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Quantum Optics Properties of QCD Vacuum

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- Quantum Squeezing and Entanglement in QCD
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Quantum QCD Vacuum Population and its Evolution

- Colour particles,
- Colour superpositions,
- Colour multiparticle states (pure separable, mixed separable and nonsepaparable (entangled),
- Mixed colourless states,
- Squeezed and entangled states ,
- Y- and Δ- glueballs,
- Jets

Squeezed States in quantum optics

[Scully, Zubairy 1997]

- Generated by the unitary <u>Squeeze operator</u>

$$|\Psi_s\rangle = \hat{S}(\xi)|\Psi\rangle \qquad \hat{S}(\xi) = \exp[\frac{1}{2}\xi^* \hat{a}^2 - \frac{1}{2}\xi \hat{a}^{\dagger 2}]$$

where $\xi = r\,exp(i\theta)$ is an arbitrary complex number,

$0 \leq r < \infty$ and $0 \leq \theta \leq 2\pi$

$$|\alpha, \xi\rangle = \hat{D}(\alpha)\hat{S}(\xi)|0\rangle$$

- <u>Squeezed Coherent State</u>
  - ✓ for $r \to 0$ we have <u>Coherent State</u> (CS),
  - ✓ for $\alpha \to 0$ we have <u>Squeezed Vacuum State</u> (SVS)

$$\begin{aligned} D(\alpha) &= \exp(\alpha a^\dagger - \alpha^* a) \\ |\alpha\rangle &= D(\alpha)|0\rangle \end{aligned}$$

- <u>Mean</u> and <u>Variance</u> of the photon number (PN) are

$$|\xi\rangle = \hat{S}(\xi)|0\rangle$$

$$\langle \hat{n} \rangle = |\alpha|^2 + \sinh^2 r$$ m of the mean PN of CS and SVS

$$\langle (\Delta \hat{n})^2 \rangle = |\alpha \cosh r - 2^* e^{i\,\theta} \sinh r|^2 + 2\sinh^2 r \cosh^2 r$$

- Condition of squeezing: $(\langle(\Delta X_i)^2\rangle < \frac{1}{4}$

- (Walls, 1983)

On possibility of squeezed and entangled color states in QCD

Gluon Squeezed States in QCD Jet

(Kuvshinov,Shaporov)

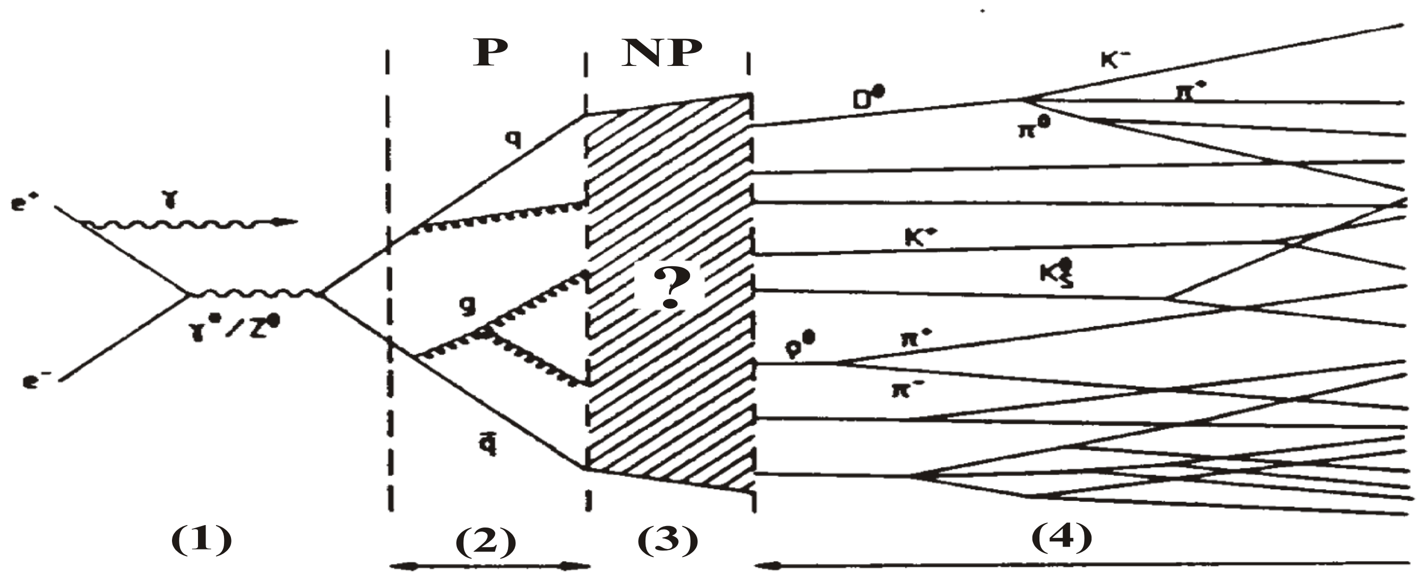
Effects which are not connected with small $\alpha_S(Q^2)$

