Soft Gluon Resummation for associated $ttH$ Production at the LHC

Vincent Theeuwes

University at Buffalo
The State University of New York

In Collaboration with: Anna Kulesza, Leszek Motyka, Tomasz Stebel

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Importance of $pp \to t\bar{t}H$

- A Higgs boson found with a mass of 125 GeV
- Precision study needed to determine if it is SM Higgs
- Direct way to access Yukawa coupling

![Graph showing the cross-section of Higgs production in various processes as a function of the center-of-mass energy.]
Current status of $pp \to t\bar{t}H$

- **QCD Corrections up to NLO** [Beenakker et al., '02] [Dawson et al., '02]
- Matched to parton showers by: aMC@NLO [Frederix et al., '11],
  PowHel [Garzelli et al., '11], Sherpa [Hoeche et al., '12], POWHEG-BOX [Hartanto et al., '14]
- **Electroweak correction** [Frixione et al., '14,'15][Zhang, '14]
- **Including top decays** [Denner, Feger, '15]
- **Absolute threshold at NLL** [Kulesza, Motyka, Stebel, VT, '15]
- **Expansion of NNLL in SCET** [Broggio et al., '15]

![Graph showing threshold resummation for $t\bar{t}H$]
Why resummation for $t\bar{t}H$?

## Gains

- NNLO corrections out of reach
- Resummation can help reduce scale uncertainty
- Good process to start:
  - Simple color structure
  - Massive particles → no final state collinear divergences
Why resummation for $t\bar{t}H$?

Gains

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  - Massive particles $\rightarrow$ no final state collinear divergences

Pitfalls

- $2 \rightarrow 3$ phase space suppressed near threshold ($\sigma \propto \beta^4$)
- Small corrections from near absolute threshold
Definition of Threshold

Q-approach

Threshold variable \( \hat{\tau}_Q = \frac{Q^2}{\hat{s}} \)

\( Q^2 \): the invariant mass final state particles

\[ 1 - \hat{\tau}_Q = 1 - \frac{Q^2}{\hat{s}} \]

\( \sim \) energy of the emitted gluons
\( \sim \) total available energy

\( \sqrt{\hat{s}} \): the partonic center of mass energy

M-approach (absolute threshold)

Threshold variable \( \hat{\tau}_M = \frac{M^2}{\hat{s}} \)

\( M \): the sum of final state masses

\[ 1 - \hat{\tau}_M = 1 - \frac{M^2}{\hat{s}} \]

\( \sim \) maximum energy of the emitted gluons
\( \sim \) total available energy

Threshold resummation for \( t\bar{t}H \)

V. Theeuwes
Logarithms

Q-approach

The IR divergences lead to logarithms:

\[
(1 - \hat{\tau}_Q)^{-1 - 2\epsilon} = -\frac{1}{2\epsilon} \delta (1 - \hat{\tau}_Q) + \left( \frac{1}{1 - \hat{\tau}_Q} \right) + 2\epsilon \left( \frac{\log(1 - \hat{\tau}_Q)}{1 - \hat{\tau}_Q} \right) + \alpha_s^n \left( \frac{\log^m(1 - \hat{\tau}_Q)}{1 - \hat{\tau}_Q} \right)
\]

In general logarithms of \(1 - \hat{\tau}_Q\)

M-approach

After integration over \(\hat{\tau}_Q\): logarithms of \(1 - \hat{\tau}_M\):

\[
\alpha_s^n \log^m(1 - \hat{\tau}_M)
\]

Logarithms become large in threshold limit: \(\hat{\tau} \to 1\)
Mellin Transform

Mellin transform is used with respect to $\tau$ (needed for factorization of phase space):

$$\tilde{\sigma}_{pp\to t\bar{t}H}(N) \equiv \int_0^1 d\tau \, \tau^{N-1} \sigma_{pp\to t\bar{t}H}(\tau, \mu_R, \mu_F)$$

$$= \sum_{i,j} \tilde{f}_{i/p}(N+1, \mu_F) \tilde{f}_{j/p}(N+1, \mu_F) \tilde{\sigma}_{ij\to t\bar{t}H}(N, \mu_R, \mu_F)$$

- $\tilde{f}_{i/p}(N+1, \mu_F)$: Mellin transform with respect to $x$
- $\tilde{\sigma}_{ij\to t\bar{t}H}(N, \mu_R, \mu_F)$: Mellin transform with respect to $\hat{\tau}$

