Matrix Element Morphing
An EFT solution for BSM sample production

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Introduction

- signal strength and kinematics depend on many parameters
- parameters do not factorize trivially into individual observables
- necessary to build a signal model taking all parameters into account simultaneously & modelling all interference effects
- use EFT as an example, but can apply same techniques to any BSM model with many parameters

Introduction

EFT analyses in Run 1

- influences of EFT parameters studied **in isolation**
- cross-section (rate) and kinematics (shape) studied **separately**

Plans for Run 2

- perform combined studies of many (all) EFT parameters
- use constraining power from rate & shape information

The Higgs Characterization Lagrangian

- interactions of scalar particle with vector boson pair

$$
\begin{aligned}
\mathscr{L}_0^V = \Big\{ & c_\alpha \kappa_{\mathrm{SM}} \big[\tfrac{1}{2} g_{HZZ}\, Z_\mu Z^\mu + g_{HWW}\, W_\mu^+ W^{-\mu}\big] \\
& - \frac{1}{4}\big[c_\alpha \kappa_{H\gamma\gamma} g_{H\gamma\gamma}\, A_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{A\gamma\gamma} g_{A\gamma\gamma}\, A_{\mu\nu} \widetilde{A}^{\mu\nu}\big] \\
& - \frac{1}{2}\big[c_\alpha \kappa_{HZ\gamma} g_{HZ\gamma}\, Z_{\mu\nu} A^{\mu\nu} + s_\alpha \kappa_{AZ\gamma} g_{AZ\gamma}\, Z_{\mu\nu} \widetilde{A}^{\mu\nu}\big] \\
& - \frac{1}{4}\big[c_\alpha \kappa_{Hgg} g_{Hgg}\, G_{\mu\nu}^a G^{a,\mu\nu} + s_\alpha \kappa_{Agg} g_{Agg}\, G_{\mu\nu}^a \widetilde{G}^{a,\mu\nu}\big] \\
& - \frac{1}{4}\frac{1}{\Lambda}\big[c_\alpha \kappa_{HZZ}\, Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{AZZ}\, Z_{\mu\nu} \widetilde{Z}^{\mu\nu}\big] \\
& - \frac{1}{2}\frac{1}{\Lambda}\big[c_\alpha \kappa_{HWW}\, W_{\mu\nu}^+ W^{-\mu\nu} + s_\alpha \kappa_{AWW}\, W_{\mu\nu}^+ \widetilde{W}^{-\mu\nu}\big] \\
& - \frac{1}{\Lambda} c_\alpha \big[\kappa_{H\partial\gamma}\, Z_\nu \partial_\mu A^{\mu\nu} + \kappa_{H\partial Z}\, Z_\nu \partial_\mu Z^{\mu\nu} + \big(\kappa_{H\partial W}\, W_\nu^+ \partial_\mu W^{-\mu\nu} + h.c.\big)\big] \Big\} X_0
\end{aligned}
$$

Morphing derivation

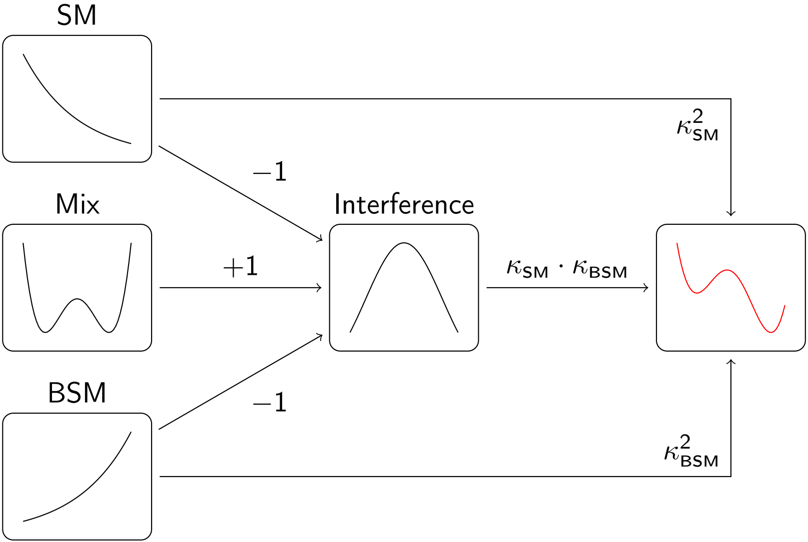
Expand calculation of diff. cross-section in polynomial of EFT op.

$$\mathcal{M}^2_{\mathrm{Mix}} = \kappa^2_{\mathrm{SM}}\mathcal{M}^2_{\mathrm{SM}} + \kappa^2_{\mathrm{BSM}}\mathcal{M}^2_{\mathrm{BSM}} + \kappa_{\mathrm{SM}}\kappa_{\mathrm{BSM}} \cdot 2\Re(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}})$$

- At LO & assuming narrow-width-approximation, one obtains polynomials of 2nd order in production and 2nd order in decay.
- determine coefficients of polynomials from MC samples generated at fixed values of BSM parameters
- use polynomial coefficients to *morph* to any other BSM parameter configuration

$$\begin{aligned}
\mathcal{M}^2_{\mathrm{Mix}(\kappa_{\mathrm{SM}}=\kappa_{\mathrm{BSM}}=c)} &= c^2(\mathcal{M}^2_{\mathrm{SM}} + \mathcal{M}^2_{\mathrm{BSM}} + 2\Re(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}})) \\
2\Re(\mathcal{M}^*_{\mathrm{SM}}\mathcal{M}_{\mathrm{BSM}}) &= \frac{1}{c^2}\mathcal{M}^2_{\mathrm{Mix}(\kappa_{\mathrm{SM}}=\kappa_{\mathrm{BSM}}=c)} - \mathcal{M}^2_{\mathrm{SM}} - \mathcal{M}^2_{\mathrm{BSM}} \\
T_{\mathrm{Mix}} &= \kappa_{\mathrm{SM}}(\kappa_{\mathrm{SM}} - \kappa_{\mathrm{BSM}})T_{\mathrm{SM}} + \kappa_{\mathrm{BSM}}(\kappa_{\mathrm{BSM}} - \kappa_{\mathrm{SM}})T_{\mathrm{BSM}} \\
&\quad + \kappa_{\mathrm{SM}}\kappa_{\mathrm{BSM}}\frac{1}{c^2}T_{\mathrm{Mix}(\kappa_{\mathrm{SM}}=\kappa_{\mathrm{BSM}}=c)}
\end{aligned}$$

