Nonlinearity of the Forward-Backward Correlation Function in the Model with String Fusion

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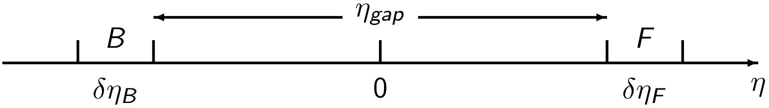
Forward-Backward Rapidity Correlations



$$\langle B \rangle_F = f(F)$$ - the FB correlation function

$$\langle B \rangle_F = a + b_{BF} F$$ - the linear regression

The correlation coefficient:

$$b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F,B)}{D_F}$$ - by correlator

$$b'_{BF} = \left. \frac{d\langle B \rangle_F}{dF} \right|_{F=\langle F \rangle}$$ - by derivative

Observables

B, F:

n_B, n_F - the extensive variables $\Rightarrow$ b_{nn}

p_{tB}, p_{tF} - the intensive variables $\Rightarrow$ $b_{p_t p_t}$

$$p_{tB} = \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}^i_{tB}| \qquad p_{tF} = \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}^i_{tF}|$$

p_{tB}, n_F - the combination of the variables $\Rightarrow$ $b_{p_t n}$

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)

The locality of strong interaction in rapidity $\Rightarrow$

Short-Range FB Correlations (SRC) (between particles from a same string)

Event-by-event variance in the number of cut pomerons (strings) $\Rightarrow$

Long-Range FB Correlations (LRC) at large η_{gap}

LRC: b_{nn}, b'_{nn} and $b_{p_t n}$, $b'_{p_t n}$

String fusion effects

pp $\rightarrow$ pA $\rightarrow$ AA - the increase of the string density in transverse plain
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993).
$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.

N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares,
Phys.Rev.Lett. **73**, 2813 (1994).
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).

More clear signal of string fusion in LR correlations involving intensive variables, e.g. the mean p_t, because they can not be explained by simple "volume" fluctuation - the event-by-event fluctuation in the number of sources (strings), and need the fluctuation in "quality" of sources.

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$$\langle n \rangle_k = \mu_0 \sqrt{k}\; S_k/\sigma_0 \;, \qquad \langle p_t^2 \rangle_k = p_0^2 \sqrt{k} \;, \qquad k = 1, 2, 3, \ldots \qquad (1)$$

global fusion (clusters)

M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$$\langle p_t^2 \rangle_{cl} = p_0^2 \sqrt{k_{cl}} \;, \qquad \langle n \rangle_{cl} = \mu_0 \sqrt{k_{cl}} S_{cl}/\sigma_0 \;, \qquad k_{cl} = k\sigma_0/S_{cl} \quad (2)$$

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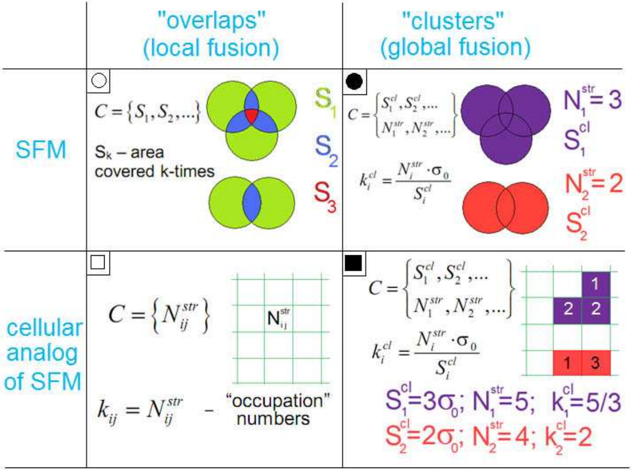
the version of SFM with the finite lattice in transverse plane

Braun M.A., Kolevatov R.S., Pajares C., V.V. Eur.Phys.J. C32 (2004) 535.

V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858.

Various versions of string fusion



Variation and fluctuation of the colour field strength in transverse plane.

Description of the configurations

String configuration (η_i - the number of string centers in i-th cell):

$$C_\eta = \{\eta_1, ..., \eta_M\} \ . \tag{3}$$

General configuration (n_i - the number of particles produced from i-th cell and p_i^j - their transverse momenta):

$$C = \left\{ C_\eta, C_n^F, C_n^B, C_p^F, C_p^B \right\} \ , \tag{4}$$

$$C_n^F = \left\{ n_1^F, ..., n_M^F \right\} \ , \quad C_n^B = \left\{ n_1^B, ..., n_M^B \right\} \ , \tag{5}$$

$$C_p^F = \left\{ p_1^{1F}, ..., p_1^{n_1^F F}; ...; p_M^{1F}, ..., p_M^{n_M^F F} \right\} \ . \tag{6}$$

Then

$$n_F = \sum_{i=1}^{M} n_i^F \ , \qquad p_F \equiv \frac{1}{n_F} \sum_{i=1}^{M} \sum_{j=1}^{n_i^F} p_i^{jF} \tag{7}$$

and the same for the p_B and n_B.