$\log^n(1 - \hat{\tau}) \Rightarrow \log^n N$ and threshold $\hat{\tau} \rightarrow 1 \sim N \rightarrow \infty$

First application for $2 \rightarrow 3$ in Mellin space
Orders of Resummation

Large logarithms $\log N \equiv L$ for $N \to \infty$

Perturbation needs to be reordered in $\alpha_s$ and $L$:

$$\tilde{\sigma} \sim \tilde{\sigma}_{LO} \times C(\alpha_s) \exp \left[ L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots \right]$$

With orders of precision: \[ \begin{array}{ccc}
\updownarrow & \updownarrow & \updownarrow \\
LL & NLL & NNLL \\
\downarrow & \downarrow & \downarrow \\
\alpha_s^n \log^{n+1}(N) & \alpha_s^n \log^n(N) & \alpha_s^{n+1} \log^n(N)
\end{array} \]

Exponential functions are universal for initial state emission

[Kodaira, Trentadue, '82][Sterman, '87][Catani, d'Emilio, Trentadue, '88][Catani, Trentadue, '89]
Color space

Need to project the matrix element onto a color basis.
Use the s-channel color basis:

\[ q\bar{q} \]
\[ 1 : \delta a_2 a_1 \delta a_3 a_4 \]
\[ 8 : T^D_{a_2 a_1} T^D_{a_3 a_4} \]

\[ gg \]
\[ 1 : \delta^A_1 A_2 \delta a_3 a_4 \]
\[ 8_S : T^D_{a_3 a_4} d^{DA} A_1 A_2 \]
\[ 8_A : iT^D_{a_3 a_4} f^{DA} A_1 A_2 \]

Basis for diagonalization of soft anomalous dimension in absolute threshold limit.
Same as for \( t\bar{t} \) production
Soft wide-angle

\[ \tilde{S}_{ij \rightarrow kl} \left( \frac{Q}{\mu N} \right) = \bar{P} \exp \left[ \int_{\mu}^{Q/N} \frac{dq}{q} \Gamma_{ij \rightarrow kl}^{+} (\alpha_s (q^2)) \right] \tilde{S}_{ij \rightarrow kl} \times P \exp \left[ \int_{\mu}^{Q/N} \frac{dq}{q} \Gamma_{ij \rightarrow kl} (\alpha_s (q^2)) \right] \]
Soft anomalous dimension matrix

\[ \Gamma^{(1)}_{q\bar{q} \rightarrow Q\bar{Q}X, 11} = -\frac{\alpha_s}{2\pi} 2C_F \left( L \beta_{34} + 1 \right) \]

\[ \Gamma^{(1)}_{q\bar{q} \rightarrow Q\bar{Q}X, 12} = \frac{\alpha_s}{2\pi} \frac{2C_F}{N_C} \Omega_3 = \frac{C_F}{2N_C} \Gamma^{(1)}_{q\bar{q} \rightarrow Q\bar{Q}X, 21} \]

\[ \Gamma^{(1)}_{q\bar{q} \rightarrow Q\bar{Q}X, 22} = \frac{\alpha_s}{2\pi} \left[ (N_C - 2C_F) \left( L \beta_{34} + 1 \right) + N_C \Lambda_3 + (8C_F - 3N_C) \Omega_3 \right] \]

\[ \Omega_3 = \frac{(T_{13} + T_{24} - T_{14} - T_{23})}{2} \]

\[ \Lambda_3 = \frac{(T_{13} + T_{24} + T_{14} + T_{23})}{2} \]

\[ T_{ij} = \log \left( \frac{m_j^2 - t_{ij}}{m_j \sqrt{s}} \right) + i\pi - \frac{1}{2} \]

with \( t_{ij} = (p_i - p_j)^2 \) and \( \beta_{34}^2 = 1 - (m_3 + m_4)^2 / s_{34} \)
Soft anomalous dimension matrix

\[\Gamma^{(1)}_{q\bar{q}\rightarrow Q\bar{Q}X,11} = -\frac{\alpha_s}{2\pi} 2C_F \left( L\beta_{34} + 1 \right)\]

