Sommerfeld quantization of the extended charge in external and own fields

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All my talk in two words:
Electron’s self-energy is complex - real energy quantization leads to the Compton length $2.4 \times 10^{-12}$ m, while imaginary energy part is quantized on the electric analog length $1.2 \times 10^{-33}$ m.

\[ E = (\sqrt{G} m + q) \varphi_0 \]

\[ = mc^2 + ieG^{-1/2}c^2 \]

\[ = (0.511 - i \cdot 1.04) \times 10^{21} \text{MeV} \]

Finite imaginary energy in the nondual physics of charged fields instead of the unphysical Coulomb divergence in the dual (particle + field) approach to reality.
The millennium problem of the Ancient Greeks
Is space empty in physical reality?

All textbooks say **Yes**, space is empty due to the laws of Newton and Coulomb $\text{div } \mathbf{E} = 0$ for dual physics of measured fields and their distant ("observed") charges

But Gustav Mie (1868-1957) said **No**, space is not empty, $\text{div } \mathbf{E} = f (|\mathbf{E}|) \neq 0$ for continuous sources in nondual physics of charged material fields !!!
A 1929 cartoon: "People slowly accustomed themselves to the idea that the physical states of space itself were the final physical reality."  
Professor Albert Einstein
We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created. Its final aim would be the explanation of all events in nature by structure laws valid always and everywhere. A thrown stone is, from this point of view, a changing field, where the states of greatest field intensity travel through space with the velocity of the stone.

There would be no place, in our new physics, for both field and matter, field being the only reality.

This new view is suggested by the great achievements of field physics, by our success in expressing the laws of electricity, magnetism, gravitation in the form of structure laws, and finally by the equivalence of mass and energy.

Our ultimate problem would be to modify our field laws in such a way that they would not break down for regions in which the energy is enormously concentrated.”


Bulyzhenkov, I.E. Relativistic Quantization of Cooper Pairs and Distributed Electrons in Rotating Superconductors, Jour. of Supercond. and Novel Magnetism 22, 627 (2009).

PHY2323 - Electricity and Magnetism

° Midterm problem, 2 year

➤ Let the charge density distribution of the astroelectron is given as \( \rho(r) = -er_e/4\pi r^2(r + r_e)^2 \), where \( e = 1.6 \times 10^{-19} C \) and \( r_e = \text{const} < < 10^{-18} m \). Compute the electric field intensity and the electric potential of this extended elementary charge everywhere in the Universe. Assume that the post Coulomb potential of the non-point charge \((-e)\) is zero at \( r = \infty \).

⊗ Continuous field and charge distributions with the spherical symmetry.

Step 1. Illustration of \( \rho(r) \).

Step 2. Right formulas: \( \nabla \varepsilon_0 \vec{E} = \rho \) and \( \vec{E} = -\nabla W \)

Step 3. Right coordinate system (spherical): \( \vec{E} = E(r) \hat{a}_r \), \( \nabla \vec{E}(r) = r^{-2} \partial_r [r^2 E(r)] = \rho(r)/\varepsilon_0 \), \( \hat{a}_r E(r) = -\hat{a}_r \partial_r W(r) \).
Continuation of the extended electron problem

Step 4. Calculations. Let \( q = (-e)/4\pi\epsilon_0 \).

\[
\partial_r (r^2 E) = \frac{q r_e}{(r + r_e)^2}, \quad r^2 E = -\frac{q r_e}{r + r_e} + C_1, \quad E(r) = -\frac{q r_e}{r^2(r + r_e)} + \frac{C_1}{r^2}
\]

\( E(r \to \infty) \to \frac{q}{r^2} \) (\( \to \) Coulomb), therefore \( C_1 \equiv q \),

\[
E(r) = \frac{q}{r^2} - \frac{q r_e}{r^2(r + r_e)} = \frac{q}{r(r + r_e)} = \frac{q}{r_e} \left( \frac{1}{r} - \frac{1}{r + r_e} \right) \text{(post-Coulomb)}
\]

\[
E(r) = \frac{q}{r(r + r_e)} = -\partial_r W(r), \quad W(r) = -\int E(r) dr = -\frac{q}{r_e} \int \left( \frac{1}{r} - \frac{1}{r + r_e} \right)
\]

\[
= \frac{q}{r_e} \ln \left( 1 + \frac{r_e}{r} \right) + C_2. \text{ Here } C_2 = 0, \text{ because } W(\infty) = 0.
\]

Step 5. Verification. \( W(r) \to \frac{q}{r} \) (\( \to \) Coulomb) for \( r \geq 10^{-18} m \).

