Applications of quantisation deformation to standard model phenomenology: flavor symmetry

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Overview

<u>1. Deformation of symmetries</u>

- Lie deformations and stability of a Lie algebra
- Deformations of enveloping algebra and quantum groups

<u>2. Example: Deformed flavor symmetry</u>

- SU(3) flavor symmetry and Gell-Mann-Okubo mass formulas
- SUq(3) flavor symmetry and deformed mass formulas
- Charge specific baryon mass formulas with deformed SUq(3) flavor symmetry
- Cabibbo angle as a function of q and baryon spin

<u>3. Conclusion</u>

- Further applications of quantum groups to HEP
- Summary

Lie-type deformations of spacetime symmetries

- Algebraic deformations provide a consistent framework within which to generalize the symmetries of spacetime and particles.

Given a Lie algebra L_0 with basis $\{X_i\}$ and bracket $[X_i, X_j]_0 = i f_{ij}^k X_k$, a one parameter deformation of L is defined through the deformed commutator

$$[X_i, X_j]_t = [X_i, X_j]_0 + \sum_{m=1}^{\infty} \phi_m(X_i, X_j) t^m,$$

Where t is the deformation parameter.

- A deformation is said to be trivial is the deformed algebra is isomorphic to L_0. An algebra is stable (rigid) if every deformation is trivial. Semi-simple algebras are stable.

Theories based on unstable Lie algebras should be deformed until a stable theory is reached. Theories based on stable Lie algebras give rise to robust physics free of fine tuning issues.

- Galilean Relativity→ Special Relativity→ (anti) de Sitter relativity

Hopf-type deformations and particle symmetries

- dS/AdS is stable (rigid) which means it can't be deformed in the category of Lie group. However it can be deformed in the category of Hopf algebras.

- A quantum group arises from a deformation of the enveloping algebra (a Hopf algebra) of a semi-simple Lie algebra.

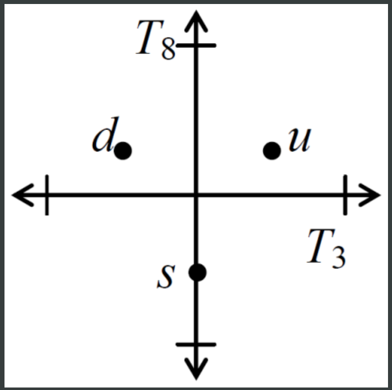
<u>Applications to SM physics:</u>

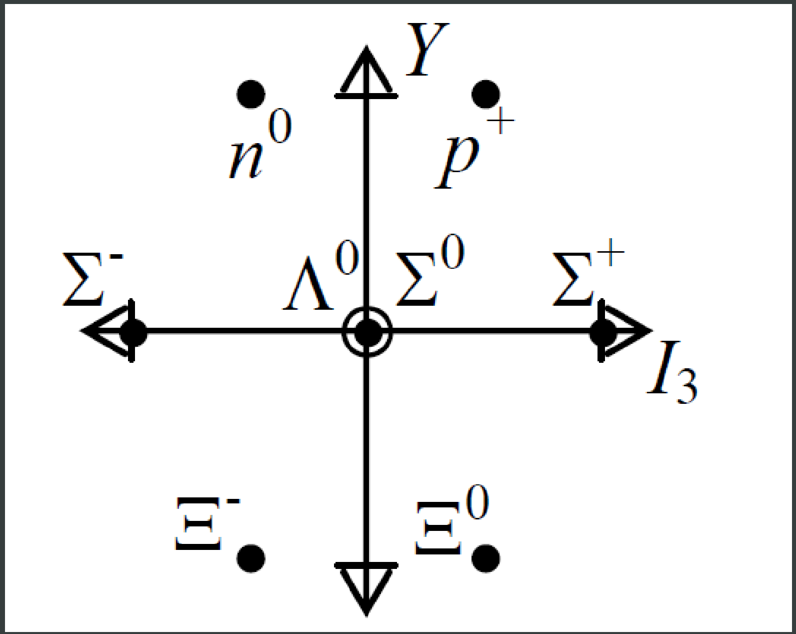
1. q-deformed gauge groups: SU(2)xU(1)→$Su_q(2)$
   - solitonic interpretation of elementary particles (Finkelstein)
   - Particle properties expressed in terms of knot invariants

