LHCHXSWG-INT-2015-001
September 11, 2015

LHC Higgs Cross Section Working Group 2 (Higgs Properties)

Higgs Basis: Proposal for an EFT basis choice for LHCHXSWG

Internal note prepared by:
André David, Michael Dührssen, Adam Falkowski, Chris Hays, and Gino Isidori.

We acknowledge contributions and feedback from:
Hermès Belusca-Maïto, Gerhard Buchalla, Alexandra Carvalho, Oscar Catà, Alejandro Celis, Benjamin Fuks, Christophe Grojean, Florian Goertz, Claudius Krause, Kentarou Mawatari, Ken Mimasu, Kin Mimouni, Marc Riembau, Tilman Plehn, Alex Pomarol, Francesco Riva, Veronica Sanz.

Note that this document is currently a work in progress. Appearing in acknowledgements does not imply full or partial endorsement. Further contributions, comments, and criticism are welcome. Please send your remarks to lhc-higgs-properties-convener@cern.ch.

1 Introduction

The LHC Higgs Cross Section Working Group is focused on various steps of the analysis chain:

Data → Fiducial cross-sections → Pseudo-observables → Model-independent EFT → BSM Models .

This note concerns model-independent interpretations of the data in the framework of effective field theory (EFT) beyond the Standard Model (SM), which is a part of the scope of the Working Group 2. The purpose of this note is to propose a common EFT language and conventions that could be universally used in LHC Higgs analyses and be implemented in numerical tools.

In the EFT approach to physics beyond the SM, the basic assumption is that the mass scale Λ of non-SM particles is larger than the electroweak scale v, $\Lambda \gg v$. If this is the case, physics at energies $E \ll \Lambda$ can be parametrized by the SM Lagrangian supplemented by new operators with canonical dimensions d larger than 4. The theory has the same field content and the same linearly realized $SU(3) \times SU(2) \times U(1)$ local symmetry as the SM.[1] The higher-dimensional operators are organized in a systematic expansion in d, where each consecutive term is suppressed by a larger power of Λ. The EFT Lagrangian can be written as

$$\mathcal{L}_{\rm EFT} = \mathcal{L}^{\rm SM} + \sum_i \frac{c_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \cdots . \tag{1.1}$$

[1] The latter assumption can be relaxed, leading to EFT with a non-linearly realized electroweak symmetry. In this note, we will not discuss these theories.

In this equation, $\mathcal{L}_{\rm SM}$ is the SM Lagrangian, which contains operators with $d \leq 4$. The remaining terms parametrize effects of heavy particles beyond the SM. Each $O_i^{(d)}$ is a gauge-invariant operator of canonical dimension d, and $c_i^{(d)}$ is the corresponding Wilson coefficient. The contribution of each $O_i^{(d)}$ to amplitudes of physical processes at the energy scale of order v scales[2] as $(v/\Lambda)^{d-4}$. Since $v/\Lambda < 1$ by construction, EFT typically describes *small* deviations from the SM predictions, except for observables that, within the SM, vanish or are suppressed by small parameters.

All dimension-5 operators that can be constructed from the SM fields violate the lepton number. Experimental constraints dictate that their coefficients must be suppressed at a level which makes them unobservable at the LHC, and for this reason d=5 operators will not be discussed here. Consequently, the leading new physics effects are expected from operators with d=6 whose contributions scale as $(v/\Lambda)^2$. We will ignore here the effects of operators with $d > 6$.

In the rest of this note, we discuss in detail the set d=6 operators that can be constructed from the SM fields. We review various possible choices of these operators (the so-called *basis*) and their phenomenological effects. Only the operators that conserve the baryon and lepton numbers are considered. On the other hand, we do not impose any flavor symmetry. Also, we include CP violating operators in our discussion.

In Section 2, to define our notation and conventions, we write down the SM Lagrangian. Two popular bases of dimension-6 operators using the manifestly $SU(2)\times U(1)$ invariant formalism are described in Section 3. In Section 4 we introduce an effective Lagrangian summarizing the new interactions of the SM mass eigenstates that arise in the presence of dimension-6 operators beyond the SM. We also derive provide a map between the couplings in that effective Lagrangian and Wilson coefficients of dimension-6 operators introduced in Section 3. In Section 5 we define a new basis of d=6 operators, the so-called Higgs basis, which is spanned by a subset of the independent couplings of the effective Lagrangian. This basis is particularly convenient for leading-order EFT analyses of LHC Higgs data.

2 Standard Model Lagrangian

The SM Lagrangian in our notation takes the form

$$\begin{aligned} \mathcal{L}^{\rm SM} &= -\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^i_{\mu\nu} - \frac{1}{4}B_{\mu\nu}B_{\mu\nu} + D_\mu H^\dagger D_\mu H + \mu_H^2 H^\dagger H - \lambda(H^\dagger H)^2 \\ &+ \sum_{f\in q,\ell} i\bar f_L \gamma_\mu D_\mu f_L + \sum_{f\in u,d,e} i\bar f_R \gamma_\mu D_\mu f_R \\ &- \left[\tilde H^\dagger \bar u_R y_u q_L + H^\dagger \bar d_R y_d V^\dagger_{\rm CKM} q_L + H^\dagger \bar e_R y_e \ell_L + {\rm h.c.}\right]. \end{aligned} \tag{2.1}$$

Here, G^a_μ, W^i_μ, and B_μ denote the gauge fields of the $SU(3)\times SU(2)\times U(1)$ local symmetry. The corresponding gauge couplings are denoted by g_s, g, g'; we also define the electromagnetic coupling $e = gg'/\sqrt{g^2+g'^2}$, and the Weinberg angle $s_\theta = g'/\sqrt{g^2+g'^2}$.

[2]Apart from the scaling with Λ, the effects of higher-dimensional operators also scale with appropriate powers of couplings in the UV theory. The latter may be important to assess the validity range of the EFT description.

The field strength tensors are defined as $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The Higgs doublet is denoted as H, and we also define $\tilde{H}_i = \epsilon_{ij} H^*_j$. It acquires the VEV $\langle H^\dagger H \rangle = v^2/2$. In the unitary gauge we have $H = (0, (v+h)/\sqrt{2})$, where h is the Higgs boson field. After electroweak symmetry breaking, the electroweak gauge boson mass eigenstates are defined as $W^\pm = (W^1 \mp i W^2)/\sqrt{2}$, $Z = c_\theta W^3 - s_\theta B$, $A = s_\theta W^3 + c_\theta B$, where $c_\theta = \sqrt{1 - s_\theta^2}$. The tree-level masses of W and Z bosons are given by $m_W = gv/2$, $m_Z = \sqrt{g^2 + g'^2} v/2$. The left-handed Dirac fermions $q_L = (u_L, V_{\rm CKM} d_L)$ and $\ell_L = (\nu_L, e_L)$ are doublets of the SU(2) gauge group, and the right-handed Dirac fermions u_R, d_R, e_R are SU(2) singlets. All fermions are 3-component vectors in the generation space, and y_f are 3×3 matrices. We work in the basis where the fermion mass matrix is diagonal with real, positive entries. In this basis, y_f are diagonal, and the fermion masses are given by $m_{f_i} = v [y_f]_{ii}/\sqrt{2}$.

For a future use, we write down the equations of motions for the gauge fields following from Eq. (2.1):

$$\begin{aligned} \partial_\nu B_{\nu\mu} &= -\frac{i g'}{2} H^\dagger \overleftrightarrow{D_\mu} H - g' j^Y_\mu , \\ \partial_\nu W^i_{\nu\mu} + g\epsilon^{ijk} W^j_\nu W^k_{\nu\mu} = D_\nu W^i_{\nu\mu} &= -\frac{i g}{2} H^\dagger \sigma^i \overleftrightarrow{D_\mu} H - g j^i_\mu , \\ D_\nu G^a_{\nu\mu} &= -g_s j^a_\mu , \end{aligned} \tag{2.2}$$

where $j^Y_\mu = \sum_f Y_f \bar{f} \gamma_\mu f$, $j^i_\mu = \bar{q} \gamma_\mu \frac{\sigma^i}{2} P_L q + \bar{\ell} \gamma_\mu \frac{\sigma^i}{2} P_L \ell$, and $j^a_\mu = \bar{q} \gamma_\mu T^a P_L q$ are the fermionic currents corresponding to the $U(1)$, $SU(2)$, and $SU(3)$ factors of the SM gauge group.

Rewriting the Lagrangian in Eq. (2.1) in terms of the mass eigenstates after electroweak symmetry breaking, one finds the following mass terms:

$$\mathcal{L}^{\rm SM}_{\rm mass} = \frac{g^2 v^2}{4} W^+_\mu W^-_\mu + \frac{(g^2 + g'^2) v^2}{8} Z_\mu Z_\mu + \sum_{f \in u,d,e} m_f \bar{f} f, \tag{2.3}$$

the gauge boson couplings to fermions:

$$\begin{aligned} \mathcal{L}^{\rm SM}_{vff} &= e A_\mu \sum_{f \in u,d,e} Q_f \bar{f} \gamma_\mu f + g_s G^a_\mu \sum_{f \in u,d} \bar{f} \gamma_\mu T^a f, \\ &+ \frac{g}{\sqrt{2}} \left(W^+_\mu \bar{u}_L \gamma_\mu V_{\rm CKM} d_L + W^+_\mu \bar{\nu}_L \gamma_\mu e_L + \text{h.c.} \right) \\ &+ \sqrt{g^2 + g'^2} Z_\mu \sum_{f \in u,d,e,\nu} \left(T^3_f \bar{f}_L \gamma_\mu f_L - s^2_\theta Q_f \bar{f} \gamma_\mu f \right), \end{aligned} \tag{2.4}$$

the couplings of a single Higgs boson to gauge bosons and fermions:

$$\mathcal{L}^{\rm SM}_h = \frac{h}{v} \left[\frac{g^2 v^2}{2} W^+_\mu W^-_\mu + \frac{(g^2 + g'^2) v^2}{4} Z_\mu Z_\mu \right] - \frac{h}{v} \sum_f m_f \bar{f} f \tag{2.5}$$

the couplings involving two or more Higgs bosons

$$\mathcal{L}^{\rm SM}_{hh} = \frac{h^2}{2v^2} \left[\frac{g^2 v^2}{2} W^+_\mu W^-_\mu + \frac{(g^2 + g'^2) v^2}{4} Z_\mu Z_\mu \right] - \frac{m_h^2}{2v} h^3 - \frac{m_h^2}{8v^2} h^4, \tag{2.6}$$

and the triple and quartic self-interactions of the vector bosons:

$$\begin{aligned}
\mathcal{L}_{\rm tgc}^{\rm SM} &= ie\left[\left(W^+_{\mu\nu}W^-_\mu - W^-_{\mu\nu}W^+_\mu\right)A_\nu + A_{\mu\nu}\,W^+_\mu W^-_\nu\right]\\
&+ igc_\theta\left[\left(W^+_{\mu\nu}W^-_\mu - W^-_{\mu\nu}W^+_\mu\right)Z_\nu + Z_{\mu\nu}\,W^+_\mu W^-_\nu\right]\\
&- g_s f^{abc}\partial_\mu G^a_\nu G^b_\mu G^c_\nu.
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
\mathcal{L}_{\rm qgc}^{\rm SM} &= \frac{g^2}{2}\left(W^+_\mu W^+_\mu W^-_\nu W^-_\nu - W^+_\mu W^-_\mu W^+_\nu W^-_\nu\right) + g^2 c_\theta^2\left(W^+_\mu Z_\mu W^-_\nu Z_\nu - W^+_\mu W^-_\mu Z_\nu Z_\nu\right)\\
&+ g^2 s_\theta^2\left(W^+_\mu A_\mu W^-_\nu A_\nu - W^+_\mu W^-_\mu A_\nu A_\nu\right)\\
&+ g^2 c_\theta s_\theta\left(W^+_\mu Z_\mu W^-_\nu A_\nu + W^+_\mu A_\mu W^-_\nu Z_\nu - 2W^+_\mu W^-_\mu Z_\nu A_\nu\right)\\
&- g_s^2 f^{abc}f^{ade}G^b_\mu G^c_\nu G^d_\mu G^e_\mu.
\end{aligned} \tag{2.8}$$

The couplings multiplying the SM interaction terms depend on a number of input parameters: m_h, m_f, $V_{\rm CKM}$, g_s, g, g', v, all of which are known with a good precision. The last 3 parameters are customarily derived from the observable Fermi constant G_F (more precisely, from the measured muon lifetime $\tau_\mu = 192\pi^3/G_F^2 m_\mu^5$), Z boson mass m_Z, and the low-energy electromagnetic coupling $\alpha(0)$. The tree-level relations between the input observables and the electroweak parameters are given by:

$$G_F = \frac{1}{\sqrt{2}v^2}, \qquad \alpha = \frac{g^2 g'^2}{4\pi(g^2+g'^2)}, \qquad m_Z = \frac{\sqrt{g^2+g'^2}v}{2}. \tag{2.9}$$

3 Bases of dimension-6 operators

A *basis* of dimension-6 operators is a complete, non-redundant set of $O_i^{(6)}$ in Eq. (1.1). Complete means that any dimension-6 operator is either a part of the basis or can be obtained from a combination of operators in the basis using equations of motion, integration by parts, field redefinitions, and Fierz transformations. Non-redundant means it is a minimal such set. Any complete basis leads to the same physical predictions concerning possible new physics effects. Several bases have been proposed in the literature, and they may be convenient for specific applications. In this section we describe two popular choices in the existing literature. Later, in Section 5, we propose a new basis choice that is particularly convenient for leading-order LHC Higgs analyses in the EFT framework.

3.1 Warsaw Basis

Historically, a complete and non-redundant set of d=6 operators was first identified in Ref. [1], and is usually referred to as the *Warsaw basis*. For our purpose, it is more convenient to work with a variant of that basis which differs from the one in Ref. [1] by the following aspects:

- We replace the operator $|H^\dagger D_\mu H|^2$ by $O_T = (H^\dagger \overleftrightarrow{D_\mu} H)^2$, where $H^\dagger \overleftrightarrow{D_\mu} H \equiv H^\dagger D_\mu H - D_\mu H^\dagger H$. The reason is that O_T is more directly connected to violation of custodial symmetry among Higgs couplings.

H^4D^2 and H^6		f^2H^3		V^3D^3	
O_H	$\left[\partial_\mu(H^\dagger H)\right]^2$	$[O_e]_{ij}$	$-\frac{\sqrt{m_i m_j}}{v}(H^\dagger H - \frac{v^2}{2})\bar{e}_i H^\dagger \ell_j$	O_{3G}	$g_s^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$
O_T	$\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$	$[O_u]_{ij}$	$-\frac{\sqrt{m_i m_j}}{v}(H^\dagger H - \frac{v^2}{2})\bar{u}_i \widetilde{H}^\dagger q_j$	$O_{3\widetilde{G}}$	$g_s^3 f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$
O_{6H}	$(H^\dagger H)^3$	$[O_d]_{ij}$	$-\frac{\sqrt{m_i m_j}}{v}(H^\dagger H - \frac{v^2}{2})\bar{d}_i H^\dagger q_j$	O_{3W}	$g^3 \epsilon^{ijk} W^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$
				$O_{3\widetilde{W}}$	$g^3 \epsilon^{ijk} \widetilde{W}^i_{\mu\nu} W^j_{\nu\rho} W^k_{\rho\mu}$

V^2H^2		f^2H^2D		f^2VHD	
O_{GG}	$\frac{g_s^2}{4} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$	$[O_{H\ell}]_{ij}$	$i\bar{\ell}_i \gamma_\mu \ell_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{eW}]_{ij}$	$g \bar{\ell}_i \sigma^k H \sigma_{\mu\nu} e_j W^k_{\mu\nu}$
$O_{\widetilde{G}\widetilde{G}}$	$\frac{g_s^2}{4} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu}$	$[O'_{H\ell}]_{ij}$	$i\bar{\ell}_i \sigma^k \gamma_\mu \ell_j H^\dagger \sigma^k \overleftrightarrow{D}_\mu H$	$[O_{eB}]_{ij}$	$g' \bar{\ell}_i H \sigma_{\mu\nu} e_j B_{\mu\nu}$
O_{WW}	$\frac{g^2}{4} H^\dagger H\, W^i_{\mu\nu} W^i_{\mu\nu}$	$[O_{He}]_{ij}$	$i\bar{e}_i \gamma_\mu \bar{e}_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{uG}]_{ij}$	$g_s \bar{q}_i \tilde{H} \sigma_{\mu\nu} T^a u_j G^a_{\mu\nu}$
$O_{\widetilde{W}\widetilde{W}}$	$\frac{g^2}{4} H^\dagger H\, \widetilde{W}^i_{\mu\nu} W^i_{\mu\nu}$	$[O_{Hq}]_{ij}$	$i\bar{q}_i \gamma_\mu q_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{uW}]_{ij}$	$g \bar{q}_i \sigma^k \tilde{H} \sigma_{\mu\nu} u_j W^k_{\mu\nu}$
O_{BB}	$\frac{g'^2}{4} H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$	$[O'_{Hq}]_{ij}$	$i\bar{q}_i \sigma^k \gamma_\mu q_j H^\dagger \sigma^k \overleftrightarrow{D}_\mu H$	$[O_{uB}]_{ij}$	$g' \bar{q}_i \tilde{H} \sigma_{\mu\nu} u_j B_{\mu\nu}$
$O_{\widetilde{B}\widetilde{B}}$	$\frac{g'^2}{4} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu}$	$[O_{Hu}]_{ij}$	$i\bar{u}_i \gamma_\mu u_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{dG}]_{ij}$	$g_s \bar{q}_i H \sigma_{\mu\nu} T^a d_j G^a_{\mu\nu}$
O_{WB}	$g g' H^\dagger \sigma^i H\, W^i_{\mu\nu} B_{\mu\nu}$	$[O_{Hd}]_{ij}$	$i\bar{d}_i \gamma_\mu d_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{dW}]_{ij}$	$g \bar{q}_i \sigma^k H \sigma_{\mu\nu} d_j W^k_{\mu\nu}$
$O_{\widetilde{W}\widetilde{B}}$	$g g' H^\dagger \sigma^i H\, \widetilde{W}^i_{\mu\nu} B_{\mu\nu}$	$[O_{Hud}]_{ij}$	$i\bar{u}_i \gamma_\mu d_j \tilde{H}^\dagger D_\mu H$	$[O_{dB}]_{ij}$	$g' \bar{q}_i H \sigma_{\mu\nu} d_j B_{\mu\nu}$

Table 1: Dimension-6 operators other than four-fermion operators in the Warsaw basis. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. For complex operators the complex conjugate operator is implicit.