= **Nonperturbative (NP)** effects in jets

Role of **NP** effects :

- confinement and hadronization
- exact YM field equations, solutions, ex. instantons,   vacuum properties
- long distances, soft collisions, diffraction
- power corrections
- NP evolution
- MC hadronization models, LPHD are not connected with QCD

Jets give example of separation between P and NP stages



Here we

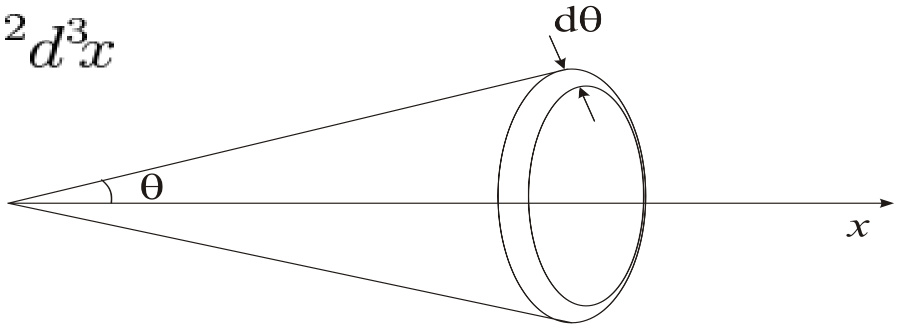
- ✓ consider **Gluon Field NP Evolution in time**
- ✓ demonstrate **Quantum Squeezing**

Gluon Evolution

Consider gluon self-interaction Hamiltonian of QCD

$$V = -g\int f_{abc}\mathbf{E}_a\mathbf{A}_b A_c^0\, d^3x + \frac{g}{2}\int f_{abc}\mathbf{B}_a[\mathbf{A}_b\mathbf{A}_c]d^3x + \\ + \frac{g^2}{2}\int (f_{abc}\mathbf{A}_b A_c^0)^2 d^3x + \frac{g^2}{8}\int (f_{abc}[\mathbf{A}_b\mathbf{A}_c])^2 d^3x$$

Take jet ring with cone angle



In terms of annihilation (creation) operators we have

$$V = \frac{k_0^4}{4(2\pi)^3}\left(1-\frac{q_0^2}{k_0^2}\right)^{3/2} g^2\,\pi\, f_{abc}f_{adf}\left\{\left(2-\frac{q_0^2}{k_0^2}\right)\left[a_{1212}^{bcdf}+a_{1313}^{bcdf}\right] + \right. \\ \left. + \; a_{2323}^{bcdf} + \frac{\sin^2\theta}{2}\left(1-\frac{q_0^2}{k_0^2}\right)\left[2a_{2323}^{bcdf} - a_{1212}^{bcdf} - a_{1313}^{bcdf}\right]\right\}\sin\theta\, d\theta.$$

Here $a_{lmlm}^{bcdf} = a_l^{b+}a_m^{c+}a_l^d a_m^f + a_l^{b+}a_m^c a_l^{d+}a_m^f + a_l^b a_m^{c+}a_l^{d+}a_m^f + h.c.$

Gluon Evolution

Hamiltonian V **has squares of operators of annihilation and creation. As it is known from QM and QO such structures in evolution Hamiltonian are nessesary condition of Squeezing States (SS) production because squeezing operator** $S(z)$ **has such operators:**

$$S(z) = \exp\left\{\frac{z^*}{2}a^2 - \frac{z}{2}(a^+)^2\right\}$$

For small time evolution t **we have final state**

$$|\,\mathrm{f}\,\rangle \simeq |\,\mathrm{in}\,\rangle - \; i\; t\; V\,|\,\mathrm{in}\,\rangle$$

***Task*: to search possibility of <u>Quantum Squeezing of Gluon States</u>**

by study NP evolution of initial state under *V*

5. Gluon SS production

To check whether final gluon state describes SS we should by analogy to quantum optics introduce operators

$$(\hat{X}_l^b)_1 = [\hat{a}_l^b + (\hat{a}_l^b)^+]/2 \quad \text{and} \quad (\hat{X}_l^b)_2 = [\hat{a}_l^b - (\hat{a}_l^b)^+]/2i$$

and to find out that dispersion of one then is smaller than that for coherent (or vacuum) state

Condition of squeezing **for fotons (Walls, 1983),**

$$\left\langle \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle = \left\langle N \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle + \frac{1}{4} < \frac{1}{4}$$

for gluons (Kuvshinov,Shaporov, Marmysh, 1999) :

$$\text{or} \qquad \left\langle N \left(\triangle (X_l^b)_{\frac{1}{2}} \right)^2 \right\rangle < 0.$$

Or in terms of a, a+:

$$\left\langle N \left(\triangle (X_l^h)_{\frac{1}{2}} \right)^2 \right\rangle = \mp \frac{i\,t}{4} \Big\{ \langle \alpha \,|\, [a_l^h(k)\,,\,[a_l^h(k)\,,\,V]\,]|\alpha\rangle - \\ - \langle \alpha \,|[\,[V, a_l^{h+}(k)], a_l^{h+}(k)]|\alpha\rangle \Big\} < 0$$