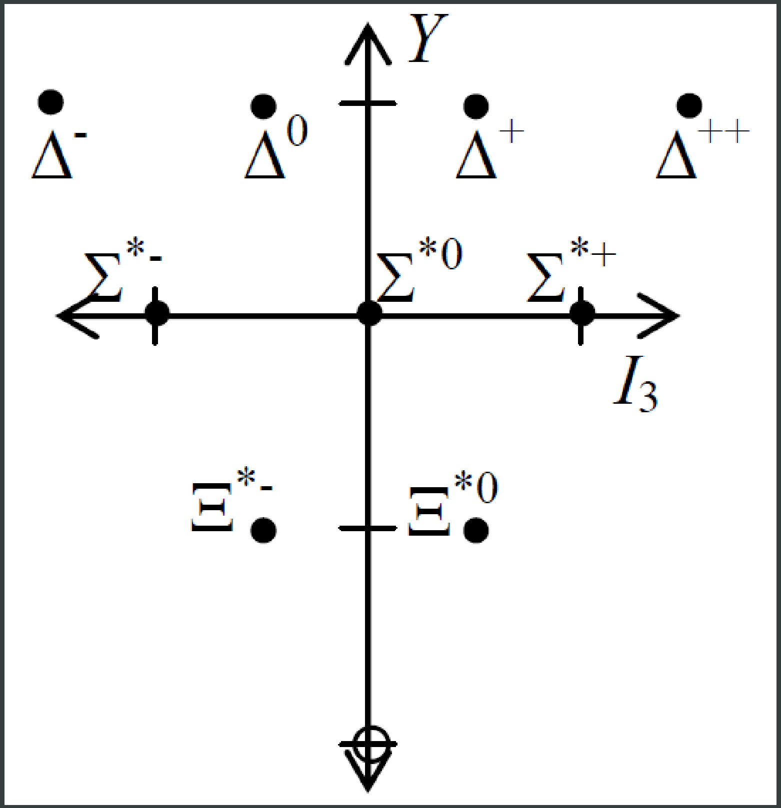
2. q-deformed flavour symmetry: SU(3)→$SU_q(3)$
   - Improved baryon mass relations
   - Connection between deformation parameter and Cabibbo angle

Robert J Finkelstein. The Elementary Particles as Quantum Knots in Electroweak Theory. *International Journal of Modern Physics A*, 22(24):4467–4480, 2007.

SU(3) flavor symmetry and Gell-Mann-Okubo mass formulas







SU(3) flavor symmetry and Gell-Mann-Okubo mass formulas

- Gell-Mann-Okubo formula

$$M = \alpha_0 + \alpha_1 S + \alpha_2 \left[I(I+1) - \frac{1}{4} S^2 \right],$$

M is the mass of a hadron within a specific multiplet,
S and I are the strangeness and isospin respectively,
$\alpha_1, \alpha_2, \alpha_3$ are free parameters.

- Octet Baryons

$$N + \Xi = \frac{3}{2}\Lambda + \frac{1}{2}\Sigma,$$

0.6%

- Decuplet Baryons

$$\Delta - \Sigma^* = \Sigma^* - \Xi^* = \Xi^* - \Omega$$

- Okubo formula (valid to 2nd order flavor symmetry breaking)

$$\Omega - \Delta = 3(\Xi^* - \Sigma^*)$$

1.4%

SUq(3) flavor symmetry and deformed mass formulas

The quantum (enveloping) algebra $SUq(n) \equiv U_q(su(n))$ corresponding to a one-parameter deformation of the universal enveloping algebra of $su(n)$, is a Hopf algebra with unit $\mathbf{1}$ and generators H_i, $X_i^{\pm}$, $i = 1, 2, ..., n-1$, defined through the commutation relations in the Cartan-Chevalley basis as

$$\begin{aligned} [H_i, H_j] &= 0 \\ [H_i, X_j^{\pm}] &= a_{ij} X_j^{\pm} \\ [X_i^{+}, X_j^{-}] &= \delta_{ij}[H_i]_q \equiv \delta_{ij}\frac{q^{H_i} - q^{-H_i}}{q - q^{-1}}, \end{aligned}$$

together with the quadratic and cubic deformed q-Serre relations

$$[X_i^{\pm}, X_j^{\pm}] = 0,\ j \neq i \pm 1\ 1 \leq i, j \leq n-1,$$

and

$$(X_i^{\pm})^2 X_j^{\pm} - [2]_q X_i^{\pm} X_j^{\pm} X_i^{\pm} + X_j^{\pm}(X_i^{\pm})^2 = 0,\ j = i \pm 1\ 1 \leq i, j \leq n-1.$$