Direct Monte-Carlo Simulations

C includes F and B:

$$P(C) \qquad\qquad \langle B\rangle_F = f(F)$$

$$\sum_C P(C)\ \ldots \Rightarrow \frac{1}{n_{sim}}\sum_{sim}\ \ldots$$

E.g. the similar MC approach, with lattice in transverse plane, was applied for the calculations of anisotropic azimuthal flows and ridge in String Fusion Model:

M.A.Braun and C.Pajares, *Eur. Phys. J. C* **71** (2011) 1558.
M.A. Braun, C. Pajares, V.V., *Nucl. Phys. A* **906** (2013) 14.
M.A. Braun, C. Pajares, V.V., *Eur. Phys. J. A* **51** (2015) 44.

The origin of the anisotropy in this approach is the quenching of produced particles in the strong color field of strings.

General formulae for LR correlations

C does not include F and B
(e.g. $C = \{C_\eta, C_n^F, C_n^B\}$ for $F = p_F$ or $C = \{C_\eta\}$ for $F = n_F$):
Braun M.A., Pajares C., V.V. Phys.Lett. B493 (2000) 54.
LRC:

$$P(F,B) = \sum_C P(C)\, P_C(F)\, P_C(B)\ . \tag{8}$$

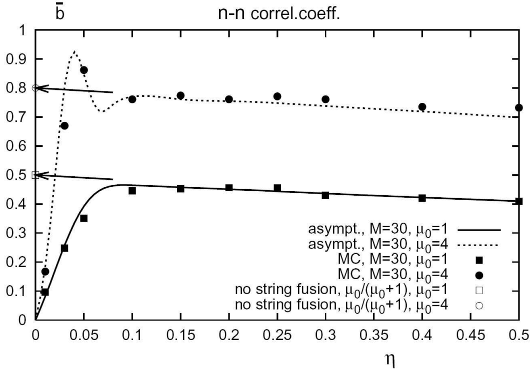
$$P(F) = \sum_C P(C)\, P_C(F)\ . \tag{9}$$

Correlation function:

$$\langle B\rangle_F \equiv \sum_B B\, P_F(B) = \frac{\sum_C \langle B\rangle_C\, P(C)\, P_C(F)}{\sum_C P(C)\, P_C(F)} \tag{10}$$

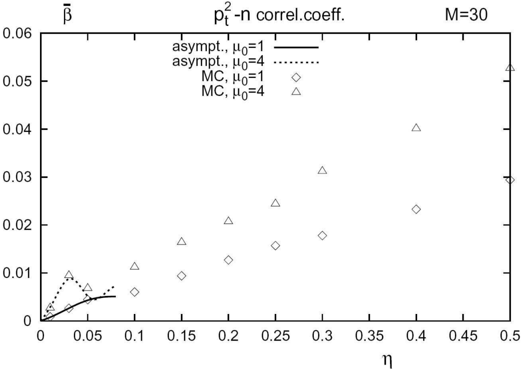
$$\langle B\rangle = \sum_C P(C)\langle B\rangle_C\ , \qquad \langle F\rangle = \sum_C P(C)\langle F\rangle_C \tag{11}$$

Previous $n-n$ results



Correlation coefficient b'_{nn}. Uniform string distribution with mean string density η per cell. M - the number of cells. μ_0 - the mean number of particle in the observation window from a decay of one string, $\mu_0 = \frac{dN}{d\eta}\delta\eta$. The poissonian fluctuations of string density and the number of particles from a string fragmentation are assumed (scaled variances: $\omega_\eta = \omega_\mu = 1$).

Previous $p_t^2 - n$ results



Correlation coefficient $b'_{pt^2 n}$ under same suggestions.
Points - MC calculations with (10) and $C = C_\eta$.
Lines - analitical results, based on the expantion at small η.
V.V., R.S. Kolevatov, arXiv:hep-ph/0305136 (2003);
Vestnik SPbU, ser.4, no.4, 11 (2004).

Without string fusion

Explicit analitical formula for b_{nn}:

$$b_{nn} = \frac{\omega_N - \lambda \overline{N}}{\frac{\omega_{\mu_F}}{\overline{\mu_F}} + \omega_N - \lambda \overline{N}} \ , \tag{12}$$

where

$$N = \sum_{i=1}^{M} \eta_i \ , \qquad \overline{N} = \eta M$$

and ω_{μ_F} and ω_N are the corresponding scaled variances:

$$\omega_{\mu_F} = D_{\mu_F}/\overline{\mu_F} \ , \qquad \omega_N = D_N/\overline{N} \ , \qquad D_N = \overline{N^2} - \overline{N}^2 \ . \tag{13}$$

$$\lambda = \frac{P(0)}{1 - P(0)} \ , \qquad P(0) = p^M(0) \tag{14}$$

Corresponds to [V.V. , arXiv:1012.0214, 2010, Proceedings of the Baldin ISHEPP XX vol.2, JINR, Dubna (2011) 10], plus it takes into account the additional condition $N \geq 1$.