- For Yukawa-type d=6 operators $H|H|^2\bar{f}f$ we subtracted v^2 from $|H|^2$in the definition, so that they do not contribute to fermion mass terms. This way we avoid tedious rotations of the fermion fields to bring them back to the mass eigenstate basis. Moreover, we isolated factor of fermion masses in the definition, for a more direct connection to minimal flavor violating scenarios. Starting with the Yukawa couplings $-H\bar{f}'_R(Y'_f + c'_f H^\dagger H/v^2)f'_L$ we can bring them to the form in Eq. (2.1) and Table 1 by defining $f'_{L,R} = U_{L,R} f_{L,R}$, $\sqrt{m_i m_j}[c_f]_{ij}/v = [U_R^\dagger c'_f U_L]_{ij}$, $Y_f = U_R^\dagger(Y'_f + c'_f/2)U_L$, where $U_{L,R}$ are unitary rotations to the mass eigenstate basis.

For other operators, we often use a different notation and normalizations than the original reference.

The Lagrangian in the Warsaw basis is given by

$$\mathcal{L}_{\rm warsaw} = \mathcal{L}^{\rm SM} + \frac{1}{\Lambda^2}\sum_i \hat{c}_i O_i, \qquad (3.1)$$

where the SM Lagrangian $\mathcal{L}^{\rm SM}$ was introduced in Section 2, Λ is the EFT expansion parameter identified with the mass scale of new particles in the UV theory, O_i are the dimension-6 operators summarized in Table 1 and Table 2, and $\hat{c}_i$ are the Wilson coefficient multiplying the operator O_i. Note that observables calculated in the EFT

$(\bar{L}L)(\bar{L}L)$ and $(\bar{L}R)(\bar{L}R)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$O_{\ell\ell}$	$(\bar{\ell}\gamma_\mu\ell)(\bar{\ell}\gamma_\mu\ell)$	O_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma_\mu e)$	$O_{\ell e}$	$(\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma_\mu e)$
O_{qq}	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$	O_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma_\mu u)$	$O_{\ell u}$	$(\bar{\ell}\gamma_\mu\ell)(\bar{u}\gamma_\mu u)$
O'_{qq}	$(\bar{q}\gamma_\mu\sigma^i q)(\bar{q}\gamma_\mu\sigma^i q)$	O_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma_\mu d)$	$O_{\ell d}$	$(\bar{\ell}\gamma_\mu\ell)(\bar{d}\gamma_\mu d)$
$O_{\ell q}$	$(\bar{\ell}\gamma_\mu\ell)(\bar{q}\gamma_\mu q)$	O_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma_\mu u)$	O_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma_\mu e)$
$O'_{\ell q}$	$(\bar{\ell}\gamma_\mu\sigma^i\ell)(\bar{q}\gamma_\mu\sigma^i q)$	O_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma_\mu d)$	O_{qu}	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma_\mu u)$
O_{quqd}	$(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)$	O_{ud}	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma_\mu d)$	O'_{qu}	$(\bar{q}\gamma_\mu T^a q)(\bar{u}\gamma_\mu T^a u)$
O'_{quqd}	$(\bar{q}^j T^a u)\epsilon_{jk}(\bar{q}^k T^a d)$	O'_{ud}	$(\bar{u}\gamma_\mu T^a u)(\bar{d}\gamma_\mu T^a d)$	O_{qd}	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma_\mu d)$
$O_{\ell equ}$	$(\bar{\ell}^j e)\epsilon_{jk}(\bar{q}^k u)$			O'_{qd}	$(\bar{q}\gamma_\mu T^a q)(\bar{d}\gamma_\mu T^a d)$
$O'_{\ell equ}$	$(\bar{\ell}^j\sigma_{\mu\nu}e)\epsilon_{jk}(\bar{q}^k\sigma^{\mu\nu}u)$				
$O_{\ell edq}$	$(\bar{\ell}^j e)(\bar{d}q^j)$				

Table 2: Four-fermion operators in the Warsaw basis [1]. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. A flavor index is implicit for each fermion field. For complex operators the complex conjugate operator is implicit.

depend only on the combination $\hat{c}_i/\Lambda^2$. Therefore, working with the low-energy EFT, it is more convenient to redefine $\hat{c}_i \to c_i\Lambda^2/v^2$. In the following we will display all the formulas using the redefined Wilson coefficients c_i.

3.2 SILH basis

Another d=6 basis choice commonly used in the literature is the SILH basis [3, 11].[3] The SILH Lagrangian is written as

$$\mathcal{L}_{\rm SILH} = \mathcal{L}^{\rm SM} + \frac{1}{v^2}\sum_i s_i O_i. \tag{3.2}$$

[3] For the sake of this note, the SILH basis is understood simply as a particular choice of a non-redundant set of d=6 operators whose Wilson coefficients are arbitrary. We do not assume any hierarchy of the Wilson coefficients motivated by particular strongly coupled UV completions that was discussed in Refs. [3, 11]. As in the case of the Warsaw basis, in this note we use a different notation and normalization than in the original references.

Compared to the Warsaw basis defined in Section 3.1, the SILH basis of dimension-6 operators introduces the following nine new operators:

$$
\begin{aligned}
O_W &= \frac{ig}{2}\left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H\right) D_\nu W^i_{\mu\nu}, \\
O_B &= \frac{ig'}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right) \partial_\nu B_{\mu\nu}, \\
O_{HW} &= ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu}, \\
O_{HB} &= ig'\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu}, \\
O_{\widetilde{HW}} &= ig\left(D_\mu H^\dagger \sigma^i D_\nu H\right) \widetilde{W}^i_{\mu\nu}, \\
O_{\widetilde{HB}} &= ig'\left(D_\mu H^\dagger D_\nu H\right) \widetilde{B}_{\mu\nu}, \\
O_{2W} &= D_\mu W^i_{\mu\nu} D_\rho W^i_{\rho\nu}, \\
O_{2B} &= \partial_\mu B_{\mu\nu} \partial_\rho B_{\rho\nu}, \\
O_{2G} &= D_\mu G^a_{\mu\nu} D_\rho G^a_{\rho\nu}.
\end{aligned}
\tag{3.3}
$$

Consequently, in order to have a non-redundant set of operators, 9 operators present in the Warsaw basis must be absent in the SILH basis. The absent ones are 4 bosonic operators O_{WW}, $O_{\widetilde{WW}}$, O_{WB}, $O_{\widetilde{WB}}$, 2 vertex operators $[O_{H\ell}]_{11}$, $[O'_{H\ell}]_{11}$, and 3 four-fermion operators $[O_{\ell\ell}]_{1221}$, $[O_{\ell\ell}]_{1122}$, $[O'_{uu}]_{3333}$. The remaining operators are the same as in the Warsaw basis, and we use the normalizations in Table 1.[4]

3.3 Map between Warsaw and SILH bases

One way to derive the translation is to first transform the operators in Eq. (3.3) to the Warsaw basis using integration by parts, Fierz transformations, and the equations of

[4] The original references do not discuss the flavor structure explicitly, and the flavor indices of the absent operators are not specified. Here, for concreteness, we made a particular though somewhat arbitrary choice of these indices.

motion Eq. (2.2). This way, one can derive the following operator equalities:

$$
\begin{aligned}
O_{HB} &= O_B - \frac{1}{4} O_{WB} - O_{BB}, \\
O_{HW} &= O_W - \frac{1}{4} O_{WB} - O_{WW}, \\
O_{\widetilde{HB}} &= -\frac{1}{4} O_{\widetilde{WB}} - O_{\widetilde{BB}}, \\
O_{\widetilde{HW}} &= -\frac{1}{4} O_{\widetilde{WB}} - O_{\widetilde{WW}}, \\
O_B &= g'^2 \left[-\frac{1}{4} O_T + \frac{1}{2} \sum_{f \in q,u,d,\ell,e} Y_f \sum_i [O_{Hf}]_{ii} \right], \\
O_W &= g^2 \left[-\frac{1}{4} O_H + O_{HD} + \frac{1}{4} \sum_{f \in q,\ell} \sum_i [O'_{Hf}]_{ii} \right], \\
O_{2B} &= g'^2 \left[-\frac{1}{4} O_T + \sum_{f \in q,u,d,\ell,e} Y_f \sum_i [O_{Hf}]_{ii} + \sum_{f_1 f_2 \in q,u,d,\ell,e} Y_{f_1} Y_{f_2} \sum_{i,j} [O_{f_1 f_2}]_{ii;jj} \right], \\
O_{2W} &= g^2 \left[-\frac{1}{4} O_H + O_{HD} + \frac{1}{2} \sum_{f \in q,\ell} \sum_i [O'_{Hf}]_{ii} \right. \\
&+ \left. \sum_{ij} \left(\frac{1}{2} [O_{\ell\ell}]_{ij;ji} - \frac{1}{4} [O_{\ell\ell}]_{ii;jj} + \frac{1}{2} [O_{\ell q}]_{ii;jj} + \frac{1}{4} [O_{qq}]_{ii;jj} \right) \right], \\
O_{2G} &= g_s^2 \sum_{i,j} \left[\frac{1}{4} [O'_{qq}]_{ij;ji} + \frac{1}{4} [O_{qq}]_{ij;ji} - \frac{1}{6} [O_{qq}]_{ii;jj} + 2 [O'_{qu}]_{ii;jj} + 2 [O'_{qd}]_{ii;jj} \right. \\
&+ \left. 2 [O'_{ud}]_{ii;jj} + \frac{1}{2} [O'_{uu}]_{ij;ji} - \frac{1}{6} [O'_{uu}]_{ii;jj} + \frac{1}{2} [O'_{dd}]_{ij;ji} - \frac{1}{6} [O'_{dd}]_{ii;jj} \right]. \qquad (3.4)
\end{aligned}
$$

The operator $O_{HD} = |H|^2 |D_\mu H|^2$ appearing above is present neither in the Warsaw nor in the SILH basis. One can remove it from the Lagrangian by rescaling the Higgs field and the Yukawa couplings as $H \to H(1 + \epsilon |H|^2/v^2)$, $y_f \to y_f(1 - \epsilon/2)$. To lowest order in ϵ, this rescaling generates the following terms in the Lagrangian

$$
\Delta \mathcal{L} = \epsilon \left(2 O_{HD} + O_H - 4\lambda O_{6H} + \sqrt{2} \sum_{f \in u,d,e} \sum_i [O_f]_{ii} \right). \qquad (3.5)
$$

Thus, to get rid of the O_{HD} operator generated by the transformation from the SILH to the Warsaw basis we need to choose $\epsilon = -g^2(s_W + s_{HW} + s_{2W})/2$. Effectively, this amount to replacing in Eq. (3.4):

$$
O_{HD} \to -\frac{1}{2} O_H + 2\lambda O_{6H} - \frac{1}{\sqrt{2}} \sum_{f \in u,d,e} \sum_i [O_f]_{ii}. \qquad (3.6)
$$

We are ready to give the translation between the Wilson coefficient in the SILH and

Warsaw basis:

$$\begin{aligned}
c_H &= s_H - \frac{3g^2}{4}\left(s_W + s_{HW} + s_{2W}\right), \\
c_T &= s_T - \frac{g'^2}{4}\left(s_B + s_{HB} + s_{2B}\right), \\
c_{6H} &= s_{6H} + 2\lambda g^2\left(s_W + s_{HW} + s_{2W}\right), \\
c_{WB} &= -\frac{1}{4}\left(s_{HB} + s_{HW}\right), \\
c_{BB} &= s_{BB} - s_{HB}, \\
c_{WW} &= -s_{HW}, \\
\tilde{c}_{WB} &= -\frac{1}{4}\left(\tilde{s}_{HB} + \tilde{s}_{HW}\right), \\
\tilde{c}_{BB} &= \tilde{s}_{BB} - \tilde{s}_{HB}, \\
\tilde{c}_{WW} &= -\tilde{s}_{HW},
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
[c_{Hf}]_{ij} &= [s_{Hf}]_{ij} + \frac{g'^2 Y_f}{2}\left(s_B + s_{HB} + 2s_{2B}\right)\delta_{ij}, \\
[c'_{Hf}]_{ij} &= [s'_{Hf}]_{ij} + \frac{g^2}{4}\left(s_W + s_{HW} + 2s_{2W}\right)\delta_{ij},
\end{aligned} \tag{3.8}$$

$$[c_f]_{ij} = [s_f]_{ij} - \delta_{ij}\frac{g^2}{\sqrt{2}}\left(s_W + s_{HW} + s_{2W}\right), \tag{3.9}$$

$$\begin{aligned}
[c_{\ell\ell}]_{iiii} &= [s_{\ell\ell}]_{iiii} + \frac{1}{4}\left(g'^2 s_{2B} + g^2 s_{2W}\right), \\
[c_{\ell\ell}]_{iijj} &= [s_{\ell\ell}]_{iijj} + \frac{1}{2}\left(g'^2 s_{2B} - g^2 s_{2W}\right), \qquad i<j, \\
[c_{\ell\ell}]_{ijji} &= [s_{\ell\ell}]_{ijji} + g^2 s_{2W}, \qquad i<j,
\end{aligned} \tag{3.10}$$

where it is implicit that $[s_{H\ell}]_{11} = [s'_{H\ell}]_{11} = [s_{\ell\ell}]_{1221} = [s_{\ell\ell}]_{1122} = 0$. For the 4-lepton operators one should take into account that $[O_{\ell\ell}]_{jiij} \equiv [O_{\ell\ell}]_{ijji}$ and $[O_{\ell\ell}]_{jjii} \equiv [O_{\ell\ell}]_{iijj}$. The translation of other 4-fermion Wilson coefficients apart from the one in Eq. (3.10) can be easily derived from Eq. (3.4), but it will not be needed in the following. For the Wilson coefficients not listed above the translation is trivial: $c_i = s_i$.

4 Phenomenological effective Lagrangian

In Section 3 we introduced d=6 operators in the $SU(2)\times U(1)$ invariant notation. At that point, the connection between the new operators and phenomenology is not obvious. In this section we relate the Wilson coefficients of dimension-6 operators to the parameters of the effective Lagrangian describing the interactions of SM mass eigenstates after electroweak symmetry breaking. The effective Lagrangian is of the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\text{SM}} + \Delta\mathcal{L}_{d=6}, \tag{4.1}$$

where $\mathcal{L}^{\text{SM}}$ is the SM Lagrangian introduced in Section 2, and $\Delta\mathcal{L}_{d=6}$, contains new interactions beyond the SM induced by the d=6 operators.[5] The effect of $\Delta\mathcal{L}_{d=6}$ is

[5]Note that, after electroweak symmetry breaking, the canonical dimensions of some interaction terms $\Delta\mathcal{L}_{d=6}$ is smaller than 6 due to insertions of the Higgs field VEV v.

either to shift the coupling strength away from the SM predictions or to introduce new tensor structures of interactions that are absent in the SM Lagrangian. A subset of these interactions is relevant to describe new physics effects in Higgs searches at the LHC.

By construction, $\mathcal{L}_{\rm eff}$ has the following features:

#1 All kinetic and mass terms are diagonal and canonically normalized. In particular, there is no kinetic mixing between the Z boson and the photon.

#2 Tree-level relations between the electroweak parameters and input observables are the same as the SM ones in Eq. (2.9). In particular, the photon and the gluon interact with fermions as in Eq. (2.4), and there is no correction to the Z boson mass term.

#3 Two-derivative self-interactions of the Higgs boson are absent.

