Calibration of the top-quark Monte-Carlo mass

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Based on work done in collaboration with:

- *Calibration of the Top-Quark Monte-Carlo Mass*
  J. Kieseler, K. Lipka and S. M. arXiv:1511.00841

- *High precision fundamental constants at the TeV scale (Procs. MITP workshop)*
  S. M., S. Weinzierl et. al. MITP workshop procs. arXiv:1405.4781

- *A new observable to measure the top-quark mass at hadron colliders*
  S. Alioli, P. Fernandez, J. Fuster, A. Irles, S. M., P. Uwer and M. Vos
  arXiv:1303.6415

- *The top quark and Higgs boson masses and the stability of the electroweak vacuum*
  S. Alekhin, A. Djouadi and S. M. arXiv:1207.0980

- *Measuring the running top-quark mass*
  U. Langenfeld, S. M. and P. Uwer arXiv:0906.5273

- Many more papers with friends . . .

Top-quark mass

What is the value of the top-quark mass ?

$$m_t = ?$$

Quantum field theory

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i}\not{D} - m_q\right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a\right)_{ij} A^a_\mu$

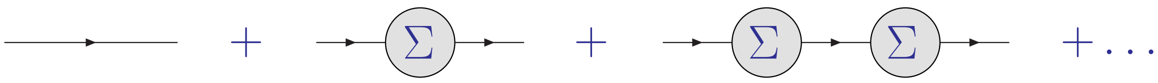
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

- Suitable observables for measurements of α_s, m_q, ...
  - comparison of theory predictions and experimental data

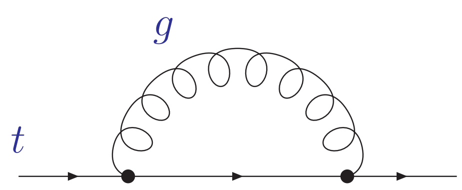
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

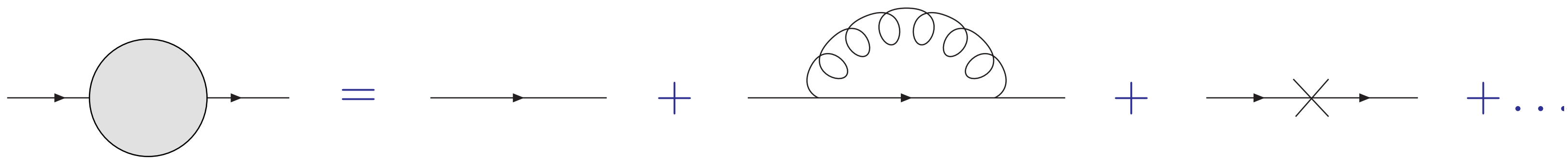
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
    - dimensional regularization $D = 4 - 2\epsilon$
    - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_q^2}\right)^{\epsilon} \left\{ (\not{p} - m_q) \left(-C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left(3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

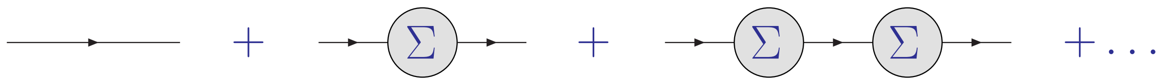
- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



$\Sigma^{\mathrm{ren}}(p, m_q)$ $\qquad\qquad$ $(Z_\psi - 1)\not{p} - (Z_m - 1)m_q$

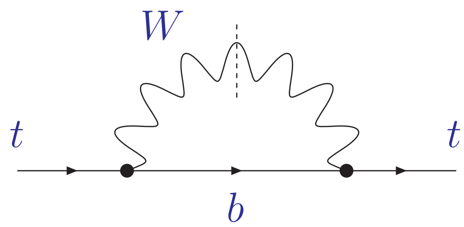
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

EW sector



- EW corrections to top-quark self-energy
  - on-shell intermediate (virtual) W-boson
  - m_t complex parameter with imaginary part $\Gamma_t = 2.0 \pm 0.7$ GeV
  - $\Gamma_t > 1$ GeV: top-quark decays before it hadronizes

Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - $m_q^{\rm ren}$ coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \to \not{p} - m_q^{\rm pole}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Bounds:
  - lattice QCD $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV Bauer, Bali, Pineda '11
  - perturbative QCD: $\Delta m_q \simeq 70$ MeV Beneke, Marquard, Nason, Steinhauser '16

$\overline{\rm MS}$ *scheme*

- $\overline{\rm MS}$ mass definition: for example one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3 C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

- $\overline{\rm MS}$ scheme induces scale dependence: $m(\mu)$

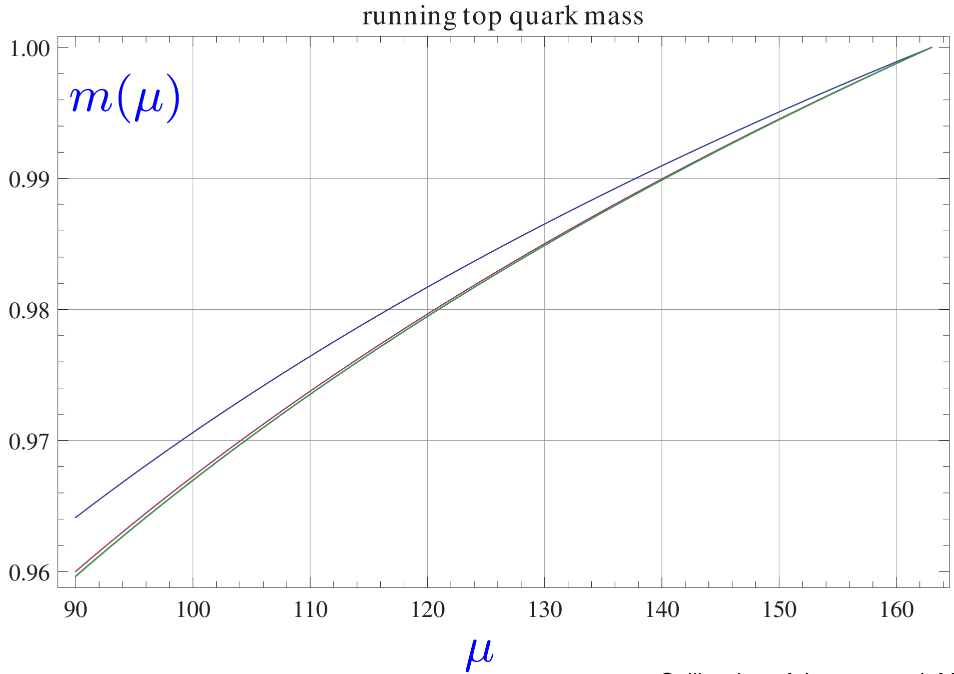
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to four loops
    Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) \;=\; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