Modified poissonian distribution in N

$$P'(0) = 0 \ , \qquad P'(N) = \frac{P(N)}{1 - P(0)} \ , \qquad N \geq 1 \ ,$$

$$P(N) \equiv e^{-\overline{N}} \frac{\overline{N}^N}{N!} \ . \tag{15}$$

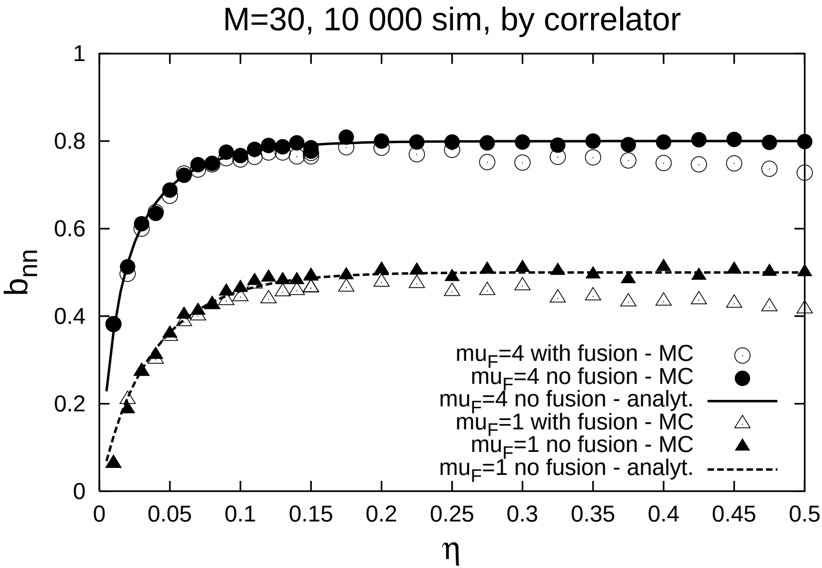
So

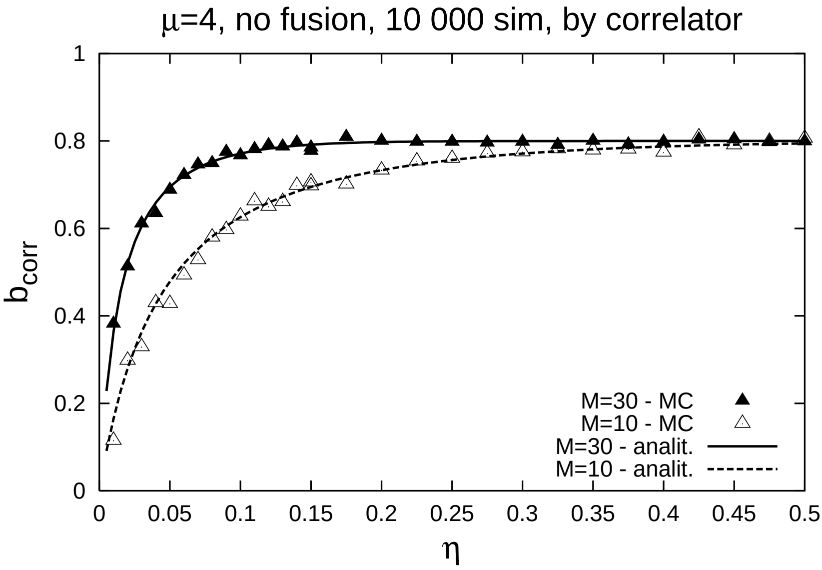
$$\langle N \rangle = \frac{\overline{N}}{1 - P(0)} \tag{16}$$

For modified poissonian distribution in number of strings we have

$$b_{nn} = \frac{1 - \lambda \eta M}{\frac{\omega_{\mu_F}}{\overline{\mu_F}} + 1 - \lambda \eta M} \ , \qquad \lambda = \frac{e^{-\eta M}}{1 - e^{-\eta M}} \ . \tag{17}$$

M dependency only at $\eta \lesssim \frac{1}{M}$.


M=30, 10 000 sim, by correlator
b_nn
η
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.2
0.4
0.6
0.8
1
mu_F=4 with fusion - MC
mu_F=4 no fusion - MC
mu_F=4 no fusion - analyt.
mu_F=1 with fusion - MC
mu_F=1 no fusion - MC
mu_F=1 no fusion - analyt.


μ=4, no fusion, 10 000 sim, by correlator
1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
bcorr
η
M=30 - MC
M=10 - MC
M=30 - analit.
M=10 - analit.