Here as initial state vector of nonperturbative evolution we use at the end of P evolution (Lupia S., Ochs W. and Wosiek J., Nucl. Phys. B 540, 405 (1999)) NBD distribution equel to superposition of products of the gluon coherent states (Poissonian distributions)

$$|\alpha\rangle = \prod_{c=1}^{8} \prod_{l=1}^{3} |\, \alpha_l^c(0)\,\rangle$$

6. Gluon SS production

➢**Terms ~A^3 (three-gluon self-interaction) don't give contribution to the squeezing condition**

$$[a_l^h(k),[a_l^h(k),V\sim A^3]]=0,\quad [[V\sim A^3,a_l^{h+}(k)],a_l^{h+}(k)]=0 \qquad f_{hhb}=0$$

Four-gluon selfinteraction is source of the squeezing effect

For example: for colour index *h=1*

$$\left\langle N\left(\triangle(X_l^1)_{\frac{1}{2}}\right)^2\right\rangle=\pm 4\pi\, u_2\, t\,\sin\theta\, d\theta\Big\{(1+u_1)\Big[\delta_{l1}(Z_{33}+Z_{22})+ +(1-\delta_{l1})Z_{11}\Big]+\delta_{l2}Z_{33}+\delta_{l3}Z_{22}+ +u_1\sin^2\theta\left[-\tfrac{1}{2}\delta_{l1}(Z_{22}+Z_{33})+\delta_{l2}(Z_{33}-\tfrac{1}{2}Z_{11})+\delta_{l3}(Z_{22}-\tfrac{1}{2}Z_{11})\right]\Big\}\neq 0$$

Here

$$Z_{mn}=\sum_{k=2}^{7}{}'\langle (X_m^k)_1\rangle\langle (X_n^k)_2\rangle (m,n=1,2,3),$$

$$\sum_{k=2}^{7}{}'(\,)=\sum_{k=2}^{3}(\,)+\frac{1}{4}\sum_{k=4}^{7}(\,),\; u_1=\left(1-\frac{q_0^2}{k_0^2}\right),\; u_2=\frac{k_0^4}{4(2\pi)^3}\frac{g^2}{2}\sqrt{u_1^3}.$$

7. Gluon SS production

Conditions for gluons (Kuvshinov,Shaporov, Marmysh, 1999)

- **We have gluon phase squeezed state if:**

$k \neq 1.\ m \neq l$

$$\langle (X^k_m)_1 \rangle < 0, \langle (X^k_m)_2 \rangle < 0$$
or
$$\langle (X^k_m)_1 \rangle > 0, \langle (X^k_m)_2 \rangle > 0$$

- **We have gluon amplitude squeezed state if:**

$$\langle (X^k_m)_1 \rangle > 0, \langle (X^k_m)_2 \rangle < 0$$
or
$$\langle (X^k_m)_1 \rangle < 0, \langle (X^k_m)_2 \rangle > 0$$

- **The conditions cover all possible cases => *gluon SS –exist***
- **The same is true for other colours**
- **Obviously, the larger are both the amplitudes of the initial gluon coherent fields with different colours and polarization indexes and coupling constant, the larger is the two-mode squeezing effect**

10.Squeezed States (Properties,Generation and Applications)

- **Properties**
- **Pure quantum state (nonclassical analog)**
- ***More organized then coherent state (entropy is small)***
- ***Can have sub-Poisson multiplicity distribution (antibunch-***
- **ing, or super-Poisson for bunching)**
- ***Pairing of photons***
- ***Can decrease quantum noise***
- ***Can be obtained from CS by nonlinear interaction with***
- **out side devices**
- ***Can be detected by interaction with controlling CS***

- **Generation of squeezed states:**

✓Nonlinear optics: χ(2) or χ(3) processes
✓Cavity-QED
✓Photon-atom interaction
✓Photonic crystals
✓Semiconductor, photon-electron (exciton) polariton interaction

•Applications of squeezed states:

✓Gravitational Waves Detection
✓Quantum Non-Demolition Measurement
✓Generation of EPR Pairs
✓Quantum Information Processing,
✓teleportation, cryptography, computing
✓In QCD due to four gluon selfinteraction

Entangled States in QO

[Kilin 2001]

- Consider the <u>Superposition</u> <u>state</u> vector for a system with two orthogonal basic states $|1\rangle$ and $|2\rangle$:

$$|\Psi\rangle = \alpha\,|1\rangle + \beta\,|2\rangle$$

$$\rho = |\Psi\rangle\,\langle\Psi| = |\alpha|^2\,|1\rangle\,\langle 1| + |\beta|^2\,|2\rangle\,\langle 2| + \alpha\beta^*\,|1\rangle\,\langle 2| + \alpha^*\beta\,|2\rangle\,\langle 1|$$

