Lectures on Particle Cosmology

Pre-SUSY School 2016

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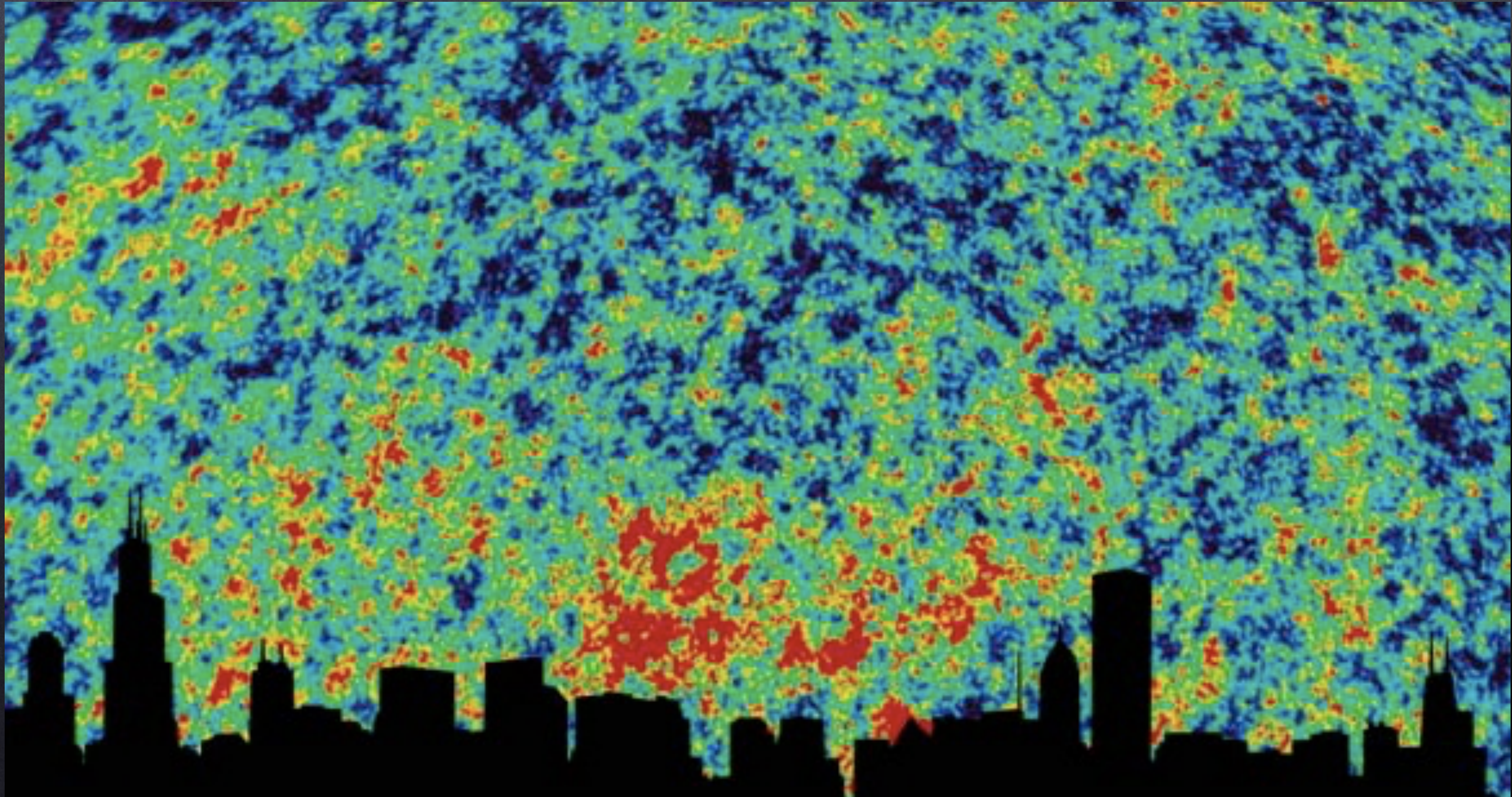