Analytical calculation of CF without fusion - 1

Correlation function:

$$\langle B\rangle_F = \frac{\sum_{N=1}\langle B\rangle_N \, P'(N)\, P_N(F)}{\sum_{N=1} \, P'(N)\, P_N(F)} \ , \tag{18}$$

with

$$\langle B\rangle_N = \mu_B N \ , \qquad \langle F\rangle_N = \mu_F N \ . \tag{19}$$

For the n-n correlation, $F = n_F$ and $B = n_B$, with poissonian $P_N(F)$:

$$P_N(F) = e^{-\mu_F N} \, \frac{(\mu_F N)^F}{F!} \ , \tag{20}$$

it gives:

$$\langle B\rangle_F = \mu_B \frac{\Phi(F+1)}{\Phi(F)} \ , \qquad \Phi(F) \equiv \sum_{N=1} P(N) N^F e^{-\mu_F N} \ . \tag{21}$$

Note the nonlinear CF dependence on μ_F ! In agreement with $\langle B\rangle_F =$ $= (F+1)\frac{P(F+1)}{P(F)}$, obtained for $\mu_F = \mu_B$ in [Braun M.A., Pajares C., V.V. Phys.Lett.B493 (2000) 54] using generating function technique.

Analytical calculation of CF without fusion - 2

For modified poissonian distribution $P'(N)$ in string number, (15), omitting constant common factor, we can use

$$\Phi(F) = \sum_{N=1} N^F \frac{a^N}{N!} = \left(a\frac{d}{da}\right)^F e^a \ . \tag{22}$$

$$\Phi(0) = e^a - 1, \qquad a \equiv \overline{N} e^{-\mu_F} = \eta M e^{-\mu_F} \ . \tag{23}$$

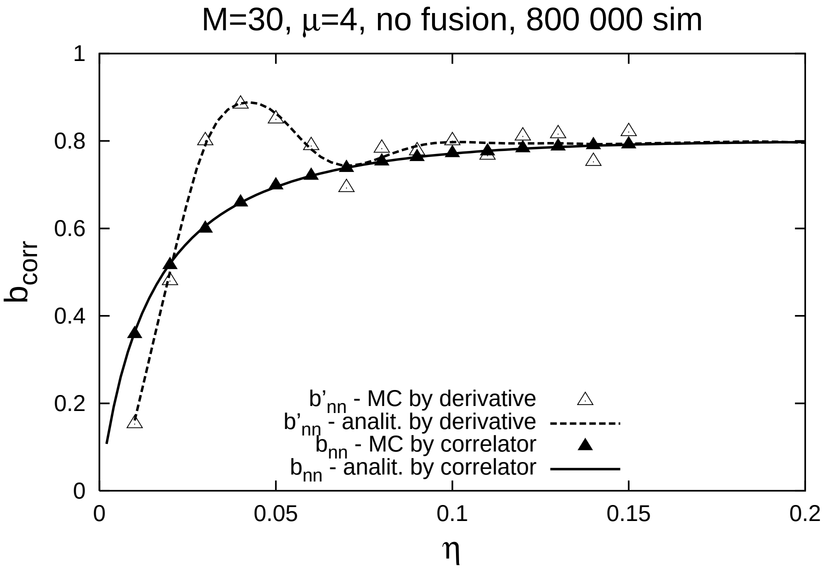
Explisit formula:

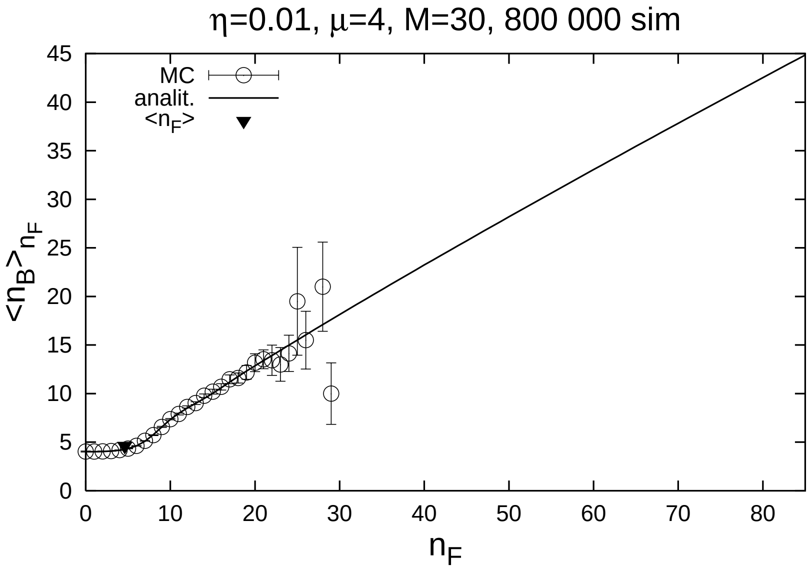
$$\Phi(F) = e^a \sum_{k=1}^{F} \frac{(-1)^k a^k}{k!} \left[\sum_{m=1}^{k} (-1)^m C_k^m m^F\right] \ . \tag{24}$$

