Theory of Hard Probes in PbPb Collisions
Jet Substructures and Cross Sections using Soft-Collinear Effective Theory

Yang-Ting Chien

Los Alamos National Laboratory, Theoretical Division, T-2
LHC Theory Initiative Fellow, MIT Center for Theoretical Physics

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Outline

- Hard Probes with jets
  - Precision jet substructure calculations
  - The need of resummation
- Soft-Collinear Effective Theory (SCET)
  - Factorization theorem
  - Renormalization group evolution
  - Medium modification by Glauber interactions
- Results and conclusions
Resolving jets and the QGP with jet substructure

**Jet quenching is a multi-scale problem**

- The strong suppression of hadron and jet cross sections has been observed more than a decade ago.
- Many models exploit the idea of parton energy loss and can explain the data.
- However, it has been clear that cross sections are not sufficient to distinguish various jet formation mechanisms.
- Jet substructure can resolve jets at different energy scales.
- It can also separate final-state, jet-medium interactions from initial state effects.
- The interference between jets and the medium is an even more complicated multi-scale problem.

**Effective field theory techniques are extremely useful**
Jet shape (Ellis, Kunszt, Soper)

Jet shapes probe the averaged energy distribution inside a jet

- The infrared structure of QCD induces Sudakov logarithms
- Fixed order calculation breaks down at small $r$
- Large logarithms of the form $\alpha_s^n \log^m r/R$ ($m \leq 2n$) need to be resummed

The necessity of resummation for jet substructure calculations

\[ \Psi_J(r, R) = \frac{\sum_{r_i < r} E_{T_i}}{\sum_{r_i < R} E_{T_i}} \]

\[ \langle \Psi \rangle = \frac{1}{N_J} \sum_J \Psi_J(r, R) \]

\[ \psi(r, R) = \frac{d\langle \Psi \rangle}{dr} \]
Soft-Collinear Effective Theory (SCET)

- Effective field theory techniques are useful whenever there is clear scale separation
- SCET separates physical degrees of freedom in QCD by a systematic expansion in power counting
  - Match SCET with QCD at the hard scale by integrating out the hard modes
  - Integrating out the off-shell modes gives collinear Wilson lines which describe the collinear radiation
  - The soft sector is described by soft Wilson lines along the jet directions
- Soft-collinear decoupling holds at leading power in the Lagrangian, which makes the factorization theorems of cross sections manifest

SCET factorizes a complicated, multi-scale problem into multiple simpler, single-scale problems
Jet shape factorization theorem (Chien et al)

- The factorization theorem for the differential cross section of the production of $N$ jets with $p_{Ti}, y_i$, the energy $E_r$ inside the cone of size $r$ in one jet, and an energy cutoff $\Lambda$ outside all the jets is the following,

$$\frac{d\sigma}{dp_{Ti}dy_i dE_r} = H(p_{Ti}, y_i, \mu)J_1^\omega(E_r, \mu)J_2^\omega(\mu) \cdots S_{1,2,\ldots}(\Lambda, \mu)$$

- For the differential jet rate

$$\frac{d\sigma}{dp_{Ti}dy_i} = H(p_{Ti}, y_i, \mu)J_1^\omega(\mu)J_2^\omega(\mu) \cdots S_{1,2,\ldots}(\Lambda, \mu)$$

- $H(p_{Ti}, y_i, \mu)$ describes the hard scattering process at high energy

- $J_1^\omega(E_r, \mu)$ describes the probability of having the amount of energy $E_r$ inside the cone of size $r$
  - $X_c$ is constrained within jets by the corresponding jet algorithm
  - $S_{1,2,\ldots}(\Lambda, \mu)$ describes how soft radiation is constrained in measurements

**Factorization theorem simplifies dramatically and has a product form**
The averaged energy inside the cone of size $r$ in jet 1 is the following,

$$\langle E_r \rangle_\omega = \frac{1}{d\sigma} \int dE_r E_r \frac{d\sigma}{dp_T d\gamma_i dE_r} = \frac{H(p_T, y, \mu) J_{E, r_1}^{\omega_1}(\mu) J_{2}^{\omega_2}(\mu) \cdots S_{1,2, \ldots}(\Lambda, \mu)}{H(p_T, y, \mu) J_{1}^{\omega_1}(\mu) J_{2}^{\omega_2}(\mu) \cdots S_{1,2, \ldots}(\Lambda, \mu)} = \frac{J_{E, r_1}^{\omega_1}(\mu)}{J_{1}^{\omega_1}(\mu)}$$

