Drell-Yan Production at NNLO+NNLL′+PS in GENEVA

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S. Alioli, C. Bauer, C. Berggren, FT, J. Walsh [PRD92 (2015), 094020]
Overview.

**GENEVA** consistently combines 3 ingredients

1. **Fully differential fixed-order calculations**
   - up to NNLO (based on N-jettiness subtractions)

2. **Higher-order resummation**
   - up to NNLL' using SCET formalism (but not restricted to it)

3. **Parton showering and hadronization to “fill out” jets**
   - using standard shower MC (currently PYTHIA8)

⇒ **NNLO+NNLL’+PS** Monte Carlo
GENEVA consistently combines 3 ingredients

1. Fully differential fixed-order calculations
   - up to NNLO (based on N-jettiness subtractions)

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   - up to NNLL\textsuperscript{'} using SCET formalism (but not restricted to it)

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\[ \Rightarrow \text{NNLO+NNLL}^\prime + \text{PS Monte Carlo} \]

Higher-order resummation

- Provides a natural link between NNLO and PS
- Is key to consistently improve perturbative accuracy outside FO region
- Allows to systematically estimating perturbative uncertainties and correlations (on event-by-event basis)
**GENEVA**

1. Physical (IR-finite, all-order) definition of events using suitable jet resolution variable $\mathcal{T}$

2. Construct resummed + FO matched MC cross sections at NNLL $\mathcal{T}$ + NNLO

**GENEVA–PYTHIA8 interface**

3. Let shower fill out jets with radiation

**PYTHIA8**

4. Hadronization

5. Additional soft interactions (MPI)
Step 1: Define Physical Events.

Jet resolution variable $\tau$ characterizes the scale of additional emission(s) (analogous to evolution variable in PS, merging scale/variable in other approaches)

- N-parton event represents an IR-finite physical (idealized) N-jet cross section fully-differential in $\Phi_N$
  - Emissions below $\tau_N^{\text{cut}}$ are unresolved (integrated over) and projected onto $\tau_{M<N}$ spectra (which are part of $\Phi_N$)
  - In the end take $\tau_N^{\text{cut}} \to 0$ (up to small IR cutoff $\Lambda_N$)
Step 1: Define Physical Events.

Jet resolution variable $\mathcal{T}$ characterizes the scale of additional emission(s) (analogous to evolution variable in PS, merging scale/variable in other approaches)

We currently use N-jettiness $\mathcal{T} \equiv \mathcal{T}_N$ [Stewart, FT, Waaelwijn ’09, ’10]

- Scales with $p^+ = E - |\vec{p}|$ of emissions (virtuality-like)
  - $e^+e^- \rightarrow 2/3$ jets: $\mathcal{T} \equiv \mathcal{T}_2$ is equivalent to thrust
  - $pp \rightarrow V + 0/1$ jets: $\mathcal{T} \equiv \mathcal{T}_0$ is equivalent to beam thrust
- Factorization and up to NNLL’ resummation in principle known for any N
Step 2: Jet Resolution Spectrum.

There are no strict boundaries $\rightarrow \mathcal{T}$ spectrum describes transition between 0-jet and $\geq 1$-jet regions

- Need consistent treatment of theory uncertainties across entire spectrum
  - quite nontrivial because it requires nontrivial correlations
    (simple factor-2-scale-variation-recipes are not good enough)

- Complete description requires consistent matching of resummation + fixed order
  - Well understood for single-differential spectra to NNLL' + NNLO
Step 2: Combining Resummation and FO.

\[
\frac{d\sigma_{0}^{MC}}{d\Phi_0} (\mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_{0}^{\text{NNLL}^{'}}}{d\Phi_0} (\mathcal{T}_0^{\text{cut}}) + \frac{d\sigma_{0}^{\text{nons}}}{d\Phi_0} (\mathcal{T}_0^{\text{cut}})
\]

\[
\frac{d\sigma_{\geq 1}^{MC}}{d\Phi_1} (\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) = \frac{d\sigma_{0}^{\text{NNLL}^{'}}}{d\Phi_0 d\mathcal{T}_0} \theta(\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}}) \mathcal{P}(\Phi_1)
\]
\[
+ \frac{d\sigma_{\geq 1}^{\text{nons}}}{d\Phi_1} (\mathcal{T}_0 > \mathcal{T}_0^{\text{cut}})
\]

Construct partonic MC cross sections that are fully-differential in $\Phi_N$ and reproduce $\text{NNLL}^{' + \text{NNLO}}_0 \mathcal{T}_0$ spectrum

- $\text{NNLL}^{'$ resummation contains full $\mathcal{O}(\alpha_s^2)$ singular contributions
  - Proper distribution of 2-loop virtuals as dictated by $\text{NNLL}^'$ resummation
- Nonsingular corrections are fixed by matching to $\text{NNLO}_0$ and $\text{NLO}_1$
  - Implementation of differential N-jettiness subtractions
Interlude: Resummation for $\mathcal{T}_0$.