#4 For each fermion pair, the coefficient of the vertex-like Higgs interaction term $\frac{h}{v} V_\mu \bar{f}\gamma_\mu f$ is equal to the vertex correction to the respective $V_\mu \bar{f}\gamma_\mu f$ interaction.

These conditions greatly simplify the connection between the parameters of the Lagrangian and collider observables. In general, dimension-6 operators can induce interaction terms that do not respect these features. However, the conditions #1-#4 can always be achieved, *without any loss of generality*, by using equations of motion, integrating by parts, and redefining the fields and couplings. Below, we discuss the required set of transformations starting from the Warsaw basis. An analogous procedure could be executed starting from the SILH basis; alternatively, the map between the SILH basis and the phenomenological effective Lagrangian can be derived using the results for the Warsaw basis obtained below together with the Warsaw-to-SILH translation given in Section 3.3,

We need to bring the Warsaw basis Lagrangian to a form that satisfies the conditions #1-#4. To begin with, the operator O_{WB} leads to a kinetic mixing between the hypercharge and SU(2) gauge bosons, $O_{WB} \to -\frac{1}{2} g g' W^3_{\mu\nu} B_{\mu\nu}$. To get rid of it, one has to use the equations of motion in Eq. (2.2):

$$\begin{aligned}
& -c_{WB}\tfrac{gg'}{2} W^3_{\mu\nu} B_{\mu\nu} = -c_{WB}\tfrac{gg'}{2}\left(-2s_\theta^2 B_\mu \partial_\nu W^3_{\nu\mu} - 2c_\theta^2 W^3_\mu \partial_\nu B_{\nu\mu} + g c_\theta^2 \epsilon^{3jk} W^j_\mu W^k_\mu B_{\mu\nu}\right) \\
\to \ & c_{WB} e^2 \left[\tfrac{(v+h)^2}{4}\left(g W^3_\mu - g' B_\mu\right)^2 - g W^3_\mu j^Y_\mu - g' B_\mu j^3_\mu - \tfrac{g^2}{2g'}\epsilon^{3jk} W^j_\mu W^k_\nu B_{\mu\nu} - g'\epsilon^{3jk} B_\mu W^j_\nu W^k_{\nu\mu}\right] \\
= \ & c_{WB} e^2 \left[\tfrac{(g^2+g'^2)(v+h)^2}{4} Z_\mu^2 - e A_\mu j^{\rm em}_\mu + \sqrt{g^2+g'^2} Z_\mu \left(j^3_\mu - c_\theta^2 j^{\rm em}_\mu\right)\right] \\
+ \ & i c_{WB}\tfrac{g^2 g'}{(g^2+g'^2)^{3/2}}\left[g^2 (g A_{\mu\nu} - g' Z_{\mu\nu}) W^+_\mu W^-_\nu - g'^2 (g A_\mu - g' Z_\mu)(W^+_{\mu\nu} W^-_\nu - W^-_{\mu\nu} W^+_\nu)\right], \quad (4.2)
\end{aligned}$$

where $j^{\rm em}_\mu = j^3_\mu + j^Y_\mu$ is the electromagnetic current. Next, the operators O_{BB}, O_{WW}, and O_{GG} change the normalization of the kinetic terms of the gauge bosons. To recover the canonical normalization we redefine the gauge fields as

$$B_\mu \to B_\mu \left(1 + \frac{c_{BB} g'^2}{4}\right), \ W^i_\mu \to W^i_\mu \left(1 + \frac{c_{WW} g^2}{4}\right), \ G^a_\mu \to G^a_\mu \left(1 + \frac{c_{GG} g_s^2}{4}\right). \quad (4.3)$$

The operator $\tilde{O}_{GG}$ contributes to the QCD θ-term which, for phenomenological reasons, should be extremely small. Therefore, we assume that this contribution if present, precisely cancels against the θ-term in the SM Lagrangian such that $|\theta_{\rm SM} + \theta_{\widetilde{GG}}| < 10^{-10}$. The operator O_H changes the normalization of the Higgs boson kinetic term, and also induces Higgs boson self-interactions that contain two derivatives. To recover the canonical normalization and remove the 2-derivative self-interactions we redefine the Higgs field as

$$h \to h\left(1 - c_H - \frac{h}{v}c_H - \frac{h^2}{3v^2}c_H\right). \tag{4.4}$$

The relation between the Higgs VEV v_0 and the mass parameter in the SM Lagrangian is affected by the O_{6H} operator:

$$v_0^2 = \frac{\mu_H^2}{\lambda}\left(1 + \frac{3}{4\lambda}c_{6H}\right), \tag{4.5}$$

while the relation between the Higgs boson mass and the quartic coupling in the SM Lagrangian is affected by both O_{6H} and O_H:

$$m_h^2 = 2v_0^2\left(\lambda - 2c_H\lambda - \frac{3}{2}c_{6H}\right). \tag{4.6}$$

We still need to ensure the condition #2 which requires that the tree-level relations between the couplings and the observables employed to determine them must be the same as in the SM. This is a non-trivial requirement, because dimension-6 operators affect the observables used to extract these parameters. We have seen that the operator O_{WB} shifts the electric charge and the Z boson mass. Similarly, the operator O_T shifts the Z boson mass term. Furthermore, one of the $O_{\ell\ell}$ operators leads to the 4-fermion coupling $v^{-2}[c_{\ell\ell}]_{1221}(\bar\nu_{\mu,L}\gamma_\rho\nu_{e,L})(\bar e_L\gamma_\rho\mu_L)$ that contributes to the muon decay at the linear level and thus effectively shifts the Fermi constant. Finally, the leptonic vertex operators $O_{H\ell}$ change the couplings of W to electrons and muons, and thus also effectively shift the Fermi constant. To undo these effects, we need to ensure that the photon and the gluon couple to the electromagnetic and strong currents as in Eq. (2.4). Furthermore, the Z boson mass term in the Lagrangian should be as in Eq. (2.3), and the tree-level $\mu \to e\bar\nu_e\nu_\mu$ decay width should be given by $\Gamma = \frac{m_\mu^5}{384\pi^3 v^4}$. This is achieved by the following redefinition of the coupling constants and the VEV:

$$\begin{aligned}
g_s &\to g_s\left(1 - c_{GG}\frac{g_s^2}{4}\right), \\
g &\to g\left(1 - c_{WW}\frac{g^2}{4} - c_{WB}\frac{g^2g'^2}{g^2-g'^2} + (c_T - \delta v)\frac{g^2}{g^2-g'^2}\right), \\
g' &\to g'\left(1 - c_{BB}\frac{g'^2}{4} + c_{WB}\frac{g^2g'^2}{g^2-g'^2} - (c_T - \delta v)\frac{g'^2}{g^2-g'^2}\right), \\
v_0 &\to v\,(1+\delta v),
\end{aligned} \tag{4.7}$$

where $\delta v = ([c'_{H\ell}]_{11} + [c'_{H\ell}]_{22})/2 - [c_{\ell\ell}]_{1221}/4$.

One last transformation is needed satisfy the condition #4. At this point, the coefficients of the contact $hVff$ and h^2Vff interactions differ from the vertex corrections to the Vff interactions by flavor universal terms depending only on the electric charge and the isospin of the fermions. It is possible to get rid of the latter using equations of motion for the gauge bosons, so as to trade them into zero- and two-derivative Higgs boson interactions with gauge bosons of the form $hV_\mu V_\mu$ and $hV_\mu \partial_\nu V_{\mu\nu}$. To this end, we add and subtract the following Lagrangian term:

$$
\begin{aligned}
\Delta\mathcal{L} &= \left(2\frac{h}{v}+\frac{h^2}{v^2}\right)[L_{\rm add}-L_{\rm add,\ eom}] \\
\mathcal{L}_{\rm add} &= \frac{g}{\sqrt{2}}\frac{g^2}{g^2-g'^2}\left(c_T-\delta v-g'^2 c_{WB}\right)\left(W^+_\mu j^-_\mu+\text{h.c.}\right) \\
&+ \sqrt{g^2+g'^2}\frac{1}{g^2-g'^2}\left((c_T-\delta v)(g^2 j^3_\mu+g'^2 j^Y_\mu)-g^2 g'^2 c_{WB}(j^3_\mu+j^Y_\mu)\right) Z_\mu
\end{aligned}
\tag{4.8}
$$

where $\mathcal{L}_{\rm add,\ eom}$ is $\mathcal{L}_{\rm add}$ with the fermionic currents j_μ eliminated in favor of bosonic terms using the equations of motion in Eq. (2.2). This step ensures the the coefficients of the vertex-like Higgs contact interactions $hVff$ and h^2Vff in the Lagrangian are proportional to the vertex correction to the SM Vff interactions.

After all these transformations, the conditions #1-#4 are satisfied. We can proceed to listing the corrections to the SM in $\Delta L_{d=6}$ in this representation. We will focus on interaction terms that are relevant for LHC phenomenology. Coefficients of all interaction terms in $\Delta L_{d=6}$ are $\mathcal{O}(1/\Lambda^2)$ in the EFT expansion, and will ignore all $\mathcal{O}(1/\Lambda^4)$ and higher contributions. To facilitate presentation, we split $\Delta L_{d=6}$ into the following parts,

$$
\Delta\mathcal{L}_{d=6}=\Delta\mathcal{L}_{\rm mass}+\Delta\mathcal{L}_{\rm vertex}+\mathcal{L}_{\rm dipole}+\Delta\mathcal{L}_{\rm tgc}+\Delta\mathcal{L}_{\rm qgc}+\Delta\mathcal{L}_{\rm h}+\mathcal{L}_{hvff}+\mathcal{L}_{hdvff}+\Delta\mathcal{L}_{h,\rm self}+\Delta\mathcal{L}_{h^2}+\mathcal{L}_{\rm other}. \tag{4.9}
$$

Below we define each term in order of appearance. In this section we give the Lagrangian in the unitary gauge when the Goldstone bosons eaten by W and Z are set to zero; see Appendix B for a generalization to the R_ξ gauge.

4.1 Quadratic terms

By construction, there are no corrections to quadratic terms of the SM mass eigenstates with the exception of the shift of the W boson mass in Eq. (2.3):

$$
\Delta\mathcal{L}_{\rm mass}=2\delta m\frac{g^2v^2}{4}W^+_\mu W^-_\mu. \tag{4.10}
$$

The relation between δm and the Wilson coefficients in the Warsaw and SILH bases is given by

$$
\begin{aligned}
\delta m &= \frac{1}{g^2-g'^2}\left[-g^2g'^2c_{WB}+g^2c_T-g'^2\delta v\right] \\
&= -\frac{g^2g'^2}{4(g^2-g'^2)}\left(s_W+s_B+s_{2W}+s_{2B}-\frac{4}{g'^2}s_T+\frac{2}{g^2}[s'_{H\ell}]_{22}\right).
\end{aligned}
\tag{4.11}
$$

4.2 Gauge boson interactions with fermions

Two types of corrections to the SM gauge boson interactions with fermions may be introduced by dimension-6 operators. One is the so-called *vertex corrections*, which shift the W and Z couplings to fermions away from the SM Lagrangian of Eq. (2.4):

$$\begin{aligned}\Delta\mathcal{L}_{\text{vertex}} &= \frac{g}{\sqrt{2}}\left(W_\mu^+\bar{\nu}_L\gamma_\mu\delta g_L^{W\ell}e_L + W_\mu^+\bar{u}\gamma_\mu\delta g_L^{Wq}d_L + W_\mu^+\bar{u}_R\gamma_\mu\delta g_R^{Wq}d_R + \text{h.c.}\right)\\ &+ \sqrt{g^2+g'^2}Z_\mu\left[\sum_{f\in u,d,e,\nu}\bar{f}_L\gamma_\mu\delta g_L^{Zf}f_L + \sum_{f\in u,d,e}\bar{f}_R\gamma_\mu\delta g_R^{Zf}f_R\right], \qquad (4.12)\end{aligned}$$

where all the δg are 3×3 Hermitian matrices in the generation space, except for δg_R^{Wq} which is a general 3×3 complex matrix. The vertex corrections to W and Z boson couplings to fermions are expressed by the Wilson coefficients in the Warsaw basis as

$$\begin{aligned}\delta g_L^{W\ell} &= c'_{H\ell} + f(1/2,0) - f(-1/2,-1),\\ \delta g_L^{Z\nu} &= \frac{1}{2}c'_{H\ell} - \frac{1}{2}c_{H\ell} + f(1/2,0),\\ \delta g_L^{Ze} &= -\frac{1}{2}c'_{H\ell} - \frac{1}{2}c_{H\ell} + f(-1/2,-1),\\ \delta g_R^{Ze} &= -\frac{1}{2}c_{He} + f(0,-1), \qquad (4.13)\end{aligned}$$

$$\begin{aligned}\delta g_L^{Wq} &= c'_{Hq}V_{\text{CKM}} + f(1/2,2/3) - f(-1/2,-1/3),\\ \delta g_R^{Wq} &= -\frac{1}{2}c_{Hud},\\ \delta g_L^{Zu} &= \frac{1}{2}c'_{Hq} - \frac{1}{2}c_{Hq} + f(1/2,2/3),\\ \delta g_L^{Zd} &= -\frac{1}{2}V_{\text{CKM}}^\dagger c'_{Hq}V_{\text{CKM}} - \frac{1}{2}V_{\text{CKM}}^\dagger c_{Hq}V_{\text{CKM}} + f(-1/2,-1/3),\\ \delta g_R^{Zu} &= -\frac{1}{2}c_{Hu} + f(0,2/3),\\ \delta g_R^{Zd} &= -\frac{1}{2}c_{Hd} + f(0,-1/3), \qquad (4.14)\end{aligned}$$

where

$$f(T^3,Q) = I_3\left[-Qc_{WB}\frac{g^2g'^2}{g^2-g'^2} + (c_T-\delta v)\left(T^3 + Q\frac{g'^2}{g^2-g'^2}\right)\right], \qquad (4.15)$$

and I_3 is the 3×3 identity matrix. The analogous expression in the SILH basis read

$$\begin{aligned}
\delta g_L^{Z\nu} &= \frac{1}{2}s'_{H\ell} - \frac{1}{2}s_{H\ell} + \hat{f}(1/2,0),\\
\delta g_L^{Ze} &= -\frac{1}{2}s'_{H\ell} - \frac{1}{2}s_{H\ell} + \hat{f}(-1/2,-1),\\
\delta g_R^{Ze} &= -\frac{1}{2}s_{He} + \hat{f}(0,-1),\\
\delta g_L^{Zu} &= \frac{1}{2}s'_{Hq} - \frac{1}{2}s_{Hq} + \hat{f}(1/2,2/3),\\
\delta g_L^{Zd} &= -\frac{1}{2}V_{\rm CKM}^\dagger s'_{Hq} V_{\rm CKM} - \frac{1}{2}V_{\rm CKM}^\dagger s_{Hq} V_{\rm CKM} + \hat{f}(-1/2,-1/3),\\
\delta g_R^{Zu} &= -\frac{1}{2}s_{Hu} + \hat{f}(0,2/3),\\
\delta g_R^{Zd} &= -\frac{1}{2}s_{Hd} + \hat{f}(0,-1/3),\\
\delta g_L^{W\ell} &= s'_{H\ell} + \hat{f}(1/2,0) - \hat{f}(-1/2,-1),\\
\delta g_L^{Wq} &= s'_{Hq}V_{\rm CKM} + \hat{f}(1/2,2/3) - \hat{f}(-1/2,-1/3),\\
\delta g_R^{Wq} &= -\frac{1}{2}s_{Hud},
\end{aligned} \tag{4.16}$$

where

$$\begin{aligned}
\hat{f}(T^3,Q) &\equiv \frac{1}{4}\left[g^2 s_{2W} + g'^2 s_{2B} + 4s_T - 2[s'_{H\ell}]_{22}\right] T^3\\
&+ \frac{g'^2}{4(g^2-g'^2)}\left[-(2g^2-g'^2)s_{2B} - g^2(s_{2W}+s_W+s_B) + 4s_T - 2[s'_{H\ell}]_{22}\right] Q.
\end{aligned} \tag{4.17}$$

Another type of gauge boson interactions with fermions, which does occur in the SM Lagrangian, are the so-called dipole interactions, We parametrize them as follows:

$$\begin{aligned}
\mathcal{L}_{\rm dipole} = -\frac{1}{4v}\Bigg[& g_s \sum_{f\in u,d} \bar{f}\sigma_{\mu\nu}T^a d_{Gf} f G^a_{\mu\nu} + e\sum_{f\in u,d,e} \bar{f}\sigma_{\mu\nu} d_{Af} f A_{\mu\nu} + \sqrt{g^2+g'^2}\sum_{f\in u,d,e} \bar{f}\sigma_{\mu\nu} d_{Zf} f Z_{\mu\nu}\\
& +\sqrt{2}g\left(\bar{d}_L\sigma_{\mu\nu} d_{Wu} u_R W^-_{\mu\nu} + \bar{u}_L\sigma_{\mu\nu} d_{Wd} d_R W^+_{\mu\nu} + \bar{\nu}_L\sigma_{\mu\nu} d_{We} e_R W^+_{\mu\nu} + \text{h.c.}\right)\\
& +g_s \sum_{f\in u,d} \bar{f}\sigma_{\mu\nu}T^a \tilde{d}_{Gf} f \widetilde{G}^a_{\mu\nu} + e\sum_{f\in u,d,e} \bar{f}\sigma_{\mu\nu} \tilde{d}_{Af} f \widetilde{A}_{\mu\nu} + \sqrt{g^2+g'^2}\sum_{f\in u,d,e} \bar{f}\sigma_{\mu\nu} \tilde{d}_{Zf} f \widetilde{Z}_{\mu\nu}\Bigg],
\end{aligned} \tag{4.18}$$

where $\sigma_{\mu\nu} = i[\gamma_\mu,\gamma_\nu]/2$, and d_{Af}, $\tilde{d}_{Af}$, d_{Zf}, $\tilde{d}_{Zf}$ are Hermitian 3×3 matrices, while d_{Wf} are general complex 3×3 matrices. The field strength tensors are defined as $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$, and $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\partial_\rho X_\sigma$. The coefficients d_{vf} are related to the

Wilson coefficients in the Warsaw basis as

$$
\begin{aligned}
d_{Gf} - i\tilde{d}_{Gf} &= -2\sqrt{2}c_{fG}, \\
d_{Af} - i\tilde{d}_{Af} &= -2\sqrt{2}\left(\eta_f c_{fW} + c_{fB}\right), \\
d_{Zf} - i\tilde{d}_{Zf} &= -\frac{2\sqrt{2}}{g^2+g'^2}\left(g^2\eta_f c_{fW} - g'^2 c_{fB}\right), \\
d_{Wf} &= -2\sqrt{2}c_{fW},
\end{aligned} \tag{4.19}
$$

where $\eta_u = +1$, $\eta_{d,e} = -1$, and the formulas in the SILH basis are the same with $c_i \to s_i$.