Morphing: The Idea



Benefits of Morphing

- computationally fast & convenient tool

Morphing

- only calculates linear sums of coefficients
- all other inputs are pre-computed once

ME Reweighting

For every configuration point

- write events to disk
- rerun analysis
- additional interpolation

- can be applied directly and without change to
  - cross sections
  - distributions (before or after detector simulation)
  - MC events
- exact continuous analytical description of rates and shapes
- even possible to **fit** κs to data & derive limits

Challenges of Morphing

- number of samples grows quickly with EFT parameters
- parameter sets and input samples need to be carefully chosen

$$N = \frac{1}{24} \cdot n_{\text{both}} \cdot (n_{\text{both}}+1) \cdot (n_{\text{both}}+2) \cdot [(n_{\text{both}}+3) + 4 \cdot (n_{\text{prod}} + n_{\text{dec}})] +$$
$$+ \frac{1}{4} \cdot [n_{\text{both}} \cdot (n_{\text{both}}+1) \cdot n_{\text{prod}} \cdot (n_{\text{prod}}+1) + n_{\text{both}} \cdot (n_{\text{both}}+1) \cdot n_{\text{dec}} \cdot (n_{\text{dec}}+1) + n_{\text{dec}} \cdot (n_{\text{dec}}+1) \cdot n_{\text{prod}} \cdot (n_{\text{prod}}+1)] +$$
$$+ \frac{1}{2} \cdot n_{\text{both}} \cdot n_{\text{dec}} \cdot n_{\text{prod}} (n_{\text{both}} + n_{\text{dec}} + n_{\text{prod}} + 3)$$

- only prod./decay, 1 BSM op. ($n_{\text{both}=0}$, $n_{\text{prod(dec)}} = 2$, $n_{\text{dec(prod)}} = 1$): **3**
- BSM VBF ($n_{\text{prod}} = 13$), SM $H \to \mu\mu$ ($n_{\text{dec}} = 1$, $n_{\text{both}} = 0$): **91**
- SM prod. ($n_{\text{prod}} = 1$, $n_{\text{both}} = 0$), BSM $H \to WW \to e\nu\mu\nu$ decay ($n_{\text{dec}} = 5$): **15** samples
- full VBF BSM $H \to ZZ \to 4\ell$ process ($n_{\text{prod}} = 4$, $n_{\text{both}} = 9$, $n_{\text{dec}} = 0$): **1605** samples

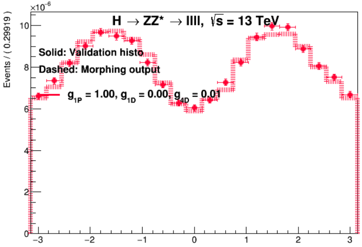
Generality of the Method

- Morphing only requires that any differential cross section can be expressed as polynomial in BSM operators
- **independent of specific generator**
- works on truth and reco-level distributions
- independent of physics process
- works on distributions and cross sections
- applicable beyond EFT

Validations

1. MG5 Truth level at $\sqrt{s} = 13\,\text{TeV}$ for Run 2 (next slides)
2. distributions based on JHU MC samples processed by the full detector simulation at $\sqrt{s} = 8$ TeV (not shown)

Example: Truth Validation using MG5 HC EFT



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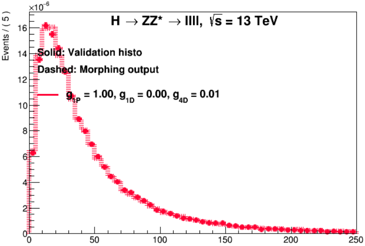
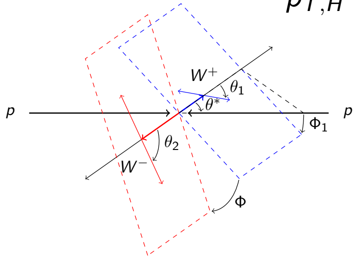


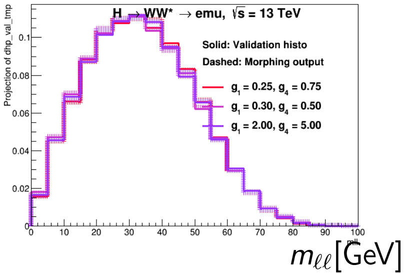
$p_{T,H}$

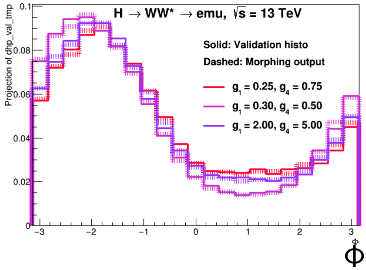
- ggF $H \to ZZ^* \to 4\ell$ @ $\sqrt{s} = 13\text{TeV}$
- Test of morphing:
  Solid: Test
  Dashed: Morphing output