Beam thrust/0-jettiness factorization in SCET [Stewart, FT, Waalewijn '09]

$$\frac{d\sigma}{d\mathcal{T}_0} = H_{ij}(\mu) \int dt_a dt_b B_i(t_a, \mu) B_j(t_b, \mu) S_{ij}\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu\right)$$

Logarithms are split apart and resummed using RGE

$$\ln^2 \frac{\mathcal{T}_0}{Q} = 2 \ln^2 \frac{Q}{\mu} - \ln^2 \frac{\mathcal{T}_0 Q}{\mu^2} + 2 \ln^2 \frac{\mathcal{T}_0}{\mu}$$

⇒ Always resums ratios of hard, beam, soft scales

$$\mu_H \sim Q, \quad \mu_B \sim \sqrt{\mathcal{T}_0 Q}, \quad \mu_S \sim \mathcal{T}_0$$

Resummation is controlled by using $\mathcal{T}_0$-dependent profile scales $\mu_i(\mathcal{T}_0)$ [Ligeti, FT, Stewart '08; Abbate et al. '10; Berger et al. '10; Gangal, Stahlhofen, FT '14]

- Can identify and estimate different sources of perturbative uncertainties using appropriate profile scale variations
- Evaluating MC cross sections for all sets of profile scales gives different weights for each event providing event-by-event pert. uncertainties
Step 3: Attaching the Parton Shower.

Since the parton shower generates perturbative emissions it should

- fill jets with radiation, i.e., provide unresolved emissions that have been integrated over and projected onto partonic events
- not change resummed jet cross sections
  - Additional showering must not change the jet $\Phi_N$ kinematics, in particular $\mathcal{T}_0$, of an event (up to small power corrections)
  - Achieved by taking $\mathcal{T}^{\text{cut}}_{0,1}$ as small as possible, first shower emission of $\Phi_1$ events done by GENEVA using $\mathcal{T}_0$-preserving phase-space map
  - Inclusive $\Phi_2$ events further showered by PYTHIA8
Step 3: Attaching the Parton Shower.

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- fill jets with radiation, i.e., provide unresolved emissions that have been integrated over and projected onto partonic events
- not change resummed jet cross sections
  - Additional showering must not change the jet $\Phi_N$ kinematics, in particular $\tau_0$, of an event (up to small power corrections)
  - Achieved by taking $\tau_0^{\text{cut}}$ as small as possible, first shower emission of $\Phi_1$ events done by GENEVA using $\tau_0$-preserving phase-space map
  - Inclusive $\Phi_2$ events further showered by PYTHIA8
Step 4: Hadronization.

**PYTHIA8 hadronization is unconstrained**

- Observed to behave as expected from field theory and factorization
  - $\mathcal{O}(1)$ effect in nonperturbative peak region at very small $\mathcal{T}$
  - Power-suppressed effect at larger $\mathcal{T}$

- With enough pert. information included, tuning becomes equivalent to extracting nonperturbative inputs from data (i.e. what it really should be)

- Can directly utilize PYTHIA8’s nonperturbative model together with higher-order resummed calculation
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**Overview**

- **Peak Region**
  - $pp \rightarrow Z/\gamma \rightarrow e^+e^-$ (7 TeV)
  - GENEVA+PYTHIA8 NNLL$_\tau$+NNLO$_0$
  - $d\sigma/d\mathcal{T}_0$ [pb/GeV]

- **Transition**
  - $pp \rightarrow Z/\gamma \rightarrow e^+e^-$ (7 TeV)
  - GENEVA+PYTHIA8 NNLL$_\tau$+NNLO$_0$
  - $d\sigma/d\mathcal{T}_0$ [pb/GeV]

- **Tail Region**
  - $pp \rightarrow Z/\gamma \rightarrow e^+e^-$ (7 TeV)
  - GENEVA+PYTHIA8 NNLL$_\tau$+NNLO$_0$
  - $d\sigma/d\mathcal{T}_0$ [pb/GeV]
Other observables: FO.

