# Quantum spin correlations in relativistic Møller scattering

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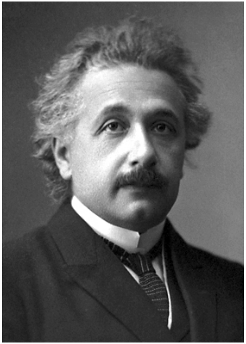
# EPR paradox



Quantum mechanical description of physical reality cannot be considered complete.

# EPR paradox

Either

- the entangled particles communicate with infinite speed (spooky action at the distance)

or

- the complete state of the system (results of all possible measurements) is precisely determined at all times

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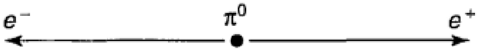
## EPR paradox

If quantum mechanics does not provide a complete description of all properties of the system it is either incomplete or the principle of local realism is not valid

# Bohm's illustration of the EPR paradox



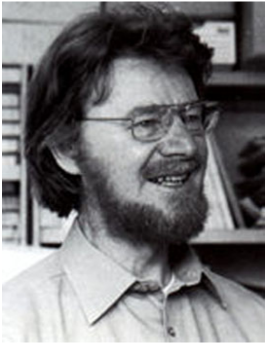

$$\pi^0 \to e^- + e^+$$

$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

# Bell inequalities

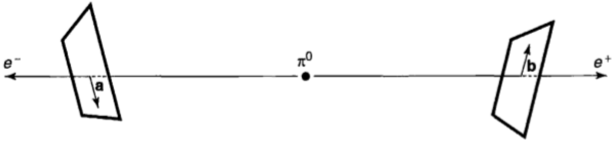
**Bell theorem**

Every theory fulfilling the principle of local realism must fulfil a certain class of inequalities.



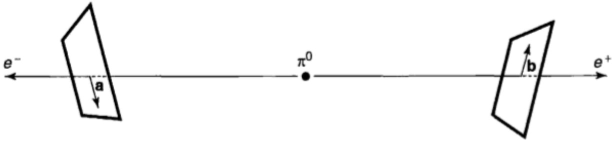
**Outcomes of measurements performed on two space-like separated systems cannot be correlated arbitrarily strong.**

# Bell inequalities




Spin projection on an arbitrary direction: $\pm\hbar/2$. After normalizing to 1, a following definition of **correlation function** is possible:

# Bell inequalities

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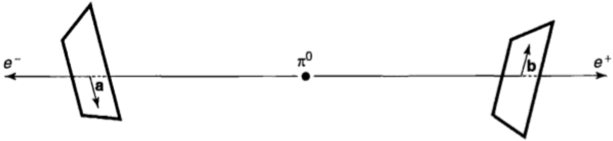
$$C(\vec{a}, \vec{b}) = P_{++} + P_{--} - P_{+-} - P_{-+}$$

# Bell inequalities

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$$C(\vec{a},\vec{b}) = P_{++} + P_{--} - P_{+-} - P_{-+}$$

Singlet state:

$$C(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b}$$

Triplet state:

$$C(\vec{a},\vec{b}) = +\vec{a}\cdot\vec{b}$$

Bohm's configuration:

$$C(\vec{a},\vec{a}) = -1$$

# Bell-type inequalities

## CHSH inequality

$$|C(\vec{a}, \vec{b}) + C(\vec{c}, \vec{b} + C(\vec{c}, \vec{d}) - C(\vec{a}, \vec{d}))| \leq 2$$

(valid in every theory of hidden variables preserving local realism)

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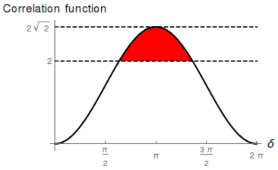



Quantum mechanics is a non–local theory and according to its predictions the Bell's inequality is violated Either quantum mechanics is incorrect (and not incomplete, as stated by EPR) or the principle of local realism is wrong

# Experiments by Aspect

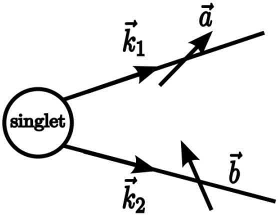
- photon pairs in singlet state
- violation of Bell (CHSH) inequalities confirmed experimentally
- local realism finally disproved

CHSH inequalities violated by 5 standard deviations.