Full similarity of GR and EM theories in nondual physics of nonempty charged space

\[
\mu_p(r) \equiv \mu_a(r) = \frac{r_0}{4\pi r^2 (r_0 + r)^2} = \frac{c^2}{4\pi G r^2 \left[1 + (rc^2/Gm)\right]^2},
\]

\[
Gm_0/c^2 = 7 \times 10^{-58} m.
\]

\[
e \equiv \int_0^\infty 4\pi r^2 \rho(r) dr = -e_0
\]

\[
E^2/4\pi = (e/r_e) \nabla E = (e/r_e) \rho(r)
\]

\[
\begin{aligned}
\mu(r) &= Mr_0/4\pi r^2 (r + r_0)^2 = w^2/4\pi G c^2 \\
w(r) &= -\nabla W(r) = -GM\hat{r}/r(r + r_0) \\
W(r) &= -c^2 \ln[(r + r_0)/r]
\end{aligned}
\]

\[
\begin{aligned}
4\pi \rho(r) &= er_e/r^2 (r + r_e)^2 = \nabla E(r) = E^2/(e/r_e) \\
E(r) &\equiv -\nabla W_e(r) = e\hat{r}/r(r + r_e) \\
W_e(r) &= (e/r_e) \ln[(r + r_e)/r] \\
E_e &= \int d^3 x \rho W_e = \int d^3 x \rho e/r_e = \int d^3 x E^2/4\pi = e^2/r_e
\end{aligned}
\]
Logarithmic potential is the strong field solution for elementary radial charges in Maxwell’s electrostatics

\[ W(r) = \frac{e}{r_e} \ln \left( 1 + \frac{r_e}{r} \right) \approx \begin{cases} \frac{e}{r}, & r_e \ll r \\ \frac{e}{c^2/\sqrt{G}}, & r \approx r_e \end{cases} \]

Half of the radial charge \( q = ie \) is within the sphere of this fundamental radius

\[ r_e = \frac{e\sqrt{G}}{c^2} = 1.38 \times 10^{-36} m \]

\[ \varphi_o \equiv \frac{c^2}{\sqrt{G}} = 1.04 \times 10^{27} \] \( B \) – the universal potential for complex self-energy of gravitational and electric charges

\[ E = (\sqrt{Gm} + q)\varphi_o = mc^2 + ieG^{-1/2}c^2 \]
Mass-energy unification with electric charge-energy:

\[ E = (\sqrt{Gm + q})\varphi_o = mc^2 + ieG^{-1/2}c^2 \]

Bulyzhenkov, *Pure field physics of continuous charges without singularities*, Proceedings of “Gravitation: 100 years after GR”, p.317, Rencontres de Moriond 2015, La Thuile, Italy


*Complex Charge Densities Unify Particles with Fields and Gravitation with Electricity*,

*Pure field electrodynamics of continuous complex charges*,
Tutorial of the 4th year course, MIPT, Moscow 2015
Point matter (leading to singularities) has been postulated from practice rather than from logic or analytical math.
By accepting continuous material space without singularities we comply with the same observations in practice.

Einstein’s curvature $R_{00} - g_{00}R/2 = 0$ at $R \neq 0$ leads to static metrics without singularities!