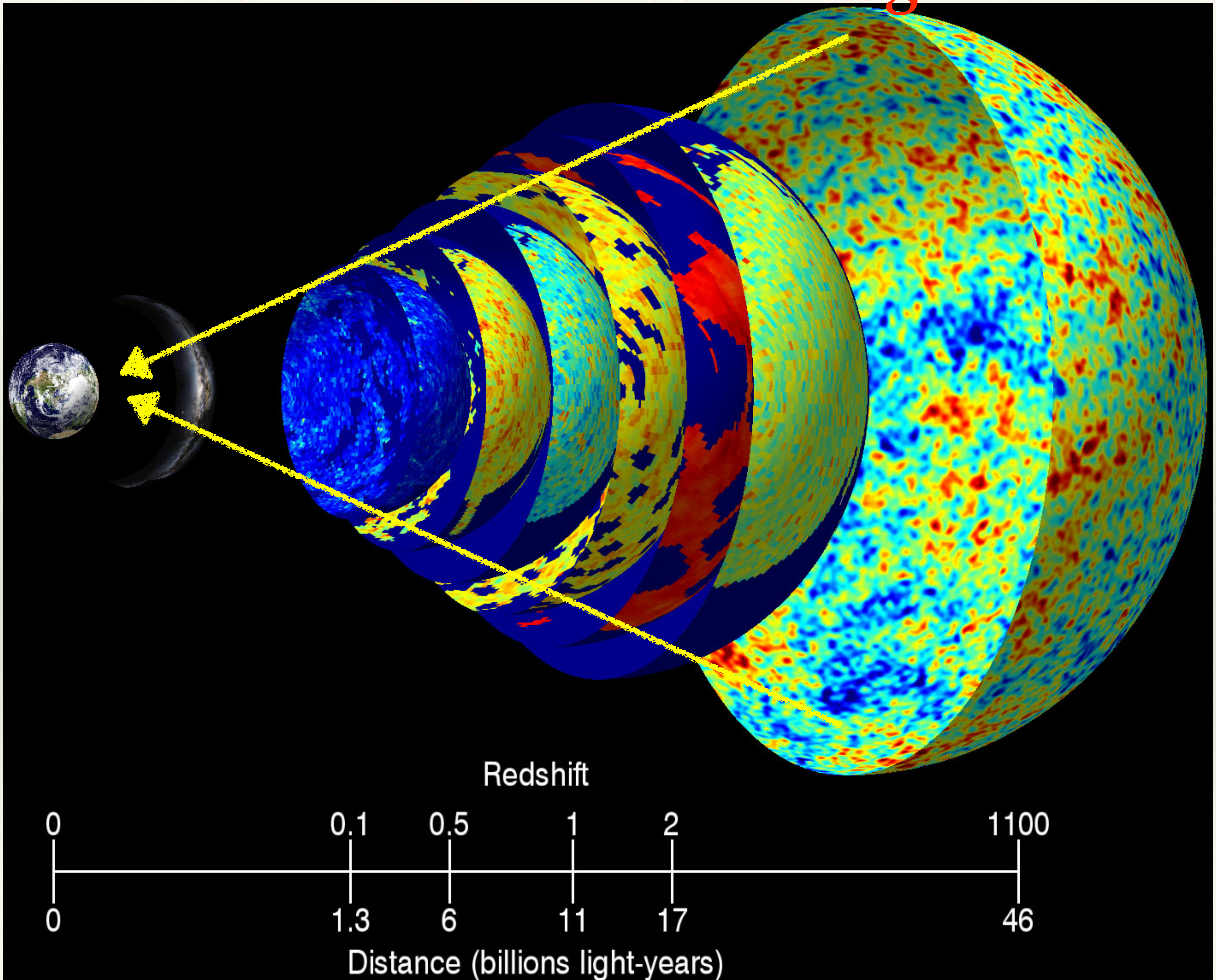
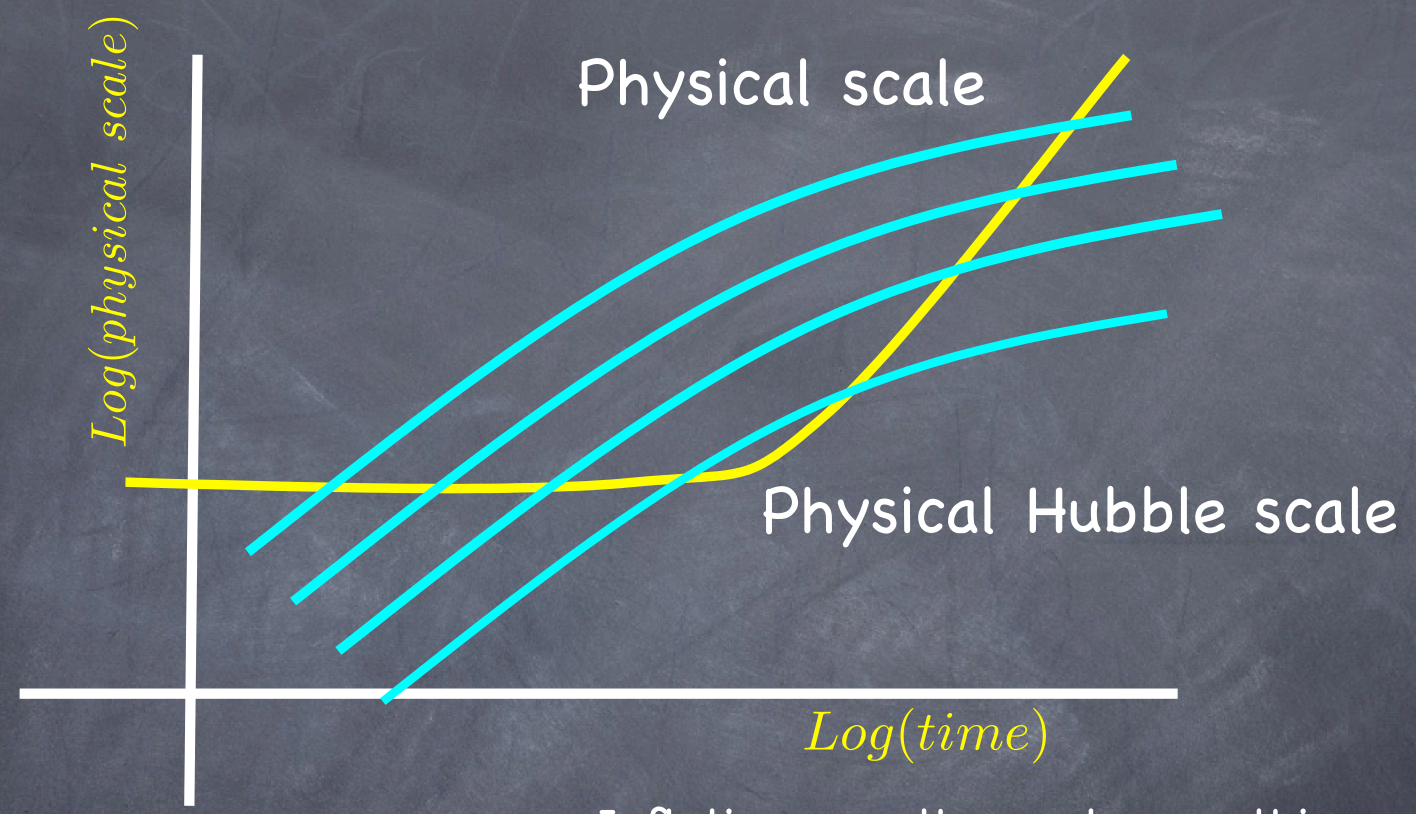
Quantum Aspects During Inflation