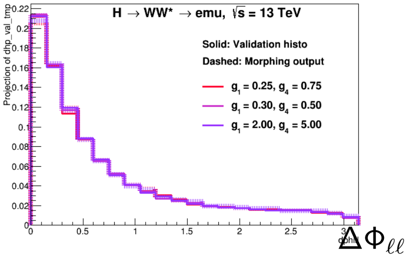
→ Successful morphing



Example: Truth Validation using MG5 HC EFT







- ggF $H \to WW^* \to \ell\nu\ell\nu$ @ $\sqrt{s} = 13$TeV
- Test of morphing:

  Solid: Test

  Dashed: Morphing output

→ Successful morphing

Propagation of Statistical Uncertainties

Obtain a physical quantity T for EFT parameters $K = (\kappa_0, \ldots, \kappa_n)$ from input distributions for parameters K_i

$$T(K; K_i) = \sum_i w(K; K_i) T(K_i)$$

For a given distribution T_i, we have

$$T(K_i) = \epsilon(K_i) \cdot \sigma(K_i) \cdot \mathcal{L}$$

The relative statistical uncertainty Σ on T is

$$\Sigma(K; K_i) = \sqrt{\sum_i w^2(K; K_i) \cdot \epsilon(K_i) \cdot \sigma(K_i) \cdot \mathcal{L}}$$

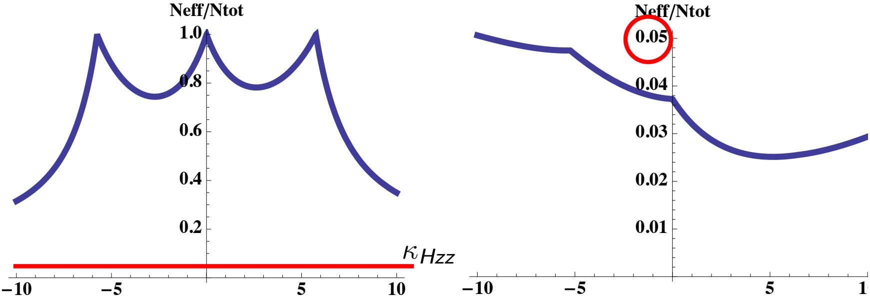
and thus highly dependent on the input samples K_i as well as the target point in EFT space K.

Effective Number of Events

- VBF $H \to \mu\mu$: Morphing with complete κ-set only varying κ_{HZZ} in two different planes of EFT-space

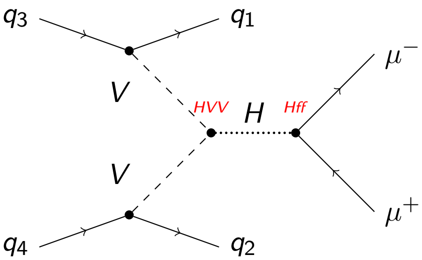
$$N_{\mathrm{tot}} = N_0 \sum_i \sigma_i |w_i| \qquad N_{\mathrm{eff}} = N_0 \sum_i \sigma_i w_i$$

$\kappa_{\mathrm{SM}} = 1,\ \kappa_H = 0,\ \kappa_A = 0$ $\qquad$ $\kappa_{\mathrm{SM}} = 1,\ \kappa_H = 1,\ \kappa_A = 1$



Study of the VBF vertex

- many operators enter VBF
- how many operators can be neglected without loss of generality?
- Minimize number of needed samples

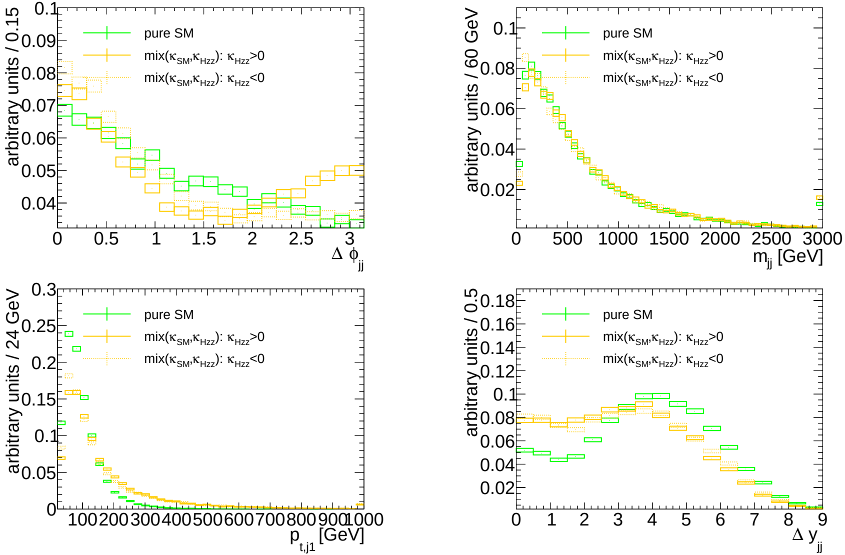


- technical choice of $H \to \mu\mu$ decay: no crossover between production and decay
- full set of 13 VBF prod. op. leads to 91 samples: still managable

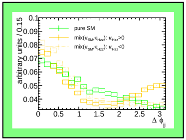
Procedure

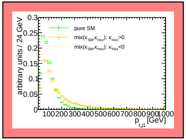
- first round of 91 samples generated
- BSM κs chosen such that one-operator pure BSM samples correspond to SM VBF cross section
- only truth so far
- validated morphing function by recovering inputs
- setup fit to SM input sample
  - learn correlations between operators
  - explore sensitivity
  - uses observables: $\Delta\phi_j j$, p_T^{j1}, m_{jj}, $\Delta\eta_{jj}$
  - understand which operators have neglegible influence on VBF