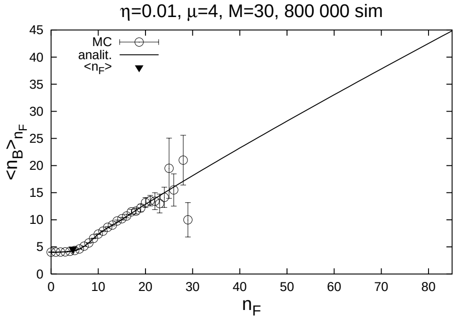
$$\Phi(F) = e^a S_F \ , \qquad S_F = \sum_{k=1}^{F} b_k^{(F)} a^k \ , \qquad F \geq 1 \ , \tag{25}$$

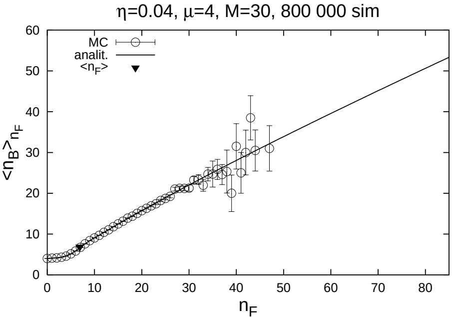
Recurrence relation: $\quad b_k^{(F+1)} = k\, b_k^{(F)} + b_{k-1}^{(F)} \ , \qquad k = 2, ..., F \ ,$

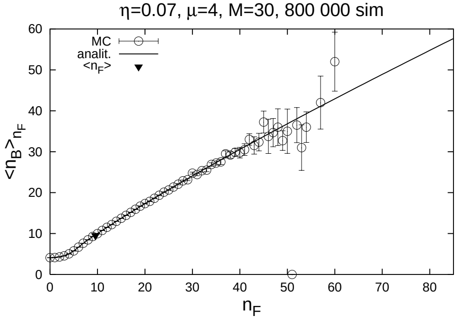
$$b_F^{(F)} = 1 \ , \qquad b_1^{(F)} = 1 \ , \qquad b_0^{(F)} = 0 \ .$$


M=30, μ=4, no fusion, 800 000 sim
b_corr
η
0
0.05
0.1
0.15
0.2
0
0.2
0.4
0.6
0.8
1
b'_nn - MC by derivative
b'_nn - analit. by derivative
b_nn - MC by correlator
b_nn - analit. by correlator


η=0.01, μ=4, M=30, 800 000 sim
MC
analit.
<n_F>
<n_B>_{n_F}
n_F
0
5
10
15
20
25
30
35
40
45
0
10
20
30
40
50
60
70
80


η=0.01, μ=4, M=30, 800 000 sim
MC
analit.
<n_F>
<n_B>_{n_F}
n_F
0
5
10
15
20
25
30
35
40
45
0
10
20
30
40
50
60
70
80


η=0.04, μ=4, M=30, 800 000 sim
MC
analit.
<n_F>
<n_B>_{n_F}
n_F
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70
80


η=0.07, μ=4, M=30, 800 000 sim
MC
analit.
<n_F>
<n_B>_{n_F}
n_F
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70
80

CF for binomial distribution in number of particles - 1

$$P_N(F) = C_{Nk}^F p^F q^{Nk-F} \ , \quad p+q=1 \ , \quad \langle F \rangle = \mu_F N = pkN \ , \quad \omega_F = q \ .$$

Correlation function:

$$\langle B \rangle_F = \frac{\mu_B \sum_{N=1} N \, P'(N) \, Nk(Nk-1)...(Nk-F+1) q^{Nk}}{\sum_{N=1} P'(N) \, Nk(Nk-1)...(Nk-F+1) q^{Nk}} \ , \tag{26}$$

$$\langle B \rangle_F = \frac{\mu_B \left(\frac{d}{dq}\right)^F \left(\frac{q}{k}\frac{d}{dq}\right) \sum_{N=1} P'(N) q^{Nk}}{\left(\frac{d}{dq}\right)^F \sum_{N=1} P'(N) q^{Nk}} \ . \tag{27}$$

For modified poissonian distribution $P'(N)$ in string number, (15), omitting constant common factor, we have

$$\langle B \rangle_F = \frac{\mu_B}{k} \frac{\left(\frac{d}{dq}\right)^F \left(q\frac{d}{dq}\right) e^{\overline{N} q^k}}{\left(\frac{d}{dq}\right)^F [e^{\overline{N} q^k} - 1]} \ . \tag{28}$$

CF for binomial distribution in number of particles - 2

For $k = 1$, $P_1(F = 0) = q$ and $P_1(F = 1) = p$,

$\Rightarrow$ Exactly linear CF for poissonian distribution in number of strings:

$$\langle B \rangle_F = q\langle B \rangle + p\,F\ , \qquad b_{nn} = b'_{nn} = p\ , \tag{29}$$

for symmetrical observation windows $\langle B \rangle = \langle F \rangle = \mu\overline{N} = pk\overline{N} = p\overline{N}$.