**The result**

Confirmation of correctness and completeness of quantum mechanics – its non–locality is not just a feature of the formalism, but a fundamental property of Nature

# Relativistic correlation function




- spin $\frac{1}{2}$
- $k_1, k_2$ – particles four–momenta
- relativistic correction to the correlation function dependent on particles momenta

$$\text{singlet} \rightarrow k_1 \; k_2$$

$$\mathcal{C}(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} \; +$$

$$+ \underbrace{\frac{\vec{k}_1 \times \vec{k}_2}{m^2 + k_1 k_2} \left[ \vec{a} \times \vec{b} + \frac{(\vec{a} \cdot \vec{k}_1)(\vec{b} \times \vec{k}_2) - (\vec{b} \cdot \vec{k}_2)(\vec{a} \times \vec{k}_1)}{(k_1^0 + m)(k_2^0 + m)} \right]}_{\text{relativistic correction}}$$

# Experiments with massive particles

- experiments with protons:

| | | |
|---|---|---|
| Lamehi, Rachti, Mittig | 1976 | Saclay (France) |
| Hamieh *et al.* | 2004 | KVI (Holland) |
| Sakai *et al.* | 2006 | RIKEN (Japan) |

- measurement of correlation function and violation of Bell inequalities for massive non–relativistic particles
- **non–relativistic quantum mechanics only (too low energies)**

# 2POL experiment

- pairs of relativistic electrons
- first measurement of relativistic corrections to the correlation function
- correlation function depends on particles momenta
- **not a Bell-test experiment**

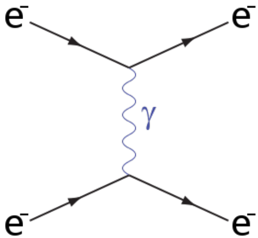
- University of Warsaw
- University of Łódź
- Technical University in Darmstadt

# Initial state preparation

- Møller scattering
- $e^- e^- \rightarrow e^- e^-$
- polarized beam scattered on an unpolarized target
- mixed state

$$\hat{\rho} = \frac{\hat{M}(\hat{\rho}_A \otimes \hat{\rho}_B)\hat{M}^\dagger}{\mathrm{Tr}\{\hat{\mathrm{M}}(\hat{\rho}_\mathrm{A} \otimes \hat{\rho}_\mathrm{B})\hat{\mathrm{M}}^\dagger\}},$$

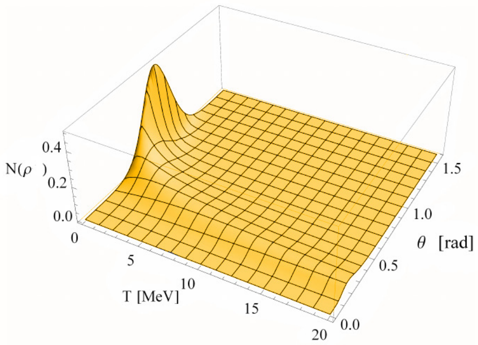
where $\hat{M}$ is the scattering amplitude.



# Entanglement of the initial state

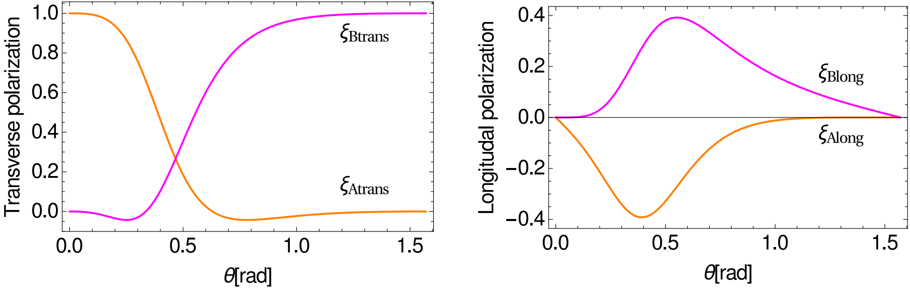
- State after the scattering can be entangled
- Negativity:

$$N(\rho) = \Sigma_i \frac{|\lambda_i| - \lambda_i}{2}$$



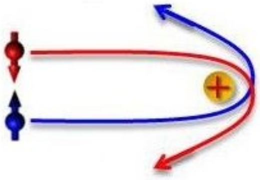
# Polarization transfer

- After the scattering the target electron gains polarization
- Unpolarized target, transversly polarized 3MeV electron beam



# Spin measurement

- Mott polarimetry
- spin–orbit coupling
- Mott scattering cross section depends on electron spin
- sensitive only to the spin projection on the direction perpendicular to the scattering plane
- backscattering (angle $\approx 120^\circ$)



# Relativistic spin operator

- Should reduce to non-relativistic form in rest frame: $\hat{\mathbf{S}} = \frac{\hat{\mathbf{W}}}{m}$,
- Should be a three-vector,

$$\left[\hat{J}_i, \hat{S}_j\right] = i\epsilon_{ijk}\hat{S}_k,$$

- Should fulfil standard commutation relations:

$$\left[\hat{S}_i, \hat{S}_j\right] = i\epsilon_{ijk}\hat{S}_k.$$

- Best possible choice is the Newton-Wigner spin operator

$$\hat{\vec{S}} = \frac{1}{m}\left(\hat{\vec{W}} - \hat{W}^0 \frac{\hat{\vec{P}}}{\hat{P}^0 + m}\right).$$