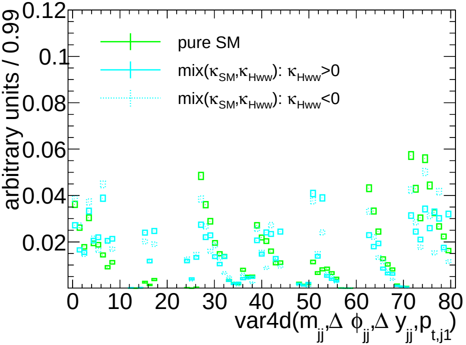
Jet kinematics depending on VBF couplings: $|\kappa_{Hzz}| \approx 3$

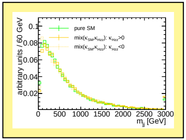


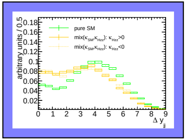
Combined distribution of four observables

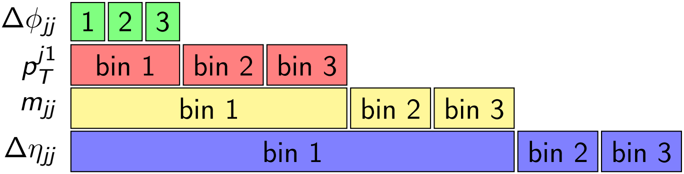












Fit result sensitivity on VBF couplings

Lambda	1000
cosa	0.70711
kHll	1.4142
kAaa	0.229290 ± 443
kAww	0.006812 ± 2.93
kAza	-3.536000 ± 862
kAzz	0.028945 ± 8.03
kHaa	-2.678200 ± 238
kHda	-0.003881 ± 0.787
kHdwI	0.007916 ± 1.61
kHdwR	-0.000145 ± 0.784
kHdz	-0.000451 ± 1.88
kHww	0.011619 ± 3.35
kHza	-0.588770 ± 96.7
kHzz	-0.016149 ± 8.48
kSM	1.422000 ± 0.0876

- fit errors give information on sensitivity
- sensitvity on $\gamma\gamma$ and $Z\gamma$ couplings small
- close to the SM, 4 couplings can be ignored for VBF without loss of generality
- other measurements will limit this operator to far smaller values
- to be tested that the $\gamma\gamma$ and $Z\gamma$ operators don't influence any other VBF observable

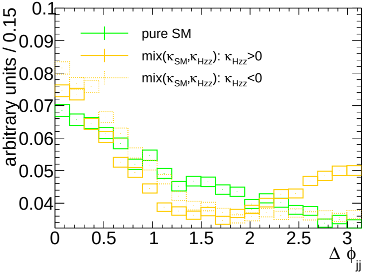
Fit result correlations of VBF couplings

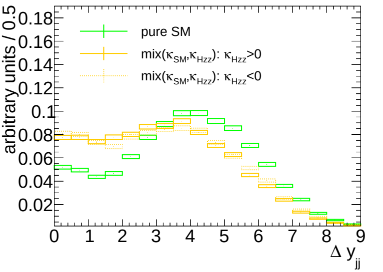
	kAaa	kAww	kAza	kAzz	kHaa	kHda	kHdwl	kHdwR	kHdz	kHww	kHza	kHzz	kSM
kAaa	1	0.14	-0.11	0.08	0.04	-0.33	0	0.11	-0.09	-0.09	0.04	0.01	-0.18
kAww	0.14	1	0.25	-0.37	-0.34	0.18	-0.14	-0.56	0.74	0.32	0.1	-0.3	-0.13
kAza	-0.11	0.25	1	-0.11	0.22	0.02	-0.1	0.15	-0.07	-0.12	0.23	-0.01	-0.04
kAzz	0.08	-0.37	-0.11	1	-0.2	0.14	0.35	0.17	-0.23	0.29	0.14	-0.42	0.16
kHaa	0.04	-0.34	0.22	-0.2	1	0.04	-0.23	0.18	-0.33	-0.46	0.07	0.24	0.02
kHda	-0.33	0.18	0.02	0.14	0.04	1	-0.28	-0.35	0.31	0.11	0.39	-0.28	0.21
kHdwl	0	-0.14	-0.1	0.35	-0.23	-0.28	1	0.02	0.03	0.29	-0.25	-0.17	0.1
kHdwR	0.11	-0.56	0.15	0.17	0.18	-0.35	0.02	1	-0.94	-0.23	-0.29	0.34	-0.01
kHdz	-0.09	0.74	-0.07	-0.23	-0.33	0.31	0.03	-0.94	1	0.35	0.2	-0.34	0.1
kHww	-0.09	0.32	-0.12	0.29	-0.46	0.11	0.29	-0.23	0.35	1	-0.21	-0.77	0.02
kHza	0.04	0.1	0.23	0.14	0.07	0.39	-0.25	-0.29	0.2	-0.21	1	-0.32	0.22
kHzz	0.01	-0.3	-0.01	-0.42	0.24	-0.28	-0.17	0.34	-0.34	-0.77	-0.32	1	-0.06
kSM	-0.18	-0.13	-0.04	0.16	0.02	0.21	0.1	-0.01	0.1	0.02	0.22	-0.06	1

- some large correlations present close to the standard model
- can likely neglect 1-2 more parameters after rotating into a paramater basis diagonal in this correlation matrix
- e. g. uncorrelated component(s) between $\kappa_{H\partial Z}$ and $\kappa_{H\partial WR}$