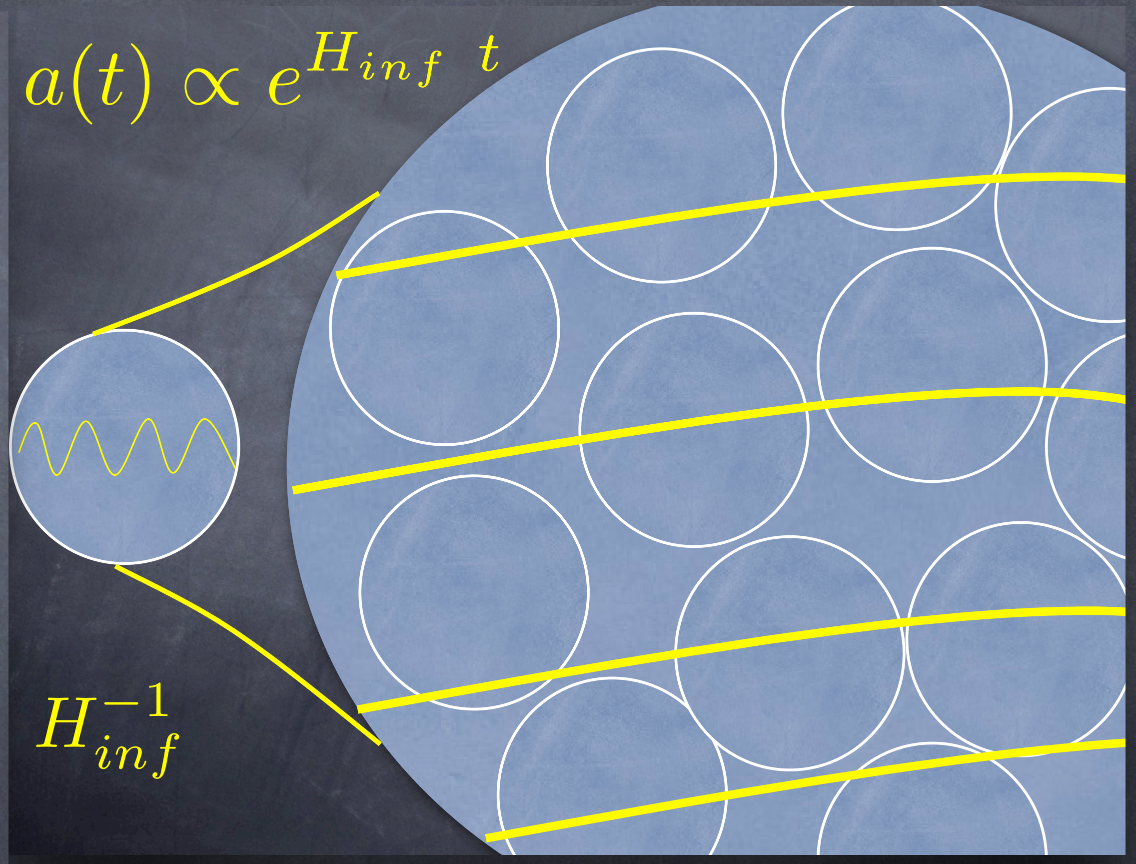
Surface of last scattering



Quantum Predictions of Inflation



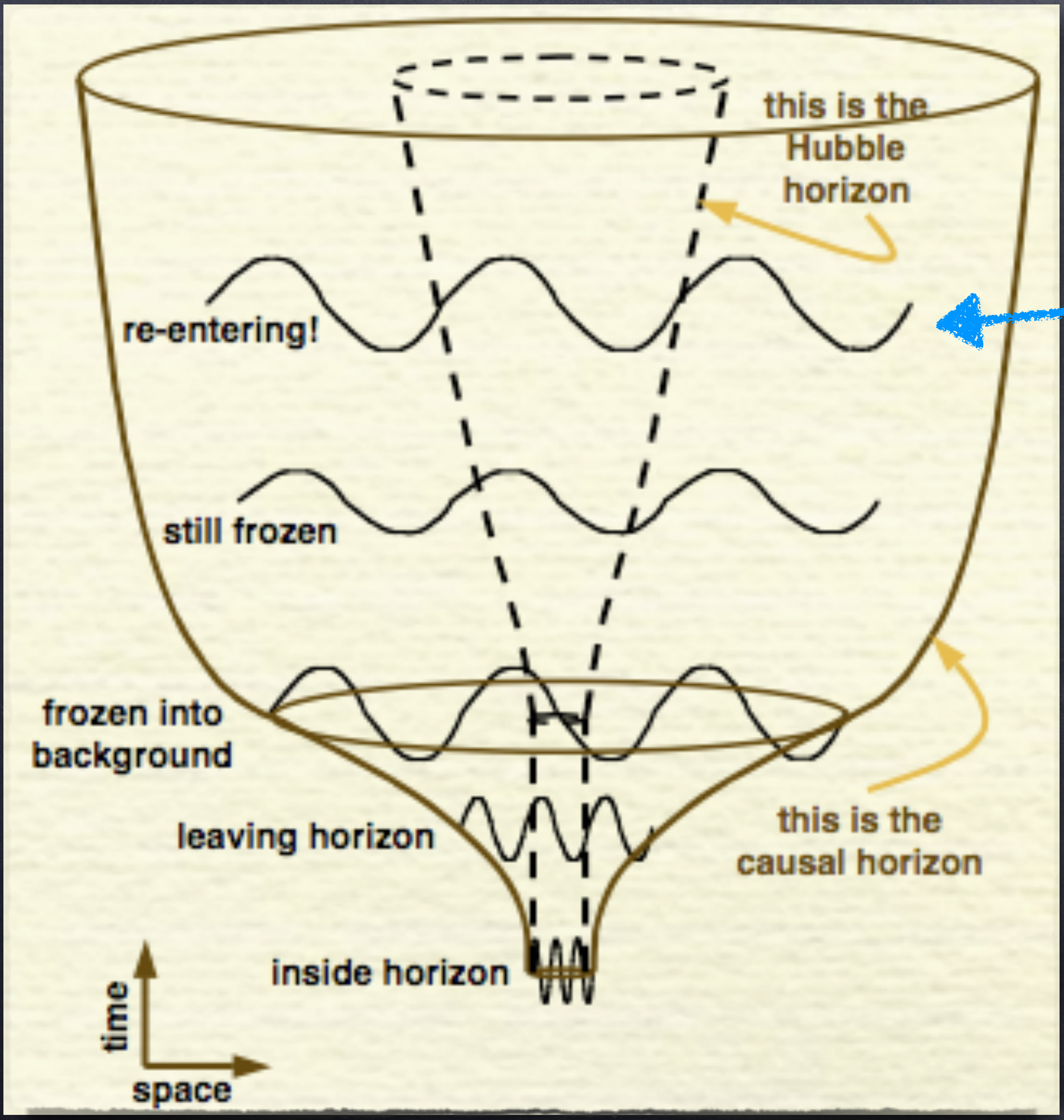
Inflation smoothes out everything, but the Quantum Fluctuations

