Calibration of the top-quark Monte-Carlo mass

Sven-Olaf Moch

Universität Hamburg

Large Hadron Collider Physics (LHCP2016) conference, Lund, June 14, 2016
Based on work done in collaboration with:

- *Calibration of the Top-Quark Monte-Carlo Mass*
  J. Kieseler, K. Lipka and S. M. [arXiv:1511.00841]

- *High precision fundamental constants at the TeV scale (Procs. MITP workshop)*
  S. M., S. Weinzierl et. al. [MITP workshop procs. arXiv:1405.4781]

- *A new observable to measure the top-quark mass at hadron colliders*

- *The top quark and Higgs boson masses and the stability of the electroweak vacuum*

- *Measuring the running top-quark mass*

- Many more papers with friends . . .
Top-quark mass

What is the value of the top-quark mass?

$m_t = ?$
Quantum field theory

QCD

• Classical part of QCD Lagrangian

\[ \mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{\text{flavors}} \bar{q}_i (i\not{\partial} - m_q)_{i,j} q_j \]

  • field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  • covariant derivative $D_{\mu,i,j} = \partial_\mu \delta_{i,j} + ig_s (t_a)_{i,j} A^a_\mu$

• Formal parameters of the theory (no observables)
  • strong coupling $\alpha_s = g_s^2/(4\pi)$
  • quark masses $m_q$

• Parameters of Lagrangian have no unique physical interpretation
  • radiative corrections require definition of renormalization scheme

Challenge

• Suitable observables for measurements of $\alpha_s, m_q, \ldots$
  • comparison of theory predictions and experimental data
**Quark mass renormalization**

- Heavy-quark self-energy $\Sigma(p, m_q)$

$$
\Sigma + \sum + \sum + \cdots = \frac{i}{p - m_q - \Sigma(p, m_q)}
$$

**QCD**

- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$
\Sigma^{(1),\text{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left( \frac{\mu^2}{m_q^2} \right)^\epsilon \left\{ (p - m_q) \left( -C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left( 3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}
$$

- Relate bare and renormalized mass parameter $m_q^{\text{bare}} = m_q^{\text{ren}} + \delta m_q$

$$
\Sigma^{\text{ren}}(p, m_q) = \Sigma + \sum + \sum + \cdots (Z_\psi - 1)p - (Z_m - 1)m_c
$$
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$

$$\Sigma + \Sigma + \Sigma + \ldots = \frac{i}{\not{p} - m_q - \Sigma(p, m_q)}$$

EW sector

- EW corrections to top-quark self-energy
  - on-shell intermediate (virtual) $W$-boson
  - $m_t$ complex parameter with imaginary part $\Gamma_t = 2.0 \pm 0.7$ GeV
  - $\Gamma_t > 1$ GeV: top-quark decays before it hadronizes
Mass renormalization scheme

Pole mass

• Based on (unphysical) concept of top-quark being a free parton
  • $m_q^{\text{ren}}$ coincides with pole of propagator at each order

\[ \not{p} - m_q - \Sigma(p, m_q) \bigg|_{\not{p}=m_q} \to \not{p} - m_q^{\text{pole}} \]

• Definition of pole mass ambiguous up to corrections $O(\Lambda_{QCD})$
  • heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $O(\Lambda_{QCD})$

• Bounds:
  • lattice QCD $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV Bauer, Bali, Pineda ’11
  • perturbative QCD: $\Delta m_q \simeq 70$ MeV Beneke, Marquard, Nason, Steinhauser ’16

\( \overline{\text{MS}} \) scheme

• \( \overline{\text{MS}} \) mass definition: for example one-loop minimal subtraction

\[ \delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3 C_F \left( \frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right) \]

• \( \overline{\text{MS}} \) scheme induces scale dependence: $m(\mu)$
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
- Mass anomalous dimension $\gamma$ known to four loops

\[
\left( \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) m(\mu) = \gamma(\alpha_s)m(\mu)
\]

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$
**Scheme transformations**

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and \( \overline{\text{MS}} \) mass
  - known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher ‘90; Chetyrkin, Steinhauser ‘99; Melnikov, v. Ritbergen ‘99; Marquard, Smirnov, Smirnov, Steinhauser ‘15
  - EW sector known to \( \mathcal{O}(\alpha_{\text{EW}}\alpha_s) \)
    Jegerlehner, Kalmykov ‘04; Eiras, Steinhauser ‘06
- example: one-loop QCD

\[
m^\text{pole} = m(\mu) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} \left( \frac{4}{3} + \ln \left( \frac{\mu^2}{m(\mu)^2} \right) \right) + \ldots \right\}
\]
Top mass from cross sections