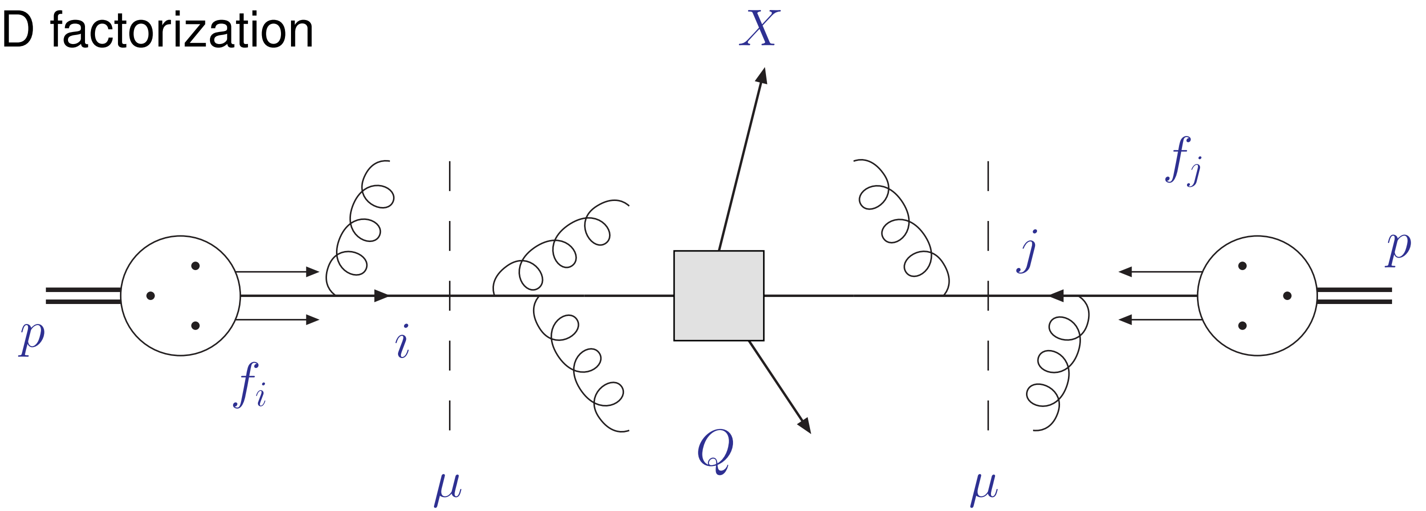
Scheme transformations

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{\mathrm{MS}}$ mass
  - known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99; Marquard, Smirnov, Smirnov, Steinhauser '15
  - EW sector known to $\mathcal{O}(\alpha_{\rm EW}\alpha_{\rm s})$ Jegerlehner, Kalmykov '04; Eiras, Steinhauser '06
  - example: one-loop QCD

$$m^{\rm pole} = m(\mu)\left\{1 + \frac{\alpha_s(\mu)}{4\pi}\left(\frac{4}{3} + \ln\left(\frac{\mu^2}{m(\mu)^2}\right)\right) + \dots\right\}$$

Top mass from cross sections

- QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

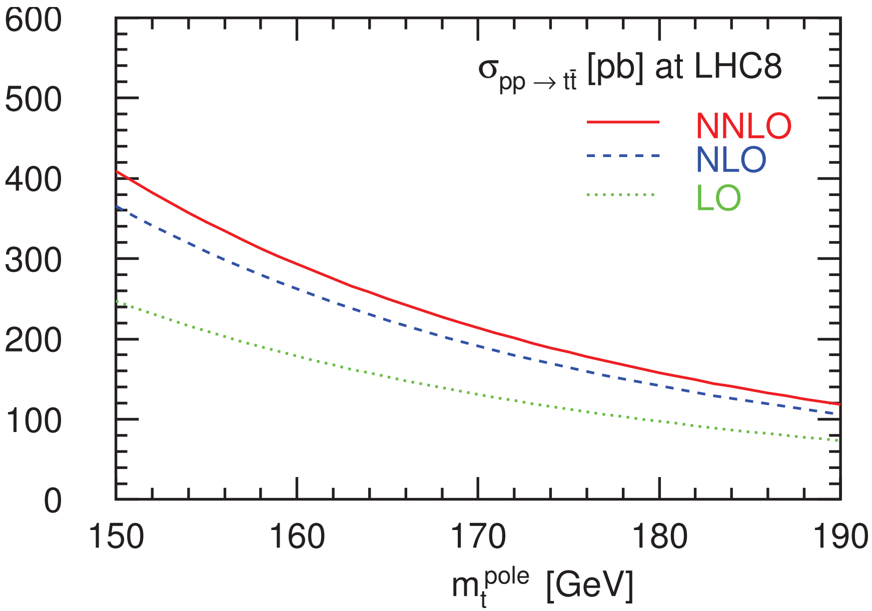
- Joint dependence on non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, masses m_X
- Total cross section: intrinsic limitation in through sensitivity $\mathcal{S} \simeq 5$