<u>Superposition state</u> should be distinguished from a <u>mixed state</u>

$$\rho_{\mathrm{mix}} = |\alpha|^2\,|1\rangle\,\langle 1| + |\beta|^2\,|2\rangle\,\langle 2|$$

- As an example of an <u>Entangled state</u>, consider the state of a composite system: <u>two-level atom-field</u>
- After a short period of interaction, the atom and the field become spatially separated.
- However, the state of the whole system remains <u>Entangled</u>:

  the <u>State of the Atom</u> is <u>strictly correlated</u> with the <u>State of the Field</u>

$$|\Psi\rangle = |\mathrm{atom}\rangle_1\,|\mathrm{field}\rangle_1 + |\mathrm{atom}\rangle_2\,|\mathrm{field}\rangle_2$$

12.Entangled States

Other example of the Entangled States is two single-photon beams with different wave vectors

$$|1_1 + 1_2\rangle = C_{\updownarrow\updownarrow}\,|\updownarrow\rangle_1\,|\updownarrow\rangle_2 + C_{\leftrightarrow\leftrightarrow}\,|\leftrightarrow\rangle_1\,|\leftrightarrow\rangle_2 + C_{\updownarrow\leftrightarrow}\,|\updownarrow\rangle_1\,|\leftrightarrow\rangle_2 + C_{\leftrightarrow\updownarrow}\,|\leftrightarrow\rangle_1\,|\updownarrow\rangle_2$$

Bell in 1964 introduced these states in relation to the EPR paradox.

Basis of the Bell states

$$|\Phi^+\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 + |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2},$$
$$|\Phi^-\rangle = (|\updownarrow\rangle_1|\updownarrow\rangle_2 - |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2)/\sqrt{2},$$
$$|\Psi^+\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2},$$
$$|\Psi^-\rangle = (|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2)/\sqrt{2},$$

Each of these Entangled States has a remarkable property:
If one photon is registered with definite polarization, the other photon immediately becomes opposite polarized.
Measurement over one particle have an instantaneous effect on the other, possibly located at a large distance.
Two-mode squeezed state is one of the example of the en-tangled states *9De Wolf , 2001)*

Entangled States

- Entangled states have another paradoxical property, which was pointed out by Schrodinger in 1935:

  Complete information about the state of the total system still does not provide complete information about the states of its parts.

  Indeed, for example: Suppose that we are going to find out the state of a particle in one of the pairs two single-photon beams.

  Then we have to average the density matrix of the pure state

$$|\Psi^-\rangle = (|\updownarrow\rangle_1 |\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1 |\updownarrow\rangle_2)/\sqrt{2}$$

  over the states of the second particle. The resulting density matrix of the first particle

  is apparently the density matrix of a mixed state, which is not maximally determinate

$$\rho^{(1)} = \mathrm{Tr}(|\Psi^-\rangle\langle\Psi^-|) = \underbrace{(|\updownarrow\rangle_1\ {}_1\langle\updownarrow| + |\leftrightarrow\rangle_1\ {}_1\langle\leftrightarrow|)/2}_{\text{mixture}}$$

Entanglement criteria

[Dodonov 2002]

- 1). <u>Index of the correlation</u>
- From the point of view of statistical mechanics and information theory, <u>Entropy</u> can be thought of as a <u>Measure of Missing Information</u>

$$S(\hat{\rho}) = -\mathrm{Tr}[\hat{\rho}\ln\hat{\rho}]$$

- For a pure state <u>S(ρpure) = 0</u> and thus repeated measurements of the state in question yield no new information. <u>S(ρmixed) > 0</u> – for a mixed state. <u>Index of the correlation</u> in terms of the total and "partial" Von Neumann Entropies (Excess entropy) can be served as a measure of entanglement

$$I_c = S_a + S_b - S, \quad I_c \geq 0,$$

2). <u>Entanglement parameter</u> [Dodonov, 2002] can be the measure of entanglement for two-mode states:

$$y = \left[\frac{|\overline{a_1 a_2^+}|^2 + |\overline{a_1 a_2}|^2}{2(\overline{a_1^+ a_1} + 1/2)(\overline{a_2^+ a_2} + 1/2)}\right]^{1/2}$$

- 3).The entanglement condition (or coefficient of correlation) of states can be verified by investigation of the conditional probability P ((*Yj/Xi)*[Kuvshinov, Marmysh, Shaporov, 2004]
- If P ((Yj/Xi) = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or P(Yj/Xi) = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ the state is entangled
- In terms of correlation coefficient
- r(X,Y) = P(X2)(P(Y2/X2) – P(Y2)/([P(X2)(1 – PX2)(PY2)(1 – P(Y2))½]½, the state is entangled when r=±1

