Vacuum Alignment and Radiatively Induced Fermi Scale

Tommi Alanne

CP³ Origins

ICNFP2016
July 13, 2016

In collaboration with A. Meroni, F. Sannino, and K. Tuominen
Phys.Rev. D93 (2016) 9, 091701

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II Raising the origin of the Fermi scale
III Goldstone-boson Higgs and Unification
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Introduction
Unbearable lightness of being

- Why is the Fermi scale so low and the Higgs so light?
  - Large hierarchy between e.g. $\Lambda_{\text{GUT}}$ or $M_{\text{Planck}}$

- Hierarchy problem could be solved by introducing new sector of strong dynamics
  - Perfect example: QCD
  - The Higgs mass is light compared to the natural scale $4\pi v_w$
  - Cannot produce fermion masses without further extensions

- Invoke some new symmetry protecting Higgs mass
  - SUSY is the time-honored example along this line
  - Requires a large amount of new, still unobserved, particles
  - New hierarchy problem with the SUSY-breaking scale
Unification

Two time-honoured schemes

<table>
<thead>
<tr>
<th>Georgi–Glashow</th>
<th>Pati–Salam</th>
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<tbody>
<tr>
<td>Unification of colour and electroweak interactions to e.g. $SU(5)$ or $SO(10)$</td>
<td>Unification of colour and lepton number to $SU(4)_{LC}$</td>
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<tr>
<td>Gauge-mediated proton decay</td>
<td>No proton decay via gauge interactions</td>
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<tr>
<td>$\Lambda_{GUT} \gtrsim 10^{15}$ GeV</td>
<td>Leptoquarks mediate rare kaon decay $K_L \rightarrow \mu^\pm e^\mp$</td>
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<tr>
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<td>$\Lambda_{GUT} \gtrsim 1.9 \times 10^6$ GeV</td>
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Hierarchy between the Unification and the Fermi scale

- Two vastly separated energy scales: $\Lambda_{\text{GUT}}$ and $v_w = 246$ GeV
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- The symmetry breaking steps are modelled via scalar sectors
  - $\langle P \rangle \sim \Lambda_{\text{GUT}}$ and $\langle H \rangle = v_w$
- The SM scalar potential: $V_{\text{SM}} = m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2$
  - Higgs mass 125 GeV $\Rightarrow \lambda_H = 0.13$
  - $m_H^2 = -\lambda_H v_w^2$

T. Alanne (CP$^3$-Origins)
Hierarchy between the Unification and the Fermi scale

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  - Higgs mass 125 GeV $\Rightarrow \lambda_H = 0.13$
  - $m_H^2 = -\lambda_H \nu_w^2$
- **But:** SM feels the GUT scalars via portal interaction $\lambda_{\text{mix}} H^\dagger H \text{Tr}[P^\dagger P]$
  - $\langle P \rangle$ induces a mass term $\sim \lambda_{\text{mix}} \Lambda_{\text{GUT}}^2$ for $H$
  - $\lambda_{\text{mix}}$ has to be highly suppressed ($\lambda_{\text{mix}} \lesssim \nu_w^2 / \Lambda_{\text{GUT}}^2$)
  - $\Rightarrow$ Huge hierarchy between $\lambda_{\text{mix}}$ and $\lambda_H$
II

Raising the origin of the Fermi scale
Emergent Fermi scale due to vacuum misalignment

- Enlarge the symmetry of the SM scalar sector to $G$
  - SSB $G \rightarrow H$ via a scalar vev $\langle \sigma \rangle$
  - This should also break $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$

If $\theta \ll 1$, then $\langle \sigma \rangle \gg v_w$
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- Embed EW gauge group into $G$
  - If $G$ large enough, possibility of different embeddings
  - Amount of EW breaking as $G \to H$ depends on the alignment of $H$ wrt EW group
  - If $\sin \theta$ gives the alignment, then $v_w = \sin \theta \langle \sigma \rangle$
  - If $\theta \ll 1$, then $\langle \sigma \rangle \gg v_w$

- Pushes origin of EWSB and new physics to higher scales!
### Elementary vs. composite Higgs

<table>
<thead>
<tr>
<th>Composite</th>
<th>Elementary</th>
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| - Physical cut-off $\Lambda \sim 4\pi f$ (compositeness scale)  
  $\Rightarrow$ Below $\Lambda$ dominant contributions to effective potential $\sim M^2 \Lambda^2$  
- Top corrections prefer SM-like vacuum $\theta = \pi/2$  
- Need extra explicit breaking to get small $\theta$  
  - [Peskin,’80 & Preskill ’81] | - Renormalisable  
  $\Rightarrow$ One-loop corrections calculable and given by the Coleman–Weinberg potential $\sim M^4 \left( \log \frac{M^2}{\mu_0^2} - C \right)$  
- Top corrections prefer Goldstone-boson-like vacuum $\theta = 0$  
- Heavy scalars important for the vacuum alignment  
  $\Rightarrow$ No extra explicit breaking needed |
Goldstone-boson Higgs and Unification
SU(4) → Sp(4) breaking pattern