- Validation against DYNNLO [Catani, Grazzini et al. '07, '09]
- True NNLO only for $p_{T\ell} < m_Z/2, \gtrsim m_Z/2$ sensitive to resummation effects due to Sudakov shoulder
Other observables: $q_T$ and $\phi^*$. Compare to analytic resummed predictions from DY$qT$ [Bozzi et al., '09, '11] (each normalized to own total cross section)

- **GENEVA** does not have formal NNLL' accuracy for variables other than $\tau_0$ itself
- Pert. improvement still clearly translates to other observables due to fully exclusive description
  - Was also observed for $e^+e^-$
  - Relies on NLL $\tau_1$ resummation and PYTHIA8 showering
  - Smaller **GENEVA** uncertainties at very small $q_T$ do not imply higher accuracy but are due to lack of uncertainties in $\tau_1$ resummation and shower interface
Other observables: $q_T$ and $\phi^*$. 

Compare to analytic resummed predictions from BDMT [Banfi et al., '12] (each normalized to own total cross section)
Essentially out-of-the-box results, no attempt at systematic tuning

- We do observe reduced sensitivity to PYTHIA8 parameters (as it should be)

- Noticeably better agreement for lower $\alpha_s(M_Z)$
  - Same as seen in $e^+e^-$ with higher-order resummation and hadronization
Essentially out-of-the-box results, no attempt at systematic tuning

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Step 5: Adding MPI.

Discussion so far was for the primary hard collision

- Addition of MPI is slightly nontrivial due to PYTHIA8 interleaved evolution
  - Shower conditions are applied to all particles identified as arising from primary hard interaction, while secondary interactions are unconstrained (requires to turn off rescattering)

- Beam thrust/0-jettiness potentially very useful for tuning MPI models
  - Primary perturbative effects are known precisely
  - Should allow to fully disentangle MPI contributions from primary soft ISR

- There has been significant progress on field-theoretic description of MPI
  - Can imagine including this in perturbative input which would then place constraint on MPI model
Traditional UE Measurements in DY.

- Overall GENEVA + PYTHIA8 agrees well PYTHIA8 in low-$p_T$ regions
  - Confirms that PYTHIA8 shower and MPI are not being spoiled by GENEVA
- Clear improvements observed toward larger transverse momenta
Summary and Outlook.

First complete matching of $\text{NNLO}+\text{NNLL}'+\text{PS}$

- Higher-order resummation of jet resolution variable provides a natural link between NNLO and PS
- Provides systematic estimate of both resummation and FO perturbative uncertainties on event-by-event basis

Current status

- $pp \rightarrow \gamma/Z$ is completed
  - $\text{NNLL'}+\text{NNLO}_0$ for $0/1$-jet resolution $T_0$
  - $\text{NLL}+\text{NLO}_1$ for $1/2$-jet resolution $T_1$
  - Interface to PYTHIA8 shower+hadronization and MPI

Plans for immediate future

- Currently working on public release
  - Spending significant effort to make the code easy to use as well as easy to extend, stay tuned ...
- $pp \rightarrow W$ at same precision is in the pipeline (likely to be part of release)
- Dedicated PYTHIA8 tune for GENEVA
- Further improve and study perturbative inputs and accuracy
Backup Slides
Uncertainties from Profile Scale Variations.

(Illustration for $gg \rightarrow H$ at $m_H = 125$ GeV)

$\Delta_{\mu_i}$: Collective overall scale variation

- Leaves all scale ratios and resummed logs invariant and thus corresponds to overall FO uncertainty (within resummed prediction)
- Reproduces usual FO scale variation in inclusive cross section
Uncertainties from Profile Scale Variations.

(Illustration for $gg \rightarrow H$ at $m_H = 125$ GeV)

$gg \rightarrow H$ (8 TeV)  
$m_H = 125$ GeV  
$T_{\text{jet}}^{\text{cm}} < T_{\text{cut}}$

$\Delta_{\text{resum}}$: Resummation scale variations
- Envelope of separately varying all profile scales for fixed $\mu_H, \mu_{FO}$ (within canonical constraints), total of six independent variations
- Directly probes size of logs and uncertainties in resummed log series
- Vanishes at large $\tau$ as resummation turns off
Perturbative Accuracy.