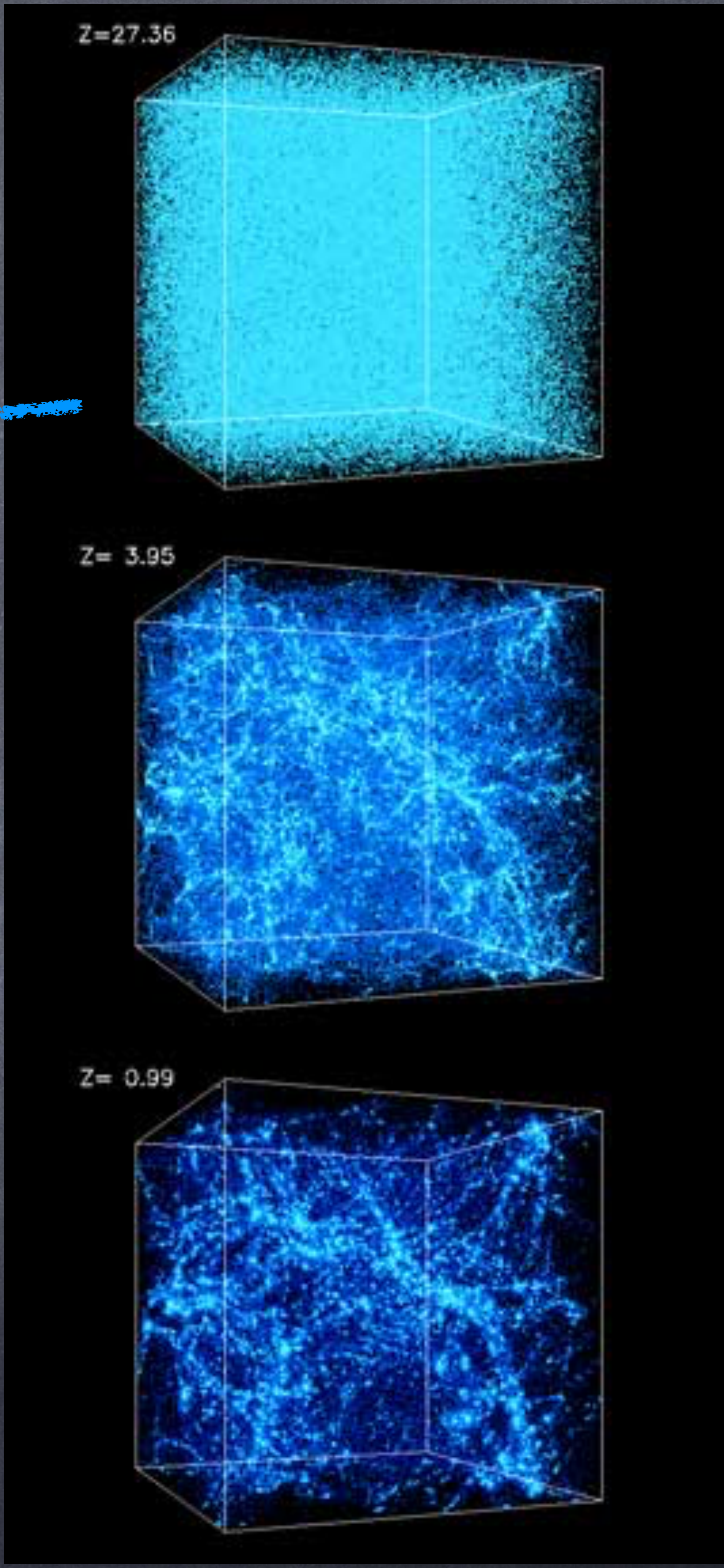


Perturbations leave the Hubble Radius during inflation & then re-enter Hubble radius at later epochs

$$\langle \delta\phi(\vec{x},t)\delta\phi(\vec{x}',t)\rangle_{vac} \sim [\mathrm{Length(t)}]^{-2} \sim \frac{1}{|\Delta\vec{x}(t)|^2} \sim H^2$$