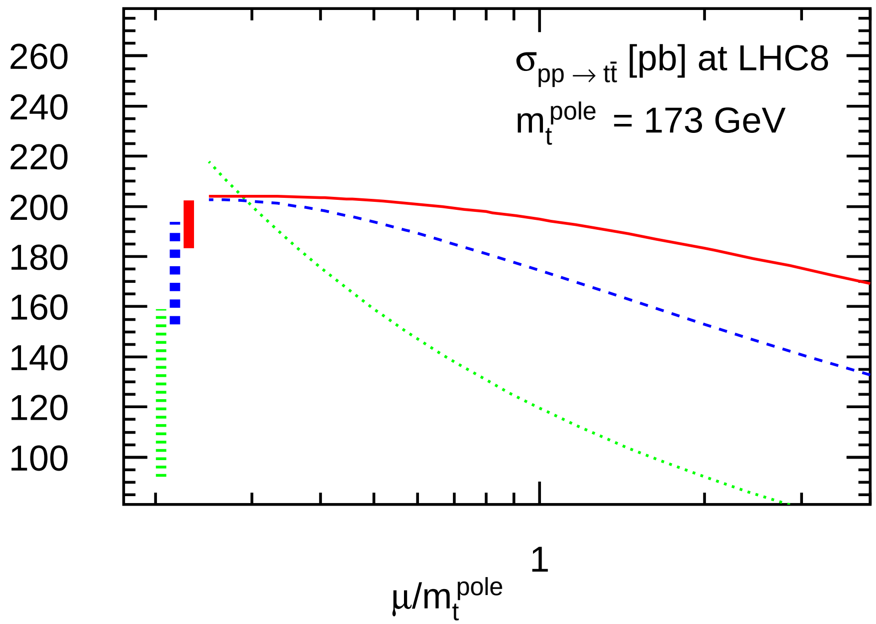
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq \mathcal{S} \times \left|\frac{\Delta m_t}{m_t}\right|$$

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13



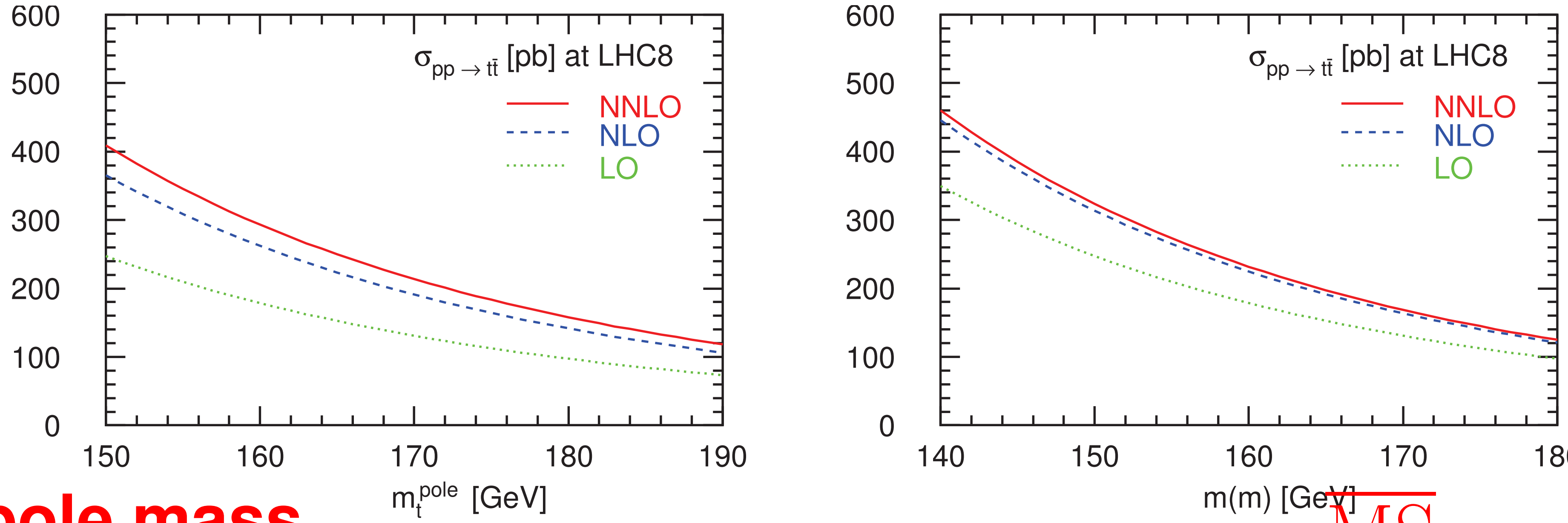


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Total cross section with running mass

Comparison pole mass vs. $\overline{\mathrm{MS}}$ mass (I)