4.3 Gauge boson self-interactions

The corrections to the cubic interactions of gauge bosons in Eq. (2.7) are parametrized as

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm tgc} &= ie\left[\delta\kappa_\gamma A_{\mu\nu}\, W_\mu^+ W_\nu^- + \tilde{\kappa}_\gamma \tilde{A}_{\mu\nu}\, W_\mu^+ W_\nu^-\right] \\
&+ igc_\theta\left[\delta g_{1,z}\left(W_{\mu\nu}^+ W_\mu^- - W_{\mu\nu}^- W_\mu^+\right) Z_\nu + \delta\kappa_z\, Z_{\mu\nu}\, W_\mu^+ W_\nu^- + \tilde{\kappa}_z\, \tilde{Z}_{\mu\nu}\, W_\mu^+ W_\nu^-\right] \\
&+ i\frac{e}{m_W^2}\left[\lambda_\gamma W_{\mu\nu}^+ W_{\nu\rho}^- A_{\rho\mu} + \tilde{\lambda}_\gamma W_{\mu\nu}^+ W_{\nu\rho}^- \tilde{A}_{\rho\mu}\right] + i\frac{gc_\theta}{m_W^2}\left[\lambda_z W_{\mu\nu}^+ W_{\nu\rho}^- Z_{\rho\mu} + \tilde{\lambda}_z W_{\mu\nu}^+ W_{\nu\rho}^- \tilde{Z}_{\rho\mu}\right] \\
&+ \frac{c_{3G}}{v^2} g_s^3 f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c + \frac{\tilde{c}_{3G}}{v^2} g_s^3 f^{abc} \widetilde{G}_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c,
\end{aligned} \tag{4.20}
$$

The couplings of electroweak gauge bosons follow the customary parametrization of Ref. [7]. The anomalous triple gauge couplings of electroweak gauge bosons are related to the Wilson coefficients in the Warsaw basis as

$$
\begin{aligned}
\delta g_{1,z} &= \frac{g^2+g'^2}{g^2-g'^2}\left(-g'^2 c_{WB} + c_T - \delta v\right), \\
\delta\kappa_\gamma &= g^2 c_{WB}, \\
\delta\kappa_z &= -2c_{WB}\frac{g^2 g'^2}{g^2-g'^2} + \frac{g^2+g'^2}{g^2-g'^2}\left(c_T - \delta v\right), \\
\lambda_\gamma &= -\frac{3}{2}g^4 c_{3W}, \\
\lambda_z &= -\frac{3}{2}g^4 c_{3W}, \\
\tilde{\kappa}_\gamma &= g^2 \tilde{c}_{WB}, \\
\tilde{\kappa}_z &= -g'^2 \tilde{c}_{WB}, \\
\tilde{\lambda}_\gamma &= -\frac{3}{2}g^4 \tilde{c}_{3W}, \\
\tilde{\lambda}_z &= -\frac{3}{2}g^4 \tilde{c}_{3W}.
\end{aligned} \tag{4.21}
$$

The analogous relations for the SILH basis read

$$
\begin{aligned}
\delta g_{1z} &= -\frac{g^2+g'^2}{4(g^2-g'^2)}\left[(g^2-g'^2)s_{HW}+g^2(s_W+s_{2W})+g'^2(s_B+s_{2B})-4s_T+2[s'_{H\ell}]_{22}\right],\\
\delta\kappa_\gamma &= -\frac{g^2}{4}\left[s_{HW}+s_{HB}\right],\\
\delta\kappa_z &= -\frac{1}{4}\left(g^2 s_{HW}-g'^2 s_{HB}\right)-\frac{g^2+g'^2}{4(g^2-g'^2)}\left[g^2(s_W+s_{2W})+g'^2(s_B+s_{2B})-4s_T+2[s'_{H\ell}]_{22}\right],\\
\lambda_z &= -\frac{3}{2}g^4 s_{3W}, \qquad \lambda_\gamma=\lambda_z,\\
\delta\tilde\kappa_\gamma &= -\frac{g^2}{4}\left[\tilde s_{HW}+\tilde s_{HB}\right],\\
\delta\tilde\kappa_z &= \frac{g'^2}{4}\left[\tilde s_{HW}+\tilde s_{HB}\right],\\
\tilde\lambda_z &= -\frac{3}{2}g^4\tilde s_{3W}, \qquad \tilde\lambda_\gamma=\tilde\lambda_z.
\end{aligned}
\tag{4.22}
$$

The quartic gauge interactions can be parametrized as

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm qgc} &= \delta g_{W^4}\frac{g^2}{2}\left(W^+_\mu W^+_\mu W^-_\nu W^-_\nu - W^+_\mu W^-_\mu W^+_\nu W^-_\nu\right)\\
&+ \delta g_{W^2Z^2}g^2c_\theta^2\left(W^+_\mu Z_\mu W^-_\nu Z_\nu - W^+_\mu W^-_\mu Z_\nu Z_\nu\right)\\
&+ \delta g_{W^2ZA}g^2c_\theta s_\theta\left(W^+_\mu Z_\mu W^-_\nu A_\nu + W^+_\mu A_\mu W^-_\nu Z_\nu - 2W^+_\mu W^-_\mu Z_\nu A_\nu\right)\\
&- \frac{g^2}{2}\frac{\lambda_{W^4}}{m_W^2}\left(W^+_{\mu\nu}W^-_{\nu\rho}-W^-_{\mu\nu}W^+_{\nu\rho}\right)\left(W^+_\mu W^-_\rho - W^-_\mu W^+_\rho\right)\\
&- g^2c_\theta^2\frac{\lambda_{W^2Z^2}}{m_W^2}\left[W^+_\mu\left(Z_{\mu\nu}W^-_{\nu\rho}-W^-_{\mu\nu}Z_{\nu\rho}\right)Z_\rho + W^-_\mu\left(Z_{\mu\nu}W^+_{\nu\rho}-W^+_{\mu\nu}Z_{\nu\rho}\right)Z_\rho\right]\\
&- e^2\frac{\lambda_{W^2A^2}}{m_W^2}\left[W^+_\mu\left(A_{\mu\nu}W^-_{\nu\rho}-W^-_{\mu\nu}A_{\nu\rho}\right)A_\rho + W^-_\mu\left(A_{\mu\nu}W^+_{\nu\rho}-W^+_{\mu\nu}A_{\nu\rho}\right)A_\rho\right]\\
&- egc_\theta\frac{\lambda_{W^2AZ}}{m_W^2}\left[W^+_\mu\left(A_{\mu\nu}W^-_{\nu\rho}-W^-_{\mu\nu}A_{\nu\rho}\right)Z_\rho + W^-_\mu\left(A_{\mu\nu}W^+_{\nu\rho}-W^+_{\mu\nu}A_{\nu\rho}\right)Z_\rho\right]\\
&- egc_\theta\frac{\lambda_{W^2ZA}}{m_W^2}\left[W^+_\mu\left(Z_{\mu\nu}W^-_{\nu\rho}-W^-_{\mu\nu}Z_{\nu\rho}\right)A_\rho + W^-_\mu\left(Z_{\mu\nu}W^+_{\nu\rho}-W^+_{\mu\nu}Z_{\nu\rho}\right)A_\rho\right]\\
&+ 3g_s^3\frac{c_{4G}}{v^2}f^{abc}f^{cde}G^a_{\mu\nu}G^b_{\nu\rho}G^d_\rho G^e_\mu + \text{CP odd},
\end{aligned}
\tag{4.23}
$$

where CP odd stands for analogous terms with $\lambda_z \to \tilde\lambda_z$, $c_{4G} \to \tilde c_{4G}$, and one of the field strength tensors replaced by the dual one. The parameters in Eq. (4.23) can be expressed by the corrections to the triple gauge couplings

$$
\begin{aligned}
\delta g_{W^4} &= \delta g_{W^2Z^2} = \delta g_{W^2ZA} = \delta g_{1,z},\\
\lambda_{W^4} &= \lambda_{W^2Z^2} = \lambda_{W^2A^2} = \lambda_{W^2AZ} = \lambda_{W^2ZA} = \lambda_z,\\
c_{4G} &= c_{3G},
\end{aligned}
\tag{4.24}
$$

and analogous formulas hold for the CP-odd couplings with $\lambda \to \tilde\lambda$ and $c \to \tilde c$.

4.4 Single Higgs couplings

This part is the most relevant one from the point of view of the LHC Higgs phenomenology. First, we define the following single Higgs boson couplings to a pair of the SM fields:

$$
\begin{aligned}
\Delta\mathcal{L}_{\rm h} \;=\; & \frac{h}{v}\Big[2\delta c_w m_W^2 W_\mu^+ W_\mu^- + \delta c_z m_Z^2 Z_\mu Z_\mu \\
& - \sum_{f\in u,d,e}\sum_{ij}\sqrt{m_{f_i}m_{f_j}}[\delta y_f]_{ij}\left[\cos\phi_{ij}^f \bar f_i f_j - i\sin\phi_{ij}^f \bar f_i\gamma_5 f_j\right] \\
& + c_{ww}\frac{g^2}{2}W_{\mu\nu}^+W_{\mu\nu}^- + \tilde c_{ww}\frac{g^2}{2}W_{\mu\nu}^+\tilde W_{\mu\nu}^- + c_{w\Box}g^2\left(W_\mu^-\partial_\nu W_{\mu\nu}^+ + \text{h.c.}\right) \\
& + c_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}A_{\mu\nu} + c_{z\gamma}\frac{e\sqrt{g^2+g'^2}}{2}Z_{\mu\nu}A_{\mu\nu} + c_{zz}\frac{g^2+g'^2}{4}Z_{\mu\nu}Z_{\mu\nu} \\
& + c_{z\Box}g^2 Z_\mu\partial_\nu Z_{\mu\nu} + c_{\gamma\Box}gg' Z_\mu\partial_\nu A_{\mu\nu} \\
& + \tilde c_{gg}\frac{g_s^2}{4}G_{\mu\nu}^a\tilde G_{\mu\nu}^a + \tilde c_{\gamma\gamma}\frac{e^2}{4}A_{\mu\nu}\tilde A_{\mu\nu} + \tilde c_{z\gamma}\frac{e\sqrt{g^2+g'^2}}{2}Z_{\mu\nu}\tilde A_{\mu\nu} + \tilde c_{zz}\frac{g^2+g'^2}{4}Z_{\mu\nu}\tilde Z_{\mu\nu}\Big],
\end{aligned}
\tag{4.25}
$$

where all the couplings above are real. The terms in the first two lines shift the SM couplings in Eq. (2.5), while the remaining terms introduce Higgs couplings to matter with a tensor structure that is absent in the SM Lagrangian. Note that, using equations of motion, we could get rid of certain 2-derivative interactions between the Higgs and gauge bosons: $hZ_\mu\partial_\nu Z_{\nu\mu}$, $hZ_\mu\partial_\nu A_{\nu\mu}$, and $hW_\mu^\pm\partial_\nu W_{\nu\mu}^\mp$. These interactions would then be traded for contact interactions of the Higgs, gauge bosons and fermions in Eq. (4.30). However, one of the defining features of our effective Lagrangian is that the coefficients of the latter couplings are equal to the corresponding vertex correction in Eq. (4.12). This form can be always obtained, without any loss of generality, starting from an arbitrary dimension-6 Lagrangian provided the 2-derivative $hV_\mu\partial_\nu V_{\nu\mu}$ are kept in the Lagrangian. Note that we work in the limit where the neutrinos are massless and the Higgs boson does not couple to the neutrinos. In the EFT context, the couplings to neutrinos induced by dimension-5 operators are proportional to neutrino masses, therefore they are far too small to have any relevance for LHC phenomenology.

The shifts of the Higgs couplings to W and Z bosons are related to the Wilson coefficients in the Warsaw and SILH basis by

$$
\begin{aligned}
\delta c_w &= -c_H - c_{WB}\frac{4g^2g'^2}{g^2-g'^2} + 4c_T\frac{g^2}{g^2-g'^2} - \delta v\frac{3g^2+g'^2}{g^2-g'^2} \\
&= -s_H - \frac{g^2g'^2}{g^2-g'^2}\left[s_W + s_B + s_{2W} + s_{2B} - \frac{4}{g'^2}s_T + \frac{3g^2+g'^2}{2g^2g'^2}[s'_{H\ell}]_{22}\right], \\
\delta c_z &= -c_H - 3\delta v \\
&= -s_H - \frac{3}{2}[s'_{H\ell}]_{22},
\end{aligned}
\tag{4.26}
$$

The Yukawa interactions are related to the Wilson coefficients in the Warsaw and

SILH basis by

$$
\begin{aligned}
[\delta y_f]_{ij} \cos\phi^f_{ij} &= \frac{1}{\sqrt{2}}\mathrm{Re}[c_f]_{ij} - \delta_{ij}\,(c_H + \delta v) \\
&= \frac{1}{\sqrt{2}}\mathrm{Re}[s_f]_{ij} - \delta_{ij}\left[s_H + \frac{1}{2}[s'_{H\ell}]_{22}\right], \\
[\delta y_f]_{ij} \sin\phi^f_{ij} &= \frac{1}{\sqrt{2}}\mathrm{Im}[c_f]_{ij} \\
&= \frac{1}{\sqrt{2}}\mathrm{Im}[s_f]_{ij}.
\end{aligned}
\tag{4.27}
$$

The two-derivative Higgs couplings to gauge bosons are related to the Wilson coefficients in the Warsaw basis by

$$
\begin{aligned}
c_{gg} &= c_{GG}, \\
c_{\gamma\gamma} &= c_{WW} + c_{BB} - 4c_{WB}, \\
c_{zz} &= \frac{g^4 c_{WW} + g'^4 c_{BB} + 4g^2 g'^2 c_{WB}}{(g^2+g'^2)^2}, \\
c_{z\Box} &= -\frac{2}{g^2}\,(c_T - \delta v)\,, \\
c_{z\gamma} &= \frac{g^2 c_{WW} - g'^2 c_{BB} - 2(g^2 - g'^2)c_{WB}}{g^2+g'^2}, \\
c_{\gamma\Box} &= \frac{2}{g^2-g'^2}\left((g^2+g'^2)c_{WB} - 2c_T + 2\delta v\right), \\
c_{ww} &= c_{WW}, \\
c_{w\Box} &= \frac{2}{g^2-g'^2}\left(g'^2 c_{WB} - c_T + \delta v\right).
\end{aligned}
\tag{4.28}
$$

and the same for the CP-odd couplings $\tilde{c}_{gg}$, $\tilde{c}_{\gamma\gamma}$, $\tilde{c}_{z\gamma}$, $\tilde{c}_{zz}$, $\tilde{c}_{ww}$, with $c \to \tilde{c}$ on the right hand side. The analogous expressions for the SILH basis read

$$
\begin{aligned}
c_{gg} &= s_{GG}, \\
c_{\gamma\gamma} &= s_{BB}, \\
c_{zz} &= -\frac{1}{g^2+g'^2}\left[g^2 s_{HW} + g'^2 s_{HB} - g'^2 s_\theta^2 s_{BB}\right], \\
c_{z\Box} &= \frac{1}{2g^2}\left[g^2(s_W + s_{HW} + s_{2W}) + g'^2(s_B + s_{HB} + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right], \\
c_{z\gamma} &= \frac{s_{HB} - s_{HW}}{2} - s_\theta^2 s_{BB}, \\
c_{\gamma\Box} &= \frac{s_{HW} - s_{HB}}{2} + \frac{1}{g^2-g'^2}\left[g^2(s_W + s_{2W}) + g'^2(s_B + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right], \\
c_{ww} &= -s_{HW}, \\
c_{w\Box} &= \frac{s_{HW}}{2} + \frac{1}{2(g^2-g'^2)}\left[g^2(s_W + s_{2W}) + g'^2(s_B + s_{2B}) - 4s_T + 2[s'_{H\ell}]_{22}\right],
\end{aligned}
\tag{4.29}
$$