Agrees with \(q\bar{q}\rightarrow Q\bar{Q}\)

\[\Gamma^{(1)}_{q\bar{q}\rightarrow Q\bar{Q}X,12} = \frac{\alpha_s}{2\pi} \frac{2C_F}{N_C} \Omega_3 = \frac{C_F}{2N_C} \Gamma^{(1)}_{q\bar{q}\rightarrow Q\bar{Q}X,21}\]

for \(p_5 \rightarrow 0\) and \(m_5 \rightarrow 0\)

\[\Gamma^{(1)}_{q\bar{q}\rightarrow Q\bar{Q}X,22} = \frac{\alpha_s}{2\pi} \left[ (N_C - 2C_F) \left( L\beta_{34} + 1 \right) + N_C \Lambda_3 + (8C_F - 3N_C) \Omega_3 \right]\]

\[\Omega_3 = (T_{13} + T_{24} - T_{14} - T_{23}) / 2\]

\[\Lambda_3 = (T_{13} + T_{24} + T_{14} + T_{23}) / 2\]

\[T_{ij} = \log \left( \frac{m_j^2 - t_{ij}}{m_j \sqrt{s}} \right) + \frac{i\pi - 1}{2}\]

with \(t_{ij} = (p_i - p_j)^2\) and \(\beta_{34}^2 = 1 - (m_3 + m_4)^2 / s_{34}\)

For \(2 \rightarrow 2:\)

\[t_{13} = t_{24}\]

\[t_{23} = t_{14}\]

\[\beta_{34} = \beta\]
Soft wide-angle emission absolute threshold

Absolute threshold limit for soft anomalous dimension:

\[ 2 \text{Re} \Gamma_{q\bar{q}\to Q\bar{Q}B,IJ,\text{thr.}}^{(1)} = \frac{\alpha_s}{\pi} \text{diag} (0, -N_C) \]

\[ 2 \text{Re} \Gamma_{g\bar{g}\to Q\bar{Q}B,IJ,\text{thr.}}^{(1)} = \frac{\alpha_s}{\pi} \text{diag} (0, -N_C, -N_C) \]

results in soft wide-angle contribution:

\[ \Delta_{I}^{\text{NLL}} = \exp [h_2(\alpha_s L, -C_I)] \]

With \( C_I \) the quadratic Casimir invariant
\[ C(\alpha_s) = 1 + \frac{\alpha_s}{\pi} C^{(1)} + \ldots \]

- Massive final state dipoles \cite{CataniDittmaierSeymourTrocsanyi02}

- Virtual contribution from PowHeg-Box \cite{HartantoJagerReinaWackeroth15} confirmed by aMC@NLO \cite{HirschiFrederixFrixioneGarzelliMaltoniPittau11}

- Include Coulomb correction \( \frac{1}{\beta_{34}} \)
Matching to Fixed Order

Resummed Cross Section

\[ \sigma^{(NLO+NLL)}(\tau) = \sigma^{(NLO)}(\tau) \]

\[ + \int_{CT} \frac{dN}{2\pi i} \tau^{-N} \tilde{T}_{g/p}(N+1) \tilde{T}_{g/p}(N+1) \]

\[ \times \left[ \tilde{\sigma}^{(NLL)}(N) - \tilde{\sigma}^{(NLL)}(N)\big|_{(NLO)} \right] \]

Matching to fixed order required to avoid double counting.
Results

[Kulesza, Motyka, Stebel, VT, '15 and in preparation]

PDFs used: MMHT2014NLO

\[ \sigma(pp \to Ht\bar{t} + X)[fb] \]
\[ \sqrt{S} = 14 \text{ TeV} \]

- \( \mu_0 = m_t + m_H/2 \)
- \( m_H = 125 \text{ GeV} \)

LO
NLO
NLO+NLL (Q)
NLO+NLL (Q w\ C)
NLO+NLL (M w\ C)

preliminary
Results

PDFs used: MMHT2014NLO

$\sigma(pp \rightarrow Ht\bar{t} + X)[\text{fb per bin}]$

$\sqrt{S} = 14 \text{ TeV}$

NLO

NLO+NLL (Q w $C$)

$\mu_0 = m_t + m_H/2$

$Q[\text{GeV}]$

$\sigma_X/\sigma_{NLO}$

preliminary
PDFs used: MMHT2014NLO

<table>
<thead>
<tr>
<th>$\sqrt{S}$ [TeV]</th>
<th>NLO [fb]</th>
<th>NLO+NLL M with $C$</th>
<th>NLO+NLL Q with $C$</th>
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<tr>
<td>13</td>
<td>$506^{+5.9%}_{-9.4%}$</td>
<td>$537^{+8.2%}_{-5.5%}$</td>
<td>1.06</td>
</tr>
<tr>
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<td>$650^{+7.9%}_{-5.7%}$</td>
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Using 7-point method:
$(\mu_F/\mu_0, \mu_R/\mu_0) = \{(0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2)\}$
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Thank you for your attention