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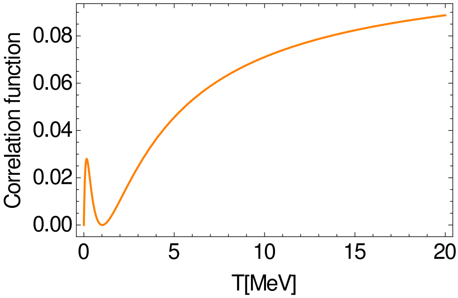
- Best possible choice is the Newton-Wigner spin operator

$$\mathbf{S} = -\frac{1}{2m}\left(-p^0\gamma^5\gamma^0\boldsymbol{\gamma} + \frac{\mathbf{p}}{p^0+m}\gamma^5\gamma^0(\mathbf{p}\cdot\gamma) + i\gamma^0(\mathbf{p}\times\gamma)\right).$$

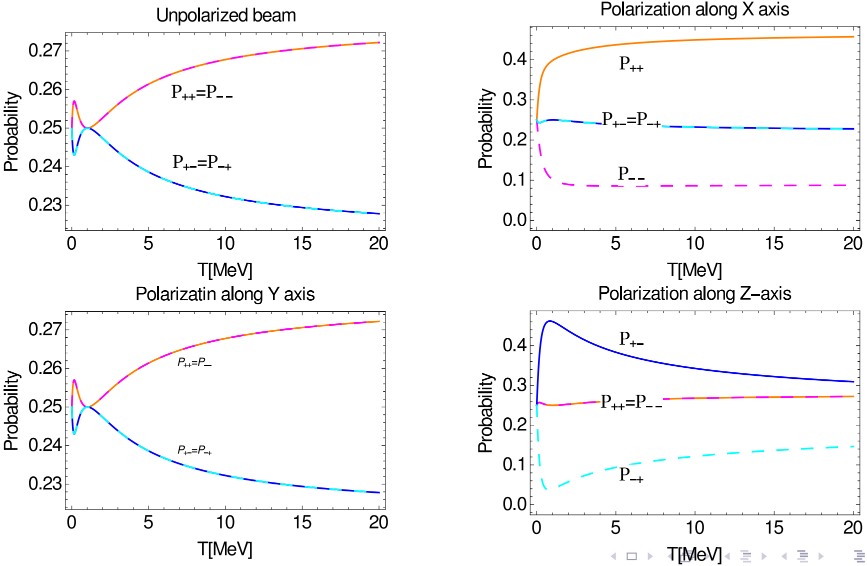
# Correlation function

- Correlation function: symmetric scattering in XZ plane, Mott scattering in YZ plane (to maximize the effect)
- No polarization dependence



# Probabilities

- Depending on beam polarization

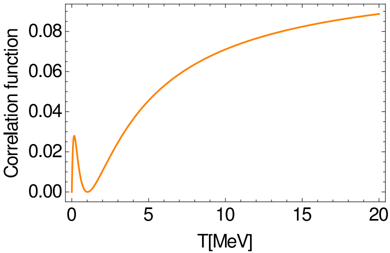


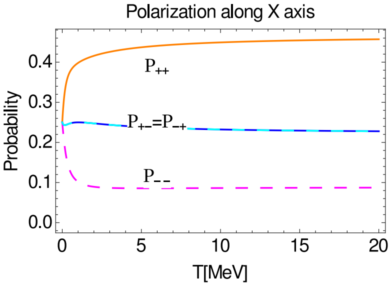
# Unpolarized target: summary

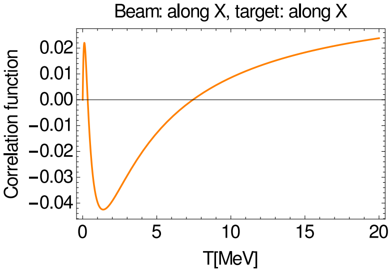
- The simplest case allowing to distinguish between non-relativistic and relativistic quantum mechanics
- Correlation function small but measurable
- Correlation function does not depend on beam polarization
- Probabilities depend on beam polarization