Next, couplings of the Higgs boson to a gauge field and two fermions (which are not present in the SM Lagrangian) can be generated by dimension-6 operators. The vertex-like contact interactions between the Higgs, electroweak gauge bosons, and fermions are parametrized as:

$$\begin{aligned}\mathcal{L}_{hvff} &= \sqrt{2}g\frac{h}{v}W_\mu^+\left(\bar{u}_L\gamma_\mu\delta g_L^{hWq}d_L + \bar{u}_R\gamma_\mu\delta g_R^{hWq}d_R + \bar{\nu}_L\gamma_\mu\delta g_L^{hW\ell}e_L\right) + \text{h.c.}\\ &+ 2\frac{h}{v}\sqrt{g^2+g'^2}Z_\mu\left[\sum_{f=u,d,e,\nu}\bar{f}_L\gamma_\mu\delta g_L^{hZf}f_L + \sum_{f=u,d,e}\bar{f}_R\gamma_\mu\delta g_R^{hZf}f_R\right], \end{aligned}\tag{4.30}$$

As discussed before, by construction, the coefficients of these interaction are equal to the corresponding vertex correction in Eq. (4.12):

$$\delta g^{hzf} = \delta g^{Zf}, \qquad \delta g^{hWf} = \delta g^{Wf}. \tag{4.31}$$

The dipole-type contact interactions of the Higgs boson are parametrized as:

$$\begin{aligned}\mathcal{L}_{\text{hdvff}} &= -\frac{h}{4v^2}\Bigg[g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a d_{hGf}fG^a_{\mu\nu} + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{hAf}fA_{\mu\nu} + \sqrt{g^2+g'^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}d_{hZf}fZ_{\mu\nu}\\ &+\sqrt{2}g\left(\bar{d}_L\sigma_{\mu\nu}d_{hWu}u_RW^-_{\mu\nu} + \bar{u}_L\sigma_{\mu\nu}d_{hWd}d_RW^+_{\mu\nu} + \bar{\nu}_L\sigma_{\mu\nu}d_{hWe}e_RW^+_{\mu\nu} + \text{h.c.}\right)\\ &+g_s\sum_{f\in u,d}\bar{f}\sigma_{\mu\nu}T^a\tilde{d}_{hGf}f\widetilde{G}^a_{\mu\nu} + e\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{hAf}f\widetilde{A}_{\mu\nu} + \sqrt{g^2+g'^2}\sum_{f\in u,d,e}\bar{f}\sigma_{\mu\nu}\tilde{d}_{hZf}f\widetilde{Z}_{\mu\nu}\Bigg],\end{aligned}\tag{4.32}$$

where d_{hAf}, $\tilde{d}_{hAf}$, d_{hZf}, $\tilde{d}_{hZf}$ are Hermitian 3×3 matrices, while d_{hWf} are general complex 3×3 matrices. The coefficients are simply related to the corresponding dipole interactions in Eq. (4.18):

$$d_{hVf} = d_{Vf}. \tag{4.33}$$

Dimension-6 operators can also induce single Higgs couplings to 3 gauge bosons, but we do not display them in this note.

4.5 Higgs boson self-couplings

Corrections to the Higgs boson self-couplings in the SM are parametrized as

$$\Delta\mathcal{L}_{h,\text{self}} = -\delta\lambda_3 v h^3 - \delta\lambda_4 h^4. \tag{4.34}$$

The relation between the cubic corrections and the Wilson coefficients in the Warsaw and SILH basis is given by

$$\begin{aligned}\delta\lambda_3 &= -\lambda\left(3c_H + \delta v\right) - c_{6H}\\ &= -\lambda\left(3s_H + \frac{1}{2}[s'_{H\ell}]_{22}\right) - s_{6H}.\end{aligned}\tag{4.35}$$

The correction to the quartic Higgs boson term in Eq. (4.34) can be expressed as

$$\delta\lambda_4 = \frac{3}{2}\delta\lambda_3 - \frac{m_h^2}{6v^2}\delta c_z. \tag{4.36}$$

Self-interactions with more than 4 fields can also arise from dimension-6 operators, but we do not display them in this note.

4.6 Couplings of two or more Higgs bosons

To describe double Higgs production at the LHC we need, apart from a subset of the single Higgs couplings introduced in Section 4.4 and the cubic Higgs self-interaction in Eq. (4.34), the interactions between two Higgs bosons and two other SM fields. They are parametrized as follows:

$$
\begin{aligned}
\Delta\mathcal{L}_{hh} \;=\;& \frac{h^2}{v^2}\left(\delta c_z^{(2)}\frac{g^2+g'^2}{2}Z_\mu Z_\mu + \delta c_w^{(2)} g^2 W^+_\mu W^-_\mu\right) - \frac{h^2}{2v^2}\sum_{f;ij}\sqrt{m_{f_i}m_{f_j}}\left[\bar f_{i,R}[y_f^{(2)}]_{ij} f_{j,L} + \text{h.c.}\right]. \\
&+ \frac{h^2}{8v^2}\left(c_{gg}^{(2)} g_s^2 G^a_{\mu\nu}G^a_{\mu\nu} + 2c^{(2)}_{ww} g^2 W^+_{\mu\nu}W^-_{\mu\nu} + c^{(2)}_{zz}(g^2+g'^2)Z_{\mu\nu}Z_{\mu\nu} + 2c^{(2)}_{z\gamma} g g' Z_{\mu\nu}A_{\mu\nu} + c^{(2)}_{\gamma\gamma} e^2 A_{\mu\nu}A_{\mu\nu}\right) \\
&+ \frac{h^2}{8v^2}\left(\tilde c_{gg}^{(2)} g_s^2 G^a_{\mu\nu}\tilde G^a_{\mu\nu} + 2\tilde c^{(2)}_{ww} g^2 W^+_{\mu\nu}\tilde W^-_{\mu\nu} + \tilde c^{(2)}_{zz}(g^2+g'^2)Z_{\mu\nu}\tilde Z_{\mu\nu} + 2\tilde c^{(2)}_{z\gamma} g g' Z_{\mu\nu}\tilde A_{\mu\nu} + \tilde c^{(2)}_{\gamma\gamma} e^2 A_{\mu\nu}\tilde A_{\mu\nu}\right) \\
&- \frac{h^2}{2v^2}\left(g^2 c^{(2)}_{w\Box}(W^+_\mu\partial_\nu W^-_{\nu\mu} + W^-_\mu \partial_\nu W^+_{\nu\mu}) + g^2 c^{(2)}_{z\Box} Z_\mu\partial_\nu Z_{\nu\mu} + g g' c^{(2)}_{\gamma\Box} Z_\mu \partial_\nu A_{\nu\mu}\right). \qquad (4.37)
\end{aligned}
$$

All double Higgs couplings arising from d=6 operators can be expressed by the single Higgs couplings:

$$
\begin{aligned}
\delta c_z^{(2)} &= \delta c_z, \qquad \delta c_w^{(2)} = \delta c_z + 3\delta m, \\
[y_f^{(2)}]_{ij} &= 3[\delta y_f]_{ij} e^{i\phi_{ij}} - \delta c_z\,\delta_{ij}, \\
c^{(2)}_{vv} &= c_{vv}, \qquad \tilde c^{(2)}_{vv} = \tilde c_{vv}, \qquad v \in \{g, w, z, \gamma\}, \\
c^{(2)}_{v\Box} &= c_{v\Box}, \qquad v \in \{w, z, \gamma\}. \qquad (4.38)
\end{aligned}
$$

Other interaction terms with two Higgs bosons involve at least 5 fields: e.g the h^2V^3 or $h^2 f f V$ contact interactions. We do not display them in this note.

4.7 Other terms

In the subsections above we wrote down interaction terms in the effective Lagrangian that are relevant for SM precision tests and for Higgs searches at the LHC. The remaining terms, which are not explicitly displayed in this note, are contained in $\mathcal{L}_{\rm other}$. They include 4-fermion terms, couplings of a single Higgs boson to 3 or more gauge bosons, dipole-like interactions of two gauge bosons and two fermions, and interaction terms with 5 or more fields. Currently, these terms are not relevant for single and double Higgs production and decay at the LHC. If phenomenological interest is presented, any of the terms in $\mathcal{L}_{\rm other}$ can be explicitly written down in this note.

5 Higgs basis

In principle, there is no theoretical obstacle to present the results of LHC Higgs analyses as constraints on the Wilson coefficients in the Warsaw or SILH basis. However, this procedure may not be the most efficient one. One difficulty is that, in those bases, one needs to consider a large number of parameters, however the LHC Higgs observables depend only on a smaller number of linear combinations of the Wilson coefficients. Another practical difficulty is that some of these linear combinations are already stringently constrained by electroweak precisions tests, such that they cannot yield observable effects at the LHC. In this section we propose a more convenient parametrization of the effective Lagrangian with d=6 operators, along the lines of the *EFT primaries* in Ref. [2].

The salient features of our proposal are the following. The goal is to parametrize the d=6 operators in a way that can be more directly connected to observable quantities in Higgs physics. We call this parametrization the *Higgs basis*. Technically, the Higgs basis can be defined as a linear transformation from the Warsaw or SILH basis into the coefficients of certain interaction terms of the mass eigenstates (in particular the W, Z, and the Higgs bosons) in the effective Lagrangian. In practice, we will define the Higgs basis by choosing a subset of the couplings multiplying interaction terms in the effective Lagrangian Eq. (4.1) defined in Section 4. We will refer to this subset as the *independent couplings*. The number of independent couplings is the same as the number of independent operators in the Warsaw or SILH basis. They define the space of all possible deformations of the SM Lagrangian in the presence of d=6 operators. The independent couplings include the single Higgs couplings to gauge bosons and fermions, such that the parameters of the Higgs basis can be easily related to LHC Higgs observables. Furthermore, the vertex corrections to the Z boson interactions with fermions are among the independent couplings so that the stringent constraints from the Z and W partial decay widths can be incorporated in a transparent way.

The number of interaction terms in the effective Lagrangian of Eq. (4.1) is larger than the number of Wilson coefficients in a dimension-6 EFT basis. Due to this fact, some of the parameters in $\Delta\mathcal{L}_{d=6}$ can be expressed by the independent couplings; we call them the *dependent* couplings. The relations between dependent and independent couplings can be inferred from the matching between the effective Lagrangian and the Warsaw or SILH basis in Section 3. These relations *hold at the level of the dimension-6 Lagrangian*, and they are in general not respected in the presence of dimension-8 and higher operators. Of course, the choice which couplings are independent and which are dependent is a subjective choice dictated by convenience. In our case, the choice of the independent couplings was motivated by their direct connection to observables constrained by electroweak precision tests and Higgs searches. However, other choices can be envisaged and may be more convenient for other applications.

5.1 Independent couplings

We select a subset of couplings in the effective Lagrangian of Eq. (4.1) that has a 1-to-1 mapping to the Wilson coefficients in the Warsaw or SILH basis (or any other dimension-6 basis). This subset of independent couplings defines the Higgs basis. It can be used on par with any other basis to describe the effect of dimension-6 operators on physical observables.

The first group of independent couplings are the ones affecting the W boson mass and the Z and W boson couplings to fermions:

$$\begin{gathered}\delta m,\ \delta g_L^{Ze},\ \delta g_R^{Ze},\ \delta g_L^{W\ell},\ \delta g_L^{Zu},\ \delta g_R^{Zu},\ \delta g_L^{Zd},\ \delta g_R^{Zd},\ \delta g_R^{Wq},\\ d_{Gu},\ d_{Gd},\ d_{Ae},\ d_{Au},\ d_{Ad},\ d_{Ze},\ d_{Zu},\ d_{Zd},\ \tilde{d}_{Gu},\ \tilde{d}_{Gd},\ \tilde{d}_{Ae},\ \tilde{d}_{Au},\ \tilde{d}_{Ad},\ \tilde{d}_{Ze},\ \tilde{d}_{Zu},\ \tilde{d}_{Zd}.\end{gathered} \tag{5.1}$$

Here the mass correction δm is defined in Eq. (4.10), the vertex corrections δg^i are defined in Eq. (4.12), and the dipole moments d_i are defined in Eq. (4.18). While they are free parameters from the EFT point of view, precision measurements constrain them to be small. In particular, most of the parameters in the first line are constrained to be $\lesssim 10^{-2} - 10^{-4}$ [10]. The remaining parameters are constrained by measurements of the magnetic and electric dipole moments. Therefore, even if combinations of dimension-6 operators defined by the independent couplings in Eq. (5.1) affect the Higgs observables, it is well-motivated to neglect them in LHC Higgs analyses whose precision is worse than the existing constraints.

The second group of independent couplings are the ones describing the interactions of the Higgs boson with the SM gauge boson, fermions, and with itself:

$$\begin{aligned} c_{gg},\ \delta c_z,\ c_{\gamma\gamma},\ c_{z\gamma},\ c_{zz},\ c_{z\Box},\ \tilde{c}_{gg},\ \tilde{c}_{\gamma\gamma},\ \tilde{c}_{z\gamma},\ \tilde{c}_{zz}, \\ \delta y_u,\ \delta y_d,\ \delta y_e,\ \sin\phi_u,\ \sin\phi_d,\ \sin\phi_\ell,\ \delta\lambda_3. \end{aligned} \tag{5.2}$$

They are defined by Eq. (4.25), except for the last one which is defined in Eq. (4.37). As opposed to the ones in Eq. (5.1), the combinations of Wilson coefficients corresponding to the independent couplings in Eq. (5.2) are weakly constrained by SM precision tests. In fact, the strongest limits on these couplings typically come from Higgs searches. An important task of the LHC collaborations is to provide model-independent limits on the parameters in Eq. (5.2).

The third group of independent couplings are related to gauge bosons self-couplings:

$$\lambda_z,\ \tilde{\lambda}_z,\ c_{3G},\ \tilde{c}_{3G}. \tag{5.3}$$

They are defined in Eq. (4.20). These couplings do not affect Higgs searches, and they are only weakly constrained by SM precision tests.

To complete the definition of the Higgs basis, one has to include the independent couplings corresponding to 4-fermion operators. We choose to parametrize them by the same set of Wilson coefficients as in the Warsaw basis:

$$\begin{aligned} c_{\ell\ell},\ c_{qq},\ c^{\prime}_{qq},\ c_{\ell q},\ c^{\prime}_{\ell q},\ c_{quqd},\ c^{\prime}_{quqd},\ c_{\ell equ},\ c^{\prime}_{\ell equ},\ c_{\ell edq}, \\ c_{\ell e},\ c_{\ell u},\ c_{\ell d},\ c_{qe},\ c_{qu},\ c^{\prime}_{qu},\ c_{qd},\ c^{\prime}_{qd},\ c_{ee},\ c_{uu},\ c_{dd},\ c_{eu},\ c_{ed},\ c_{ud},\ c^{\prime}_{ud}. \end{aligned} \tag{5.4}$$

The parameters c_{ff} have 4 flavor indices. The non-trivial question of which combination of flavor indices constitutes an independent set was worked out in Ref. [8]. In the Higgs basis we take the same choice of independent 4-fermion couplings as in that reference, with one exception. As explained in the next subsection, in the Higgs basis the coupling $[c_\ell]_{1221}$ is a dependent coupling that can be expressed by δm and δg^i. Therefore $[c_\ell]_{1221}$ is not among the independent couplings defining the Higgs basis.

5.2 Dependent couplings

The remaining couplings in the effective Lagrangian are called the dependent couplings because, at the level of a dimension-6 EFT Lagrangian, they can be expressed by the independent couplings defining the Higgs basis. To obtain the relations between the dependent and independent couplings one can use the matching between the Warsaw basis and the effective Lagrangian worked out in Section 3.1. The procedure is to solve for the Warsaw basis Wilson coefficients in terms of the independent couplings and eliminate the former from the expressions for the dependent couplings.