It can be also easily checked using generating function technique.

For $k = 2$, $P_1(F = 0) = q^2$, $P_1(F = 1) = 2qp$ and $P_1(F = 2) = p^2$,

introducing $x \equiv q\sqrt{\overline{N}}$, we find

$$\langle B \rangle_F = \mu_B \frac{E(F)}{D(F)} = \mu_B \frac{Q_F}{P_F}\ , \tag{30}$$

$$E(F) = \left(\frac{d}{dx}\right)^F [x^2 e^{x^2}] \equiv e^{x^2} Q_F$$

$$D(F) = \left(\frac{d}{dx}\right)^F e^{x^2} \equiv e^{x^2} P_F \ \text{ for } \ F \geq 1\ , \qquad D(0) = e^{x^2} - 1$$

CF for binomial distribution in number of particles - 3

$$Q_F = \sum_{k=0}^{F+2} c_k^{(F)} x^k \ , \qquad P_F = \sum_{k=0}^{F} d_k^{(F)} x^k \ , \qquad F \geq 1 \ . \qquad (31)$$

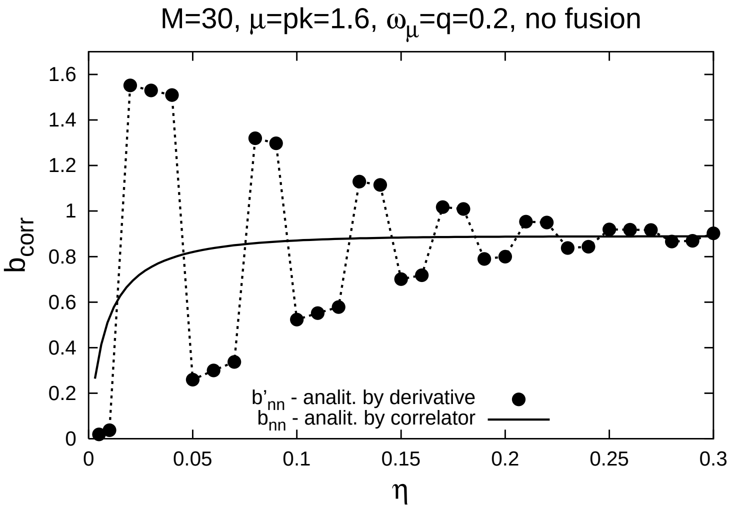
Recall, that $x \equiv q\sqrt{\overline{N}}$.

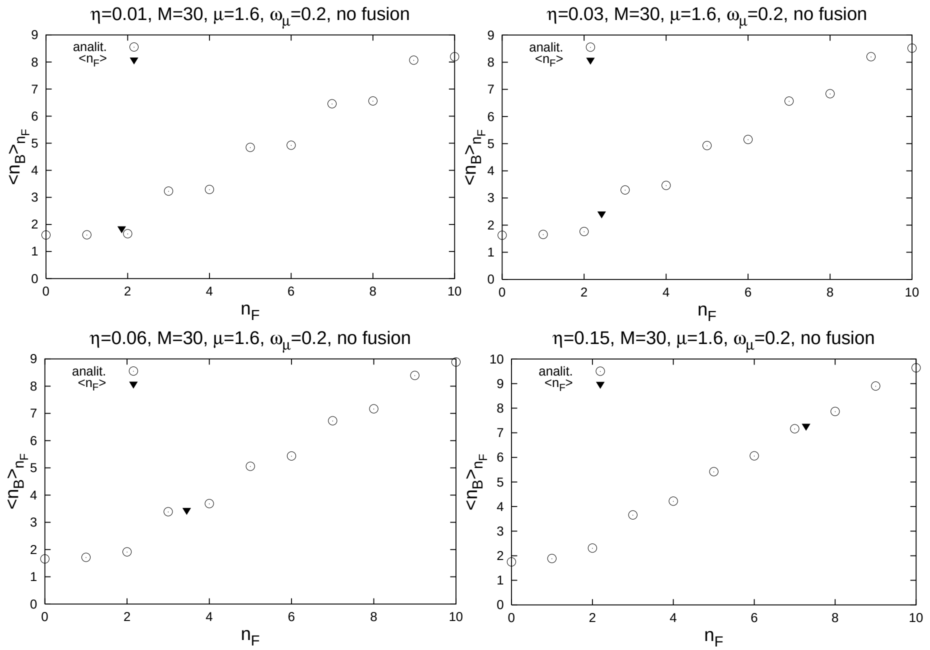
The same recurrence relation for Q_F and P_F:

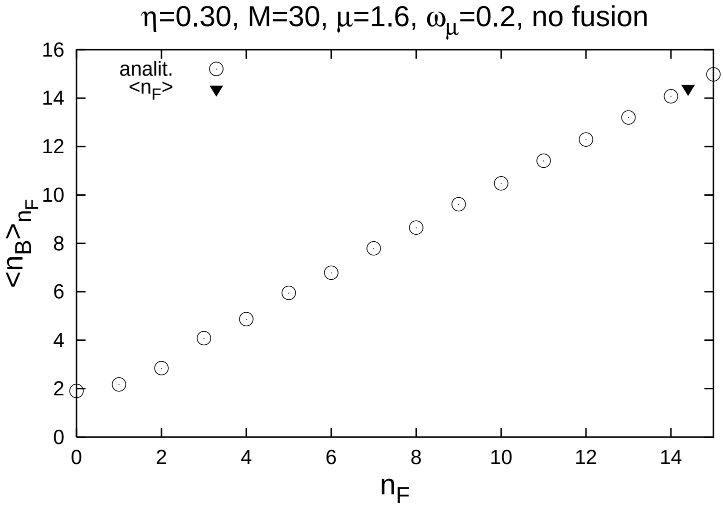
$$c_k^{(F+1)} = (k+1)\, c_{k+1}^{(F)} + 2\, c_{k-1}^{(F)} \ , \qquad k = 1, ..., F+1 \ ,$$

$$c_{F+3}^{(F+1)} = 2\, c_{F+2}^{(F)} \ , \qquad c_{F+2}^{(F+1)} = 2\, c_{F+1}^{(F)} \ , \qquad c_0^{(F+1)} = c_1^{(F)} \ ,$$

but with different initial conditions.


M=30, μ=pk=1.6, ω$_μ$=q=0.2, no fusion
b$_{corr}$
η
b'$_{nn}$ - analit. by derivative
b$_{nn}$ - analit. by correlator
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
0.05
0.1
0.15
0.2
0.25
0.3


η=0.01, M=30, μ=1.6, ω_μ=0.2, no fusion
η=0.03, M=30, μ=1.6, ω_μ=0.2, no fusion
η=0.06, M=30, μ=1.6, ω_μ=0.2, no fusion
η=0.15, M=30, μ=1.6, ω_μ=0.2, no fusion
analit.
<n_F>
<n_B>_{n_F}
n_F


η=0.30, M=30, μ=1.6, ω_μ=0.2, no fusion
analit.
<n_F>
<n_B>_{n_F}
n_F
0
2
4
6
8
10
12
14
16

Mean p_t - multiplicity correlation - 1

Due to string fusion:

$$\langle p_{tB} \rangle_{n_F} = \frac{\sum_C \langle p_{tB} \rangle_C \, P(C) \, P_C(n_F)}{\sum_C P(C) \, P_C(n_F)} \tag{32}$$

Recall that

$$C = \left\{ C_\eta, C_n^F, C_n^B, C_p^F, C_p^B \right\} \ . \tag{33}$$

Averaging over C_p^B gives

$$\langle p_{tB} \rangle_{C_\eta C_n^B} = \left\langle \frac{\sum_{i=1}^M n_i^B \overline{p}_{tB}(\eta_i)}{\sum_{i=1}^M n_i^B} \right\rangle_{C_\eta C_n^B} \ , \tag{34}$$

where

$$\overline{p}_{tB}(\eta_i) = p_0 \sqrt[4]{\eta_i} \ .$$

Mean p_t - multiplicity correlation - 2

.*Braun M.A., Kolevatov R.S., Pajares C., V.V.* Eur.Phys.J.C32(2004)535

For poissonian distribution in n_i^B averaging over C_n^B gives:

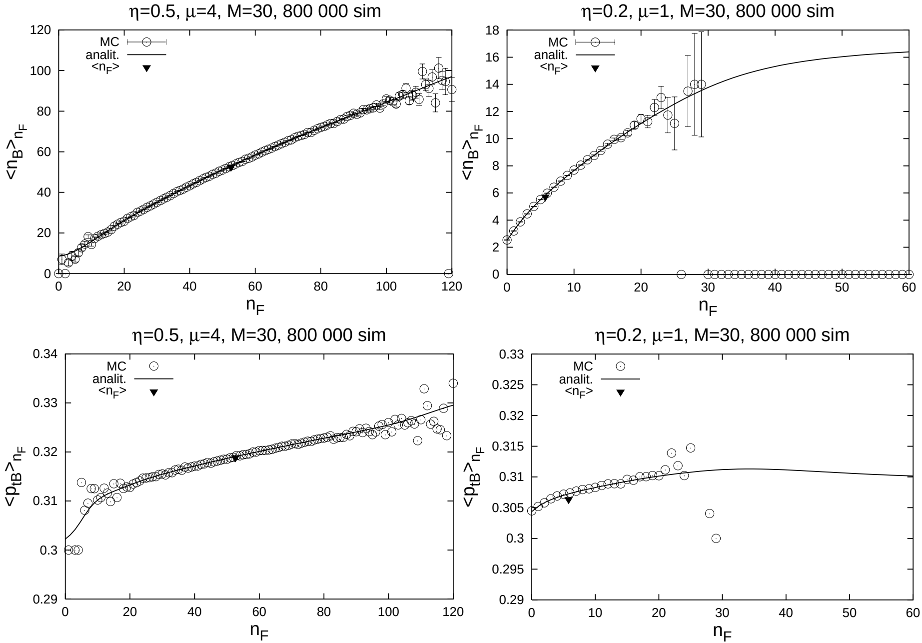
$$\langle p_{tB} \rangle_{C_\eta} = \left\langle \frac{\sum_{i=1}^{M} \overline{n}_i^B(\eta_i) \overline{p}_{tB}(\eta_i)}{\sum_{i=1}^{M} \overline{n}_i^B(\eta_i)} \right\rangle_{C_\eta} , \qquad (35)$$