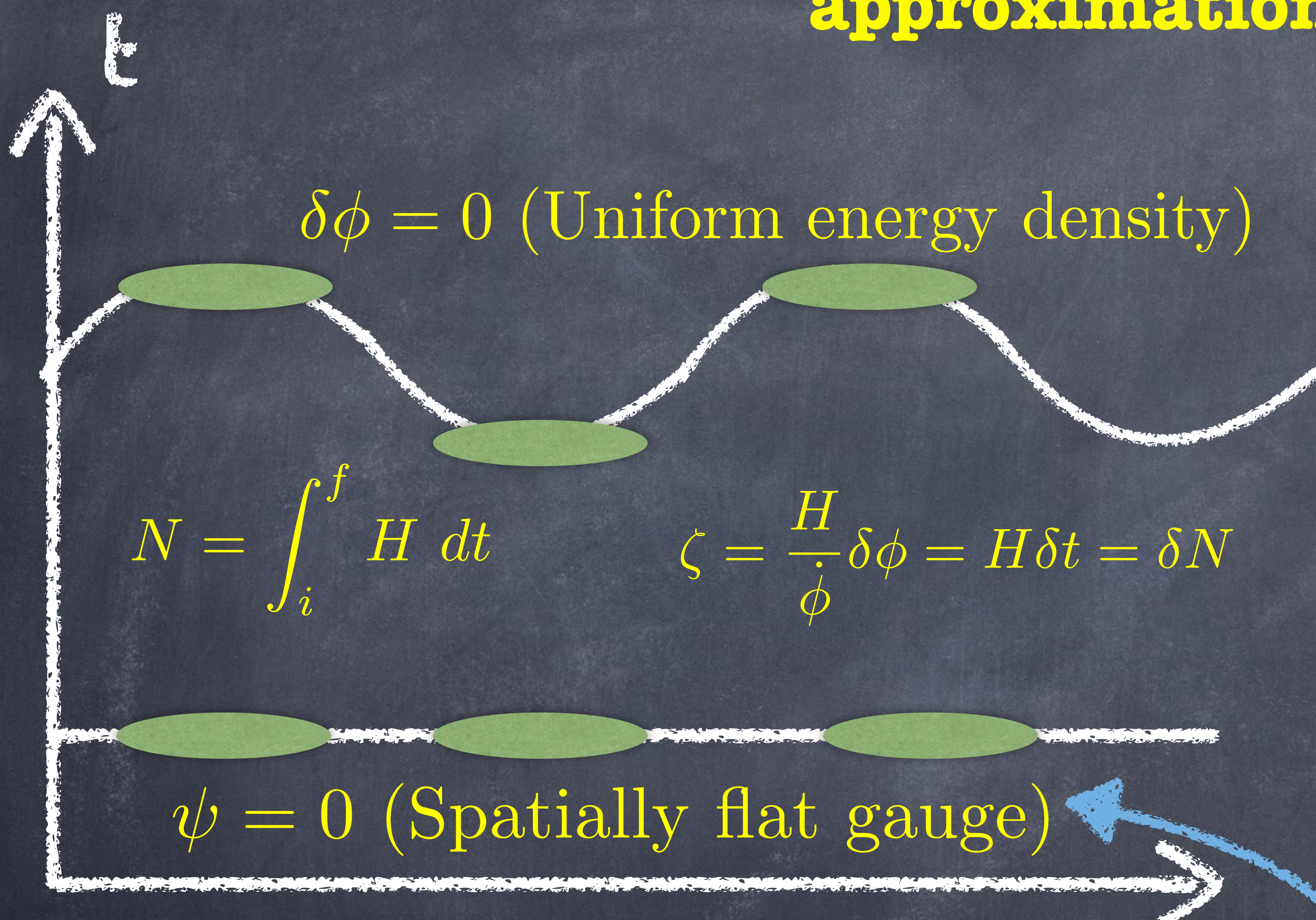
Quantum Predictions of Inflation





Quantum Predictions of Inflation

Separate Universe : Long wavelength approximation



$\delta\phi = 0$ (Uniform energy density)

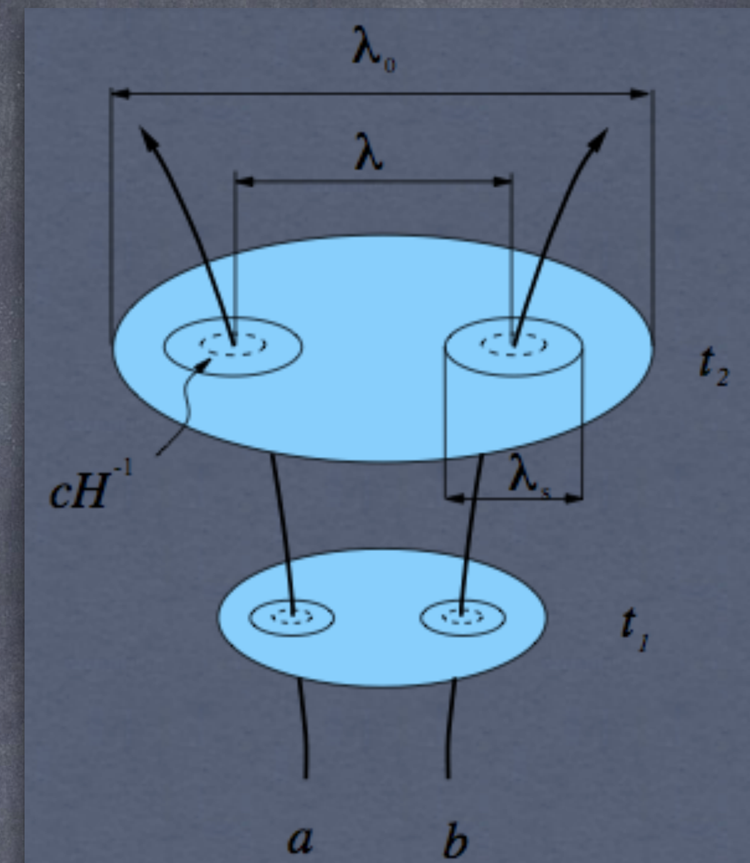
$$N = \int_i^f H \; dt \qquad \zeta = \frac{H}{\dot\phi}\delta\phi = H\delta t = \delta N$$

$\psi = 0$ (Spatially flat gauge)

$$g_{\mu\nu}(x) = g^b_{\mu\nu}(t) + \delta g_{\mu\nu}(x)\,; \qquad \Phi(x) = \phi(t) + \varphi(x)$$

$$\mathcal{R} \equiv \psi + \frac{H}{\dot\phi}\varphi$$

Comoving Gauge **Constant Curvature gauge**



Wands et al., astro-ph/0003278v2

$$\zeta = \delta N$$

$$\delta N = N'\delta\phi_* + \frac{1}{2}N''\delta\phi_*^2$$

Mukhanov, Feldman Brandenberger Review (1992)

Quantum Perturbations

Constant Curvature Gauge : $\psi = 0$ (Spatially flat gauge)

