Soft Gluon Resummation for associated $t\bar{t}H$ Production at the LHC

Vincent Theeuwes

University at Buffalo
The State University of New York

In Collaboration with: Anna Kulesza, Leszek Motyka, Tomasz Stebel

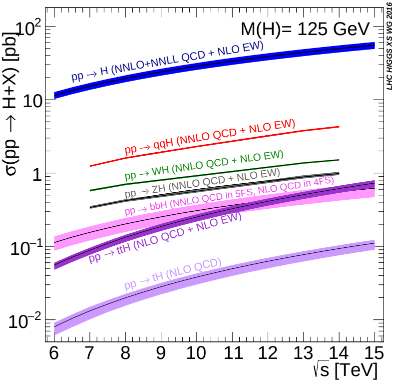
arXiv:1509.02780

LHCP, 14-06-2016

University at Buffalo
The State University of New York

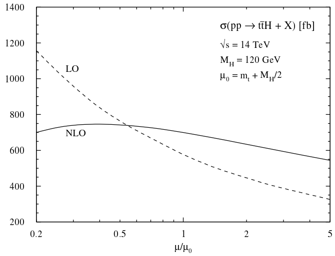
Importance of $pp \to t\bar{t}H$

- A Higgs boson found with a mass of $125\ \mathrm{GeV}$
- Precision study needed to determine if it is SM Higgs
- Direct way to access Yukawa coupling



Current status of $pp \to t\bar{t}H$

- QCD Corrections up to NLO *[Beenakker et al. , '02] [Dawson et al. , '02]*
- Matched to parton showers by: aMC@NLO *[Frederix et al. , '11]*, PowHel *[Garzelli et al. , '11]*, Sherpa *[Hoeche et al., '12]*, POWHEG-BOX *[Hartanto et al. , '14]*
- Electroweak correction *[Frixione et al. , '14,'15][Zhang, '14]*
- Including top decays *[Denner, Feger, '15]*
- Absolute threshold at NLL *[Kulesza, Motyka, Stebel, VT, '15]*
- Expansion of NNLL in SCET *[Broggio et al., '15]*



[Beenakker et al. , '02]

Why resummation for $t\bar{t}H$?

Gains

- NNLO corrections out of reach
- Resummation can help reduce scale uncertainty
- Good process to start:
    - Simple color structure
    - Massive particles $\to$ no final state collinear divergences

University at Buffalo
The State University of New York

Why resummation for $t\bar{t}H$?

Gains

- NNLO corrections out of reach
- Resummation can help reduce scale uncertainty
- Good process to start:
  - Simple color structure
  - Massive particles $\to$ no final state collinear divergences

Pitfalls

- $2 \to 3$ phase space supressed near threshold ($\sigma \propto \beta^4$)
- Small corrections from near absolute threshold

University at Buffalo
The State University of New York

Definition of Threshold

Q-approach

Threshold variable $\hat{\tau}_Q = \frac{Q^2}{\hat{s}}$

Q^2: the invariant mass final state particles

$$1 - \hat{\tau}_Q = 1 - \frac{Q^2}{\hat{s}}$$

$$\sim \frac{\text{energy of the emitted gluons}}{\text{total available energy}}$$

M-approach (absolute threshold)

Threshold variable $\hat{\tau}_M = \frac{M^2}{\hat{s}}$

M: the sum of final state masses

$$1 - \hat{\tau}_M = 1 - \frac{M^2}{\hat{s}}$$

$$\sim \frac{\text{maximum energy of the emitted gluons}}{\text{total available energy}}$$

$\sqrt{\hat{s}}$: the partonic center of mass energy

University at Buffalo
The State University of New York

Logarithms

Q-approach

The IR divergences lead to logarithms:

$$(1-\hat{\tau}_Q)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\delta\left(1-\hat{\tau}_Q\right) + \left(\frac{1}{1-\hat{\tau}_Q}\right)_+ - 2\epsilon\left(\frac{\log(1-\hat{\tau}_Q)}{1-\hat{\tau}_Q}\right)_+$$

$$\boxed{\alpha_s^n\left(\frac{\log^m(1-\hat{\tau}_Q)}{1-\hat{\tau}_Q}\right)_+}$$