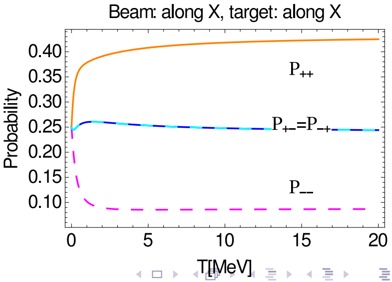
# Polarized target

- Target polarized in 8%



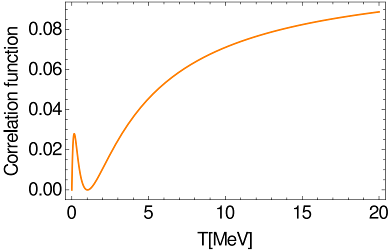



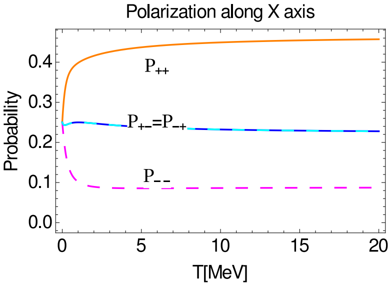



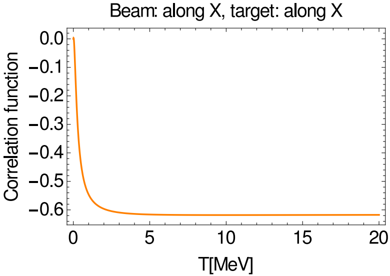



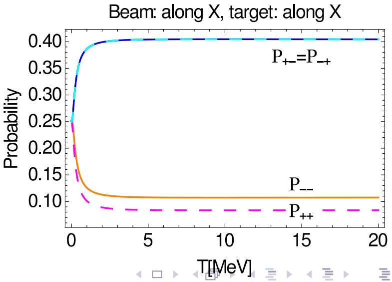
# Polarized target

- Target polarized in 80%



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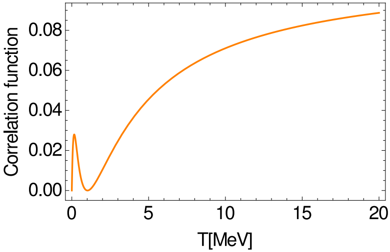
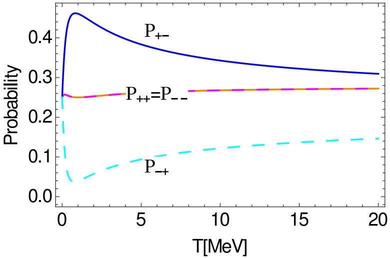
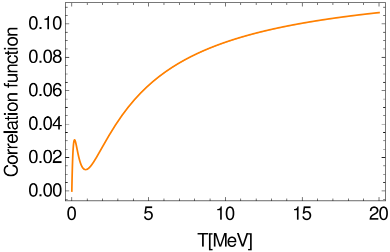



Polarization along Z–axis




Beam: along Z, target: along Z

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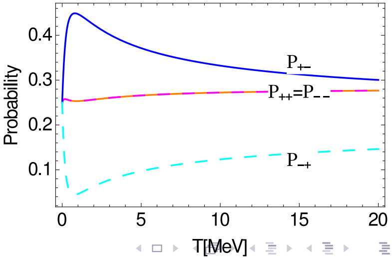

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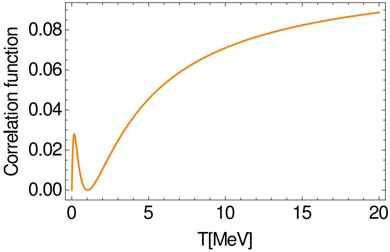


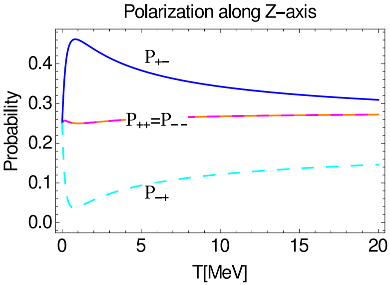


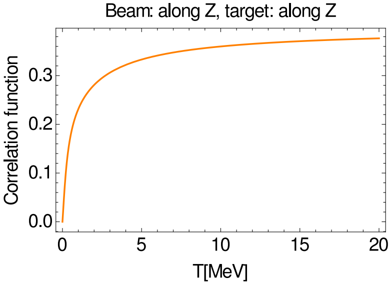


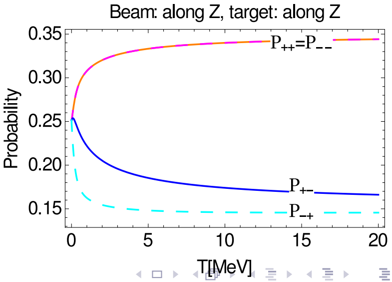

# Conclusions

- No EPR type experiment has been performed so far in relativistic regime
- We calculated the correlation function and the corresponding probabilities for the simplest case possible to arrange experimentally (a pair of electrons originating from Møller scattering of a polarized electron beam off a stationary unpolarized target)
- The correlation function is small but measurable
- The correlation function does not depend on beams polarization
- The probabilities depend on beam polarization
- In case of a polarized target, both correlation function and the probabilities depend on the polarization of the beam and the target
- The absolute value of the correlation function in case of degree of target polarization possible to obtain in real experiment is not much higher than in case of unpolarized target

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