Gluon Entangled State

- By **analogy with Index of the correlation [Dodonov]**

we used as the measure of entanglement for gluon states the next coefficient:

$$y = \left[\frac{|\overline{a_l^h a_l^{g+}}|^2 + |\overline{a_l^h a_l^g}|^2}{2(\overline{a_l^{h+} a_l^h} + 1/2)(\overline{a_l^{g+} a_l^g} + 1/2)} \right]^{1/2}$$

$$\overline{a_l^h a_l^{g+}} = \langle a_l^h a_l^{g+} \rangle - \langle a_l^h \rangle \langle a_l^{g+} \rangle$$

$$0 \le y < 1$$

Gluon Entangled State

- Measure of entanglement is proportional to the squeezing coefficient (at small squeezing)

$$y = \sqrt{2}r$$

- <u>Entanglement</u> imposes additional restrictions on the <u>squeezing parameter</u>

- In particular, for the collinear gluons we have $$0 < r < \frac{1}{\sqrt{2}}$$

$$0 < \left| t\frac{\alpha_s \pi}{2k_0}(f_{ahb}f_{ahc} + f_{agb}f_{agc} + f_{ahb}f_{agc} + f_{agb}f_{ahc})\sum_{l_1 \neq l} |\alpha_{l_1}^b||\alpha_{l_1}^c|\sin(\gamma_{l_1}^b + \gamma_{l_1}^c) \right| < \frac{1}{\sqrt{2}}$$

- Thus, by analogy with quantum optics as a result of four-gluon self-interaction we obtain two-
- mode gluon squeezed states which are also entangled

Quantum system and Environment, Decoherence

- **Interactions of some quantum system with the environment can be effectively represented by additional stochastic terms in the Hamiltonian of the system.**
- **The density matrix of the system is obtained by averaging with respect to degrees of freedom of environment**
- **Interactions with the environment result in decoherence and relaxation of quantum superpositions. Information on the initial state of the quantum system is lost after suficiently large time** (**Haken; Haake; Peres [4-7]**)
- **Quantum decoherence** is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition.
- **D.**occurs when a system interacts with its environment in a thermodynamically irreversible way
- **D.** can be viewed as the loss of information from a system due to the environment (often modeled as a heat bath)
- **Dissipation** is a decohering process by which the populations of quantum states are changed due to entanglement with a bath
- **Relaxation** usually means the return of a perturbed system into equilibrium. Each relaxation process can be characterized by a **relaxation time** τ.

Stochastic QCD Vacuum as Closed Environment for Colour States

1. Quantum system and Environment, Decoherence

(**Haken; Haake; Peres**)

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2. Stochastic QCD vacuum

(Ambjorn; Simonov; Dosch)

- The model of QCD stochastic vacuum is one of the popular phenomenological models which explains quark confinement (WL decreasing) , string tensions and field congurations around static charges

- Only the second correlators are important and the other are negligible (which are important in coherent vacuum where all correlators are important) (Simonov [11]) (Gauss domination) It has been conrmed by lattice calculation Shevchenko, Simonov [43]. The most important evidence for this is Casimir scaling [39].

- It is based on the assumption that one can calculate vacuum expectation values of gauge invariant quantities as expectation values with respect to some well-behaved stochastic gauge field

- It is known that such vacuum provides confining properties, giving rise to QCD strings with constant tension at large distances

3.Stochastic QCD vacuum as environment

(Kuvshinov, Kuzmin, Buividovich)

- We consider QCD stochastic vacuum as the environment for colour quantum particles and average all their quantities over external QCD stochastic vacuum implementations(degrees of freedom)

- Instead of considering nonperturbative dynamics of Yang-Mills fields one introduces external environment and average over its implementations

As a consequence we obtain:

- decoherence, relaxation of quantum superpositions

- information loss and confinement of colour states phenomenon

- white objects can be obtained as white mixtures of states described by the density matrix as a result of evolution of the particles in the QCD stochastic vacuum as environment

4. Colour particles evolution and decoherence

(Kuvshinov, Kuzmin, Buividovich)

Consider propagation of heavy spinless colour particle along some fixed path γ. The amplitude is obtained by parallel transport

$$\partial_\mu |\phi\rangle = i\hat{A}_\mu |\phi\rangle \qquad |\phi(\gamma)\rangle = \hat{P}\exp\left(i\int_\gamma \hat{A}_\mu dx^\mu\right)|\phi_{in}\rangle \qquad (1)$$

In order to consider mixed states we introduce the colour density matrix taking into account both colour particle and QCD stochastic vacuum (environment)

$$\rho(loop, \gamma\bar{\gamma}) = \langle \phi(\gamma)\rangle\langle\phi(\gamma)|\rangle \qquad (2)$$

Here we average over all implementations of stochastic gauge field (environment degrees of freedom) and obtain decoherence due to interaction with environment. In the model of QCD stochastic vacuum only expectation values of path ordered exponents over closed paths are defined.

Closed path corresponds to a process in which the particle-antiparticle pair is created, propagate and finally annihilated.