- $J_{E, r}^{\omega}(\mu) = \int dE_r E_r J^{\omega}(E_r, \mu)$ is referred to as the jet energy function
- Huge cancelation between the hard, unmeasured jet and soft functions
  - The jet shape is insensitive to the details of the underlying hard scattering process as well as the other part of the event
- The integral jet shape, averaged over all jets, is the following

$$\langle \Psi \rangle = \frac{1}{\sigma_{\text{total}}} \sum_{i=q,g} \int_{PS} d\beta d\gamma \frac{d\sigma}{dp_T d\gamma} \Psi_i^\omega, \text{ where } \Psi_\omega = \frac{J_{E, r}(\mu)/J(\mu)}{J_{E,R}(\mu)/J(\mu)} = \frac{J_{E, r}(\mu)}{J_{E,R}(\mu)}$$

The jet shape is within the class of collinear observables and is relatively insensitive to the soft radiation
Scale hierarchy and renormalization group evolution

\[
\frac{dJ^q_{E,r}(r, R, \mu)}{d \ln \mu} = \left[ -C_F \Gamma_{\text{cusp}} \ln \frac{\omega^2 \tan^2 \frac{R}{2}}{\mu^2} - 2\gamma_q \right] J^q_{E,r}(r, R, \mu)
\]

\[
\frac{dJ^g_{E,r}(r, R, \mu)}{d \ln \mu} = \left[ -C_A \Gamma_{\text{cusp}} \ln \frac{\omega^2 \tan^2 \frac{R}{2}}{\mu^2} - 2\gamma_g \right] J^g_{E,r}(r, R, \mu)
\]

- $\langle E_r \rangle_\omega$ and $\Psi_\omega$ are renormalization group invariant

\[
\Psi_\omega = \frac{J_{E,r}(\mu)}{J_{E,R}(\mu)} = \frac{J_{E,r}(\mu_{j_r})}{J_{E,R}(\mu_{j_R})} U_J(\mu_{j_r}, \mu_{j_R})
\]

- Identify the natural scale $\mu_{j_r}$ to eliminate large logarithms in $J_{E,r}(\mu_{j_r})$

- The RG evolution kernel $U_J(\mu_{j_r}, \mu_{j_R})$ resums the large logarithms

\[\text{RG evolution between } \mu_{j_r} \text{ and } \mu_{j_R} \text{ resums } \log \frac{\mu_{j_r}}{\mu_{j_R}} = \log \frac{r}{R}\]
Baseline jet shape calculations

- Compare CMS $pp$ data at 2.76 and 7 TeV
- Bands are theory uncertainties estimated by varying $\mu_{j_r}$ and $\mu_{j_R}$
- The shape difference for jets reconstructed using different algorithms is significant
- In the region $r \approx R$, higher fixed order calculations and power corrections are more prominent

- For low $p_T$ jets, power corrections have significant contributions
Multiple scattering in a medium

- Coherent multiple scattering and induced bremsstrahlung are the qualitatively new ingredients in the medium parton shower

- Interplay between several characteristic scales:
  - Debye screening scale $\mu$
  - Parton mean free path $\lambda$
  - Radiation formation time $\tau$

- From thermal field theory and lattice QCD calculations, an ensemble of quasi particles with debye screened potential and thermal masses is a reasonable parameterization of the medium properties

\[
\frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2q_\perp} = \frac{\mu^2}{\pi(q^2_\perp + \mu^2)^2}
\]

Parton splitting and induced bremsstrahlung interfere in the jet formation
SCET with Glauber gluons (SCET\(_G\))

- Glauber gluon is the relevant mode for medium interactions
- SCET\(_G\) was extended from SCET (Idilbi et al, Vitev et al)
- Glauber gluons are generated from the colored charges in the QGP providing transverse momentum transfer
- Given a medium model, we can use SCET\(_G\) to consistently couple the medium to jets

- Because of the collinear nature, the jet shape can be calculated using only the splitting functions

\[
J_{E,r}^i(\mu) = \sum_{j,k} \int_{PS} dx dk_\perp \left[ \frac{dN_{i\rightarrow jk}^{vac}}{dx d^2k_\perp} + \frac{dN_{i\rightarrow jk}^{med}}{dx d^2k_\perp} \right] E_r(x, k_\perp)
\]