Infinitely extended particle

High mass density: $R \neq 0$

Very low mass density: $R \neq 0$

High mass density: $R \neq 0$

\[ \gamma_{ij}(x) \equiv \frac{g_{oi}(x)g_{oj}(x)}{g_{oo}(x)} - g_{ij}(x) \Rightarrow \begin{cases} \neq \delta_{ij}, \text{warped empty 3D space with holes} \\ = \delta_{ij}, \text{inherent metric symmetries for Euclidean nonempty 3D space in curved 4D} \end{cases} \]

\[ mn[x, X_e(t)] = m \frac{r_o}{4\pi [x - X_e(t)]^2 [||x - X_e(t)|| + r_o]^2} \int d^3x n[x, X_e(t)] = 1 \]
Nondual Classical Electrodynamics of charged fields

\[
\begin{align*}
\nabla_\nu F^{\nu\mu}(x) &\equiv 4\pi j^\mu(x)/c \equiv 4\pi \rho(x) u^\mu(x) \\
\gamma_\mu(x) \nabla_\nu F^{\nu\mu}(x) &\equiv 4\pi \rho(x) \\
\nabla_\nu F^{\nu\mu}(x) &\equiv u^\mu(x) u^\lambda(x) \nabla_\nu F^{\nu\lambda}(x).
\end{align*}
\]

Maxwell’s equations in purely field terms for 4-momentum of electric energy flows

\[
\begin{align*}
\left[\nabla_\lambda F_{\mu\nu}(x) + \nabla_\mu F_{\nu\lambda}(x) + \nabla_\nu F_{\lambda\mu}(x)\right] \varphi_o / 4\pi c &\equiv 0 \\
\left[\delta_\lambda^\mu - u^\mu(x) u^\lambda(x)\right] \nabla_\nu F^{\nu\lambda}(x) \varphi_o / 4\pi c &\equiv 0.
\end{align*}
\]

Einstein’s equations takes the same form for non-relativistic energy flows. Therefore, magnetic monopoles in Classical Electrodynamics and same gravimagnetic monopoles in General Relativity have equal theoretical rights to be discussed or to be searched in experiments.
Fluxoid quantization for the point particle

\[-\frac{1}{c} \partial_\mu \chi_e = P_\mu \equiv g_{\mu\nu} \left( mc \frac{dx^\nu}{ds} + \frac{q}{c} A^\nu \right) \equiv \{ P_o; P_i \} \]

\[= \left\{ \left[ \frac{mc \sqrt{g_{00}}}{\sqrt{1 - v^2 c^{-2}}} + \frac{g_{0\nu} q A^\nu}{c} \right] ; \left[ -m(\gamma_{ij} v^j + \sqrt{g_{00}} g_{i} c) \right. \frac{1}{\sqrt{1 - v^2 c^{-2}}} + \frac{g_{i\nu} q A^\nu}{c} \right\} \]

\[\frac{1}{c} \oint_{d\tau = 0} dx^\mu \nabla_\mu \chi_s = \oint_{d\tau = 0} \left[ -P_i dx^i - P_o dx^o \right] \equiv \]

\[\oint_{d\tau = 0} \left[ \frac{m(\gamma_{ij} v^j + c \sqrt{g_{00}} g_i)}{\sqrt{1 - v^2 c^{-2}}} - \frac{q(g_{i0} A^o + g_{ij} A^j)}{c} \right] dx^i \]

\[= \oint_{d\tau = 0} \left( \frac{mc \sqrt{g_{00}}}{\sqrt{1 - v^2 c^{-2}}} + \frac{q(g_{00} A^o + g_{0j} A^j)}{c} \right) g_i dx^i \]

\[\equiv \oint_{d\tau = 0} \left[ \frac{q A^j}{c} + \frac{mv^j}{\sqrt{1 - v^2 c^{-2}}} \right] \gamma_{ij} dx^i = \pm 2\pi N \hbar. \]

\[m \delta(x - X_k) \]

\[v^i \equiv dx^i / d\tau \]

\[v^2 \equiv \gamma_{ij} v^i v^j \]

\[\gamma_{i}(x) \equiv g_i g_j g_{00} - g_{ij} \]

\[g_i \equiv -g_{oi} / g_{00} \]

\[d\tau = 0 \]

\[dx^o = g_i dx^i \]

\[\gamma_{ij}(x) = \delta_{ij} \]
External magnetic flux through instantaneous loops is ultimately formed by electron’s Coulomb energy in external electric fields, while mechanical flux - by real self-energy $m_0c^2$. The inertial mass $m_0$ might originate, due to 4D Sommerfeld quantization rule, from real energy self-interactions between imaginary densities of the elementary electric charge (after their violation of the spherical symmetry).