In general logarithms of $1-\hat{\tau}_Q$

M-approach

After integration over $\hat{\tau}_Q$: logarithms of $1-\hat{\tau}_M$:

$$\boxed{\alpha_s^n \log^m(1-\hat{\tau}_M)}$$

Logarithms become large in threshold limit: $\hat{\tau} \to 1$

University at Buffalo
The State University of New York

Mellin Transform

Mellin transform is used with respect to τ (needed for factorization of phase space):

$$\begin{aligned} \tilde{\sigma}_{pp \to t\bar{t}H}(N) &\equiv \int_0^1 d\tau\, \tau^{N-1} \sigma_{pp \to t\bar{t}H}(\tau, \mu_R, \mu_F) \\ &= \sum_{i,j} \tilde{f}_{i/p}(N+1, \mu_F) \tilde{f}_{j/p}(N+1, \mu_F) \tilde{\hat{\sigma}}_{ij \to t\bar{t}H}(N, \mu_R, \mu_F) \end{aligned}$$

- $\tilde{f}_{i/p}(N+1, \mu_F)$: Mellin transform with respect to x
- $\tilde{\hat{\sigma}}_{ij \to t\bar{t}H}(N, \mu_R, \mu_F)$: Mellin transform with respect to $\hat{\tau}$

$\log^n(1-\hat{\tau}) \Rightarrow \log^n N$ and threshold $\hat{\tau} \to 1 \sim N \to \infty$

First application for $2 \to 3$ in Mellin space

University at Buffalo
The State University of New York

Orders of Resummation

Large logarithms $\log N \equiv L$ for $N \to \infty$

Perturbation needs to be reordered in α_s and L:

$$\boxed{\tilde{\sigma} \sim \tilde{\sigma}_{LO} \times \mathcal{C}(\alpha_s) \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots\right]}$$

With orders of precision:

	LL	NLL	NNLL
	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$\alpha_s^n \log^{n+1}(N)$	$\alpha_s^n \log^n(N)$	$\alpha_s^{n+1} \log^n(N)$

Exponential functions are universal for initial state emission

[Kodaira, Trentadue, '82][Sterman, '87][Catani, d'Emilio, Trentadue, '88][Catani, Trentadue, '89]

University at Buffalo
The State University of New York

Color space

Need to project the matrix element onto a color basis.
Use the s-channel color basis:

$q\bar{q}$

$$\mathbf{1}: \quad \delta_{a_2 a_1}\delta_{a_3 a_4}$$

$$\mathbf{8}: \quad T^D_{a_2 a_1} T^D_{a_3 a_4}$$

gg

$$\mathbf{1}: \quad \delta^{A_1 A_2}\delta_{a_3 a_4}$$

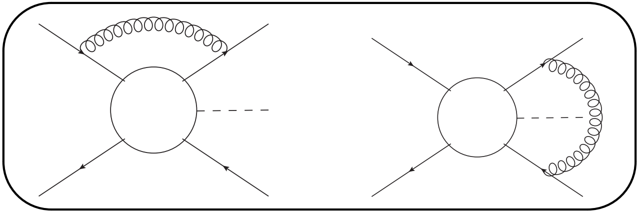
$$\mathbf{8_S}: \quad T^D_{a_3 a_4} d^{D A_1 A_2}$$

$$\mathbf{8_A}: \quad i T^D_{a_3 a_4} f^{D A_1 A_2}$$

Basis for diagonalization of soft anomalous dimension in absolute threshold limit.
Same as for $t\bar{t}$ production

University at Buffalo
The State University of New York

Soft wide-angle



[Kidonakis et al., '97-'01]

$$\tilde{S}_{ij\to kl}\left(\frac{Q}{\mu N}\right) = \bar{\mathrm{P}}\exp\left[\int_{\mu}^{Q/N}\frac{dq}{q}\Gamma^{\dagger}_{ij\to kl}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]\tilde{S}_{ij\to kl}$$

$$\times \mathrm{P}\exp\left[\int_{\mu}^{Q/N}\frac{dq}{q}\Gamma_{ij\to kl}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]$$