With the help of (1) and (2) we can obtain expression for density matrix [1,3]:

$$\rho(loop,\gamma\bar{\gamma}) = N_c^{-1} + (|\phi_{in}\rangle\langle\phi_{in}| - N_c^{-1})W_{adj}(loop,\gamma\bar{\gamma}) \qquad (3)$$

and Wilson loop in fundamental representation is [3]

$$W_{fund}(loop,\gamma\bar{\gamma}) = \langle T_r \hat{P} \exp\left(\int\limits_{loop,\gamma\bar{\gamma}} i\hat{A}_\mu dx^\mu\right)\rangle \qquad (4)$$

WL decays exponentially with the area spanned on loop in confinement region, then

$$\rho(loop,\gamma\bar{\gamma}) = N_c^{-1} + (\rho_{in} - N_c^{-1})\exp(-\sigma_{adj}RT) \qquad (5)$$

where $\sigma_{adj} = \sigma_{fund} G_{adj} G_{fund}^{-1}$ is string tension in the adjoint representation, G_{adj}, G_{fund} - eigenvalues of quadratic Casimir operators. Under Gaussian dominance string tension is

$$\sigma_{fund} = \frac{g^2}{2} l_{corr}^2 F^2 \qquad (6)$$

g is coupling constant, l_{corr} – correlation length in the QCD stochastic vacuum, F - average of the second cumulant of curvature tensor (Dosch; Simonov[12,13])

5.Decoherence rate, Purity, Von Neumann entropy

The decoherence rate of transition from pure colour states to white mixture can be estimated on the base of purity (Haake[8])

P=Tr ρ [2]

$$P = N_c^{-1} + (1 - N_c^{-1}) \exp(-2\sigma_{fund} G_{adj} G_{fund}^{-1} RT) \qquad (7)$$

When RT tends to 0, P → 1, that corresponds to pure state. When composition RT tends to infinity the purity tends to 1/Nc, that corresponds to the white mixture

The rate of purity decrease is

$$T_{dec}^{-1} = -2\sigma_{fund} G_{adj} G_{fund}^{-1}$$

Left side of the equation is the characteristic time of decoherence proportional to QCD string tension

The information of quark colour states is lost due to interaction between quarks and confining non-Abelian gauge fields

Von Neumann entropy: $S = -\mathrm{Tr}\,(\hat{\rho} \ln \hat{\rho})$ = 0 for the initial state and S = ln Nc for large RT

It can inferred from (3) and (7) : the stronger is particle-antiparticle pair coupled by QCD string or the larger is the distance between particle and antiparticle the quicker information about colour state is losing as a result of interaction with the QCD stochastic vacuum. Thus mixed white states can be obtained as a result of decoherence process

7.Interaction of Colour Superposition with QCD Vacuum

(Kuvshinov, Bagashov)

When the initial (pure) colour state is a superposition of colour states

$$|\phi_{in}\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$$

The corresponding density matrix is

$$\hat{\rho}_{in} = |\phi_{in}\rangle\langle\phi_{in}| \qquad \hat{\rho}_{in} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix}$$

After integration and averaging $\hat{\rho}(y) = \langle\langle\hat{\rho}_1(y)\rangle\rangle = N_c^{-1}\hat{I} + (\hat{\rho}_{in} - N_c^{-1}\hat{I})W_{adj}(L)$

When RT→∞ $W_{adj}(L) = \exp(-\sigma_{adj}RT)$

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix} \to \begin{pmatrix} N_c^{-1} & 0 & 0 \\ 0 & N_c^{-1} & 0 \\ 0 & 0 & N_c^{-1} \end{pmatrix}$$

Density matrix becomes diagonal ρ_{out}=diag (1/Nc)

8.Purity, Von Neumann Entropy(Superposition)

PurityEntropy $\mathrm{P} = N_c^{-1} + (1 - N_c^{-1}) W_{adj}^2(L)$

$S=(1-N^{-1}_{C})(1-\ln\frac{Wadj(L)}{Nc})$

- **For the initial stateRT →0: purity P → 1 -pure state, entropy S→O**
- **Asymptotically RT→∞: P=N_C^{-1}-fully mixed state,entropy S=lNc**

-Interaction of an arbitrary colour superposition with the QCD stochastic vacuum at large distances leads to an emergence of a mixed state
-With equal probabilities for different colours
-Without any non-diagonal terms in the corresponding density matrix ρout=diag (1/Nc)

11.Interaction of Np multiparticle states with QCDV
Density matrix, Purity, Von Neumann Entropy (TPS)

$$\hat{\rho}(y) = N_c^{-N_p}\hat{I} + (\hat{\rho}_{in} - N_c^{-N_p}\hat{I})W_{adj}(L) \qquad \hat{\rho}(L : RT \to \infty) = N_c^{-N_p}\hat{I}$$

$$\mathrm{P} = N_c^{-N_p} + (1 - N_c^{-N_p})W_{adj}^2(L)$$

$$S = -\mathrm{Tr}\left(N_c^{-N_p}\hat{I}\,\ln\left(N_c^{-N_p}\hat{I}\right)\right) = \mathrm{Tr}\left(N_c^{-N_p}\hat{I}N_p\ln N_c\right) = N_c^{N_p}N_c^{-N_p}N_p\ln N_c =$$

$$= N_p\ln N_c$$

12. Purity, Von Neumann Entropy

State:	pure separable	mixed separable	pure entangled
P (purity)	1	$\frac{1}{N_c^{N_p}} \leq P < 1$	1
S (entropy)	0	$0 < S \leq N_p \ln N_c$	0