SUq(3) flavor symmetry and deformed mass formulas

Here a_{ij} is an element of the Cartan matrix

$$a_{ij} = \begin{cases} 2 & j = i \\ -1 & j = i \pm 1 \\ 0 & \text{otherwise.} \end{cases}$$

The q-number

$$[N]_q = \frac{q^N - q^{-N}}{q - q^{-1}}$$

is defined for both operators and real numbers (Here we will only have to deal with integer values of N). The definition of the algebra is completed by the Hermiticity properties

$$(H_i)^\dagger = H_i, \qquad (X_i^\pm)^\dagger = X_i^\mp.$$

Michio Jimbo. A q-difference analogue of U (g) and the Yang-Baxter equation. *Letters in Mathematical Physics*, 10(1):63–69, 1985.

R. Jaganathan. Some introductory notes on quantum groups, quantum algebras, and their applications. *arXiv preprint math-ph/0105002*, 2001.

SUq(3) flavor symmetry and deformed mass formulas

- General procedure

1. Embed the algebra $U_q(su(3))$ into dynamical $U_q(su(4))$ and further into $U_q(su(5))$ (in order to lift $\Lambda - \Sigma$ mass degeneracy),
2. Construct the mass operator $\hat{M}$ invariant under isospin + hypercharge deformed $U_q(su(2))$ from operators representing basis elements of $U_q(su(5))$,
3. Evaluate the matrix elements $M_{B_i} =< B_i|\hat{M}|B_i >$ involving M_0 and flavor symmetry breaking parameters α and β as well as the deformation parameter q in the framework of the Gel'fand-Tsetlin formalism,
4. Exclude the unknown constants M_0, α, and β from the expressions for baryon multiplet masses to obtain q-deformed mass relations.

AM Gavrilik, II Kachurik, and AV Tertychnyj. Representations of the $u\_q(u\_\{4,1\})$ and a q-polynomial that determines baryon mass sum rules. *arXiv preprint hep-ph/9504233*, 1995.

AM Gavrilik and NZ Iorgov. Quantum groups as flavor symmetries: account of nonpolynomial su (3)-breaking effects in baryon masses. *arXiv preprint hep-ph/9807559*, 1998.

SUq(3) flavor symmetry and deformed mass formulas

Octet Baryons

$$N + \frac{1}{[2]_{q_n} - 1}\Xi = \frac{[3]_{q_n}}{[2]_{q_n}}\Lambda + \left(\frac{[2]_{q_n}}{[2]_{q_n} - 1} - \frac{[3]_{q_n}}{[2]_{q_n}}\right)\Sigma$$

$$q = q_n = e^{i\pi/n} \text{ for integer } n$$

- Deformed mass formulas depends on the deformation parameter q_n.
- Infinite sequence of mass formulas, one for each integer n.
- Best fit to data when n=7. Error of only 0.06%

$$N + \frac{\Xi}{[2]_{q_7} - 1} = \frac{\Lambda}{[2]_{q_7} - 1} + \Sigma$$

$$[3]_{q_7} = \frac{[2]_{q_7}}{[2]_{q_7} - 1}$$

SUq(3) flavor symmetry and deformed mass formulas

Decuplet Baryons

$$(\Sigma^* - \Delta + \Omega - \Xi^*) = [2]_q(\Xi^* - \Sigma^*)$$

- Holds to first order flavor symmetry breaking (like the equal spacing rule).
- Rearranging into a form reminiscent of Okubo formula gives formula that holds to second order flavor symmetry breaking

$$\Omega - \Delta = (1 + [2]_q)(\Xi^* - \Sigma^*)$$

- Good fit to data for a range of values for $q=q_n$. n=14 considered by Gavrilik
- Solving for n yields n=15.7, so n=16 would give a better fit

SUq(3) flavor symmetry and deformed mass formulas

Octet-Decuplet mass relation

$$([2]_{q_{14}})^2 = q_{14}^2 + 2 + q_{14}^{-2} = q_7 + q_7^{-1} + 2 = [2]_{q_7} + 2.$$

- Solve both the octet and decuplet formula for $[2]_q$ to obtain a relation between the octet and decuplet baryon masses.

$$\frac{\Omega - \Xi^* + \Sigma^* - \Delta}{\Xi^* - \Sigma^*} = \left(3 + \frac{\Xi - \Lambda}{\Sigma - N}\right)^{1/2}$$