The medium induced splitting functions are calculated numerically using SCET\(_G\) with the Bjorken-expanded hydrodynamic QGP model
Landau-Pomeranchuk-Migdal effect

- The hierarchy between $\tau$ and $\lambda$ determines the degree of coherence between multiple scatterings

\[ \tau = \frac{x \omega}{(q_\perp - k_\perp)^2} \quad \text{v.s.} \quad \lambda \]

- $\tau \gg \lambda$: destructive interference
- $\tau \ll \lambda$: Bethe-Heitler incoherence limit

- Medium induced splitting functions in SCET$_G$ (Ovanesyan et al)

\[
\frac{dN_{q\rightarrow qg}^{\text{med}}}{dxd^2k_\perp} = \frac{C_F \alpha_s}{\pi^2} \frac{1}{x} \int_0^L d\Delta z \frac{1}{\lambda} \int d^2q_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}}{d^2q_\perp} \frac{2k_\perp \cdot q_\perp}{k_\perp^2 (q_\perp - k_\perp)^2} \left[ 1 - \cos \left( \frac{(q_\perp - k_\perp)^2 \Delta z}{x \omega} \right) \right]
\]

- $\frac{dN^{\text{med}}}{dxd^2k_\perp} \rightarrow \text{finite as } k_\perp \rightarrow 0$: the LPM effect

- $\frac{dN^{\text{vac}}}{dxd^2k_\perp} \rightarrow \frac{1}{k_\perp} \text{ as } k_\perp \rightarrow 0$

Large angle bremsstrahlung takes away energy, resulting in jet energy loss and the modification of jet shapes
Jet substructure Soft Collinear Effective Theory

Medium interactions

Conclusions

Jet shapes in heavy ion collisions

- Jet shapes get modified through the modification of jet energy functions

\[ \Psi(r) = \frac{J_{E,r}^{vac} + J_{E,r}^{med}}{J_{E,R}^{vac} + J_{E,R}^{med}} = \frac{\Psi_{vac}(r) J_{E,r}^{vac} + J_{E,r}^{med}}{J_{E,R}^{vac} + J_{E,R}^{med}} \]

- Large logarithms in \( \Psi_{vac}(r) = J_{E,r}^{vac} / J_{E,R}^{vac} \) have been resummed
- There are no large logarithms in \( J_{E,r}^{med} \) due to the LPM effect
- The RG evolution of medium-modified jet energy functions is unchanged

- However, with the use of small \( R \)'s in heavy ion collisions, there is significant jet energy loss which leads to the suppression of jet production cross sections

- Jet-by-jet shapes are averaged with the jet cross sections

\[
\frac{1}{\left<N_{\text{bin}}\right>} \frac{d\sigma_{\text{CNM}}^k}{d\eta dp_T} = \sum_{ijX} \int dx_1 dx_2 f_i^A(x_1, \mu_{\text{CNM}}) f_j^A(x_2, \mu_{\text{CNM}}) \frac{d\sigma_{ij \rightarrow kX}}{dx_1 dx_2 d\eta dp_T}
\]

\[
\frac{1}{\left<N_{\text{bin}}\right>} \frac{d\sigma_{\text{med}}^i}{d\eta dp_T} \bigg|_{p_T} = \frac{1}{\left<N_{\text{bin}}\right>} \frac{d\sigma_{\text{CNM}}^i}{d\eta dp_T} \bigg|_{p_T} \frac{1}{1 - \epsilon_i}
\]

- Cold nuclear matter effects are characterized by \( \mu_{\text{CNM}} \)
The plots are the ratios between the jet cross sections and differential jet shapes in lead-lead and proton-proton collisions.

Jet shapes are insensitive to cold nuclear matter effects.

Jet shape modifications are due to the following two effects:

- **Gluon jets are more suppressed which increases the quark jet fraction**
- **Jet-by-jet the shape is broadened**
The plots shows the dependence of jet cross section suppressions on centrality, jet rapidity and jet radius.
Results

- Predictions for jet shapes and cross sections at 5 TeV for inclusive and photon-tagged jets
Conclusions

- Jet substructure in proton and heavy ion collisions can be calculated within the same framework
  - Promising agreement with data and phenomenological applications
  - Stay tuned before Hard Probes 2016 for pA and AA jet fragmentation function and jet mass distribution
- Take-home message: the modification of jet substructure is a combination of cross section suppression and jet-by-jet broadening