Dowling, S.M. '13



pole mass

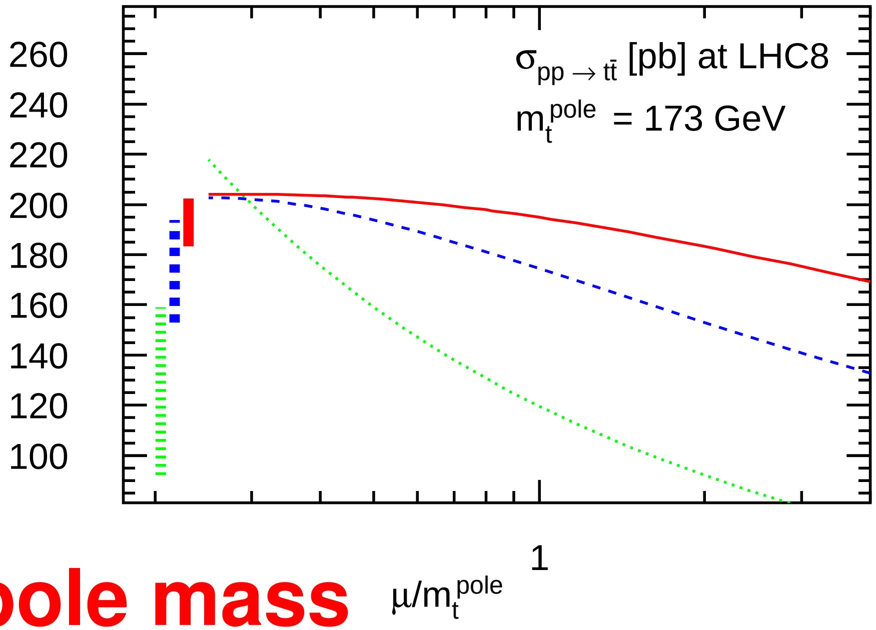
$\overline{\mathrm{MS}}$ mass

- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity from scale variation

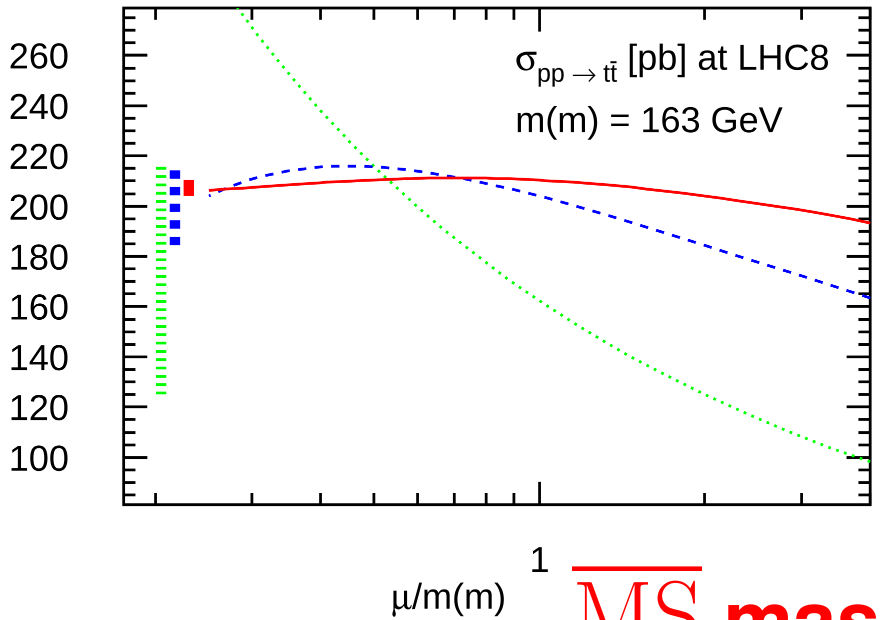
Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (II)

Dowling, S.M. '13



pole mass



$\overline{\text{MS}}$ mass

- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity from scale variation

Top-quark mass determination

- Cross section measurement ATLAS arXiv:1406.5375

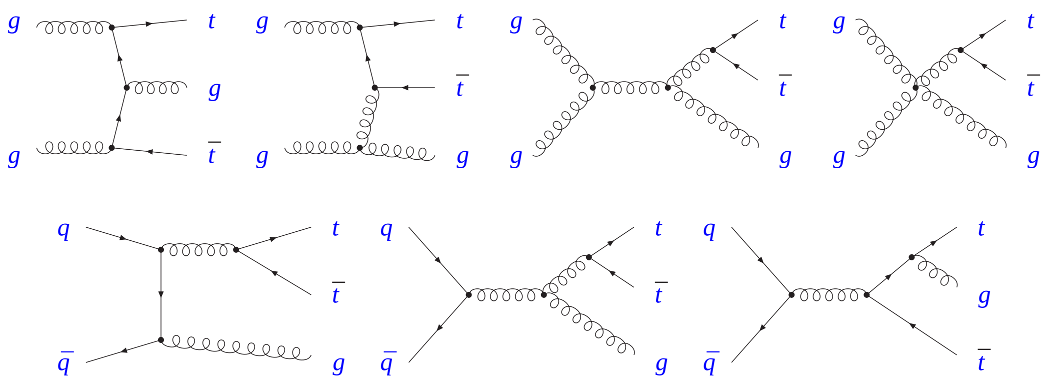
$$\sigma_{t\bar{t}} = 242.4 \pm 9.5 \text{ pb}$$

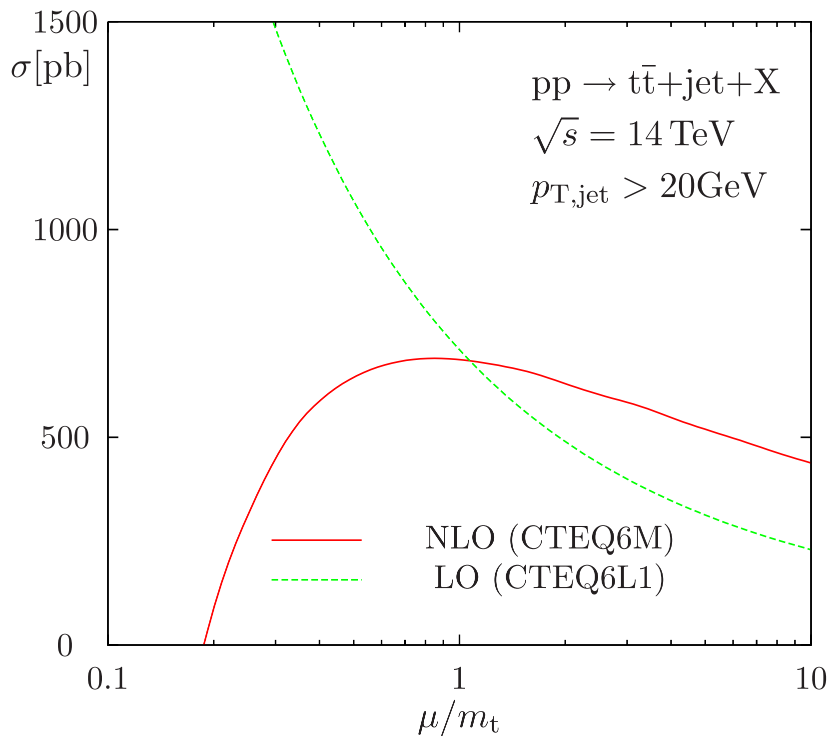
	$m^{\text{pole}} + \Delta^{\text{exp}} + \Delta^{\text{th+PDF}}$	$m(m) + \Delta^{\text{exp}} + \Delta^{\text{th+PDF}}$	$m^{\text{pl}}_{\text{1lp}}$	$m^{\text{pl}}_{\text{2lp}}$	$m^{\text{pl}}_{\text{3lp}}$
ABM12	$166.4 \pm 1.3 \pm 2.1$	$159.1 \pm 1.2 \pm 1.2$	166.2	167.8	168.4
CT14	$173.8 \pm 1.3 \pm 2.2$	$165.9 \pm 1.3 \pm 1.3$	173.5	175.4	176.0
MMHT	$173.7 \pm 1.3 \pm 2.0$	$165.8 \pm 1.3 \pm 1.0$	173.4	175.2	175.9
NNPDF3.0	$173.5 \pm 1.3 \pm 2.0$	$165.6 \pm 1.3 \pm 1.0$	173.2	175.0	175.7

- m_t from total cross section sensitive to PDFs
  - pole mass from $\overline{MS}$ mass $m_t(m_t)$ gives spread
    $m^{\text{pole}} = 168.4 \ldots 176.0$ GeV
- Scale uncertainty from range $m_t/2 \le \mu_r, \mu_f \le 2m_t$ GeV

Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$

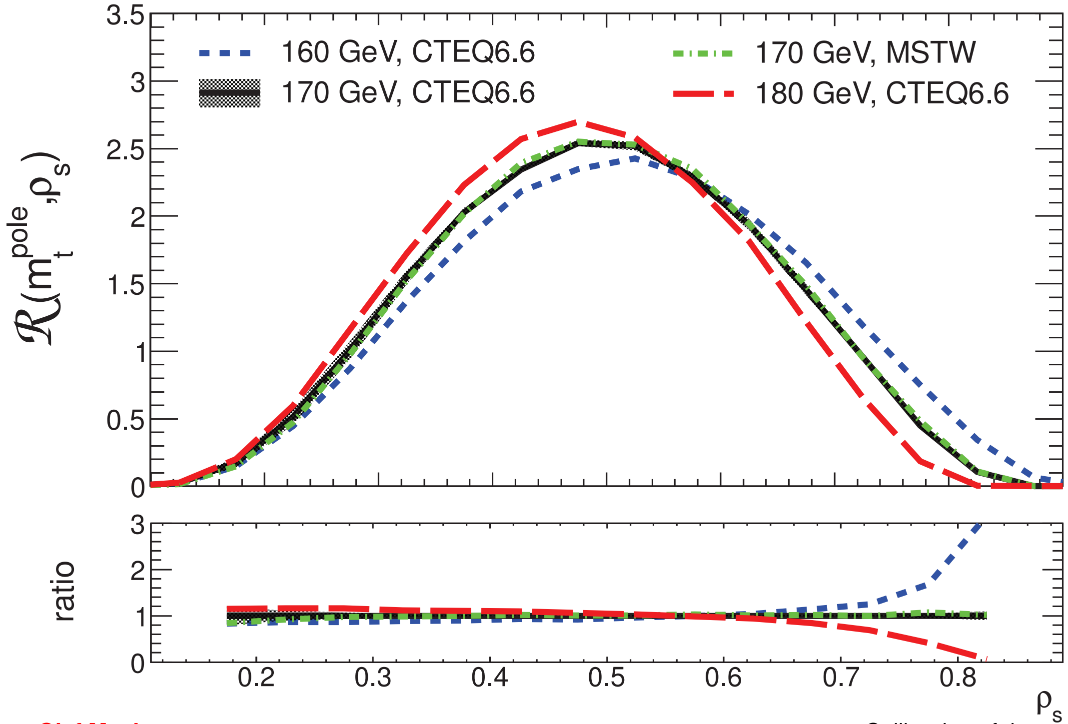
Top mass with $t\bar{t}$ + jet-samples

- Normalized-differential $t\bar{t} + \mathrm{jet}$ cross section

Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos '13

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

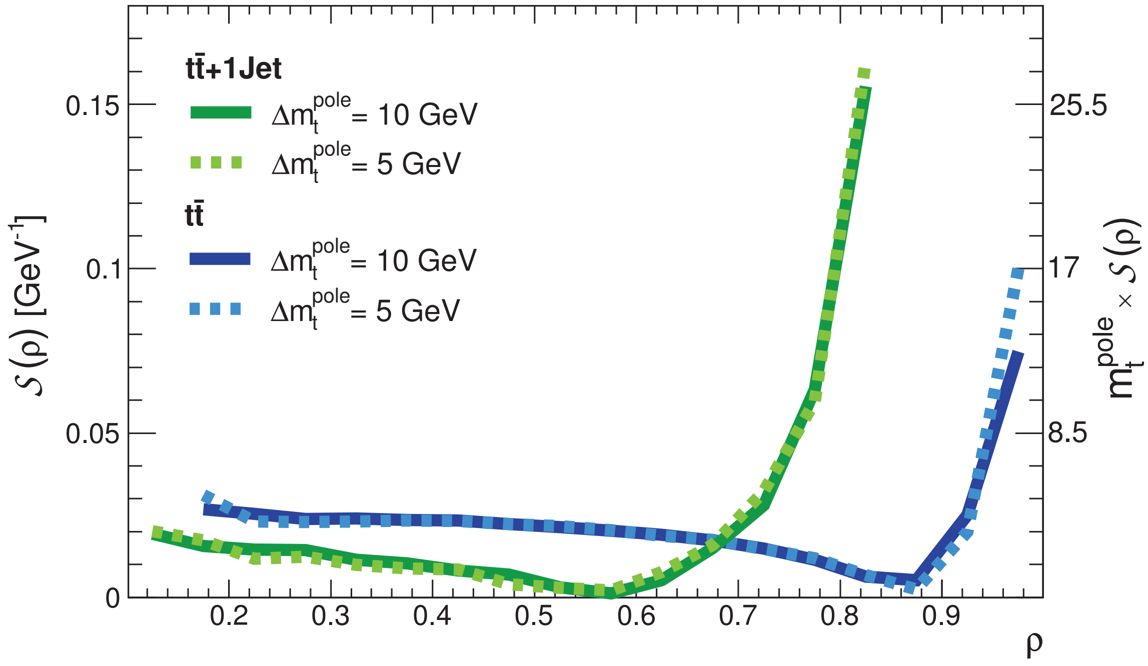
  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t} + 1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$
- Significant mass dependence for $0.4 \le \rho_s \le 0.5$ and $0.7 \le \rho_s$