Summary

- plan for Run 2: Higgs coupling measurements
- combine rate and shape information within EFT framework
- new method for modelling BSM EFT effects
  - contuinous
  - analytical
  - fast
- first application: detailed and complete study of EFT VBF production
- next steps
  - similar study of WH, ZH and $H \to VV$
  - first look into most complex cases of VBF, VH or $H \to VV$


arbitrary units / 0.15
pure SM
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}>0
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}<0
$\Delta\ \phi_{jj}$


arbitrary units / 0.5
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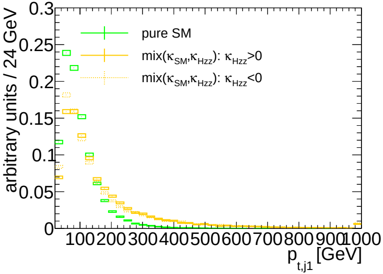
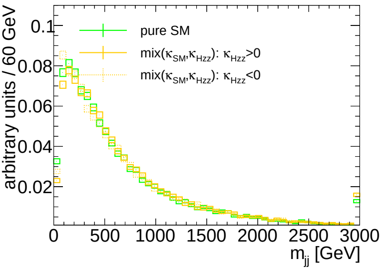
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}>0
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}<0
$\Delta\ y_{jj}$


arbitrary units / 24 GeV
pure SM
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}>0
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}<0
$p_{t,j1}$ [GeV]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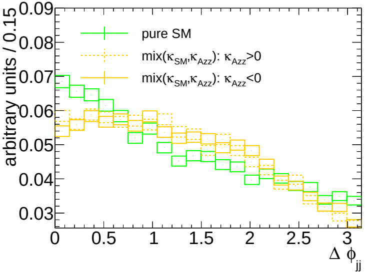
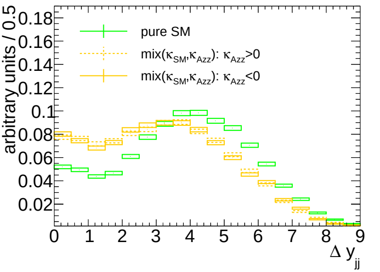
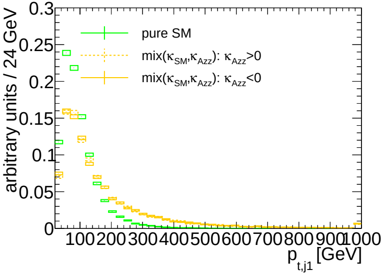
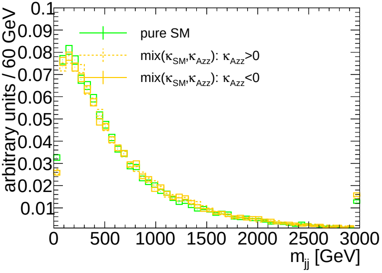
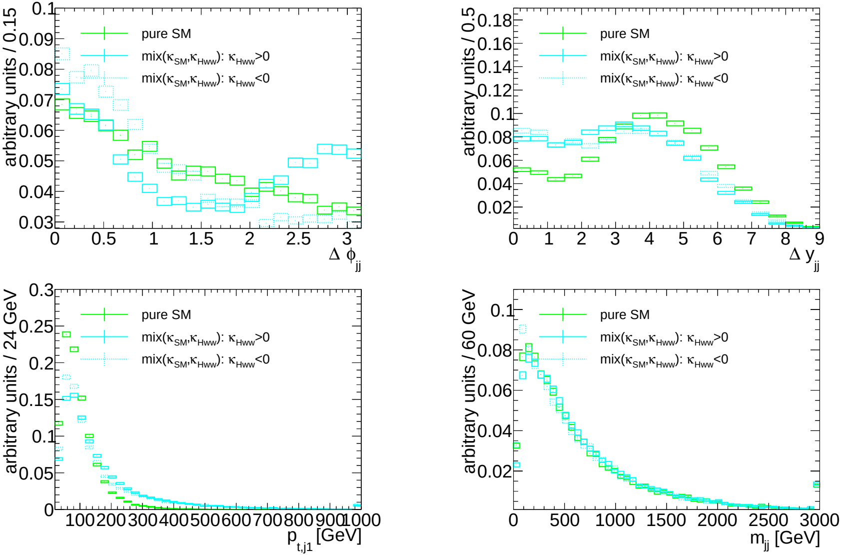
mix(κ_{SM},κ_{Hzz}): κ_{Hzz}<0
m_{jj} [GeV]


arbitrary units / 0.15
pure SM
mix(κ_{SM},κ_{Azz}): κ_{Azz}>0
mix(κ_{SM},κ_{Azz}): κ_{Azz}<0
$\Delta\ \phi_{jj}$

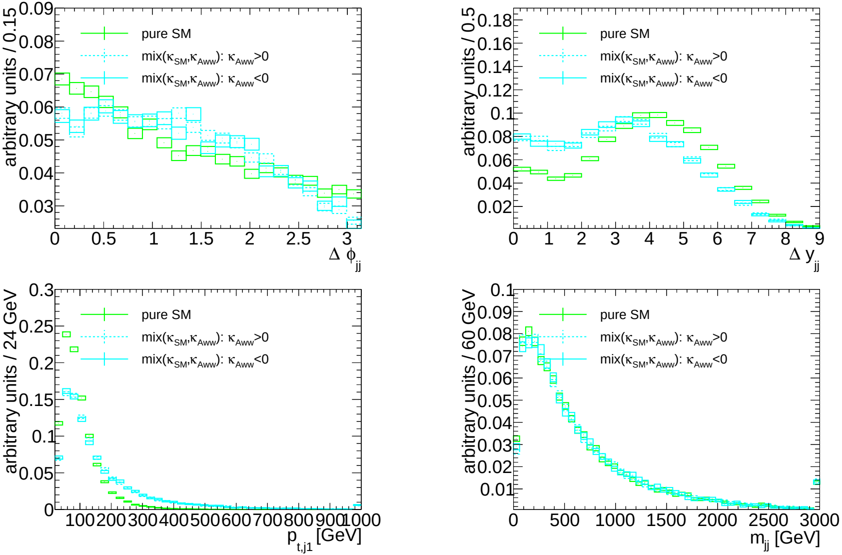

arbitrary units / 0.5
pure SM
mix(κ_{SM},κ_{Azz}): κ_{Azz}>0
mix(κ_{SM},κ_{Azz}): κ_{Azz}<0
$\Delta\ y_{jj}$