(Notation: \( \tau = \mathcal{T}/Q, \ L = \ln \tau, \ L_{\text{cut}} = \ln \tau_{\text{cut}} \))

\[
\frac{\sigma(\tau_{\text{cut}})}{\sigma_B} = \begin{array}{cccc}
\mathrm{LL}_\sigma & \mathrm{NLL}_\sigma & \mathrm{NLL'}_\sigma & \mathrm{NNLL}_\sigma \\
1 & & & \mathrm{LO}_N \\
\end{array}
\]

\[
+ \alpha_s \left[ \frac{c_{11}}{2} L_{\text{cut}}^2 + c_{10} L_{\text{cut}} + c_{1,-1} + F_1(\tau_{\text{cut}}) \right] \mathrm{NLO}_N
\]

\[
+ \alpha_s^2 \left[ \vdots + \vdots + \vdots + \vdots \right] \mathrm{NLO}_N
\]

\[
\frac{1}{\sigma_B} \frac{d\sigma}{d\tau} = \alpha_s/\tau \left[ c_{11} L + c_{10} \ + \tau f_1(\tau) \right] \mathrm{LO}_{N+1}
\]

\[
+ \alpha_s^2/\tau \left[ c_{23} L^3 + c_{22} L^2 + c_{21} L + c_{20} + \tau f_2(\tau) \right] \mathrm{NLO}_{N+1}
\]

\[
+ \alpha_s^3/\tau \left[ \vdots + \vdots + \vdots + \vdots \right] \mathrm{NLO}_{N+1}
\]

Lowest perturbative accuracy at all \( \mathcal{T} \) requires \((N)\mathrm{LL}_\sigma + \mathrm{LO}_{N+1}\)

\(\rightarrow\) Provided by ME/PS: CKKW, MLM (except PS might not get full \(\mathrm{NLL}_\sigma\))

\(\rightarrow\) \(\mathrm{LO}_N\) is naturally part of \(\mathrm{LL}_\sigma\) and so automatically included
Perturbative Accuracy.

(Notation: $\tau = \mathcal{T}/Q$, $L = \ln \tau$, $L_{\text{cut}} = \ln \tau_{\text{cut}}$)

$$\frac{\sigma(\tau_{\text{cut}})}{\sigma_B} =$$

$$1 + \frac{\alpha_s}{2} L_{\text{cut}}^2 + c_{10} L_{\text{cut}} + c_{1,-1} + F_1(\tau_{\text{cut}})$$

$$+ \frac{\alpha_s^2}{2} L_{\text{cut}}^3 + c_{23} L_{\text{cut}}^2 + c_{22} L_{\text{cut}} + c_{21} L_{\text{cut}} + c_{20} + \tau f_2(\tau)$$

NLO + PS matching (MC@NLO, POWHEG) adds full NLO$_N$ to $\sigma(\tau_{\text{cut}})$

→ Improves accuracy for $\sigma(\tau_{\text{cut}} \sim 1)$ to NLO

→ Does not improve accuracy of spectrum
Perturbative Accuracy.

(Notation: $\tau = T/Q$, $L = \ln \tau$, $L_{\text{cut}} = \ln \tau_{\text{cut}}$)

$$\frac{\sigma(\tau_{\text{cut}})}{\sigma_B} = \frac{LL_\sigma}{1} + \alpha_s \left[ \frac{c_{11}}{2} L_{\text{cut}}^2 + c_{10} L_{\text{cut}} + c_{1,-1} \right] + F_1(\tau_{\text{cut}})$$

$$+ \alpha_s^2 \left[ \ldots + \ldots + \ldots + \ldots \right]$$

$$\frac{1}{\sigma_B} \frac{d\sigma}{d\tau} = \frac{\alpha_s}{\tau} \left[ c_{11} L + c_{10} \right] + \tau f_1(\tau)$$

$$+ \alpha_s^2 \left[ c_{23} L^3 + c_{22} L^2 + c_{21} L + c_{20} + \tau f_2(\tau) \right]$$

$$+ \alpha_s^3 \left[ \ldots + \ldots + \ldots + \ldots \right]$$

Relative $O(\alpha_s)$ accuracy at all $\mathcal{T}$ requires $\text{NNLL}_\sigma + \text{NLO}_{N+1}$

$\rightarrow \text{NLO}_N$ is now naturally part of $\text{NLL'}_\sigma$ and automatically included

$\rightarrow$ similarly $\text{NNLO}_N$ is naturally part of $\text{NNLL'}_\sigma$
Resummation Order Counting.

Resummation is really performed in the exponent of the cross section with counting $\alpha_s L \sim 1$

$$\sigma \sim [1 + \alpha_s + \alpha_s^2 + \cdots] \exp \left[ \sum_n \alpha_s^n L^{n+1} (1 + \alpha_s + \alpha_s^2 + \cdots) \right]$$

$$\sim \text{LL + NLL + NNLL + \cdots}$$

Default conventions:

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<tr>
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<th>Fixed-order corrections</th>
<th>Resummation input</th>
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