- QCD factorization

\[ \sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X) \]

- Joint dependence on non-perturbative parameters: parton distribution functions \( f_i \), strong coupling \( \alpha_s \), masses \( m_X \)

- Total cross section: intrinsic limitation in through sensitivity \( S \simeq 5 \)
  \[ \left| \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq S \times \left| \frac{\Delta m_t}{m_t} \right| \]
Total cross section

Exact result at NNLO in QCD
Czakon, Fiedler, Mitov ‘13

\[ \sigma_{pp \to tt} \text{[pb]} \text{ at LHC8} \]

- NNLO perturbative corrections (e.g. at LHC8)
  - \( K \)-factor (NLO \( \to \) NNLO) of \( \mathcal{O}(10\%) \)
  - scale stability at NNLO of \( \mathcal{O}(\pm 5\%) \)

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Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (I)

Dowling, S.M. ‘13

- NNLO cross section with running mass significantly improved
- Good apparent convergence of perturbative expansion
- Small theoretical uncertainty from scale variation

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Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (II)

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# Top-quark mass determination

- Cross section measurement \( \text{ATLAS arXiv:1406.5375} \)
  \[
  \sigma_{t\bar{t}} = 242.4 \pm 9.5 \text{ pb}
  \]

<table>
<thead>
<tr>
<th>Method</th>
<th>(m_{\text{pole}} + \Delta_{\text{exp}} + \Delta_{\text{th+PDF}})</th>
<th>(m(m) + \Delta_{\text{exp}} + \Delta_{\text{th+PDF}})</th>
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<th>(m_{2lp}^{\text{pl}})</th>
<th>(m_{3lp}^{\text{pl}})</th>
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<tbody>
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<td>ABM12</td>
<td>166.4 ± 1.3 ± 2.1</td>
<td>159.1 ± 1.2 ± 1.2</td>
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<tr>
<td>CT14</td>
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<td>MMHT</td>
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</tr>
</tbody>
</table>

- \(m_t\) from total cross section sensitive to PDFs
  - pole mass from \(\overline{MS}\) mass \(m_t(m_t)\) gives spread
    \[
    m_{\text{pole}} = 168.4 \ldots 176.0 \text{ GeV}
    \]
  - Scale uncertainty from range \(m_t/2 \leq \mu_r, \mu_f \leq 2m_t\) GeV
Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ [Dittmaier, Uwer, Weinzierl '07-'08]
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero

- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$

Graphical representation of Feynman diagrams corresponding to the processes described.

- $pp \rightarrow t\bar{t} + \text{jet} + X$
  - $\sqrt{s} = 14$ TeV
  - $p_{T,\text{jet}} > 20$GeV

Graph showing the cross-section $\sigma$ [pb] as a function of $\mu/m_t$ with curves for LO (CTEQ6L1) and NLO (CTEQ6M).
Top mass with $t\bar{t} + \text{jet}$-samples

- Normalized-differential $t\bar{t} + \text{jet}$ cross section

$$R(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{jet}}} \frac{d\sigma_{t\bar{t}+1\text{jet}}}{d\rho_s}(m_t, \rho_s)$$

- variable $\rho_s = \frac{2m_0}{\sqrt{s}}$ with invariant mass of $t\bar{t} + 1\text{jet}$ system and fixed scale $m_0 = 170$ GeV

- Significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$
Mass sensitivity of $t\bar{t} + \text{jet}$-samples

- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, ...
- Increased sensitivity for system $t\bar{t} + \text{jet}$ compared

\[
\left| \frac{\Delta \mathcal{R}}{\mathcal{R}} \right| \approx (m_t S) \times \left| \frac{\Delta m_t}{m_t} \right|
\]
LHC analyses

- High precision data in threshold region $0.7 \leq \rho_s$
- Cancellation of systematics in the normalized distribution
- ATLAS measurement ATLAS arxiv:1507.01769
  top-quark pole mass $m_{\text{pole}} = 173.7 \pm 1.5(\text{stat.}) \pm 1.4(\text{syst.})^{+1.0}_{-0.5}(\text{theo.})$

• CMS measurement CMS-PAS-TOP-13-006
  top-quark pole mass $m_{\text{pole}} = 169.9 \pm 1.1(\text{stat.})^{+2.5}_{-3.1}(\text{syst.})^{+3.6}_{-1.6}(\text{theo.})$