- Assumes that $q=q_{14}$ for octet baryons and $q=q_{21}$ for decuplet baryons.
- Error or around 1.5%

Electromagnetic contributions to baryon masses

- In the standard theory as well as in the deformed octet and decuplet formulas considered, the baryon masses used are the averages of the baryons masses within a specific isospin multiplet

$$\frac{|\Xi^- - \Xi^0|}{\Xi^-} = 0.005 = 0.5\%$$
$$\frac{|\Sigma^- - \Sigma^+|}{\Sigma^-} = 0.007 = 0.7\%$$

$$\frac{|\Xi^{*-} - \Xi^{*0}|}{\Xi^{*-}} = 0.002 = 0.2\%$$
$$\frac{|\Sigma^{*-} - \Sigma^{*+}|}{\Sigma^{*-}} = 0.003 = 0.3\%$$

- Mass splittings within isoplets comparable errors in the deformed octet and decuplet formulas.
- For deformed relations to be meaningful we must take into account the EM contributions to baryon masses.

Electromagnetic contributions to baryon masses

- Electromagnetic contributions to baryon masses determined within QCD general parametrization scheme in spin-flavor space.
- To zeroth order, the electromagnetic contributions given in terms of four parameters.

$$\begin{aligned}
\delta_0 p &= \mu + \frac{5}{9}\nu + \eta + \rho \qquad \delta_0 n = \frac{2}{3}\mu \\
\delta_0 \Lambda &= \frac{2}{3}\mu + \frac{1}{9}\nu \qquad \delta_0 \Sigma^+ = \mu + \frac{5}{9}\nu + \eta + \rho \\
\delta_0 \Sigma^- &= \frac{1}{3}\mu + \frac{1}{9}\nu + \eta + \frac{1}{3}\rho \qquad \delta_0 \Sigma^0 = \frac{2}{3}\mu + \frac{1}{3}\nu \\
\delta_0 \Xi^0 &= \frac{2}{3}\mu \qquad \delta_0 \Xi^- = \frac{1}{3}\mu + \frac{1}{9}\nu + \eta + \frac{1}{3}\rho.
\end{aligned}$$

G Morpurgo. Field theory and the nonrelativistic quark model: A parametrization of the baryon magnetic moments and masses. *Physical Review D*, 40(9):2997, 1989.

Electromagnetic contributions to baryon masses

Octet Baryons

$$\delta_0 p + \delta_0 \Xi^0 = \frac{3}{4}\delta_0\Lambda^0 + \frac{1}{2}(2\delta_0\Sigma^+ - \delta_0\Sigma^0)$$

- The standard Gell-Mann-Okubo formula becomes

$$\frac{1}{2}(p + \Xi^0) + T = \frac{1}{4}(3\Lambda + 2\Sigma^+ - \Sigma^0)$$

$T = \Xi^{*-} - \frac{1}{2}(\Omega^- + \Sigma^{*-}) = 5.18\mathrm{MeV}$ is a decuplet correction

- Equal electromagnetic contributions on both sides.
- Error of around 0.13%. Roughly a factor of 4 reduction in error compared to standard GMO formula.

G Morpurgo. Electromagnetic mass differences of the octet and decuplet baryons. *Physical Review D*, 45(5):1686, 1992.

Electromagnetic contributions to baryon masses

<u>Decuplet Baryons</u>

- Equal spacing rule only valid at first order flavor symmetry breaking. At second order, only the Okubo relation continues to hold.

$$\Omega - \Delta = 3(\Xi^* - \Sigma^*)$$

- Applying the same parametrization as for octet baryons one finds

$$\delta_0\Omega = \delta_0\Sigma^{-*} = \delta_0\Xi^{-*} = \delta_0\Delta^-$$

- Charge specific decuplet formula with equal electromagnetic contributions is:

$$\Omega^- - \Delta^- = 3(\Xi^{*-} - \Sigma^{*-})$$

- Accurate to about 0.67%. A factor of about two reduction in error compared to Okubo formula.