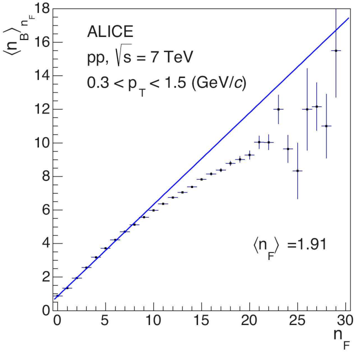
with $\quad \overline{n}_i^B(\eta_i) = \mu_B \sqrt{\eta_i}.$

$$\langle p_{tB} \rangle_{n_F} = \frac{\sum_{C_\eta} \langle p_{tB} \rangle_{C_\eta} P(C_\eta)\, P_{C_\eta}(n_F)}{\sum_{C_\eta} P(C_\eta)\, P_{C_\eta}(n_F)} \qquad (36)$$

For poissonian distribution in n_i^F:

$$P_{C_\eta}(n_F) = P_{\langle n_F \rangle}(n_F) = e^{-\langle n_F \rangle} \frac{\langle n_F \rangle^{n_F}}{n_F!} , \qquad \langle n_F \rangle = \mu_F \sum_{i=1}^{M} \sqrt{\eta_i} \qquad (37)$$

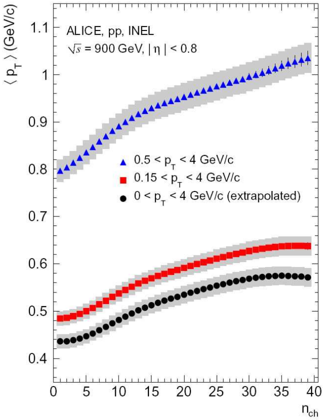

η=0.5, μ=4, M=30, 800 000 sim
MC
analit.
<nF>
<nB>nF
nF
η=0.2, μ=1, M=30, 800 000 sim
MC
analit.
<nF>
<nB>nF
nF
η=0.5, μ=4, M=30, 800 000 sim
MC
analit.
<nF>
<ptB>nF
nF
η=0.2, μ=1, M=30, 800 000 sim
MC
analit.
<nF>
<ptB>nF
nF



ALICE Collab., JHEP 05 (2015) 097. pp 7 TeV/c

Explanation of the saturation of the *n*-*n* CF at large multiplicities for bulk particle production:

The *n*-*n* long-range FBC are dominated mainly by fluctuations in the number of initial strings (the so-called "volume" fluctuation). The influence of the string fusion is additional small effect. So at fixed maximal number of initial strings the strength of *n*-*n* long-range FBC saturates, due to independent string decays in forward and backward hemispheres.



ALICE Collab., Phys.Lett.B693 (2010) 53-68. pp 900 GeV/c

Explanation of the decrease of the p_t-n CF at large multiplicities:

for bulk particle production: at fixed maximal number of initial strings the fluctuation of their distribution in transverse plane, compared with the average distribution, due to string fusion effects increases the mean transverse momentum and decreases the multiplicity.

Nonlinearity of the Correlation Function

- Important. Due to huge statistics at LHC we can measure the distribution in F in wide region - far from $\langle F \rangle$.
  (For F in vicinity of $\langle F \rangle$ one can always assume a linear regression.)
- Nonlinearity is greater for large observation rapidity interval $\delta\eta_F$
  (Deviation from the poissonian distribution in F.
  V.V. Nucl. Phys. A 939 (2015) 21.)
- Nonlinear correlation function $\langle B \rangle_F$ containes more physical information, which can not be reduced to one number - the correlation coefficient.
  $\Rightarrow$ The value of the correlation coefficient starts to depend on its definition ($b_{BF} \neq b'_{BF}$), whereas the correlation function, $\langle B \rangle_F$, is uniquely defined.

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