University at Buffalo
The State University of New York

Soft anomalous dimension matrix

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,11} = -\frac{\alpha_s}{2\pi} 2C_F \left(L_{\beta_{34}} + 1\right)$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,12} = \frac{\alpha_s}{2\pi} \frac{2C_F}{N_C} \Omega_3 = \frac{C_F}{2N_C} \Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,21}$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,22} = \frac{\alpha_s}{2\pi} \left[(N_C - 2C_F)\left(L_{\beta_{34}} + 1\right) + N_C \Lambda_3 + (8C_F - 3N_C)\,\Omega_3\right]$$

$$\Omega_3 = (T_{13} + T_{24} - T_{14} - T_{23})/2$$

$$\Lambda_3 = (T_{13} + T_{24} + T_{14} + T_{23})/2$$

$$T_{ij} = \log\left(\frac{m_j^2 - t_{ij}}{m_j \sqrt{s}}\right) + \frac{i\pi - 1}{2}$$

with $t_{ij} = (p_i - p_j)^2$ and $\beta_{34}^2 = 1 - (m_3 + m_4)^2/s_{34}$

University at Buffalo
The State University of New York

Soft anomalous dimension matrix

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,11} = -\frac{\alpha_s}{2\pi} 2C_F \left(L_{\beta_{34}} + 1\right)$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,12} = \frac{\alpha_s}{2\pi} \frac{2C_F}{N_C} \Omega_3 = \frac{C_F}{2N_C} \Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,21}$$

$$\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}X,22} = \frac{\alpha_s}{2\pi} \left[(N_C - 2C_F)\left(L_{\beta_{34}} + 1\right) + N_C \Lambda_3 + (8C_F - 3N_C)\,\Omega_3\right]$$

Agrees with $q\bar{q} \to Q\bar{Q}$

for $p_5 \to 0$ and $m_5 \to 0$

$$\Omega_3 = \left(T_{13} + T_{24} - T_{14} - T_{23}\right)/2$$

$$\Lambda_3 = \left(T_{13} + T_{24} + T_{14} + T_{23}\right)/2$$

$$T_{ij} = \log\left(\frac{m_j^2 - t_{ij}}{m_j \sqrt{s}}\right) + \frac{i\pi - 1}{2}$$

with $t_{ij} = (p_i - p_j)^2$ and $\beta_{34}^2 = 1 - (m_3 + m_4)^2/s_{34}$

$\Downarrow$ $\Downarrow$

For $2 \to 2$: $t_{13} = t_{24}$, $t_{23} = t_{14}$ $\quad$ $s_{34} = \hat{s}$, $\beta_{34} = \beta$

University at Buffalo
The State University of New York

Soft wide-angle emission absolute threshold

Absolute threshold limit for soft anomalous dimension:

$$2\mathcal{R}e\Gamma^{(1)}_{q\bar{q}\to Q\bar{Q}B,IJ,\text{thr.}} = \frac{\alpha_\text{s}}{\pi}\text{diag}\left(0, -N_C\right)$$

$$2\mathcal{R}e\Gamma^{(1)}_{gg\to Q\bar{Q}B,IJ,\text{thr.}} = \frac{\alpha_\text{s}}{\pi}\text{diag}\left(0, -N_C, -N_C\right)$$

results in soft wide-angle contribution:

$$\Delta_I^{\text{NLL}} = \exp\left[h_2(\alpha_\text{s}L, -C_I)\right]$$

With C_I the quadratic Casimir invariant

University at Buffalo
The State University of New York

Hard Matching Coefficient (Schematically)

$$\mathcal{C}(\alpha_{\rm s}) = 1 + \frac{\alpha_{\rm s}}{\pi}\mathcal{C}^{(1)} + \dots$$

- Massive final state dipoles *[Catani,Dittmaier,Seymour,Trócsányi,'02]*
- Virtual contribution from PowHeg-Box *[Hartanto,Jäger,Reina,Wackeroth,'15]* confirmed by aMC@NLO *[Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau,'11]*
- Include Coulomb correction $\frac{1}{\beta_{34}}$