- The breaking SU(4) → Sp(4) achieved by a scalar $M$ in $6_A \in SU(4)$
  - Leaves behind 5 GB’s, $\Pi$;
  - These decompose as $(2,2) + (1,1)$ under $SU(2)_L \times SU(2)_R$
    ⇒ Allows for SM-like Higgs bi-doublet of GB’s
  - Additional EW-singlet GB ⇒ possible DM candidate
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- Different alignments between EW group and Sp(4)
  - GB-like vacuum $E_{GB}$ leaves EW intact
  - Higgs-like vacuum $E_H$ breaks EW → $U(1)_Q$
  - In general, a superposition of these $E = \cos \theta E_{GB} + \sin \theta E_H$
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- Composite-Higgs scenario of SU(4) → Sp(4) breaking already studied
  [Cacciapaglia & Sannino 2014]
  - $SU(2)_{TC}$ gauge group with 2 Dirac fermions
Embed $SU(2)_L \times SU(2)_R$ into $SU(4)$
  - Gauge the EW symmetry $\Rightarrow$ Breaks $SU(4)$ explicitly

As $M$ acquires vev, the EW bosons get masses

$$m^2_W = \frac{1}{4}g^2v^2\sin^2\theta, \quad \text{and} \quad m^2_Z = \frac{1}{4}(g^2 + g'^2)v^2\sin^2\theta$$

Elementary scalars $\Rightarrow$ masses for SM fermions without further dynamics

For the vacuum analysis, the top quark is dominant

The top gets mass as $M$ acquires vev, $m_t = \frac{y_t}{\sqrt{2}}v\sin\theta$

The vacuum angle $\theta$ is a priori a free parameter
One-loop potential

- The true vacuum is determined by quantum corrections
- Calculate the one-loop Coleman–Weinberg potential
  \[ V^{(1)}(\Phi) = \frac{1}{64\pi^2} \text{Str} \left[ M^4(\Phi) \left( \log \frac{M^2(\Phi)}{\mu_0^2} - C \right) \right] \]
- The EW and top sectors break the global SU(4) symmetry at one-loop level
  - Picks a preferred value for the vacuum angle \( \theta \)
  - Gives mass to the pseudo-Goldstone boson \( \Pi_4 \)
    \( \Rightarrow \) pGB Higgs
Simplest unification scenario

- Global symmetry of the scalar sector $\text{SU}(4)_\chi$
  $\Rightarrow$ The natural unification scenario is à la Pati–Salam
    - Unify colour with lepton number
      $\Rightarrow$ $\text{SU}(4)_{\text{LC}}$ of leptocolour
      $\Rightarrow$ The full symmetry $G = \text{SU}(4)_\chi \times \text{SU}(4)_{\text{LC}}$

- The simplest realisation to illustrate the idea
  - $M$ breaks $\text{SU}(4)_\chi \rightarrow \text{Sp}(4)_\chi$
  - Add another scalar multiplet, $P$, to break the leptocolour
Results

- Fix $\Lambda_{\text{GUT}} = \langle P \rangle = 2.5 \cdot 10^6$ GeV (above the experimental bound)
- Is it possible to find parameters that
  1. give the correct EW spectrum ($\nu \sin \theta = \nu_w$)
  2. produce the correct Higgs mass?
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Is it possible to find parameters that
1. give the correct EW spectrum ($\nu \sin \theta = \nu_w$)
2. produce the correct Higgs mass?

Yes!
- Typically $\nu \sim \Lambda_{\text{GUT}}$
- All quartic couplings are small ($\lesssim 0.01$) but no large hierarchy between them
- The mass parameters of the same order
Conclusions

- Vast hierarchy between the Fermi and the unification scale
- No hierarchy problem if the Fermi scale generated radiatively
  - Extended global symmetry & vacuum misalignment $\Rightarrow \nu_w = \nu \sin \theta$
  - Vacuum alignment different for elementary and composite scalars
- Viable realisation connecting unification and Fermi scalers within the Pati–Salam framework
  - If $\theta \ll 1$, possible that $\nu \sim \Lambda_{\text{GUT}}$
  - Quartic scalar couplings tiny ($\sim \sin^2 \theta$), but of the same order
Thank you!