We start with the dependent couplings in Eq. (4.25) describing the single Higgs boson interactions with matter. They can be expressed in terms of the independent couplings as[6]

$$
\begin{aligned}
\delta c_w &= \delta c_z + 4\delta m, \\
c_{ww} &= c_{zz} + 2s_\theta^2 c_{z\gamma} + s_\theta^4 c_{\gamma\gamma}, \\
\tilde{c}_{ww} &= \tilde{c}_{zz} + 2s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_{\gamma\gamma}, \\
c_{w\Box} &= \frac{1}{g^2 - g'^2}\left[g^2 c_{z\Box} + g'^2 c_{zz} - e^2 s_\theta^2 c_{\gamma\gamma} - (g^2 - g'^2) s_\theta^2 c_{z\gamma}\right], \\
c_{\gamma\Box} &= \frac{1}{g^2 - g'^2}\left[2g^2 c_{z\Box} + (g^2 + g'^2) c_{zz} - e^2 c_{\gamma\gamma} - (g^2 - g'^2) c_{z\gamma}\right].
\end{aligned} \tag{5.5}
$$

The coefficients of W-boson dipole interactions in Eq. (4.18) are related to those of the Z and the photon as

$$
\eta_f d_{wf} = d_{zf} - i\tilde{d}_{zf} + s_\theta^2 (d_{Af} - i\tilde{d}_{Af}), \tag{5.6}
$$

where $\eta_u = 1$ and $\eta_{d,e} = -1$. The coefficients of the dipole-like Higgs couplings in Eq. (4.32) are simply related to the corresponding dipole moments:

$$
d_{hvf} = d_{vf}, \quad \tilde{d}_{hvf} = \tilde{d}_{vf}, \qquad v \in \{g, w, z, \gamma\}. \tag{5.7}
$$

The correction to the quartic Higgs boson term in Eq. (4.34) is given by

$$
\delta\lambda_4 = \frac{3}{2}\delta\lambda_3 - \frac{m_h^2}{6v^2}\delta c_z. \tag{5.8}
$$

Coefficients of all interaction terms with two Higgs bosons in Eq. (4.37) are dependent couplings. The can be expressed in terms of the independent couplings as:

$$
\begin{aligned}
\delta c_z^{(2)} &= \delta c_z, \qquad \delta c_w^{(2)} = \delta c_z + 3\delta m, \\
[y_f^{(2)}]_{ij} &= 3[\delta y_f]_{ij} e^{i\phi_{ij}} - \delta c_z\, \delta_{ij}, \\
c_{vv}^{(2)} &= c_{vv}, \qquad \tilde{c}_{vv}^{(2)} = \tilde{c}_{vv}, \qquad v \in \{g, w, z, \gamma\}, \\
c_{v\Box}^{(2)} &= c_{v\Box}, \qquad v \in \{w, z, \gamma\}.
\end{aligned} \tag{5.9}
$$

The dependent vertex corrections are expressed in terms of the independent ones as

$$
\delta g_L^{Z\nu} = \delta g_L^{Ze} + \delta g_L^{W\ell}, \qquad \delta g_L^{Wq} = \delta g_L^{Zu} V_{\rm CKM} - V_{\rm CKM}\delta g_L^{Zd}. \tag{5.10}
$$

Note that we choose the W couplings to leptons (rather than the Z couplings to neutrinos) as our independent couplings, because in the flavor non-universal case the former are more directly constrained by experiment (in particular, in leptonic W decays measured at LEP).

[6] The relation between c_{ww}, $\tilde{c}_{ww}$ and other parameters can also be viewed as a consequence of the accidental custodial symmetry at the level of the dimension-6 operators [11].

Next, all but two triple gauge couplings in Eq. (4.20) are dependent couplings expressed in terms of the independent couplings as

$$\begin{aligned}
\delta g_{1,z} &= \frac{1}{2(g^2-g'^2)}\left[c_{\gamma\gamma}e^2 g'^2 + c_{z\gamma}(g^2-g'^2)g'^2 - c_{zz}(g^2+g'^2)g'^2 - c_{z\Box}(g^2+g'^2)g^2\right] \\
\delta\kappa_\gamma &= -\frac{g^2}{2}\left(c_{\gamma\gamma}\frac{e^2}{g^2+g'^2} + c_{z\gamma}\frac{g^2-g'^2}{g^2+g'^2} - c_{zz}\right), \\
\tilde\kappa_\gamma &= -\frac{g^2}{2}\left(\tilde c_{\gamma\gamma}\frac{e^2}{g^2+g'^2} + \tilde c_{z\gamma}\frac{g^2-g'^2}{g^2+g'^2} - \tilde c_{zz}\right), \\
\delta\kappa_z &= \delta g_{1,z} - t_\theta^2\delta\kappa_\gamma, \qquad \tilde\kappa_z = -t_\theta^2\tilde\kappa_\gamma, \\
\lambda_\gamma &= \lambda_z, \quad \tilde\lambda_\gamma = \tilde\lambda_z.
\end{aligned} \tag{5.11}$$

Note that $\delta g_{1,z}$, $\delta\kappa_\gamma$, and $\tilde\kappa_\gamma$ are *dependent* couplings here, unlike in Ref. [2]. Our motivation is that the Higgs basis should be parametrized such that the connection with Higgs observables is the simplest. However, for the sake of studying WW and WZ production a different set of independent couplings would be more convenient. For example, one could choose the independent couplings as $\delta g_{1,z}$, $\delta\kappa_\gamma$, λ_z, $\tilde\kappa_\gamma$, $\tilde\lambda_z$, and consider $c_{z\Box}$, c_{zz}, and $\tilde c_{zz}$ as dependent couplings expressed in terms of this set.

The corrections to quartic gauge boson self-couplings in Eq. (4.23) are all dependent. They can be expressed by corrections to triple gauge couplings as

$$\begin{aligned}
\delta g_{W^4} &= \delta g_{W^2Z^2} = \delta g_{W^2ZA} = \delta g_{1,z}, \\
\lambda_{W^4} &= \lambda_{W^2Z^2} = \lambda_{W^2A^2} = \lambda_{W^2AZ} = \lambda_{W^2ZA} = \lambda_z, \\
c_{4G} &= c_{3G},
\end{aligned} \tag{5.12}$$

Finally, we discuss how the Wilson coefficient $[c_{\ell\ell}]_{1221}$ of the 2-electron-2-muon operator is expressed by the independent couplings. One feature of the effective Lagrangian Eq. (4.1) is that the tree-level relations between the SM electroweak parameters and input observables are not affected by new physics. On the other hand, one of the four-fermion couplings in the Lagrangian,

$$\mathcal{L}_{4f}^{D=6} \supset [c_{\ell\ell}]_{1221}(\bar\ell_{1,L}\gamma_\rho\ell_{2,L})(\bar\ell_{2,L}\gamma_\rho\ell_{1,L}) \tag{5.13}$$

does affect the relation between the parameter v and the muon decay width from which $G_F = 1/\sqrt{2}v^2$ is determined:

$$\frac{\Gamma(\mu\to e\nu\nu)}{\Gamma(\mu\to e\nu\nu)_{\rm SM}} \approx 1 + 2[\delta g_L^{We}]_{11} + 2[\delta g_L^{We}]_{22} - 4\delta m - [c_{\ell\ell}]_{1221}. \tag{5.14}$$

Therefore, the muon decay width is unchanged with respect to the SM when $[c_{\ell\ell}]_{1221}$ is related to δm and δg as

$$[c_{\ell\ell}]_{1221} = 2\delta[g_L^{We}]_{11} + 2[\delta g_L^{We}]_{22} - 4\delta m. \tag{5.15}$$

In other words, due to the fact that we defined δm as an independent coupling in the Higgs basis , $[c_{\ell\ell}]_{1221}$ has to be a dependent coupling. Of course, one could equivalently choose $[c_{\ell\ell}]_{1221}$ to define the Higgs basis, and remove δm from the list of independent couplings.

5.3 Summary and comments

In summary, the Higgs basis is parametrized by the independent couplings in Eqs. (5.1), (5.2), (5.3), (5.4). In total, the Higgs basis, as any complete basis at the dimension-6 level, is parametrized by 2499 independent real couplings [8]. One should not, however, be intimidated by this number. The point is that a much smaller subset in Eq. (5.2) is adequate for EFT analyses of Higgs data at leading order in new physics parameters. For example, to describe single Higgs production and decay processes in full generality one needs 10 bosonic and $2 \times 3 \times 3 \times 3 = 54$ fermionic couplings. Furthermore, 31 of these couplings are CP-odd, therefore they affect the Higgs signal strength measurement only at the quadratic level, while flavor off-diagonal Yukawa couplings only affect exotic Higgs decays. In the limit where fermionic couplings respect the minimal flavor violation paradigm, 9 parameters are enough to describe leading order EFT corrections to the existing Higgs signal strength measurements at the LHC. In the Higgs basis, these 9 parameters are:

$$c_{gg},\ \delta c_z,\ c_{\gamma\gamma},\ c_{z\gamma},\ c_{zz},\ c_{z\Box},\ \delta y_u,\ \delta y_d,\ \delta y_e. \tag{5.16}$$

We conclude with a number of comments.

- The Higgs basis is particularly well suited for data analyses performed using tree-level (LO) EFT calculations. On the other hand, existing one-loop EFT calculations have been performed in the Warsaw basis, therefore the Warsaw basis is currently the most natural choice as far as analyses beyond LO are concerned. In order to facilitate the transition between the two bases, and in order to provide a proper definition of the Higgs basis, the complete mapping between these two bases is provided. It is straightforward to extend this mapping to any other complete basis, and we provide a detailed mapping also in the case of the *SILH basis*, that is particularly useful within specific model-dependent approaches. At the same time, the independent couplings can be easily connected to Higgs *pseudo-observables* at the amplitude level, as defined e.g. in Ref. [9].

- The choice of independent couplings in the Higgs basis is made such that the constraints from the Z and W partial decay widths (measured with a per-mille precision by the LEP experiment) can be easily incorporated. These are among the most stringent constraints on EFT parameters, and they have an important impact on possible signals in Higgs searches. In particular, assuming vertex corrections are flavor blind, all the independent couplings in Eq. (5.1) are constrained to be smaller than $O(10^{-3})$ (for the leptonic vertex corrections and $\delta m \equiv \delta m_W/m_W$), or $O(10^{-2})$ (for the quark vertex corrections) [4, 6, 12]. Dropping the assumption of flavor blindness, all the leptonic, bottom and charm quark vertex corrections are still constrained (assuming only $d \leq 6$ operators contribute to the precision observables) at the level of $O(10^{-2})$ or better [10]. In the LHC environment, experimental sensitivity is typically not sufficient to probe these parameters with a comparable accuracy. If that is indeed the case, the electroweak constraints on Z and W boson couplings to fermions can be imposed when analyzing LHC data, especially in the context of Higgs physics. Other precision observables, such as WW production or off-shell fermion scattering, lead to less stringent constraints that are not discussed in this note (see e.g. [4, 5, 6] for a recent discussion).

- The relations between independent and dependent couplings in Eqs. (5.5), (5.6), (5.7), (5.8), (5.9), (5.10), (5.11), (5.12), (5.15) are consequences of the *linear* realization of electroweak symmetry breaking at the level of dimension-6 EFT operators. *They are an essential part of the definition of the Higgs basis.* If the independent and dependent couplings were unrelated, then $\mathcal{L}_{\rm Higgs\,Basis}$ would not be a dimension-6 basis but would belong to a more general class of theories. Such theories are outside of the scope of this note.

- Customarily, the SM electroweak parameters are extracted from $\alpha(0)$, m_Z and G_F. One could also use m_W instead of G_F, as suggested in Ref. [4]. This formalism leads to the same relations between the independent and dependent couplings as written down here, except that $\delta m = 0$ by definition, and that $[c_{\ell\ell}]_{1221}$ becomes an independent coupling. The downside of this formalism is that the SM predictions for all observables would have to be recalculated, as all existing high-precision calculations use G_F as an input.

- The number of independent couplings in Eq. (5.2) relevant for Higgs observables is still large. At the early stages of the LHC run-2 it may be reasonable to employ simplified analyses with a smaller number of parameters. There are several motivated assumptions about the underlying UV theory that reduce the number of parameters:

  - *Flavor universality*, in which case the matrices $m_f \delta y_f$ and $\sin\phi_f$ reduce to a single number for each $f = u, d, e$.
  - *Minimal flavor violation*, in which case the dominant entries in δy_f are $[\delta y_u]_{33}$ and $[\delta y_d]_{33}$, while other diagonal entries are suppressed by the respective mass square ratio.
  - *CP conservation*, in which case all CP-odd couplings vanish: $\tilde{c}_i = 0 = \sin\phi_f$.
  - *Custodial symmetry*, in which case $\delta m = 0$.[7]

  We stress that independent couplings should not be arbitrarily set to zero without an underlying symmetry assumption. Furthermore, the relations between the dependent and independent couplings should be consistently imposed, so as to preserve the weak $SU(2)$ local symmetry.

- The independent couplings are formally of order v^2/Λ^2, where Λ is the scale of new physics. For completeness, it is important to define the range of independent couplings such that the EFT description is valid. The rule of thumb is that this is the case when the dimensionless independent couplings are $\lesssim 1$; a more sophisticated discussion of this issue will be performed in another document.

[7] Custodial symmetry implies several relations between Higgs couplings to gauge bosons: $\delta c_w = \delta c_z$, $c_{w\Box} = c_\theta^2 c_{z\Box} + s_\theta^2 c_{\gamma\Box}$, $c_{ww} = c_{zz} + 2 s_\theta^2 c_{z\gamma} + s_\theta^4 c_\gamma$, and $\tilde{c}_{ww} = \tilde{c}_{zz} + 2 s_\theta^2 \tilde{c}_{z\gamma} + s_\theta^4 \tilde{c}_\gamma$. The last three are satisfied automatically at the level of dimension-6 Lagrangian, while the first one is true for $\delta m = 0$, see Eq. (5.5).

A More dictionaries

In this section we quote the linear transformation between the parameters defining the Higgs basis and the Wilson coefficients in several other bases of dimension-6 operators utilized in the literature.[8] For simplicity, we assume here (unlike in the rest of this note) that the parameters are flavor blind. Moreover, we give the dictionary only for the subset of the Higgs basis parameters that can give observable contributions to single Higgs and electroweak diboson processes, given the constraints from electroweak precision tests. That set consists of 10 CP-even and 8 CP-odd parameters:

$$c_{gg},\ \delta c_z,\ c_{\gamma\gamma},\ c_{z\gamma},\ c_{zz},\ c_{z\Box},\ \delta y_u,\ \delta y_d,\ \delta y_e,\ \lambda_z, \tag{A.1}$$

$$\tilde{c}_{gg},\ \tilde{c}_{\gamma\gamma},\ \tilde{c}_{z\gamma},\ \tilde{c}_{zz},\ \sin\phi_u,\ \sin\phi_d,\ \sin\phi_e,\ \tilde{\lambda}_z. \tag{A.2}$$

The dictionaries below allow one to translate results of any complete EFT Higgs analyses into constraints on the Higgs basis parameters (and, by consequence, between any pair of bases), as long as the full likelihood function in the space of Wilson coefficients is given.

A.1 SILH' basis

The original SILH basis of Ref. [3] includes operators O_{2W}, O_{2B} and O_{2G}, which lead to 4-derivative corrections to the kinetic terms of the gauge fields. This may be inconvenient for some applications. A simple fix is to remove these operators in favor of the Warsaw basis 4-fermion operators $[O_{\ell\ell}]_{1221}$, $[O_{\ell\ell}]_{1122}$, and $[O'_u]_{3333}$. This construction was used in Ref. [4] and we refer to it as the *SILH' basis*. One advantage of this choice is that electroweak precision constraints take a particularly simple form. Namely, the vanishing of the vertex correction δg and the W mass correction δm corresponds to setting $s_T = [s_{\ell\ell}]_{1221} = s_{Hf} = s'_{Hf} = 0$, and $s_B = -s_W$.

The CP even Higgs basis parameters in Eq. (A.1) are related to the Wilson coefficients in the SILH' basis by

$$\begin{aligned}
c_{gg} &= s_{GG}, \\
\delta c_z &= -s_H + \frac{3}{4}[s_{\ell\ell}]_{1221}, \\
c_{\gamma\gamma} &= s_{BB}, \\
c_{z\gamma} &= \frac{s_{HB} - s_{HW}}{2} - s_\theta^2 s_{BB}, \\
c_{zz} &= -c_\theta^2 s_{HW} - s_\theta^2 s_{HB} - s_\theta^4 s_{BB}, \\
c_{z\Box} &= \frac{1}{2}\left(s_W + s_{HW}\right) + \frac{g'^2}{2g^2}\left(s_B + s_{HB}\right) - \frac{2}{g^2} s_T - \frac{1}{2g^2}[s_{\ell\ell}]_{1221}, \\
\delta y_f \cos\phi_f &= \frac{1}{\sqrt{2}}\mathrm{Re}[s_f] - s_H + \frac{1}{4}[s_{\ell\ell}]_{1221}, \qquad j \in \{u, d, e\}, \\
\lambda_z &= -\frac{3}{2} g^4 s_{3W}.
\end{aligned} \tag{A.1}$$

[8] On request, translation to other bases may be added in the future.