Mass sensitivity of $t\bar{t} + \mathrm{jet}$*-samples*

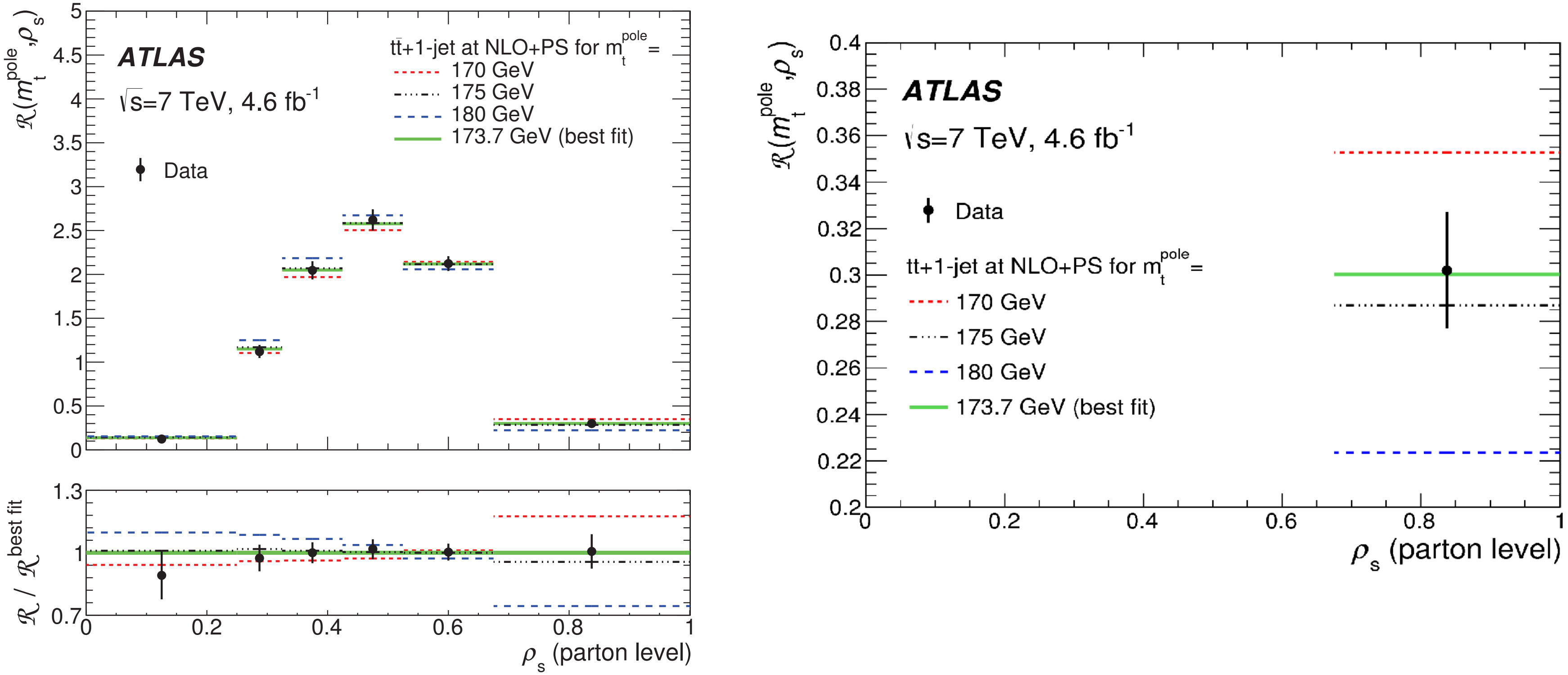
- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .
- Increased sensitivity for system $t\bar{t} + \mathrm{jet}$ compared

$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$



LHC analyses

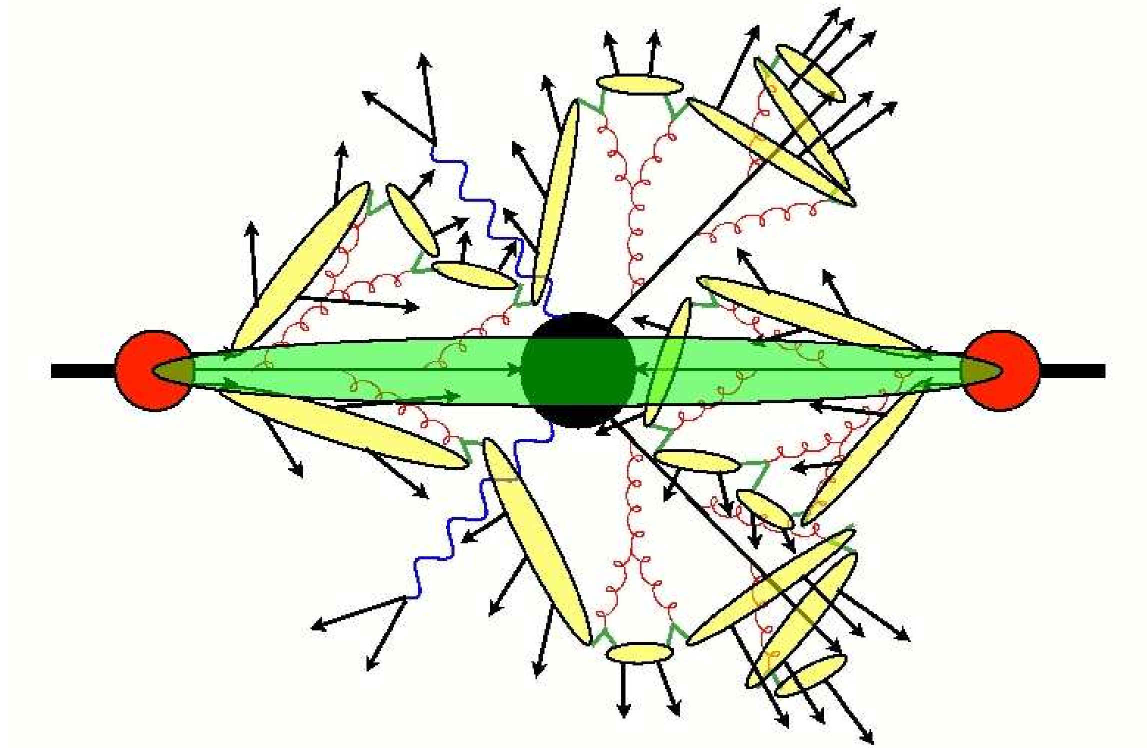
- High precision data in threshold region $0.7 \leq \rho_s$
- Cancellation of systematics in the normalized distribution
- ATLAS measurement ATLAS arxiv:1507.01769
  top-quark pole mass $m_{\rm pole} = 173.7 \pm 1.5(\text{stat.}) \pm 1.4(\text{syst.})^{+1.0}_{-0.5}(\text{theo.})$



- CMS measurement CMS-PAS-TOP-13-006
  top-quark pole mass $m_{\rm pole} = 169.9 \pm 1.1(\text{stat.})^{+2.5}_{-3.1}(\text{syst.})^{+3.6}_{-1.6}(\text{theo.})$

Monte Carlo mass

- Hard interaction and parton emission in QCD followed by hadronization
- Top-quark decays on shell (e.g. leptonic decay $t \to bW \to bl\bar{\nu}_l$)



[picture by B.Webber]