$$\zeta = -\frac{H}{\dot{\phi}}\varphi \equiv -\frac{H}{\dot{\phi}}\delta\phi \equiv -H\delta t(x)$$

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2}\langle\zeta\zeta\rangle = \left(\frac{H}{\dot{\phi}}\right)^2 \frac{k^3}{2\pi^2}\langle\delta\phi\delta\phi\rangle \equiv \left(\frac{H}{\dot{\phi}}\right)^2 \mathcal{P}_{\delta\phi}(k)$$

Power Spectrum

$$\ddot{\delta\phi}_{\mathbf{k}} + 3H\dot{\delta\phi}_{\mathbf{k}} + \left(V''(\phi) + \frac{\mathbf{k}^2}{a^2}\right)\delta\phi_{\mathbf{k}} = 0,$$

$$\mathrm{d}\tau \equiv \frac{\mathrm{d}t}{a},$$

$$\psi \equiv a\delta\phi,$$

$$\delta\phi_{\mathbf{k}} \equiv \int \frac{\mathrm{d}^3\mathbf{x}}{(2\pi)^{\frac{3}{2}}}\delta\phi(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}.$$

$$\int_\tau^0 \mathrm{d}\tau = \int_t^{t_e}\frac{\mathrm{d}t}{a} = \int_a^{a_e}\frac{\mathrm{d}a}{a^2H} \simeq \frac{1}{H}\int_a^{a_e}\frac{\mathrm{d}a}{a^2} \approx \frac{1}{aH}.$$

