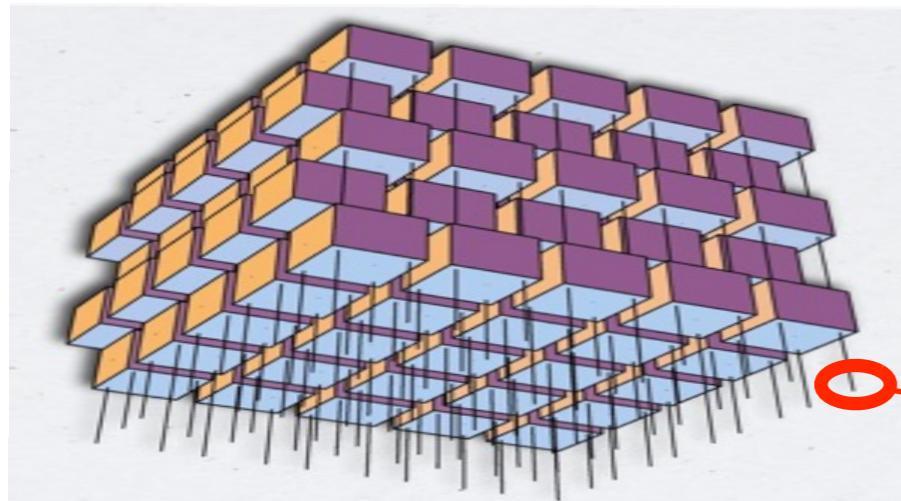
No spin-statistic theorem

S. Bravyi, A. Kitaev, Annals of Physics, 298, 210 (2002)

G. M. D'Ariano, F. Manessi, P. Perinotti, A. Tosini, e-print arXiv:1307.7902

T. Farrelly, A. J. Short, Phys. Rev. A 89, 012302 (2014)

Spin and statistics



elementary system
field local algebra
 $\{\psi^{(i)}(x), \psi^{(i)\dagger}(x)\}_{i=1,\dots,N}$

fermionic?
qubit?
bosonic?

No spin-statistic theorem

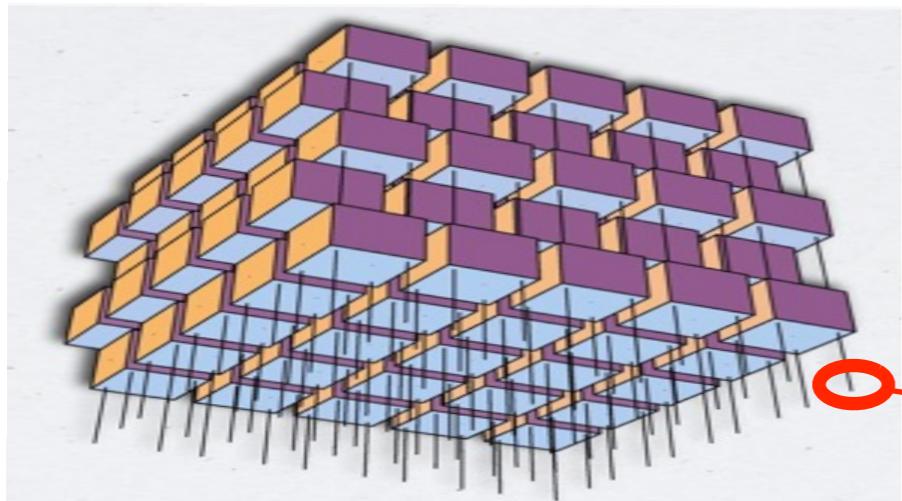
Bosons \rightarrow infinite information in one site

S. Bravyi, A. Kitaev, Annals of Physics, 298, 210 (2002)

G. M. D'Ariano, F. Manessi, P. Perinotti, A. Tosini, e-print arXiv:1307.7902

T. Farrelly, A. J. Short, Phys. Rev. A 89, 012302 (2014)

Spin and statistics



elementary system
field local algebra
 $\{\psi^{(i)}(x), \psi^{(i)\dagger}(x)\}_{i=1,\dots,N}$

fermionic?
qubit?
bosonic?