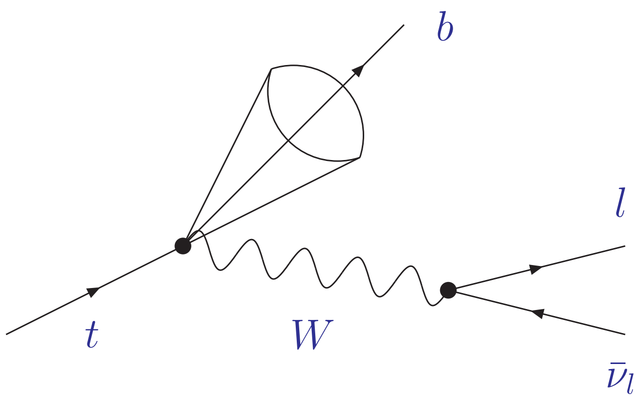
- Intuition: Monte Carlo mass identified with pole mass due to kinematics

$$m_q^2 \,=\, E_q^2 - p^2$$

- Caveat: heavy quarks in QCD interact with potential due to gluon field

Kinematic reconstruction

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$

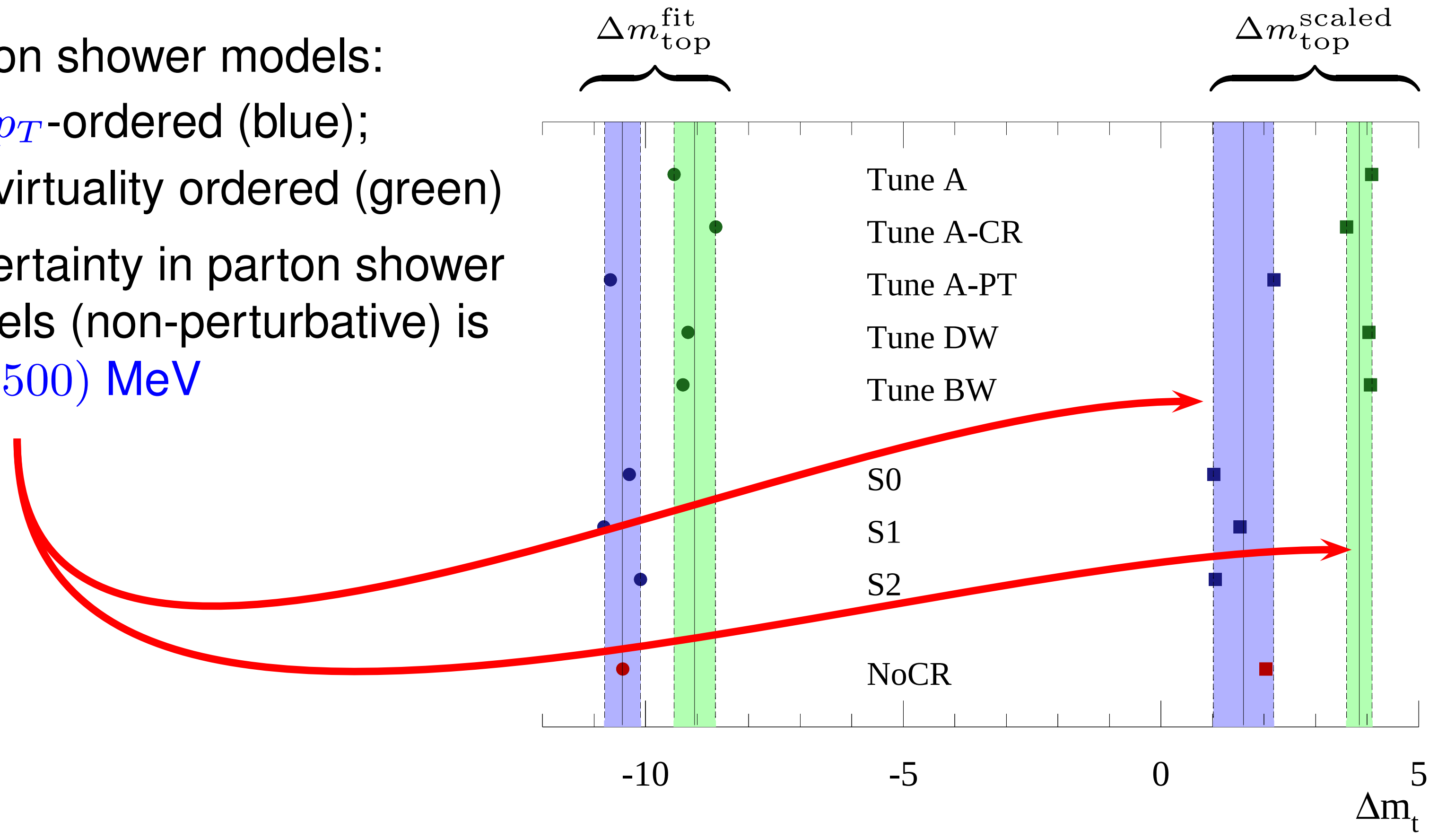


Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - uncertainties from variation of Monte Carlo parameters

Non-perturbative corrections

- Simulation of top mass measurement Skands, Wicke ’07
  - test of different Monte Carlo tunes for non-perturbative physics / colour reconnection
  - calibration offsets before/after scaling with jet energy scale corrections
- Parton shower models:
  - p_T-ordered (blue);
  - virtuality ordered (green)
- Uncertainty in parton shower models (non-perturbative) is $\mathcal{O}(\pm 500)$ MeV



Calibration of Monte-Carlo Mass (I)

Idea Kieseler, Lipka, S.M. '15

- Simultaneous fit of $m^{\rm MC}$ and observable $\sigma(m_t)$ sensitive to m_t, e.g., total cross section, differential distributions, ...
- Observable σ does not rely on any prior assumptions about relation between m_t and $m^{\rm MC}$
- Extraction of m_t from $\sigma(m_t)$ calibration of $m^{\rm MC}$, e.g. pole mass

$$\Delta_m = m_t^{\rm pole} - m^{\rm MC}$$

Implementation [J. Kieseler, DESY-THESIS-2015-054]

- Confront N^d reconstructed events to N^p simulated ones
  - model parameters $\vec{\lambda}$