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Calibration of the top-quark Monte-Carlo mass – p.16
Monte Carlo mass

- Hard interaction and parton emission in QCD followed by hadronization
- Top-quark decays on shell (e.g. leptonic decay $t \rightarrow bW \rightarrow bl\bar{\nu}_l$)

Intuition: Monte Carlo mass identified with pole mass due to kinematics

$$m_q^2 = E_q^2 - p^2$$

Caveat: heavy quarks in QCD interact with potential due to gluon field
Kinematic reconstruction

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - \( m_t^2 = (p_{\text{W-boson}} + p_{b-jet})^2 \)

Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - uncertainties from variation of Monte Carlo parameters
Non-perturbative corrections

- Simulation of top mass measurement Skands, Wicke ’07
  - test of different Monte Carlo tunes for non-perturbative physics / colour reconnection
  - calibration offsets before/after scaling with jet energy scale corrections
- Parton shower models:
  - $p_T$-ordered (blue);
  - virtuality ordered (green)
- Uncertainty in parton shower models (non-perturbative) is $\mathcal{O}(\pm 500)$ MeV
**Calibration of Monte-Carlo Mass (I)**

**Idea**  
Kieseler, Lipka, S.M. ‘15

- Simultaneous fit of $m^{MC}$ and observable $\sigma(m_t)$ sensitive to $m_t$, e.g., total cross section, differential distributions, …
- Observable $\sigma$ does not rely on any prior assumptions about relation between $m_t$ and $m^{MC}$
- Extraction of $m_t$ from $\sigma(m_t)$ calibration of $m^{MC}$, e.g. pole mass  
  \[ \Delta m = m_{\text{pole}}^{t} - m^{MC} \]

**Implementation**  
[J. Kieseler, DESY-THESIS-2015-054]

- Confront $N^d$ reconstructed events to $N^p$ simulated ones
  - model parameters $\vec{\lambda}$

\[
N^p = L \cdot \epsilon(m^{MC}, \vec{\lambda}) \cdot \sigma \cdot n^p(m^{MC}, \vec{\lambda}) + \underbrace{N^{bg}(\vec{\lambda})}_{\text{background}}
\]

- shape of distribution constrains $m^{MC}$, normalization determines $\sigma$
Top-Quark Monte-Carlo Mass (II)

Likelihood fit [J. Kieseler, DESY-THESIS-2015-054]

- Correlations between $m_{MC}$ and $\sigma$ present in $\epsilon(m_{MC}, \bar{\lambda})$
- minimize in $m_{MC}$ dependence in efficiency
- Reduce contribution of $m_{MC}$ to total uncertainty of $\sigma$
- constrain $m_{MC}$ in predicted events $n_p(m_{MC}, \bar{\lambda})$

- Cross section measurement CMS at $\sqrt{s} = 8$ TeV: $\sigma_{t\bar{t}} = 243.9 \pm 9.3$ pb

J. Kieseler, DESY-THESIS-2015-054

- Calibration of $m_{MC}$ with uncertainty of approximately 2 GeV on $\Delta_m = m_{t}^{\text{pole}} - m_{MC}$ possible
Results

• Determination of $m_t(m_t)$ or of $m_t^{\text{pole}}$ from total cross section at NNLO
  • conversion of $m_t(m_t)$ to pole mass $m_t^{\text{pole},c}$
  • Results strongly dependent on parton luminosity and $\alpha_s$

<table>
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<tr>
<th></th>
<th>$\alpha_s(M_Z)$</th>
<th>$m_t(m_t)$ [GeV]</th>
<th>$m_t^{\text{pole}}$ [GeV]</th>
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• Calibration of $\overline{\text{MS}}$ mass ($\tilde{\Delta}_m$), pole mass ($\Delta_m^p$), and pole mass from conversion ($\Delta_m^{p,c}$)

<table>
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<tr>
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<th>$\tilde{\Delta}_m$ [GeV]</th>
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Summary

Top-quark mass

- Top-quark mass is parameter of Standard Model Lagrangian
- Measurements of $m_t$ require careful definition of observable
- Quality of perturbative expansion depends on scheme for top-quark mass
- Monte-Carlo mass $m_{MC}$ needs calibration with data
  - current calibration of $m_{MC}$ with uncertainty of approximately 2 GeV

Future tasks

- Joint effort theory and experiment