University at Buffalo
The State University of New York

Matching to Fixed Order

Resummed Cross Section

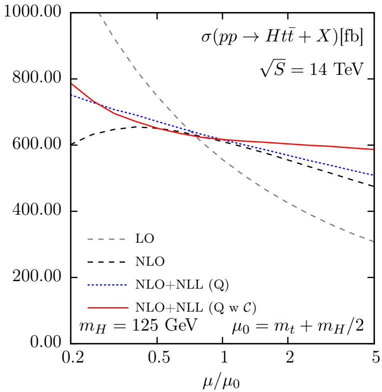
$$\sigma^{(\mathrm{NLO+NLL})}(\tau) = \sigma^{(\mathrm{NLO})}(\tau) + \int_{\mathrm{CT}} \frac{dN}{2\pi i} \tau^{-N} \tilde{f}_{g/p}(N+1) \tilde{f}_{g/p}(N+1) \times \left[\tilde{\hat{\sigma}}^{(\mathrm{NLL})}(N) - \tilde{\hat{\sigma}}^{(\mathrm{NLL})}(N)\big|_{(\mathrm{NLO})} \right]$$

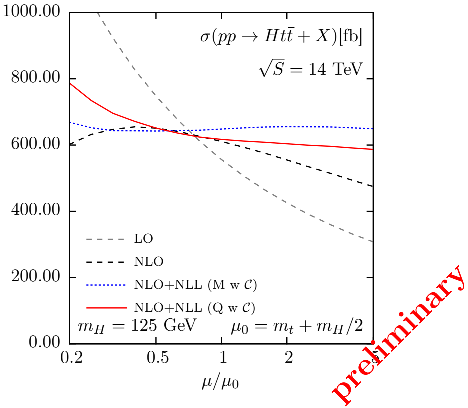
Matching to fixed order required to avoid double counting.

Results

[Kulesza, Motyka, Stebel, VT, '15 and in preperation]

PDFs used: MMHT2014NLO



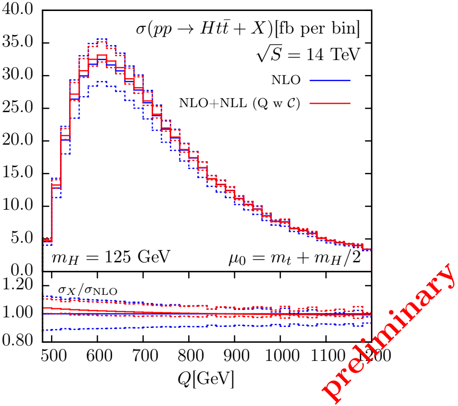


University at Buffalo
The State University of New York

Results

[Kulesza, Motyka, Stebel, VT, in preperation]

PDFs used: MMHT2014NLO



Results

[Kulesza, Motyka, Stebel, VT, '15 and in preperation]

PDFs used: MMHT2014NLO

$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL M with $\mathcal{C}$		NLO+NLL Q with $\mathcal{C}$	
		Value [fb]	K-factor	Value [fb]	K-factor
13	$506^{+5.9\%}_{-9.4\%}$	$537^{+8.2\%}_{-5.5\%}$	1.06	$512^{+5.1\%}_{-6.2\%}$	1.01
14	$613^{+6.2\%}_{-9.4\%}$	$650^{+7.9\%}_{-5.7\%}$	1.06	$619^{+5.2\%}_{-6.4\%}$	1.01

preliminary

Using 7-point method:
$(\mu_F/\mu_0, \mu_R/\mu_0) = \{(0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2)\}$

University at Buffalo
The State University of New York

Results

[Kulesza, Motyka, Stebel, VT, '15 and in preperation]

PDFs used: MMHT2014NLO

$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL M with $\mathcal{C}$		NLO+NLL Q with $\mathcal{C}$	
		Value [fb]	K-factor	Value [fb]	K-factor
13	$506^{+5.9\%}_{-9.4\%}$	$537^{+8.2\%}_{-5.5\%}$	1.06	$512^{+5.1\%}_{-6.2\%}$	1.01
14	$613^{+6.2\%}_{-9.4\%}$	$650^{+7.9\%}_{-5.7\%}$	1.06	$619^{+5.2\%}_{-6.4\%}$	1.01

preliminary

Using 7-point method:

$$(\mu_F/\mu_0, \mu_R/\mu_0) = \{(0.5,0.5),(0.5,1),(1,0.5),(1,1),(1,2),(2,1),(2,2)\}$$

Thank you for your attention

University at Buffalo
The State University of New York