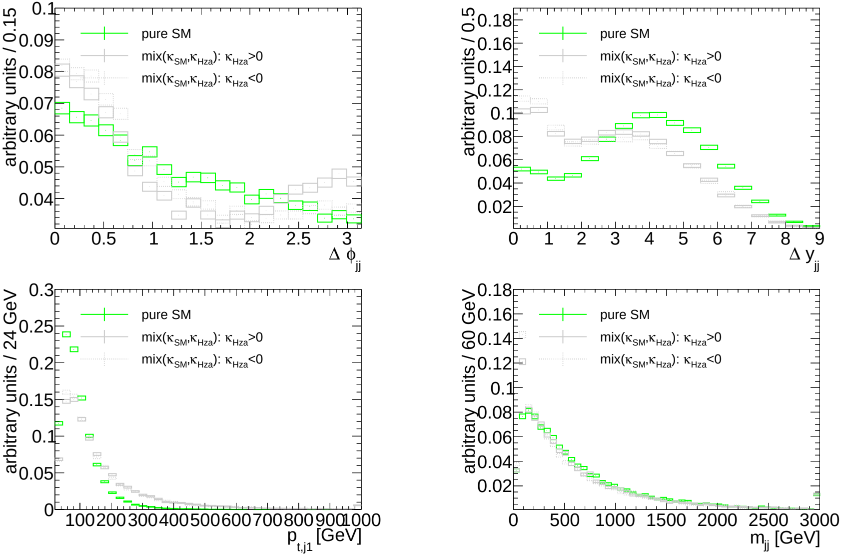

arbitrary units / 24 GeV
pure SM
mix(κ_{SM},κ_{Azz}): κ_{Azz}>0
mix(κ_{SM},κ_{Azz}): κ_{Azz}<0
$p_{t,j1}$ [GeV]


arbitrary units / 60 GeV
pure SM
mix(κ_{SM},κ_{Azz}): κ_{Azz}>0
mix(κ_{SM},κ_{Azz}): κ_{Azz}<0
m_{jj} [GeV]

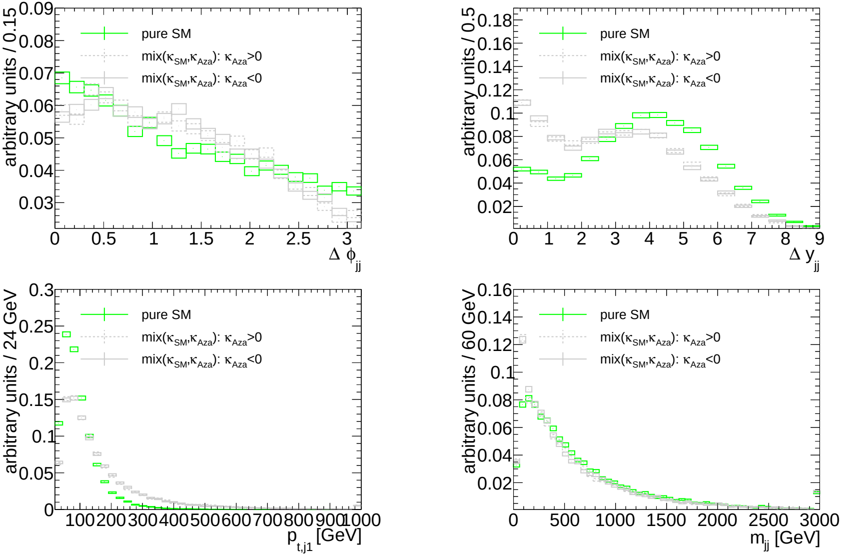


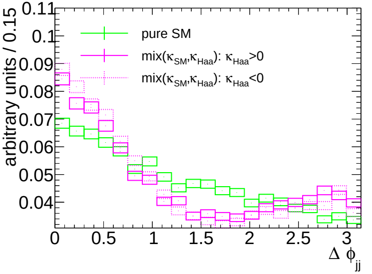
pure SM
mix(κSM,κHww): κHww>0
mix(κSM,κHww): κHww<0
arbitrary units / 0.15
Δ φjj
arbitrary units / 0.5
Δ yjj
arbitrary units / 24 GeV
pt,j1 [GeV]
arbitrary units / 60 GeV
mjj [GeV]


arbitrary units / 0.15
pure SM
mix(κSM,κAww): κAww>0
mix(κSM,κAww): κAww<0
Δ φjj
arbitrary units / 0.5
Δ yjj
arbitrary units / 24 GeV
pt,j1 [GeV]
arbitrary units / 60 GeV
mjj [GeV]

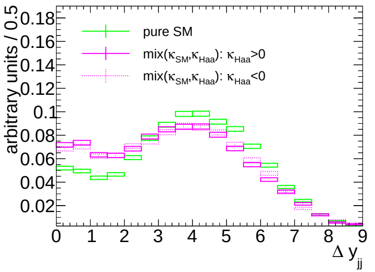


pure SM
mix(κSM,κHza): κHza>0
mix(κSM,κHza): κHza<0
arbitrary units / 0.15
Δ φjj
arbitrary units / 0.5
Δ yjj
arbitrary units / 24 GeV
pt,j1 [GeV]
arbitrary units / 60 GeV
mjj [GeV]