No spin-statistic theorem

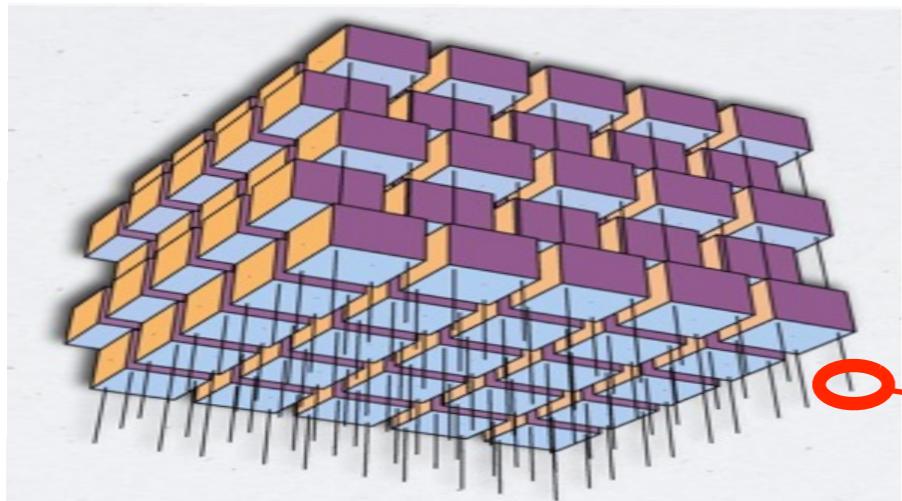
Bosons \rightarrow infinite information in one site

S. Bravyi, A. Kitaev, Annals of Physics, 298, 210 (2002)

G. M. D'Ariano, F. Manessi, P. Perinotti, A. Tosini, e-print arXiv:1307.7902

T. Farrelly, A. J. Short, Phys. Rev. A 89, 012302 (2014)

Spin and statistics

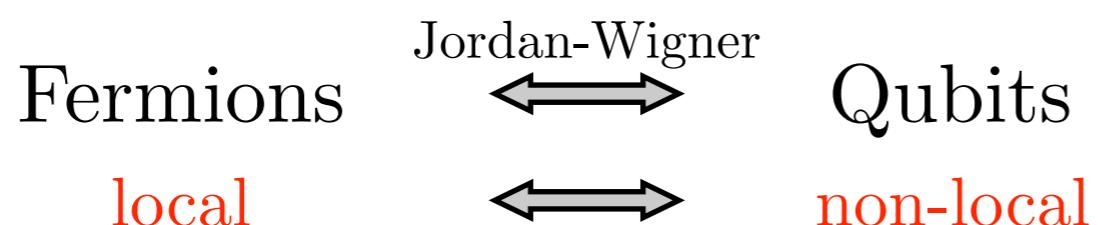


elementary system
field local algebra
 $\{\psi^{(i)}(x), \psi^{(i)\dagger}(x)\}_{i=1,\dots,N}$

fermionic?
qubit?
bosonic?