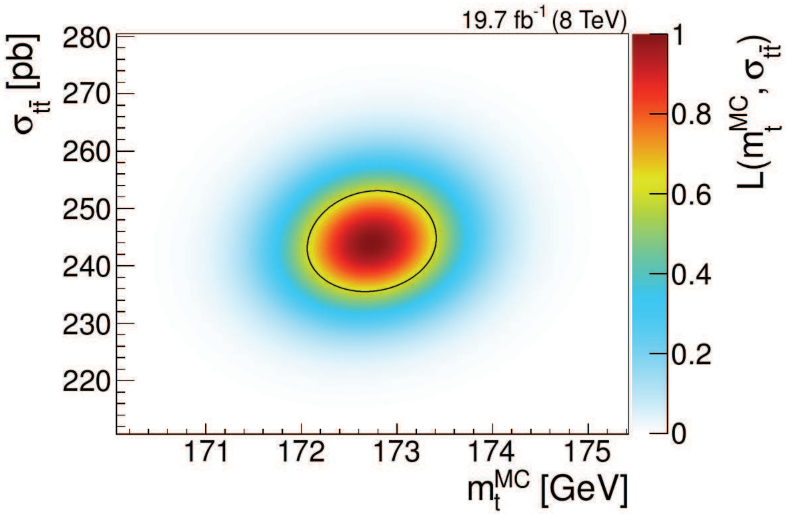
$$N^p = \mathcal{L} \cdot \underbrace{\epsilon(m^{\rm MC}, \vec{\lambda})}_{\text{efficiency}} \cdot \underbrace{\sigma}_{\text{observable}} \cdot \underbrace{n^p(m^{\rm MC}, \vec{\lambda})}_{\text{predicted shape contribution}} + \underbrace{N^{bg}(\vec{\lambda})}_{\text{background}}$$

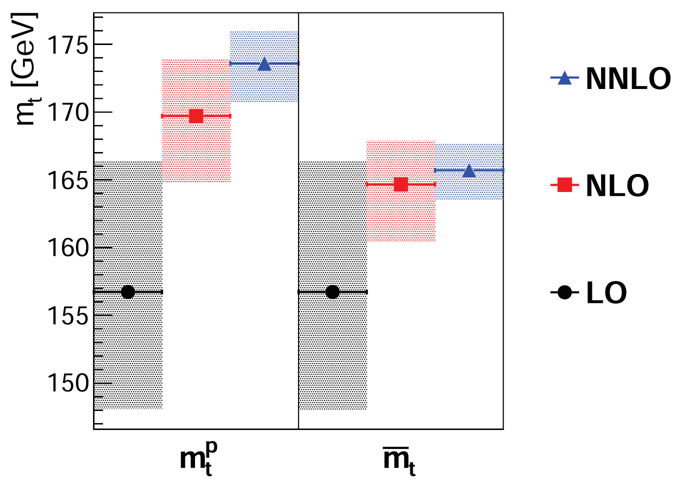
  - shape of distribution constrains $m^{\rm MC}$, normalization determines σ

Top-Quark Monte-Carlo Mass (II)

Likelihood fit [J. Kieseler, DESY-THESIS-2015-054]

- Correlations between m^{MC} and σ present in $\epsilon(m^{\mathrm{MC}}, \vec{\lambda})$
  - minimize in m^{MC} dependence in efficiency
- Reduce contribution of m^{MC} to total uncertainty of σ
  - constrain m^{MC} in predicted events $n^p(m^{\mathrm{MC}}, \vec{\lambda})$





- Cross section measurement CMS at $\sqrt{s} = 8$ TeV: $\sigma_{t\bar{t}} = 243.9 \pm 9.3$ pb
  J. Kieseler, DESY-THESIS-2015-054
- Calibration of m^{MC} with uncertainty of approximately 2 GeV on $\Delta_m = m_t^{\mathrm{pole}} - m^{\mathrm{MC}}$ possible

Top-Quark Monte-Carlo Mass (III)

Results

- Determination of $m_t(m_t)$ or of $m_t^{\rm pole}$ from total cross section at NNLO
  - conversion of $m_t(m_t)$ to pole mass $m_t^{\rm pole,c}$
- Results strongly dependent on parton luminosity and α_s

	$\alpha_s(M_Z)$	$m_t(m_t)$ [GeV]	$m_t^{\rm pole}$ [GeV]	$m_t^{\rm pole,c}$ [GeV]
ABM12	0.113	$158.4\pm^{1.2}_{1.9}$	$166.6\pm^{1.6}_{1.9}$	$168.0\pm^{1.3}_{2.1}$
NNPDF3.0	0.118	$165.2\pm^{1.1}_{1.7}$	$174.0\pm^{1.4}_{1.7}$	$175.1\pm^{1.2}_{1.9}$
MMHT2014	0.118	$165.4\pm^{1.1}_{1.9}$	$174.3\pm^{1.4}_{1.8}$	$175.3\pm^{1.3}_{2.1}$
CT14	0.118	$165.5\pm^{1.5}_{2.0}$	$174.4\pm^{1.8}_{2.0}$	$175.4\pm^{1.7}_{2.2}$

- Calibration of $\overline{\rm MS}$ mass ($\bar{\Delta}_m$), pole mass (Δ_m^p), and pole mass from conversion ($\Delta_m^{p,c}$)

	$\bar{\Delta}_m$ [GeV]	Δ_m^p [GeV]	$\Delta_m^{p,c}$ [GeV]
ABM12	$-14.3\pm^{1.4}_{2.0}$	$-6.1\pm^{1.7}_{2.0}$	$-4.7\pm^{1.5}_{2.2}$
NNPDF3.0	$-7.6\pm^{1.3}_{1.9}$	$1.3\pm^{1.6}_{1.9}$	$2.4\pm^{1.5}_{2.0}$
MMHT2014	$-7.3\pm^{1.3}_{2.1}$	$1.5\pm^{1.6}_{2.0}$	$2.6\pm^{1.5}_{2.2}$
CT14	$-7.2\pm^{1.7}_{2.1}$	$1.6\pm^{1.9}_{2.1}$	$2.7\pm^{1.8}_{2.3}$

Summary

Top-quark mass

- Top-quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Quality of perturbative expansion depends on scheme for top-quark mass
- Monte-Carlo mass m^{MC} needs calibration with data
  - current calibration of m^{MC} with uncertainty of approximately 2 GeV

Future tasks

- Joint effort theory and experiment