$\downarrow$ RT$\rightarrow\infty$

State:	pure separable	mixed separable	pure entangled
P (purity)	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$
S (entropy)	$N_p \ln N_c$	$N_p \ln N_c$	$N_p \ln N_c$

Purity decreases, Entropy increases

Stability of Movement of Gauge Fields and Source Fields

- **1). Classical level**
- **1.Order to Chaos transition, critical energy, Higgs mass**

- SU(2) Yang-Mills field system has unstable movement under any values of parameters, **chaotic solutions of Yang-Mills** (**S.G. Matinyan (1981); G. K. Savvidy, (1983)**), **possible chaos onset (Kawabe)**
-
- Higgs fields and quantum fluctuations of gauge fields induce to regularization of dynamics of system of Yang-Mills, to appear of areas of stable and the regularized motion **(Berman; Matinyan; Salashnich; Muller;); Kuvshinov, Kuzmin, Petrov**).
- **It was shown that Higgs bosons and its vacuum quantum fluctuations regularize the system and lead to the emergence of order-chaos transition at some critical energy (Matinyan Kuvshinov, Kuzmin [18-21])**

$$E_c = \frac{3\mu^4}{64\pi^2}\exp\left(1 - \frac{\lambda}{g^4}\right)$$

$$E_C = \frac{3\mu^4}{32\pi^2}\exp\left(2\alpha_w - \frac{2\lambda}{g^4}\beta_w\right)\left(1 + \frac{1}{2\cos^4\theta_w}\right)\left(1 - 7e^{-2}\right)$$

$$\alpha_w = \frac{2\ln\cos\theta_w}{1 + 2\cos^4\theta_w}; \quad \beta_w = \frac{32\pi^2\cos^4\theta_w}{9\left(1 + 2\cos^4\theta_w\right)}$$

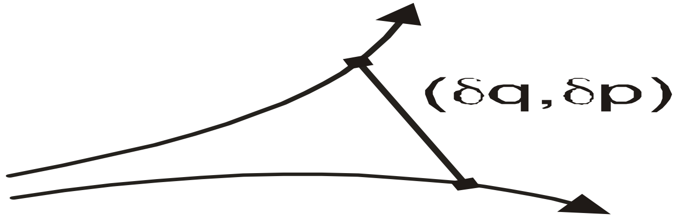
- **Here μ is mass of Higgs boson, λ is its self interaction coupling constant, g is coupling constant gauge and Higgs fields**
- **In the region of confinement there exists the point of order -chaos transition where the fidelity decreased exponentially and which is equal to string tension .**
- **This connects the properties of stochastic QCD vacuum and Higgs boson mass and self interaction coupling constant**

2.Generalized Toda criterion (N- number degrees of freedom)

(Kuvshinov,Kuzmin)

Hamiltonian:

$$H = \frac{1}{2}\vec{p}^{\,2} + V(\vec{q}),$$



$$\vec{p} = (p_{1,\dots,}p_N), \qquad \vec{q} = (q_{1,\dots,}q_N)$$

Linearized Hamilton equations:

$$\frac{d}{dt}\begin{pmatrix}\delta\vec{q}\\ \delta\vec{p}\end{pmatrix} = G\begin{pmatrix}\delta\vec{q}\\ \delta\vec{p}\end{pmatrix}, \; G \equiv \begin{pmatrix} 0 & I \\ -\Sigma & 0\end{pmatrix}, \qquad \Sigma \equiv \left(\frac{\partial^2 V}{\partial q_i \partial q_j}\right)_{\vec{q}_0}$$

$\lambda_i \;, \; i = \overline{1, 2N}$ eigenvalues of G matrix

$\xi_i \;, \; i = \overline{1, N}$ eigenvalues of Σ matrix

3.Generalized Toda criterion

If there exist i, that $\mathrm{Re}\,\lambda_i \neq 0$

If there exist j, that $\xi_j \geq 0$, then

we have local instability and chaos

In other case movement is regular and stable

In the case of two degrees jf freedom , the criterium transfer into the known Toda critetion (M. Toda, Phys.Lett.A vol.48 N5 (1974) p.335)

2). Quantum level

1. Fidelity

The stability of quantum motion of the particles is described by fidelity f (Peres[40], Prosen[41], Cheng[42]). The definition of fidelity is similar with Wilson loop definition in QCD (Kuvshinov, Kuzmin [14]). Using the analogy between the theory of gauge fields and the theory of holonomic quantum computation (Reineke ; Kuvshinov,Kuzmin, Buividovich [9,14,15]) we can define the fidelity of quark motion (the scalar product of state vectors for perturbed and unperturbated motion) (or two density matrices) as an integral over the closed loop, with particle traveling from point x to the point y

$$\boxed{f = \left\langle \left(\langle \phi_{in} | \hat{P} \exp\left(\int i \hat{A}_\mu dx^\mu \right) | \phi_{in} \rangle \right) \right\rangle \quad (8)}$$

The final expression for the fidelity of the particle moving stochastic vacuum is

$$f = \exp\left(-\frac{1}{2} g^2 l_{corr}^2 F^2 S_\gamma \right) \qquad (9)$$

Thus, fidelity for colour particle moving along contour decays exponentially with the surface spanned over the contour, the decay rate being equal to the string tension (6)

Motion becomes more and more unstable when $S_\gamma \rightarrow \infty$.