Bunch-Davis Vacuum (Quantum Initial Condition)

$$\psi''_{\mathbf{k}} + \left(\mathbf{k}^2 - \frac{2}{\tau^2}\right)\psi_{\mathbf{k}} = 0$$

$$\hat{\psi}_{\mathbf{k}} \propto \frac{e^{-ik\tau}}{\sqrt{2k}}.$$

$$t \to 0 \text{ or } \tau \to -\infty$$

$$P_{\delta\phi}(k) = \frac{H^2}{2k^3}\left(1 + \frac{k^2}{a^2H^2}\right)$$

Quantum Predictions of Inflation

Scalar Perturbations

Compute the Power spectrum at : $k = aH$

$$P_\zeta = \frac{H^4}{\dot{\phi}^2}\frac{1}{k^3}$$

Scale Dependence (Spectral Tilt) :

$$P_\zeta \sim \frac{1}{k^3}\left(\frac{k}{k_0}\right)^{n_s-1}$$

$$n_s - 1 = \frac{d\ln(k^3 P_k)}{d\ln k} = \left.\frac{d\ln\left(H^4/\dot{\phi}^2\right)}{d\ln k}\right|_{k\sim aH}$$

$$n_s - 1 = -6\epsilon + 2\eta$$

Note : $d\ln k = d\ln(aH) = dN \sim H dt$

Gravitational Waves

$$P_{grav}(k) = \left.\frac{2}{M_{Pl}^2}\left(\frac{H}{2\pi}\right)^2\right|_{k=aH} \qquad r \equiv \frac{P_{grav}}{P_\zeta} = 16\epsilon, \quad n_t = \frac{d\ln P_{grav}(k)}{d\ln k} \sim -2\epsilon$$

Summary of all relevant expressions

Slow Roll parameters

$$\varepsilon_V = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 ; \; \eta_V = M_P^2\left(\frac{V''}{V}\right)$$
$$\xi_V^2 = M_P^4\left(\frac{V'V'''}{V^2}\right) ; \; \sigma_V^3 = M_P^6\left(\frac{V'^2V''''}{V^3}\right)$$

Observables in terms of Slow Roll

$$\begin{aligned}
A_{\rm s} &\approx \frac{V}{24\pi^2 M_{\rm pl}^4 \varepsilon_V},\\
n_{\rm s}-1 &\approx 2\eta_V - 6\varepsilon_V,\\
{\rm d}n_{\rm s}/{\rm d}\ln k &\approx 16\varepsilon_V\eta_V - 24\varepsilon_V^2 - 2\xi_V^2,\\
{\rm d}^2 n_{\rm s}/{\rm d}\ln k^2 &\approx -192\varepsilon_V^3 + 192\varepsilon_V^2\eta_V - 32\varepsilon_V\eta_V^2\\
&\quad - 24\varepsilon_V\xi_V^2 + 2\eta_V\xi_V^2 + 2\sigma_V^3,\\
A_{\rm t} &\approx \frac{2V}{3\pi^2 M_{\rm pl}^4},\\
n_{\rm t} &\approx -2\varepsilon_V,\\
{\rm d}n_{\rm t}/{\rm d}\ln k &\approx 4\varepsilon_V\eta_V - 8\varepsilon_V^2.
\end{aligned}$$

Scalar & Tensor Amplitudes

$$\mathcal{P}_s(k) = \frac{1}{8\pi^2}\frac{H^2}{\varepsilon_V}\bigg|_{k=aH} \qquad \mathcal{P}_s(k_*) = A_s \quad k_* = 0.05\ {\rm Mpc}^{-1}$$
$$= A_{\rm s}\left(\frac{k}{k_*}\right)^{n_{\rm s}-1+\frac{1}{2}{\rm d}n_{\rm s}/{\rm d}\ln k\ln(k/k_*)+\frac{1}{6}{\rm d}^2n_{\rm s}/{\rm d}\ln k^2(\ln(k/k_*))^2+\ldots}$$
$$\mathcal{P}_{\rm t}(k) = \frac{2H^2}{\pi^2}\bigg|_{k=aH}$$
$$= A_{\rm t}\left(\frac{k}{k_*}\right)^{n_{\rm t}+\frac{1}{2}{\rm d}n_{\rm t}/{\rm d}\ln k\ln(k/k_*)+\ldots},$$

$$r(k=k_*) = \frac{\mathcal{P}_t(k_*)}{\mathcal{P}_s(k_*)}.$$

Scalar-Tensor Ratio

$$r = 16\varepsilon_V\frac{\left[1-(\mathcal{C}_E+1)\varepsilon_V\right]^2}{\left[1-(3\mathcal{C}_E+1)\varepsilon_V+\mathcal{C}_E\eta_V\right]^2}$$
$$\simeq 16\varepsilon_V,$$

$$\mathcal{C}_E = 4(\ln 2+\gamma_E)-5 \qquad \gamma_E = 0.5772$$

Chatterjee, Mazumdar, 1409.4442

An Example: Consistency Relationship

$$r \equiv \frac{P_{grav}}{P_\zeta} = 16\epsilon, \quad n_t = \frac{d\ln P_{grav}(k)}{d\ln k} \sim -2\epsilon$$