\[
\Delta \int_{d\tau=0} \gamma_{ij} dx^i \left( \frac{m_0c^2 (dx^j/d\tau)}{\sqrt{1 - \gamma_{kl} (dx^k/d\tau) (dx^l/d\tau)}} + (-ie_o) A^j_{ext} \right)
\]

\[
= \Delta \int_{d\tau=0} \gamma_{ij} dx^i \left( \frac{m_0c^2 dx^j}{\sqrt{d\tau^2 - \gamma_{kl} dx^k dx^l}} + (-ie_o) \sum_{k=1}^{\infty} \frac{\varphi_k dx^j}{\sqrt{d\tau^2 - \gamma_{pl} dx^p dx^l}} \right)
\]

\[
= \Delta \int_{d\tau=0} \gamma_{ij} dx^i \left( \frac{m_0c^2 dx^j}{i \sqrt{\gamma_{kl} dx^k dx^l}} + (-ie_o) \sum_{k=1}^{\infty} \frac{\varphi_k dx^j}{i \sqrt{\gamma_{pl} dx^p dx^l}} \right)
\]

\[
= \Delta \left( -im_0c^2 l_0 + (-e_o) \sum_{k=1}^{\infty} \frac{(-ie_k) l_k}{|r_k - r_o|} \right) = 2\pi i\hbar c \Delta N
\]
1967 Sakharov’s idea – inertia is a residual property of vacuum electromagnetic fields

**London mechanic+magnetic fluxoid**

\[
\int_{d\tau=0} \delta_{ij} dx^i \left( \frac{mc^2 dx^j}{i \sqrt{\delta_{kl} dx^k dx^l}} + (-ie_o) A^j_{ext} \right) = 2\pi i \Delta N \hbar c
\]

Dualism of continuous fields and discrete states belongs to London’s fluxoid quantization (for Cooper pairs take 2m, 2e_o)

The real mass \( m \) is required for closed path quantization of the imaginary electric charge \( q = -ie_o \). No closed paths for an infinite EM wave (photon) – zero masses for it. If the Universe is finite, \( R_U = c/H \), then \( \lambda \leq 2\pi R_U \) and \( m_{\lambda} c^2 \geq \hbar c / R_U = \hbar H = 1.05 \times 10^{-34} \times 2.2 \times 10^{-18} = 2.32 \times 10^{-52} \text{J} = 1.45 \times 10^{-33} \text{eV} \).
Relativistic quantization for the extended charge

\[ -\frac{1}{c} \partial_\mu \chi(x, x') = n(x, x') g_{\mu\nu} \left( mc \frac{dx^\nu}{ds} + \frac{q_e}{c} A^\nu(x) \right) \equiv \{ nP_0(x); nP_i(x) \} \]

\[ \equiv \left\{ \left[ \frac{nm c \sqrt{g_{oo}}}{\sqrt{1 - v^2 c^{-2}}} + \frac{g_{ov} n q_e A^\nu}{c} \right]; \left[ -nm (\gamma_{ij} v^j + \sqrt{g_{oo}} g_{ic}) \frac{\sqrt{g_{oo}} g_{ic}}{\sqrt{1 - v^2 c^{-2}}} + \frac{g_{iv} n q_e A^\nu}{c} \right] \right\} \]

\[ n(x, x') = r_o / 4\pi (x - x')^2 [||x - x'| + r_o]^2 \]

\[ r_o = q / \varphi_o \]

\[ q = ie \]

\[ A_{self}^o(x, x') = -\varphi_o ln[1 + (q / \varphi_o ||x - x'||)] \]