The CP odd Higgs basis parameters in Eq. (A.2) are related to the Wilson coefficients in the SILH' basis by

$$\begin{aligned}
\tilde{c}_{gg} &= \tilde{s}_{GG}, \\
\tilde{c}_{\gamma\gamma} &= \tilde{s}_{BB}, \\
\tilde{c}_{z\gamma} &= \frac{\tilde{s}_{HB} - \tilde{s}_{HW}}{2} - s_\theta^2 \tilde{s}_{BB}, \\
\tilde{c}_{zz} &= -c_\theta^2 \tilde{s}_{HW} - s_\theta^2 \tilde{s}_{HB} - s_\theta^4 \tilde{s}_{BB}, \\
\delta y_f \sin\phi_f &= \frac{1}{\sqrt{2}} \mathrm{Im}[s_f].
\end{aligned} \tag{A.2}$$

A.2 HISZ basis

We consider a subset of bosonic operators introduced by Hagiwara et al. (HISZ) in Ref. [7]:

$$\begin{aligned}
\hat{O}_{H,2} &= \frac{1}{2}\left(\partial_\mu (H^\dagger H)\right)^2, \\
\hat{O}_{GG} &= -\frac{g_s^2}{32\pi^2} H^\dagger H G^a_{\mu\nu} G^a_{\mu\nu}, \\
\hat{O}_{WW} &= H^\dagger W_{\mu\nu} W_{\mu\nu} H, \\
\hat{O}_{BB} &= H^\dagger B_{\mu\nu} B_{\mu\nu} H, \\
\hat{O}_{W} &= D_\mu H^\dagger W_{\mu\nu} D_\nu H, \\
\hat{O}_{B} &= D_\mu H^\dagger B_{\mu\nu} D_\nu H, \\
\hat{O}_{WWW} &= \mathrm{Tr}\left[W_{\mu\nu} W_{\nu\rho} W_{\rho\mu}\right],
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
O_{\widetilde{GG}} &= -\frac{g_s^2}{32\pi^2} H^\dagger H G^a_{\mu\nu} \widetilde{G}^a_{\mu\nu}, \\
\hat{O}_{\widetilde{WW}} &= H^\dagger W_{\mu\nu} \widetilde{W}_{\mu\nu} H, \\
\hat{O}_{\widetilde{BB}} &= H^\dagger B_{\mu\nu} \widetilde{B}_{\mu\nu} H, \\
\hat{O}_{\widetilde{W}} &= D_\mu H^\dagger \widetilde{W}_{\mu\nu} D_\nu H, \\
\hat{O}_{\widetilde{WWW}} &= \mathrm{Tr}\left[W_{\mu\nu} W_{\nu\rho} \widetilde{W}_{\rho\mu}\right],
\end{aligned} \tag{A.4}$$

where the electroweak field strength tensors are related to the one used in this note via:[9]

$$B_{\mu\nu} = -\frac{i}{2} g' B_{\mu\nu}, \qquad \hat{W}_{\mu\nu} = -\frac{i}{2} g \sigma^i W^i_{\mu\nu}. \tag{A.5}$$

We also consider the Yukawa operators

$$\hat{O}_u = \left(H^\dagger H - \frac{v^2}{2}\right) \bar{q}_L \tilde{H} \frac{m_u}{v} u_R,\ \hat{O}_d = \left(H^\dagger H - \frac{v^2}{2}\right) \bar{q}_L H \frac{m_d}{v} d_R,\ \hat{O}_e = \left(H^\dagger H - \frac{v^2}{2}\right) \bar{\ell}_L H \frac{m_e}{v} e_R, \tag{A.6}$$

[9] The additional minus sign in Eq. (A.5) is due to the fact that the covariant derivatives in Refs. [7] are defined with the opposite sign to that used here. This amounts to rescaling the gauge fields as $W_\mu \to -W_\mu$, $B_\mu \to -B_\mu$ in the translation.

where m_f are 3×3 diagonal fermion mass matrices. The dimension-6 Lagrangian is given by

$$\mathcal{L}_{\rm HISZ}^{\rm D=6} = \frac{1}{\Lambda^2}\left[\sum_i f_i \hat{O}_i + \sum_j \left(f_j \hat{O}_j + \text{h.c.}\right) + \ldots\right], \tag{A.7}$$

where the first sum goes over the bosonic operators in Eq. (A.3) and Eq. (A.4), the second sum goes over the fermionic operators in Eq. (A.6), and the dots stands for remaining operators that complete the dimension-6 basis. The CP-even operators from this set (except $\hat{O}_{WWW}$) are used by SFitter [13] to describe constraints on dimension-6 operators from LHC Higgs data. Ref. [14] proposes to use the HISZ operators $\hat{O}_W$, $\hat{O}_B$, $\hat{O}_{WWW}$, $\hat{O}_{\widetilde{W}}$, and $\hat{O}_{\widetilde{WWW}}$ to describe constraints on dimension-6 operators from the pair production of electroweak gauge bosons.

The CP even Higgs basis parameters in Eq. (A.1) are related to the Wilson coefficients in the HISZ basis by

$$\begin{aligned}
c_{gg} &= -\frac{1}{8\pi^2} f_{GG}\frac{v^2}{\Lambda^2}, \\
\delta c_z &= -\frac{1}{2} f_{H,2}\frac{v^2}{\Lambda^2}, \\
c_{\gamma\gamma} &= \left(-f_{WW} - f_{BB}\right)\frac{v^2}{\Lambda^2}, \\
c_{z\gamma} &= \left(\frac{1}{4}f_W - \frac{1}{4}f_B - c_\theta^2 f_{WW} + s_\theta^2 f_{BB}\right)\frac{v^2}{\Lambda^2}, \\
c_{zz} &= \left(\frac{c_\theta^2}{2}f_W + \frac{s_\theta^2}{2}f_B - c_\theta^4 f_{WW} - s_\theta^4 f_{BB}\right)\frac{v^2}{\Lambda^2}, \\
c_{z\Box} &= \left(-\frac{1}{4}f_W - \frac{s_\theta^2}{4c_\theta^2}f_B\right)\frac{v^2}{\Lambda^2}, \\
\delta y_j \cos\phi_j &= \left(-\frac{1}{2}f_{H,2} - \frac{\text{Re}f_j}{\sqrt{2}}\right)\frac{v^2}{\Lambda^2}, \qquad j \in \{u, d, e\}, \\
\lambda_z &= \frac{3g^4}{8}\frac{v^2}{\Lambda^2} f_{WWW},
\end{aligned} \tag{A.8}$$

The CP odd Higgs basis parameters in Eq. (A.2) are related to the Wilson coefficients in the HISZ basis by

$$\begin{aligned}
\tilde{c}_{gg} &= -\frac{1}{8\pi^2}\tilde{f}_{GG}\frac{v^2}{\Lambda^2}, \\
\tilde{c}_{\gamma\gamma} &= \left(-\tilde{f}_{WW} - \tilde{f}_{BB}\right)\frac{v^2}{\Lambda^2}, \\
\tilde{c}_{z\gamma} &= \left(\frac{1}{4}\tilde{f}_W - c_\theta^2\tilde{f}_{WW} + s_\theta^2\tilde{f}_{BB}\right)\frac{v^2}{\Lambda^2}, \\
\tilde{c}_{zz} &= \left(\frac{c_\theta^2}{2}\tilde{f}_W - c_\theta^4\tilde{f}_{WW} - s_\theta^4\tilde{f}_{BB}\right)\frac{v^2}{\Lambda^2}, \\
\delta y_j \sin\phi_j &= \left(\frac{\text{Im}f_j}{\sqrt{2}}\right)\frac{v^2}{\Lambda^2}, \qquad j \in \{u, d, e\},
\end{aligned} \tag{A.9}$$

For completeness, we also give the relation between the anomalous TGCs and the HISZ basis Wilson coefficients:

$$\begin{aligned}
\delta g_{1z} &= \frac{g^2+g'^2}{8} f_W \frac{v^2}{\Lambda^2} \\
\delta\kappa_\gamma &= \frac{g^2}{8}\left(f_W + f_B\right)\frac{v^2}{\Lambda^2}, \qquad \delta\tilde{\kappa}_\gamma = \frac{g^2}{8}\tilde{f}_W \frac{v^2}{\Lambda^2} \\
\lambda_z &= \frac{3g^4}{8} f_{WWW}\frac{v^2}{\Lambda^2}, \qquad \tilde{\lambda}_z = \frac{3g^4}{8}\tilde{f}_{WWW}\frac{v^2}{\Lambda^2}.
\end{aligned} \tag{A.10}$$

Inverting the transformations, the relation between the Wilson coefficients in the HISZ basis and the Higgs basis parameters reads

$$\begin{aligned}
f_{GG}\frac{v^2}{\Lambda^2} &= -8\pi^2 c_{gg}, \\
f_{H,2}\frac{v^2}{\Lambda^2} &= -2\delta c_z, \\
f_W\frac{v^2}{\Lambda^2} &= -\frac{4}{g^2-g'^2}\left[g^2 c_{z\Box} + g'^2 c_{zz} - s_\theta^2 e^2 c_{\gamma\gamma} - s_\theta^2 (g^2-g'^2) c_{z\gamma}\right], \\
f_B\frac{v^2}{\Lambda^2} &= \frac{4}{g^2-g'^2}\left[g^2 c_{z\Box} + g^2 c_{zz} - c_\theta^2 e^2 c_{\gamma\gamma} - c_\theta^2 (g^2-g'^2) c_{z\gamma}\right], \\
f_{WW}\frac{v^2}{\Lambda^2} &= -\frac{1}{g^2-g'^2}\left[2g^2 c_{z\Box} + (g^2+g'^2) c_{zz} - s_\theta^2 g'^2 c_{\gamma\gamma}\right], \\
f_{BB}\frac{v^2}{\Lambda^2} &= \frac{1}{g^2-g'^2}\left[2g^2 c_{z\Box} + (g^2+g'^2) c_{zz} - c_\theta^2 g^2 c_{\gamma\gamma}\right], \\
f_{WWW}\frac{v^2}{\Lambda^2} &= \frac{8}{3g^4}\lambda_z,
\end{aligned} \tag{A.11}$$

$$f_j \frac{v^2}{\Lambda^2} = \sqrt{2}\delta c_z - \sqrt{2}\delta y_j e^{-i\phi_j}, \qquad j \in \{u,d,e\}, \tag{A.12}$$

$$\begin{aligned}
\tilde{f}_{GG}\frac{v^2}{\Lambda^2} &= -8\pi^2 \tilde{c}_{gg}, \\
\tilde{f}_W\frac{v^2}{\Lambda^2} &= -\frac{4}{g^2-g'^2}\left[g'^2 \tilde{c}_{zz} - s_\theta^2 e^2 \tilde{c}_{\gamma\gamma} - s_\theta^2 (g^2-g'^2)\tilde{c}_{z\gamma}\right], \\
\tilde{f}_{WW}\frac{v^2}{\Lambda^2} &= -\frac{1}{g^2-g'^2}\left[(g^2+g'^2)\tilde{c}_{zz} - s_\theta^2 g'^2 \tilde{c}_{\gamma\gamma}\right], \\
\tilde{f}_{BB}\frac{v^2}{\Lambda^2} &= \frac{1}{g^2-g'^2}\left[(g^2+g'^2)\tilde{c}_{zz} - c_\theta^2 g^2 c_{\gamma\gamma}\right], \\
\tilde{f}_{WWW}\frac{v^2}{\Lambda^2} &= \frac{8}{3g^4}\tilde{\lambda}_z.
\end{aligned} \tag{A.13}$$

B Goldstone bosons and gauge fixing

In the main body of this note we worked in the unitary gauge where the Goldstone boson degrees of freedom in the Higgs doublet are set to zero. This is enough for the sake of tree-level EFT calculations. However, if the necessity arises to extend the calculations to a loop level, retrieving the Goldstone degrees of freedom is convenient, as this allows one to perform the standard gauge fixing procedure. This is done in this appendix.

We parametrize the Higgs doublet as

$$H = \begin{pmatrix} iG_+ \\ \frac{1}{\sqrt{2}}(v + h - iG_3) \end{pmatrix} \tag{B.1}$$

where $G_\pm$ and G_3 are three Goldstone fields, that will be eaten by the W and Z bosons. In the Higgs basis, derivation of the Goldstone boson couplings follows exactly the same algorithm as the one applied before to derive the Lagrangian for physical fields: we first derive these couplings in the Warsaw basis, and then perform the field and coupling redefinitions that take us to the Higgs basis. Of course, all the Goldstone boson couplings are dependent ones, that is they can be expressed by the independent couplings defining the Higgs basis. As an illustration, below we display a subset of these couplings that are relevant for the 1-loop calculation of $h \to VV^*$. These are

1. Goldstone kinetic terms and their mixing with the electroweak gauge fields.

2. Cubic interactions with one Higgs boson and one or two Goldstone fields.

3. Cubic interactions with one or two Goldstone fields and one electroweak gauge field.

4. Quartic interactions with one or two Goldstone fields and two electroweak gauge fields.

The relevant part of the Lagrangian is parametrized as

$$\mathcal{L}_G = \mathcal{L}_G^{\rm kin} + \mathcal{L}_G^{\rm S^3} + \mathcal{L}_G^{\rm S^2V} + \mathcal{L}_G^{\rm SV^2} + \mathcal{L}_G^{\rm SVdV} + \mathcal{L}_G^{\rm S^2V^2} + \mathcal{L}_G^{\rm S^2dV^2}. \tag{B.2}$$

where

$$\mathcal{L}_G^{\rm kin} = \partial_\mu G_+ \partial_\mu G_- + \frac{1}{2}(\partial_\mu G_3)^2 - \beta_{cW}\frac{gv}{2}\left(\partial_\mu G_+ W_\mu^- + \text{h.c.}\right) - \frac{\sqrt{g^2+g'^2}v}{2}\partial_\mu G_3 Z_\mu, \tag{B.3}$$

$$\mathcal{L}_G^{\rm S^3} = -\frac{m_h^2}{v}\beta_{hcc} h G_+ G_- - \frac{m_h^2}{2v}\beta_{h33} h G_3 G_3 \tag{B.4}$$

$$\begin{aligned}
\mathcal{L}_G^{\rm S^2V} &= \beta_{hcW}\frac{g}{2}\partial_\mu h\left(G_+ W_\mu^- + \text{h.c.}\right) + \beta_{h3z}\frac{\sqrt{g^2+g'^2}}{2}\partial_\mu h G_3 Z_\mu \\
&+ i\beta_{3cW}\frac{g}{2}\partial_\mu G_3\left(G_+ W_\mu^- - \text{h.c.}\right) - \beta_{3hz}\frac{\sqrt{g^2+g'^2}}{2}\partial_\mu G_3 h Z_\mu \\
&+ ie\left(\partial_\mu G_+ G_- - \text{h.c.}\right)A_\mu + i\beta_{ccZ}\frac{g^2-g'^2}{2\sqrt{g^2+g'^2}}\left(\partial_\mu G_+ G_- - \text{h.c.}\right)Z_\mu \\
&- \beta_{chW}\frac{g}{2}\left(\partial_\mu G_+ W_\mu^- + \text{h.c.}\right)h - i\beta_{c3W}\frac{g}{2}\left(\partial_\mu G_+ W_\mu^- - \text{h.c.}\right)G_3,
\end{aligned} \tag{B.5}$$

$$\mathcal{L}_G^{\rm SV^2} = i\beta_{cWA}\frac{egv}{2}\left(G_+ W_\mu^- - \text{h.c.}\right)A_\mu - i\beta_{cWZ}\frac{c_\theta g'^2 v}{2}\left(G_+ W_\mu^- - \text{h.c.}\right)Z_\mu, \tag{B.6}$$

$$\mathcal{L}_G^{\rm SVdV} = i\eta_{cWA}\frac{eg}{2v}\left(G_+W^-_{\mu\nu} - {\rm h.c.}\right)A_{\mu\nu} - i\eta_{cWA}\frac{eg'}{2v}\left(G_+W^-_{\mu\nu} - {\rm h.c.}\right)Z_{\mu\nu} + ({\rm CP\text{–}odd}). \tag{B.7}$$

$$\begin{aligned}
\mathcal{L}_G^{\rm S^2V^2} &= G_+G_-\left(e^2A_\mu A_\mu + \beta_{ccAZ}\frac{e(g^2-g'^2)}{\sqrt{g^2+g'^2}}A_\mu Z_\mu + \beta_{ccZZ}\frac{(g^2-g'^2)^2}{4(g^2+g'^2)}Z_\mu Z_\mu + \beta_{ccWW}\frac{g^2}{2}W^+_\mu W^-_\mu\right)\\
&+ G_3G_3\left(\beta_{33WW}\frac{g^2}{4}W^+_\mu W^-_\mu + \beta_{33ZZ}\frac{g^2+g'^2}{8}Z_\mu Z_\mu\right)\\
&+ i\beta_{chWA}\frac{eg}{2}\left(G_+W^-_\mu - {\rm h.c.}\right)hA_\mu - \beta_{c3WA}\frac{eg}{2}\left(G_+W^-_\mu + {\rm h.c.}\right)G_3A_\mu\\
&- i\beta_{chWZ}\frac{eg'}{2}\left(G_+W^-_\mu - {\rm h.c.}\right)hZ_\mu + \beta_{c3WZ}\frac{eg'}{2}\left(G_+W^-_\mu + {\rm h.c.}\right)G_3Z_\mu\\
&+ \eta'_{ccWW}g_L^2\left(G_+G_+W^-_\mu W^-_\mu + {\rm h.c.}\right),
\end{aligned} \tag{B.8}$$

$$\begin{aligned}
\mathcal{L}_G^{\rm S^2dV^2} &= G_+G_-\left(\eta_{ccA^2}e^2A_{\mu\nu}A_{\mu\nu} + \eta_{ccAZ}gg'A_{\mu\nu}Z_{\mu\nu} + \eta_{ccZ^2}(g^2+g'^2)Z_{\mu\nu}Z_{\mu\nu} + \eta_{ccW^2}g^2W^+_{\mu\nu}W^-_{\mu\nu}\right)\\
&+ G_3G_3\left(\eta_{33AA}e^2A_{\mu\nu}A_{\mu\nu} + \eta_{33AZ}gg'A_{\mu\nu}Z_{\mu\nu} + \eta_{33ZZ}(g^2+g'^2)Z_{\mu\nu}Z_{\mu\nu} + \eta_{33WW}g^2W^+_{\mu\nu}W^-_{\mu\nu}\right)\\
&+ \eta_{c3WA}eg\left(G_+W^-_{\mu\nu} + {\rm h.c.}\right)G_3A_{\mu\nu} + \eta_{c3WZ}eg'\left(G_+W^-_\mu + {\rm h.c.}\right)G_3Z_{\mu\nu} + ({\rm CP\text{–}odd}).
\end{aligned} \tag{B.9}$$