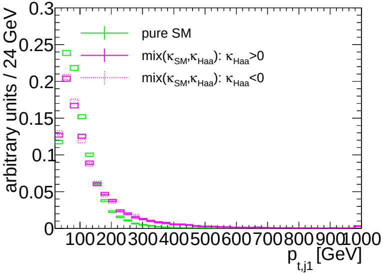

pure SM
mix(κSM,κAza): κAza>0
mix(κSM,κAza): κAza<0
arbitrary units / 0.15
Δ ϕjj
arbitrary units / 0.5
Δ yjj
arbitrary units / 24 GeV
pt,j1 [GeV]
arbitrary units / 60 GeV
mjj [GeV]



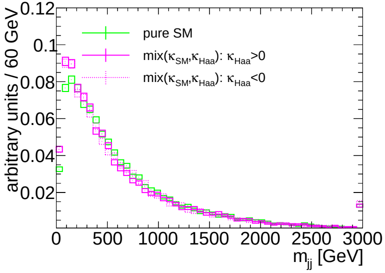
arbitrary units / 0.15
pure SM
mix(κSM,κHaa): κHaa>0
mix(κSM,κHaa): κHaa<0
Δ φjj



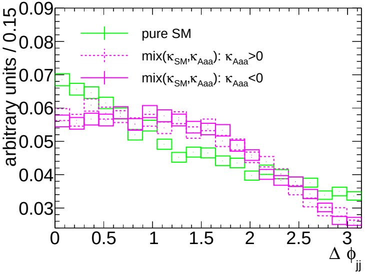
arbitrary units / 0.5
pure SM
mix(κSM,κHaa): κHaa>0
mix(κSM,κHaa): κHaa<0
Δ yjj

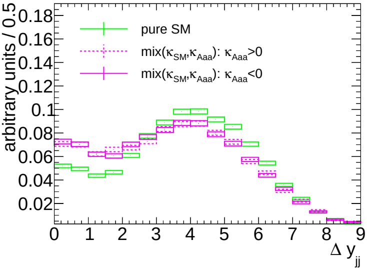


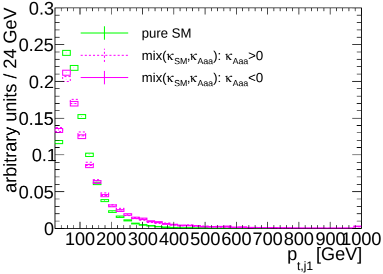
arbitrary units / 24 GeV
pure SM
mix(κSM,κHaa): κHaa>0
mix(κSM,κHaa): κHaa<0
pt,j1 [GeV]

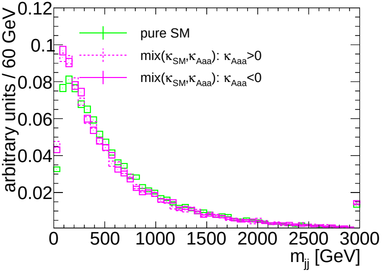


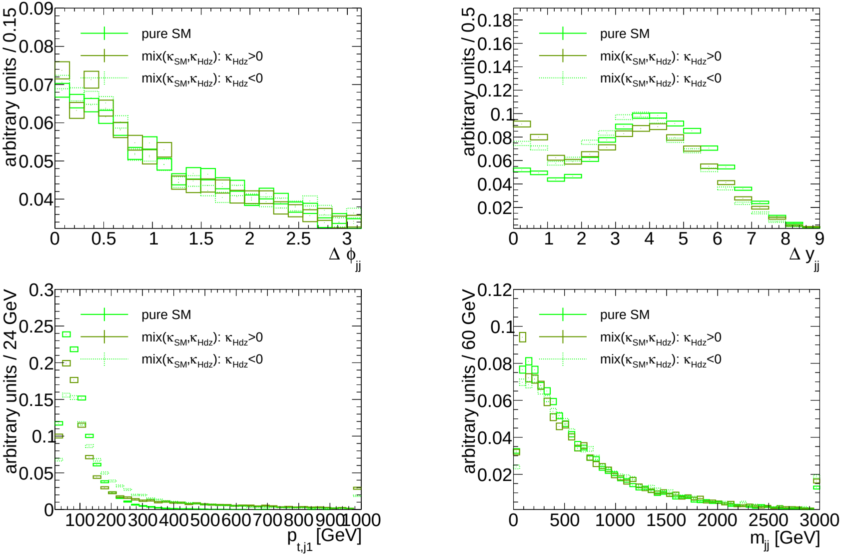
arbitrary units / 60 GeV
pure SM
mix(κSM,κHaa): κHaa>0
mix(κSM,κHaa): κHaa<0
mjj [GeV]

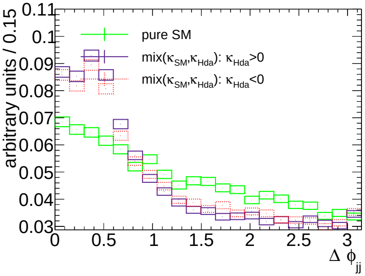

arbitrary units / 0.15
pure SM
mix(κSM,κAaa): κAaa>0
mix(κSM,κAaa): κAaa<0
Δ φjj

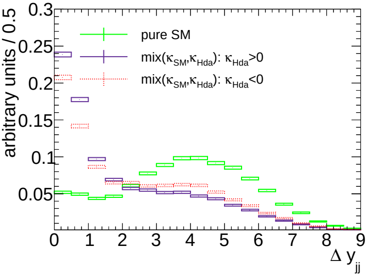

arbitrary units / 0.5
pure SM
mix(κSM,κAaa): κAaa>0
mix(κSM,κAaa): κAaa<0
Δ yjj

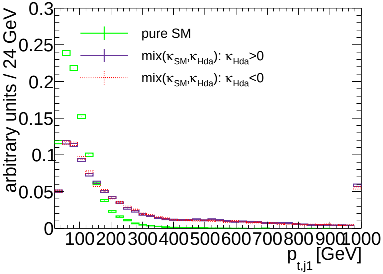

arbitrary units / 24 GeV
pure SM
mix(κSM,κAaa): κAaa>0
mix(κSM,κAaa): κAaa<0
pt,j1 [GeV]