\[ \varphi_o \equiv c^2 / \sqrt{G} = 1.04 \times 10^{27} \text{B} \]
\[
\int_{d\tau = 0} \int d^3 x' dx'^\mu \nabla_\mu \chi(x, x') \equiv \int_{d\tau = 0} \int d^3 x' cn(x, x') \left[ -P_i dx^i - P_0 dx^0 \right]
\]
\[
\equiv \int_{d\tau = 0} \int d^3 x' n(x, x') \left[ \frac{mc(\gamma_{ij} v^j + c \sqrt{g_{oo} g_i})}{\sqrt{1 - v^2 c^{-2}}} - q_e (g_{io} A^o + g_{ij} A^j) \right] dx^i
\]
\[
- \int_{d\tau = 0} \int d^3 x' n(x, x') \left( \frac{mc^2 \sqrt{g_{oo}}}{\sqrt{1 - v^2 c^{-2}}} + q_e (g_{oo} A^o + g_{oj} A^j) \right) g_i dx^i
\]
\[
\equiv \int_{d\tau = 0} \int d^3 x' n(x, x') \left[ \frac{mc v^i}{\sqrt{1 - v^2 c^{-2}}} + q_e (A_{ext}^i + \varphi w^i) \right] \gamma_{ij} dx^j
\]
\[
\equiv \int_{d\tau = 0} \frac{\gamma_{ij} dx^i dx^j}{\sqrt{c^2 d\tau^2 - dl^2}} \int d^3 x' n(x, x') \left[ mc^2 + q \varphi(x, x') \right]
\]
\[
+ \int_{d\tau = 0} \gamma_{ij} dx^i A_{ext}^j \int d^3 x' q_e n(x, x')
\]
\[
= -il (mc^2 + q_e \varphi_0) + q_e \Phi_{ext} = \pm 2\pi \hbar c N \pm 2\pi i \hbar c K
\]
\[
A^j = A_{ext}^j + A_{self}^j
\]
\[
A_{self}^j = A_{self}^0(x, x') u^j = \varphi(x, x') dx^j / cd\tau \sqrt{1 - v^2 c^{-2}}
\]
Compton length electric analog $l_{\text{min}}$

$$\Phi_{\text{self}} \equiv \int d\tau \int d^3 x' n(x, x') A^j_{\text{self}}(x, x')$$

$$= i l \varphi_o = \hbar c \Delta N/q$$

$$l_{\text{min}} \equiv 2\pi r_h = 1.19 \times 10^{-33} m$$

$$r_h \equiv \frac{\hbar c}{|q\varphi_o|} = \frac{\hbar \sqrt{G}}{ce_o} = \begin{cases} 1.89 \times 10^{-34} m & \text{for the electron} \\ 9.46 \times 10^{-35} m & \text{for the Coper pair} \end{cases}$$

$$\lambda \equiv 2\pi \hbar / m_o c = 2.43 \times 10^{-12} m$$
NEW NONDUAL PHYSICS OF STRONG MATERIAL FIELDS

$$r_o \equiv |ie/\varphi_o| \Rightarrow e_o \sqrt{G/c^2} = 1.38 \times 10^{-36} m$$

Fundamental scale of the extended electron, half of the elementary charge inside $r_o$

$$\begin{align*}
\frac{r_h}{r_o} &= \frac{\hbar c}{e_o^2} \equiv \alpha^{-1} = 137, \\
\frac{r_h \cdot r_o}{\sqrt{G}} &= \frac{\hbar G}{c^3} \equiv l_p^2 = (1.6 \times 10^{-35} m)^2
\end{align*}$$

Basic constants in nondual physics of charged material fields are $r_o$ and $r_h$, but not Sommerfeld’s $\alpha$ and Planck’s $l_p$
Conclusions.

There is no spatial scale in reality for an assumed transition from nondual field physics of the quantum microworld to dual physics of the “observed” macroworld.

Classical Electrodynamics and General Relativity should be redesigned ASAP in nondual terms of material fields with high and low densities of relevant energy flows.