Charge specific baryon mass formulas with deformed SUq(3) flavor symmetry

<u>Octet Baryons</u>

- Take the deformed octet mass formula and balance EM contributions to masses one obtains

$$p+\frac{\Xi^0}{[2]_{q_7}-1}=\frac{\Lambda^0}{[2]_{q_7}-1}+(2\Sigma^+-\Sigma^0)$$

- n=7 still the best fit to data (assumed in the above formula)

LHS=2577.87MeV RHS=2577.33MeV

- Error of about 0.02%

Charge specific baryon mass formulas with deformed SUq(3) flavor symmetry

Decuplet Baryons

- Applying EM correction to decuplet formula gives

$$\Omega^- - \Delta^- = ([2]_q + 1)(\Xi^{*-} - \Sigma^{*-})$$

- Accuracy depends on value of $q=q_n$. n=14 no longer a very good choice.
- Solving for n gives n=22 as best fit. We choose n=21 however (because 21=3x7).
- For n=21 we get:

$$\mathrm{LHS}=440.45\mathrm{MeV} \qquad \mathrm{RHS}=440.10\mathrm{MeV}$$

- Error of about 0.08%

Charge specific baryon mass formulas with deformed SUq(3) flavor symmetry

New Octet-Decuplet mass relation

- Choosing n=21 we obtain

$$[2]_{q_{21}}^3 = (q_{21} + q_{21}^{-1})^3 = [2]_{q_7} + 3[2]_{q_{21}}$$

- Solving the charge specific octet and decuplet formulas for $[2]_q$ and applying the above relation we get a new octet-decuplet formula

$$\left(\frac{\Omega^- - \Delta^-}{\Xi^{*-} - \Sigma^{*-}} - 1\right)^3 - 3\left(\frac{\Omega^- - \Delta^-}{\Xi^{*-} - \Sigma^{*-}} - 1\right) = \left(1 + \frac{\Xi^0 - \Lambda^0}{2\Sigma^+ - \Sigma^0 - p}\right)$$

- Error of around 1% (compared to 1.5% for non charge specific deformed formula).

The Cabibbo angle as a function of q and baryon spin

- Relationship between q and the Cabibbo angle suggested by Gavrilik

$$q = e^{i\theta_{\mathbf{8}}},\ \theta_{\mathbf{8}} = \tfrac{\pi}{7} \text{ for octet baryons.}$$
$$q = e^{i\theta_{\mathbf{10}}},\ \theta_{\mathbf{10}} = \tfrac{\pi}{14} \text{ for decuplet baryons.}$$

$$\theta_{\mathbf{10}} = \theta_C,\ \ \theta_{\mathbf{8}} = 2\theta_C \qquad \theta_C = \tfrac{\pi}{14}$$

- With n=21 for the charge specific deformed decuplet formula we instead obtain the relation

$$\theta_C = \frac{1}{2}\theta_{\mathbf{8}} = \frac{3}{2}\theta_{\mathbf{10}} = \frac{\pi}{14}$$

- The Cabibbo angle is now a formula of the deformation parameter q and the spin of the baryons

$$\theta_C = -iS\ln q.$$

Alexandre Gavrilik. Can the Cabibbo mixing originate from noncommutative extra dimensions? In *Noncommutative Structures in Mathematics and Physics*, pages 343–355. Springer, 2001.

Outlook of further applications of quantum groups to particle symmetries

Cabibbo angle and CKM matrix

- Value of $\pi/14$ slightly outside the experimental range
- Could replace the standard trigonometric functions by their q-deformed version
- For a suitable value of q the experimental data might agree with a value of $\pi/14$ for the Cabibbo angle

Neutrino oscillations

- Use q-deformed trigonometric function in PMNS matrix
- If Cabibbo angle can be related to a deformation, perhaps θ_{13} can too. θ_{13} confirmed as nonzero. Allows for CP violation.

Baryon/Meson interactions and decays

- Calculate correction to lifetimes, branching ratios, etc.

Summary

Octet Baryons

	GMO	Charge specific	deformed	Charge specific + deformed
LHS (MeV)	2257.20	2263.53	2585.87	2577.95
RHS (MeV)	2270.10	2266.57	2584.45	2577.40
Error (%)	0.57	0.13	0.06	0.02
$\lvert LHS - RHS \rvert$	12.9	3.1	1.4	0.5

Decuplet Baryons

	Okubo	Charge specific	deformed	charge specific + deformed)
LHS (MeV)	440.45	440.45	440.45	440.45
RHS (MeV)	446.49	443.40	439.05	440.15
Error (%)	1.4	0.67	0.32	0.07
$\lvert LHS - RHS \rvert$	6.0	3.0	1.4	0.3

- Mass relations relies on complex q. Fitting of Cabibbo angle gives real q. Likely need to consider multiple deformation parameters.
- Complex q for internal symmetries? Real q for deformed spacetime?

THANK YOU