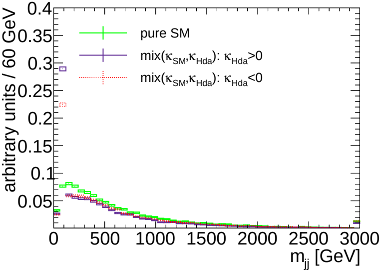

arbitrary units / 60 GeV
pure SM
mix(κSM,κAaa): κAaa>0
mix(κSM,κAaa): κAaa<0
mjj [GeV]

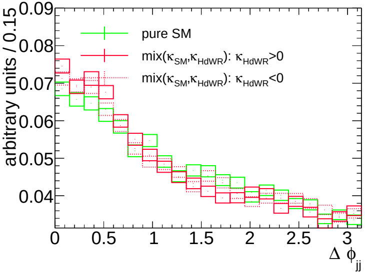

pure SM
mix(κSM,κHdz): κHdz>0
mix(κSM,κHdz): κHdz<0
arbitrary units / 0.15
Δ ϕjj
arbitrary units / 0.5
Δ yjj
arbitrary units / 24 GeV
pt,j1 [GeV]
arbitrary units / 60 GeV
mjj [GeV]

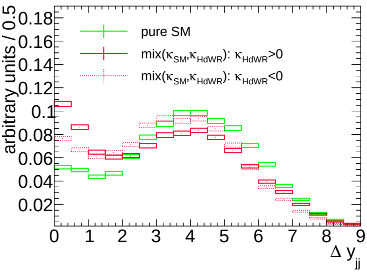

arbitrary units / 0.15
pure SM
mix(κSM,κHda): κHda>0
mix(κSM,κHda): κHda<0
Δ φjj

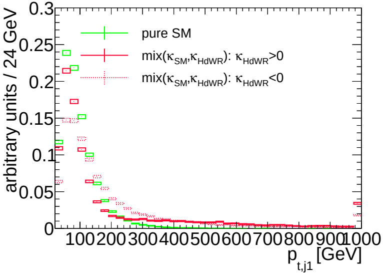

arbitrary units / 0.5
pure SM
mix(κSM,κHda): κHda>0
mix(κSM,κHda): κHda<0
Δ yjj

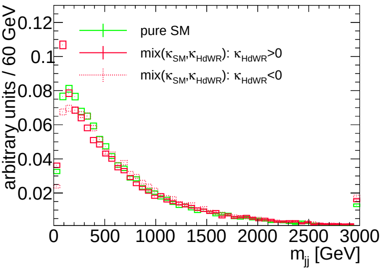

arbitrary units / 24 GeV
pure SM
mix(κSM,κHda): κHda>0
mix(κSM,κHda): κHda<0
pt,j1 [GeV]

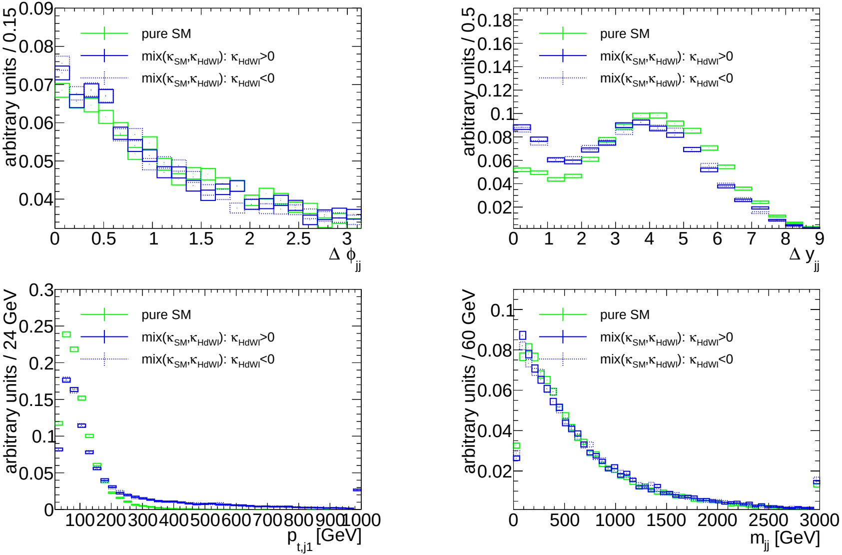

arbitrary units / 60 GeV
pure SM
mix(κSM,κHda): κHda>0
mix(κSM,κHda): κHda<0
mjj [GeV]


arbitrary units / 0.15
pure SM
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}>0
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}<0
$\Delta\,\phi_{jj}$


arbitrary units / 0.5
pure SM
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}>0
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}<0
$\Delta\,y_{jj}$


arbitrary units / 24 GeV
pure SM
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}>0
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}<0
$p_{t,j1}$ [GeV]


arbitrary units / 60 GeV
pure SM
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}>0
mix(κ_{SM},κ_{HdWR}): κ_{HdWR}<0
m_{jj} [GeV]


pure SM
mix(κSM,κHdWl): κHdWl>0
mix(κSM,κHdWl): κHdWl<0
arbitrary units / 0.15
Δ φjj
arbitrary units / 0.5
Δ yjj
arbitrary units / 24 GeV
pt,j1 [GeV]
arbitrary units / 60 GeV
mjj [GeV]