Above, "CP-odd" stands for analogous terms with $V_{\mu\nu} \to \tilde{V}_{\mu\nu}$, and $\eta \to \tilde{\eta}$. Note the Goldstone kinetic terms in Eq. (B.3) are assumed to be canonically normalized. To achieve this, one needs to rescale the neutral Goldstone field as

$$G_3 \to G_3\left(1 + c_T + 2c_T\frac{h}{v}\right). \tag{B.10}$$

Moreover, the Lagrangian in Eq. (B.2) does not contain 2-derivative cubic scalar self-interactions. To ensure this feature, the Higgs boson field redefinition in Eq. (4.4) has to be generalized to

$$h \to h\left(1 - c_H - c_H\frac{h}{v} - c_H\frac{h^2}{3v^2}\right) - c_H\frac{2G_+G_- + G_3G_3}{v} - 2c_T\frac{G_3G_3}{v}. \tag{B.11}$$

The above field redefinitions are in addition to the steps described in Section 3.1. These include the gauge coupling rescaling and the use of the equations of motion (that are modified to include the Goldstone fields). The final step is to transform the couplings from the Warsaw to the Higgs basis using the dictionary provided in Section 3.1. At the end of the day, the coefficients in the Goldstone Lagrangian of Eq. (B.2) take the form

$$\beta_{cW} = 1 + \delta m, \tag{B.12}$$

$$\begin{aligned}
\beta_{hcc} &= 1 + g^2c_{w\Box} + \delta c_z + 2\delta m,\\
\beta_{h33} &= 1 + g^2c_{z\Box} + \delta c_z,
\end{aligned} \tag{B.13}$$

$$
\begin{aligned}
\beta_{hcW} &= 1 + g^2 c_{w\Box} + \delta c_z + 3\delta m, \\
\beta_{h3Z} &= 1 + g^2 c_{z\Box} + \delta c_z, \\
\beta_{3cW} &= 1 - 2g^2 c_{w\Box} + \frac{3}{2} g^2 c_{z\Box} - 3\delta m, \\
\beta_{3hZ} &= 1 + \delta c_z, \\
\beta_{ccZ} &= 1 + \frac{g^2 + g'^2}{2(g^2 - g'^2)} \left(-g^2 c_{z\Box} + 4\delta m\right), \\
\beta_{chW} &= 1 + \delta c_z + 3\delta m, \\
\beta_{c3W} &= 1 - \frac{g^2}{2} c_{z\Box} + \delta m,
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
\beta_{cWA} &= 1 + \delta m, \\
\beta_{cWZ} &= 1 + \frac{g^2(g^2 + g'^2)}{2g'^2} \left(c_{z\Box} - c_{w\Box}\right) - \frac{2g^2 + g'^2}{g'^2} \delta m,
\end{aligned} \tag{B.15}
$$

$$
\eta_{cWA} = \eta_{cWZ} = c_{zz} - \frac{g^2 - g'^2}{g^2 + g'^2} c_{z\gamma} - e^2 c_{\gamma\gamma}, \tag{B.16}
$$

$$
\begin{aligned}
\beta_{ccAZ} &= 1 + \frac{g^2 + g'^2}{2(g^2 - g'^2)} \left(-g^2 c_{z\Box} + 4\delta m\right), \\
\beta_{ccZZ} &= 1 + \frac{(g^2 + g'^2)^2}{(g^2 - g'^2)^2} \left(-\frac{g^2(g^2 - g'^2)}{g^2 + g'^2} c_{z\Box} + 3g^2 c_{w\Box} + 2\delta c_z + 2\frac{5g^4 + 6g^2 g'^2 + g'^4}{(g^2 + g'^2)^2} \delta m\right), \\
\beta_{ccWW} &= 1 + 2g^2 c_{z\Box} + 2\delta c_z + 2\delta m, \\
\beta_{33ZZ} &= 1 + 2g^2 c_{z\Box} + 2\delta c_z, \\
\beta_{33WW} &= 1 + g^2(c_{w\Box} + c_{z\Box}) + 2\delta c_z + 4\delta m, \\
\beta_{chWA} &= 1 + \delta c_z + 3\delta m, \\
\beta_{c3WA} &= 1 - \frac{g^2}{2} c_{z\Box} + \delta m, \\
\beta_{chWZ} &= 1 + \frac{3}{2} \frac{g^2(g^2 + g'^2}{g'^2} \left(c_{z\Box} - c_{w\Box}\right) + \delta c_z - 3\frac{2g^2 + g'^2}{g'^2} \delta m, \\
\beta_{c3WZ} &= 1 + \frac{g^4}{2g'^2} c_{z\Box} - \frac{g^2(g^2 + g'^2)}{2g'^2} c_{w\Box} - \frac{2g^2 + g'^2}{g'^2} \delta m, \\
\eta'_{ccWW} &= \frac{g^2}{2} \left(c_{w\Box} - c_{z\Box}\right) + \delta m,
\end{aligned} \tag{B.17}
$$

$$
\begin{aligned}
\eta_{ccAA} &= c_{zz} - \frac{g^2 - g'^2}{g^2 + g'^2} c_{z\gamma} + \frac{(g^2 - g'^2)^2}{4(g^2 + g'^2)} c_{\gamma\gamma}, \\
\eta_{33AA} &= \frac{1}{8} c_{\gamma\gamma}, \\
\eta_{ccAZ} &= \frac{g^2 - g'^2}{g^2 + g'^2} c_{zz} - \frac{g^4 - 6g^2 g'^2 + g'^4}{2(g^2 + g'^2)^2} c_{z\gamma} - \frac{e^2(g^2 - g'^2)}{(g^2 + g'^2)^2} c_{\gamma\gamma}, \\
\eta_{33AZ} &= \frac{c_{z\gamma}}{4}, \\
\eta_{ccZZ} &= \frac{(g^2 - g'^2)^2}{4(g^2 + g'^2)^2} c_{zz} - \frac{e^2(g^2 - g'^2)}{(g^2 + g'^2)^2} c_{z\gamma} + \frac{e^4}{(g^2 + g'^2)^2} c_{\gamma\gamma}, \\
\eta_{33ZZ} &= \frac{c_{zz}}{8}, \\
\eta_{ccWW} &= \frac{1}{2} c_{zz} + s_\theta^2 c_{z\gamma} + \frac{s_\theta^4}{2} c_{\gamma\gamma}, \\
\eta_{33WW} &= \frac{1}{4} c_{zz} + \frac{s_\theta^2}{2} c_{z\gamma} + \frac{s_\theta^4}{4} c_{\gamma\gamma}, \\
\eta_{c3WA} &= -\frac{1}{2} c_{zz} + \frac{g^2 - g'^2}{2(g^2 + g'^2)} c_{z\gamma} + \frac{e^2}{2(g^2 + g'^2)} c_{\gamma\gamma}, \\
\eta_{c3WZ} &= \frac{1}{2} c_{zz} - \frac{g^2 - g'^2}{2(g^2 + g'^2)} c_{z\gamma} - \frac{e^2}{2(g^2 + g'^2)} c_{\gamma\gamma}.
\end{aligned}
\tag{B.18}
$$

With the Goldstone bosons degrees of freedom present in the Lagrangian, gauge fixing can be implemented as in any gauge theory. Below we show how to implement the linear R_ξ gauge. For the electroweak sector, we introduce the following gauge fixing Lagrangian

$$
\mathcal{L}_{\rm gf} = -\frac{1}{2\xi} \left[F_A^2 + F_Z^2 + 2F_+ F_- \right], \tag{B.19}
$$

where

$$
\begin{aligned}
F_A &= \partial_\mu A_\mu \left(1 + e^2 c_{WB}\right) + \partial_\mu Z_\mu c_{WB} \frac{gg'(g^2 - g'^2)}{g^2 + g'^2}, \\
F_Z &= \partial_\mu Z_\mu - \xi \frac{\sqrt{g^2 + g'^2} v}{2} G_3 \left(1 - 2c_T + e^2 c_{WB}\right), \\
F_\pm &= \partial_\mu W_\mu^\pm - \xi \frac{gv}{2} G_\pm.
\end{aligned}
\tag{B.20}
$$

Above, the electroweak parameters g, g', v and the Goldstone fields $G_\pm$, G_3 are the ones before the rescaling in Eq. (4.7) and Eq. (B.10). After the rescaling and going to the Higgs basis the quadratic terms in the gauge fixing Lagrangian become

$$
\mathcal{L}_{\rm gf} = -\frac{1}{2\xi} \left[(\partial_\mu A_\mu)^2 + \left(\partial_\mu Z_\mu - \xi \frac{\sqrt{g^2 + g'^2} v}{2} G_3 \right)^2 + 2 \left| \partial_\mu W_\mu^+ - \xi \frac{gv}{2} (1 + \delta m) G_+ \right|^2 \right]. \tag{B.21}
$$

This way, the kinetic mixing between the Goldstone bosons and massive vector bosons in Eq. (B.3) is canceled after introducing the gauge fixing term. At the same time, the

Goldstone bosons acquire the gauge dependent masses:

$$m_{G_\pm} = \sqrt{\xi}\frac{gv}{2}\left(1+\delta m\right) \equiv \sqrt{\xi}m_W, \qquad m_{G_3} = \sqrt{\xi}\frac{\sqrt{g^2+g'^2}v}{2} \equiv \sqrt{\xi}m_Z. \tag{B.22}$$

To derive Eq. (B.21) one needs to take into account that the gauge fixing term affects the equations of motion used in Eq. (4.2) and Eq. (4.8) to bring the Warsaw basis Lagrangian to the prescribed form of phenomenological effective Lagrangian. Due to this, the gauge fixing term affects not only quadratic terms in the Lagrangian, but also yields new interactions terms of the Goldstone bosons, Higgs boson, and gauge fields.

Finally, the ghost Lagrangian can be obtained by the usual Fadeev-Popov procedure. In the R_ξ gauge introduced above

$$\mathcal{L}_{\rm ghost} = -\sum_{n\in(+,-,Z,\gamma)}\left[\bar{c}_+\frac{\partial\delta F_+}{\partial\alpha_n} + \bar{c}_-\frac{\partial\delta F_-}{\partial\alpha_n} + \bar{c}_Z\frac{\partial\delta F_Z}{\partial\alpha_n} + \bar{c}_\gamma\frac{\partial\delta F_A}{\partial\alpha_n}\right]c_n, \tag{B.23}$$

where δF is the variation of the gauge fixing term under the infinitesimal $SU(2)\times U(1)$ gauge symmetry transformations parametrized by α_n. Since the F's in Eq. (B.20) contain the original (unrescaled) gauge and Goldstone fields, their gauge transformations are the same as in the SM:

$$\begin{aligned}
\delta A_\mu &= \partial_\mu\alpha_\gamma + ie\left(W^-_\mu\alpha^+ - W^+_\mu\alpha^-\right),\\
\delta Z_\mu &= \partial_\mu\alpha_Z + igc_\theta\left(W^-_\mu\alpha^+ - W^+_\mu\alpha^-\right),\\
\delta W^+_\mu &= \partial_\mu\alpha_+ - ig\alpha_+\left(c_\theta Z_\mu + s_\theta A_\mu\right) + ig\left(c_\theta\alpha_Z + s_\theta\alpha_\gamma\right)W^+_\mu,
\end{aligned} \tag{B.24}$$

$$\begin{aligned}
\delta h &= -\frac{\sqrt{g^2+g'^2}}{2}G_3\alpha_Z - \frac{g}{2}\left(G_+\alpha_- + G_-\alpha_+\right),\\
\delta G_3 &= \frac{\sqrt{g^2+g'^2}}{2}\left(v+h\right)\alpha_Z - \frac{ig}{2}\left(G_+\alpha_- - G_-\alpha_+\right),\\
\delta G_+ &= \frac{g}{2}\left(v+h-iG_3\right)\alpha_+ + ieG_+\alpha_\gamma + i\frac{g^2-g'^2}{2\sqrt{g^2+g'^2}}G_+\alpha_Z.
\end{aligned} \tag{B.25}$$

At this point the ghost kinetic and mass terms are not diagonal. To this end one needs to perform the transformation

$$\begin{aligned}
\bar{c}_Z &\to \bar{c}_Z\left(1+\delta\kappa_\gamma\right),\\
c_\gamma &\to c_\gamma\left(1-s^2_\theta\delta\kappa_\gamma\right) - c_Z\frac{g'(g^2-g'^2)}{g'(g^2+g'^2)},\\
c_Z &\to c_z\left(1-\delta g_{1,z} + s^2_\theta\delta\kappa_\gamma\right).
\end{aligned} \tag{B.26}$$

After this transformation the ghost kinetic and mass terms become diagonal and the kinetic terms are canonically normalized. Their gauge dependent masses of the ghosts are given by

$$m_{c_\pm} = \sqrt{\xi}\frac{gv}{2}\left(1+\delta m\right) \equiv \sqrt{\xi}m_W, \quad m_{c_Z} = \sqrt{\xi}\frac{\sqrt{g^2+g'^2}v}{2} \equiv \sqrt{\xi}m_Z, \quad m_{c_\gamma} = 0. \tag{B.27}$$

References

[1] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **1010** (2010) 085, [arXiv:1008.4884].

[2] R. S. Gupta, A. Pomarol, and F. Riva, *BSM Primary Effects*, *Phys. Rev.* **D91** (2015), no. 3 035001, [arXiv:1405.0181].

[3] G. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, *The Strongly-Interacting Light Higgs*, *JHEP* **0706** (2007) 045, [hep-ph/0703164].

[4] A. Pomarol and F. Riva, *Towards the Ultimate SM Fit to Close in on Higgs Physics*, *JHEP* **1401** (2014) 151, [arXiv:1308.2803].

[5] M. Trott, *On the consistent use of constructed observables*, *JHEP* **02** (2015) 046, [arXiv:1409.7605].

[6] A. Falkowski and F. Riva, *Model-independent precision constraints on dimension-6 operators*, *JHEP* **1502** (2015) 039, [arXiv:1411.0669].

[7] K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, *Low-energy effects of new interactions in the electroweak boson sector*, *Phys.Rev.* **D48** (1993) 2182–2203.

[8] R. Alonso, E. E. Jenkins, A. V. Manohar, and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **1404** (2014) 159, [arXiv:1312.2014].

[9] M. Gonzalez-Alonso, A. Greljo, G. Isidori, and D. Marzocca, *Pseudo-observables in Higgs decays*, *Eur. Phys. J.* **C75** (2015), no. 3 128, [arXiv:1412.6038].

[10] A. Efrati, A. Falkowski, and Y. Soreq, *Electroweak constraints on flavorful effective theories*, *JHEP* **07** (2015) 018, [arXiv:1503.07872].

[11] R. Contino, M. Ghezzi, C. Grojean, M. Mühlleitner, and M. Spira, *Effective Lagrangian for a light Higgs-like scalar*, *JHEP* **1307** (2013) 035, [arXiv:1303.3876].

[12] J. Ellis, V. Sanz, and T. You, *The Effective Standard Model after LHC Run I*, arXiv:1410.7703.

[13] T. Corbett, O. J. P. Eboli, D. Goncalves, J. Gonzalez-Fraile, T. Plehn, and M. Rauch, *The Higgs Legacy of the LHC Run I*, arXiv:1505.05516.

[14] C. Degrande, N. Greiner, W. Kilian, O. Mattelaer, H. Mebane, et al., *Effective Field Theory: A Modern Approach to Anomalous Couplings*, *Annals Phys.* **335** (2013) 21–32, [arXiv:1205.4231].