No spin-statistic theorem

Bosons ~~infinite information in one site~~

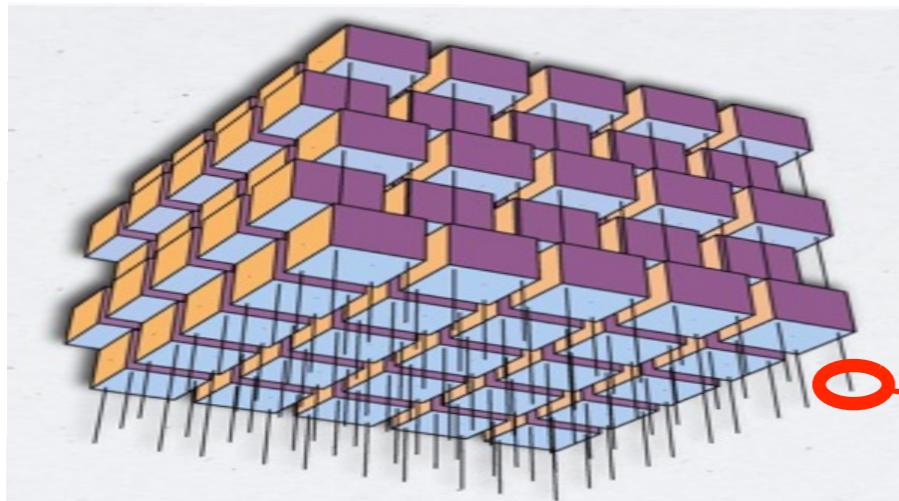


S. Bravyi, A. Kitaev, Annals of Physics, 298, 210 (2002)

G. M. D'Ariano, F. Manessi, P. Perinotti, A. Tosini, e-print arXiv:1307.7902

T. Farrelly, A. J. Short, Phys. Rev. A 89, 012302 (2014)

Spin and statistics

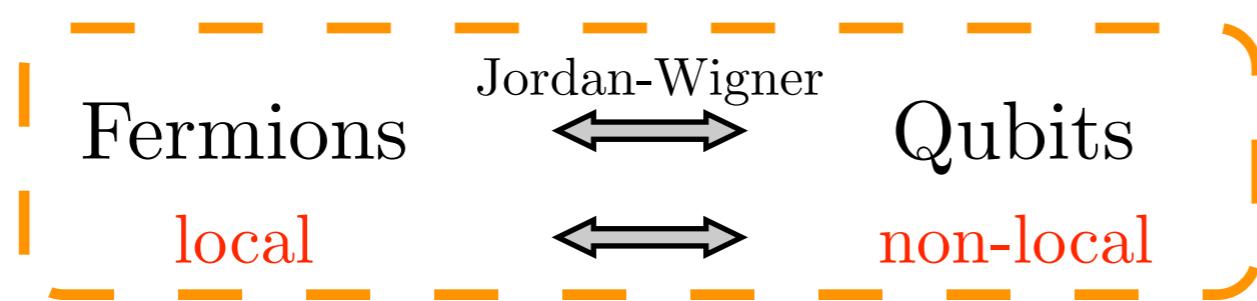


elementary system
field local algebra
 $\{\psi^{(i)}(x), \psi^{(i)\dagger}(x)\}_{i=1,\dots,N}$

fermionic?
qubit?
bosonic?

No spin-statistic theorem

Bosons ~~infinite information in one site~~



Fermionic
theory

S. Bravyi, A. Kitaev, Annals of Physics, 298, 210 (2002)

G. M. D'Ariano, F. Manessi, P. Perinotti, A. Tosini, e-print arXiv:1307.7902

T. Farrelly, A. J. Short, Phys. Rev. A 89, 012302 (2014)

QCA theory of light

Vacuum electrodynamics

$$\vec{E}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^1(p) e^{ipx} + \vec{u}_2(p) a^2(p) e^{ipx}) - h.c.$$

$$\vec{B}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^2(p) e^{ipx} - \vec{u}_2(p) a^1(p) e^{ipx}) - h.c.$$

Coulomb
gauge

QCA theory of light

Vacuum electrodynamics

$$\vec{E}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^1(p) e^{ipx} + \vec{u}_2(p) a^2(p) e^{ipx}) - h.c.$$

$$\vec{B}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^2(p) e^{ipx} - \vec{u}_2(p) a^1(p) e^{ipx}) - h.c.$$

Coulomb
gauge

$$\nabla \times B(x, t) = \partial_t E(x, t)$$

vacuum

$$-\nabla \times E(x, t) = \partial_t B(x, t)$$

Maxwell
equations

$$\vec{u}_i \cdot \vec{p} = 0$$

QCA theory of light

Vacuum electrodynamics

$$\vec{E}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^1(p) e^{ipx} + \vec{u}_2(p) a^2(p) e^{ipx}) - h.c.$$

$$\vec{B}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^2(p) e^{ipx} - \vec{u}_2(p) a^1(p) e^{ipx}) - h.c.$$

Coulomb
gauge

$$\nabla \times B(x, t) = \partial_t E(x, t)$$

vacuum

$$-\nabla \times E(x, t) = \partial_t B(x, t)$$

Maxwell
equations

$$\vec{u}_i \cdot \vec{p} = 0$$

$$[a_p^r, a_q^{s\dagger}] = (2\pi^3) \delta_{rs} \delta_{p-q}$$

bosonic
commutation
relations

$$[a_p^r, a_q^s] = [a_p^{r\dagger}, a_q^{s\dagger}] = 0$$

QCA theory of light

$$\vec{E}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^1(p) e^{ipx} + \vec{u}_2(p) a^2(p) e^{ipx}) - h.c.$$

$$\vec{B}(x) = -i \int \frac{d^3 p}{(2\pi)^3} \sqrt{\frac{\epsilon_p}{2}} (\vec{u}_1(p) a^2(p) e^{ipx} - \vec{u}_2(p) a^1(p) e^{ipx}) - h.c.$$