❖ Sometimes fidelity is defined in another way (Hubner [34], Uhlmann [35], Kuvshinov, Bagashov [33])

$$F(\omega,\tau) = \mathrm{Tr}\,\left(\sqrt{\sqrt{\omega}\tau\sqrt{\omega}}\right) \rightarrow F = \mathrm{Tr}\,\left(\hat{\rho}_{in}\sqrt{N_c^{-1} + (1 - N_c^{-1})W_{adj}(L)}\right)$$

(Square root of probability of transition from the state with density matrix ω to state with d.m. τ; ϱin to ϱout) The fidelity decreases For two random paths in Minkowski space, which are close to each other, the expression for the fidelity is similar, but now the averaging is performed with respect to all random paths which are close enough. And the final expression is

$$f = \exp\left(-\frac{1}{2}g^2 l_{corr}\int_{\gamma_1} dx^{\upsilon} F_{\chi\alpha}\widetilde{F}_{\upsilon\beta}\upsilon^{\chi}\langle\delta\chi^{\alpha}\delta\chi^{\beta}\rangle\right) \qquad (10)$$

where δχ - is the deviation of the path γ2 from the path γ1, υ is the four-dimensional velocity and lcorr is the correlation length of perturbation of the particle path expressed in units of world line length. If unperturbed path is parallel to the time axis in Minkowski space, the particle moves randomly around some point in three dimensional space. The fidelity in this case decays exponentially with time.

❖ Thus, we have connection between confinement and instability of colour particle motion

3.Quantum Chaos Criterion

(Kuvshinov, Kuzmin, PL, 2002)

Two-point connected Green function :

$$G_{ik}(x,y) = -\left.\frac{\delta^2 W[\vec{J}]}{\delta J_i(x)\delta J_k(y)}\right|_{\vec{J}=0}$$

Chaos criterion :

- Chaos: Green function exponentially (or faster) tends to zero under |x – y| → ∞,$(x - y)^2>0$, $(x^0 – y^0) > 0$.
- Order: Green function oscilates and slowly decreases in this limit.

Correspondence with classical chaos criterion:

$$G_i(t_1 - t_2) = \frac{i}{2}\mathrm{Re}\left(\frac{\exp\{-\lambda_i(t_1 - t_2)\}}{\lambda_i}\right),\ t_1 > t_2$$

When $\tau q \gg \Delta tcl$ (dynamical localization)

a) If classical motion is locally unstable (chaotic) then according Toda criterion there is real eigenvalue λi. Therefore Green function exponentially goes to zero for some I when (t1 − t2) → +∞. Opposite is also true. If Green function exponentially goes to zero under the condition (t1 − t2) → +∞ for some i, then there exists real eigenvalue of the stability matrix and thus classical motion is locally unstable.

b) If all eigenvalues of the stability matrix G are pure imaginary, that corresponds classically stable motion, then in the limit (t1 − t2) → +∞ Green function oscillates as a sine. Opposite is also true. If for any i Green functions oscillate in the limit (t1−t2) → +∞ then {λi} are pure imaginary for any i and classical motion is stable and regular.

Thus, proposed quantum chaos criterion coincides with Toda criterion in the semi-classical limit (corresponding principle)

Conclusion

- Quantum squeezinq and entanglement of gluons are possible under nonperturbative (A^4) evolution of colour particles in QCD vacuum
- Vacuum of quantum chromodynamics can be considered as environment (in the sense of quantum optics) for colour particles
- In the case of stochastic (not coherent) QCD vacuum (only correlators of the second order are important) in confinement region (Wilson loop decays exponentially) we have decoherence of pure colour states into a mixed white states with purity which decays exponentially (decay rate =string tension)
- Density matrix, Purity and Fidelity for colour particles are depended on Wilson loop averaged through QCD vacuum degrees of freedom
- For multiparticle states (pure separable, mixed separable and nonsepaparable (entangled) when $RT \to \infty$ we obtain diagonalization of density matrix, decreasing of purity and fidelity, increasing of Von Neumann entropy
- Instable dynamics of classical Yang-Mills fields can be regularized by Higgs fields and quantum fields fluctuations. Critical point of order-chaos transition appears
- Instability (chaoticity) of quantum colour particle motion in confinement region corresponds to the fidelity which drops exponentially
- Quantum chaos criterium is formulated in terms of two-point connected Green function

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Thank you for the attention!