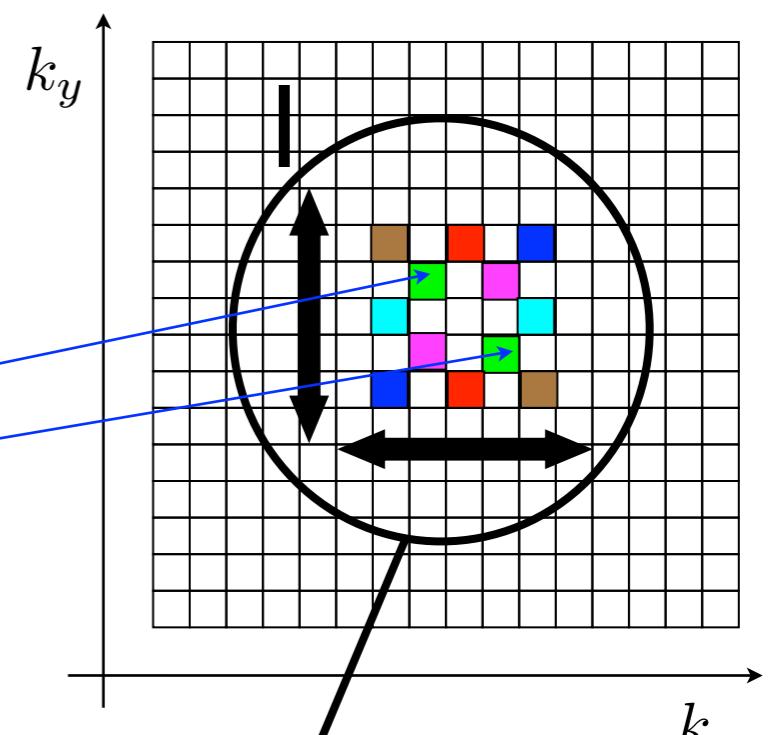
$$a^1(p) = \xi(p) - \eta(p)$$

$$a^2(p) = -i(\xi(p) + \eta(p))$$

$$\xi(p) = \sum_{n \in I} \frac{1}{\sqrt{N}} \nu_{\frac{p}{2} - n\delta} \mu_{\frac{p}{2} + n\delta}$$

similarly for $\eta(p)$

Massless Weyl fermions



$$\text{Vol} = N\delta^3$$

QCA theory of light

$$a^1(p) = \xi(p) - \eta(p)$$

$$a^2(p) = -i(\xi(p) + \eta(p))$$

$$\xi(p) = \sum_{n \in I} \frac{1}{\sqrt{N}} \nu_{\frac{p}{2} - n\delta} \mu_{\frac{p}{2} + n\delta}$$

$$\eta(p) = \sum_{n \in I} \frac{1}{\sqrt{N}} \nu'_{\frac{p}{2} - n\delta} \mu'_{\frac{p}{2} + n\delta}$$

ν, ν' Weyl particles

μ, μ' Weyl anti-particles

Evolution: 3D Weyl automaton

QCA theory of light

$$a^1(p) = \xi(p) - \eta(p)$$

$$a^2(p) = -i(\xi(p) + \eta(p))$$

$$\xi(p) = \sum_{n \in I} \frac{1}{\sqrt{N}} \nu_{\frac{p}{2}-n\delta} \mu_{\frac{p}{2}+n\delta}$$

$$\eta(p) = \sum_{n \in I} \frac{1}{\sqrt{N}} \nu'_{\frac{p}{2}-n\delta} \mu'_{\frac{p}{2}+n\delta}$$

ν, ν' Weyl particles

μ, μ' Weyl anti-particles

Evolution: 3D Weyl automaton



$$E(x, t) \simeq \tilde{E}(x, t) + A\sigma^2 t \mathcal{M}_E$$

$$\nabla \times \tilde{B}(x, t) = \partial_t \tilde{E}(x, t)$$

$$B(x, t) \simeq \tilde{B}(x, t) + A\sigma^2 t \mathcal{M}_B$$

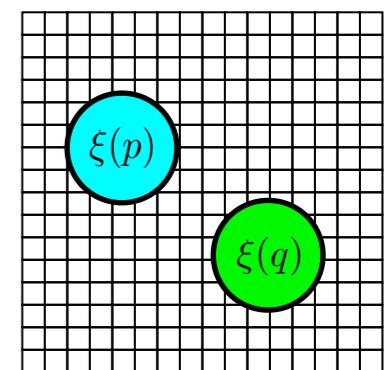
$$-\nabla \times \tilde{E}(x, t) = \partial_t \tilde{B}(x, t)$$

$$[\xi(p), \xi^\dagger(q)] \simeq \delta(p - q) \left(1 + \frac{1}{N} \hat{n} \right)$$

quasi-bosons

similarly for $\eta(p)$

number operator



Processing of Quantum Information

What kind of computer?

“Could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? [...] I’m not sure whether Fermi particles could be described by such a system.” R. Feynman

R. P. Feynman, Int. J. Theo. Phys. 21, 467 (1982)

The simulation is possible. The two models are computationally equivalent but we have a non-local encoding

S. Bravy and A. Kitaev, Ann. Phys. 298, 210–226 (2002)

G. M. D’Ariano, F. Manessi, PP, and A. Tosini, Int. J. Mod. Phys. A, in press

G. M. D’Ariano, F. Manessi, PP, and A. Tosini, arXiv:1307.7902 (2013)

Fermionic Quantum Computer

Dirac QW vs Dirac evolution

$$\mathbf{U}(k) = \exp(-i\mathbf{H}_A(k))$$

$$\mathbf{H}_A(k) \xrightarrow{m, k \rightarrow 0} \mathbf{H}_D(k) + O(m^2 k)$$
$$\mathbf{H}_D(k) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix}$$

Dispersion relation

$$\cos^2(\omega_A) = (1 - m^2) \cos^2(k)$$

$$\omega_A \xrightarrow{m, k \rightarrow 0} \omega_D \left(1 - \frac{m^2}{6} \frac{k^2 - m^2}{k^2 + m^2} \right)$$
$$\omega_D^2 = k^2 + m^2$$

Dirac QW vs Dirac evolution

$$\mathbf{U}(k) = \exp(-i\mathbf{H}_A(k))$$

$$\mathbf{H}_A(k) \xrightarrow{m, k \rightarrow 0} \mathbf{H}_D(k) + O(m^2 k)$$

$$\mathbf{H}_D(k) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix}$$

Dispersion relation

$$\cos^2(\omega_A) = (1 - m^2) \cos^2(k)$$

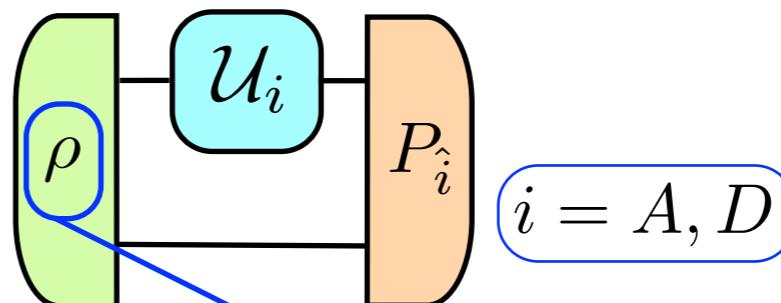
$$\omega_A \xrightarrow{m, k \rightarrow 0} \omega_D \left(1 - \frac{m^2}{6} \frac{k^2 - m^2}{k^2 + m^2} \right)$$

$$\omega_D^2 = k^2 + m^2$$

Discrimination between black boxes

$$\mathcal{U}_A = \exp(-iH_A t) \text{ Automaton}$$

$$\mathcal{U}_D = \exp(-iH_D t) \text{ Dirac}$$



$\rho \in \mathcal{S}_{\bar{k}, \bar{N}}$ less than \bar{N} particles
momentum smaller than \bar{k}

$$p_{err} = \frac{1}{2} \left(p(A|D) + p(D|A) \right) \geq \frac{1}{2} \left(1 - \frac{1}{6} m^2 \bar{k} \bar{N} t \right)$$

AB, G. M. D'Ariano, A. Tosini, Ann. of Phys 354, 244 (2015).

AB, G. M. D'Ariano, A. Tosini, Phys. Rev. A 88, 032301 (2013).