Assumptions: Matter & Gravity both Quantum

$$p_k'' + \left(k^2 - \frac{a''}{a}\right) p_k = 0 \qquad p_k(\tau) = -\alpha_k \sqrt{\frac{2}{k\pi}} e^{-ik\tau} + \beta_k \sqrt{\frac{2}{k\pi}} e^{ik\tau}$$

$$|\alpha_k|^2 - |\beta_k|^2 = \frac{\pi}{4} \qquad p_k(\tau) \to \frac{1}{\sqrt{2k}} e^{-ik\tau} \quad \text{for} \quad k\tau \to -\infty$$

$$\alpha_k = -\frac{\sqrt{\pi}}{2}, \qquad \beta_k = 0 \qquad \mathcal{P}_T^{\text{quantum}} = \frac{16 H^2}{\pi M_p^2}$$

Bunch-Davis vacuum

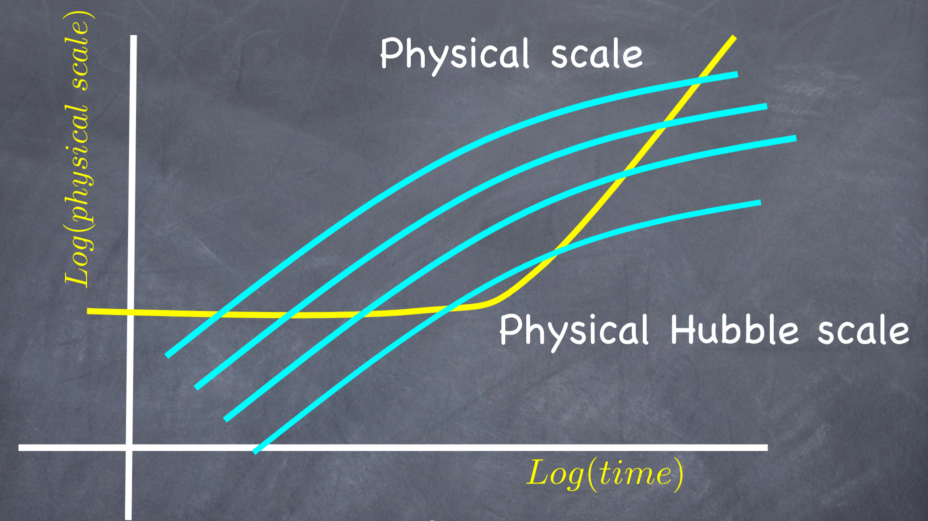
Assuming Gravity is Classical

$$\alpha_{\mathbf{k},\lambda} = -\beta^*_{-\mathbf{k},\lambda} \qquad \mathcal{P}_T^{\text{classical}} = \frac{64\,|\alpha_{\mathbf{k},\lambda} + \beta_{\mathbf{k},\lambda}|^2 H^2}{M_p^2 \pi^2}$$

Ashorioon, Bhupal-Dev, Mazumdar, 1211.4678

Evolution of the Perturbations

Number of e-foldings of Inflation



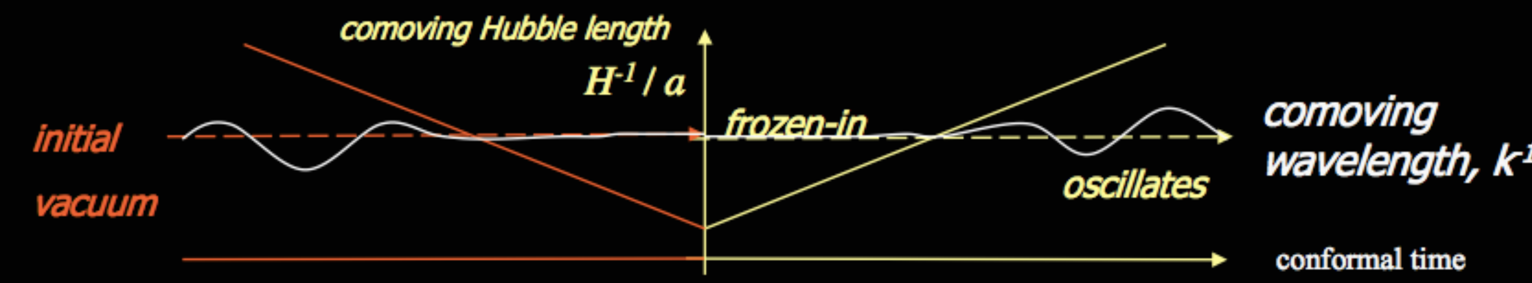
Inflation smoothes out everything, but the Quantum Fluctuations

Perturbations leave the Hubble Radius during inflation & then re-enter Hubble radius at later epochs

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + (k/a)^2\,\delta\varphi = 0$$

Characteristic timescales for waves, comoving wavemode k

- *small-scales k > aH* *under-damped oscillator*
- *large-scales k < aH* *over-damped oscillator*



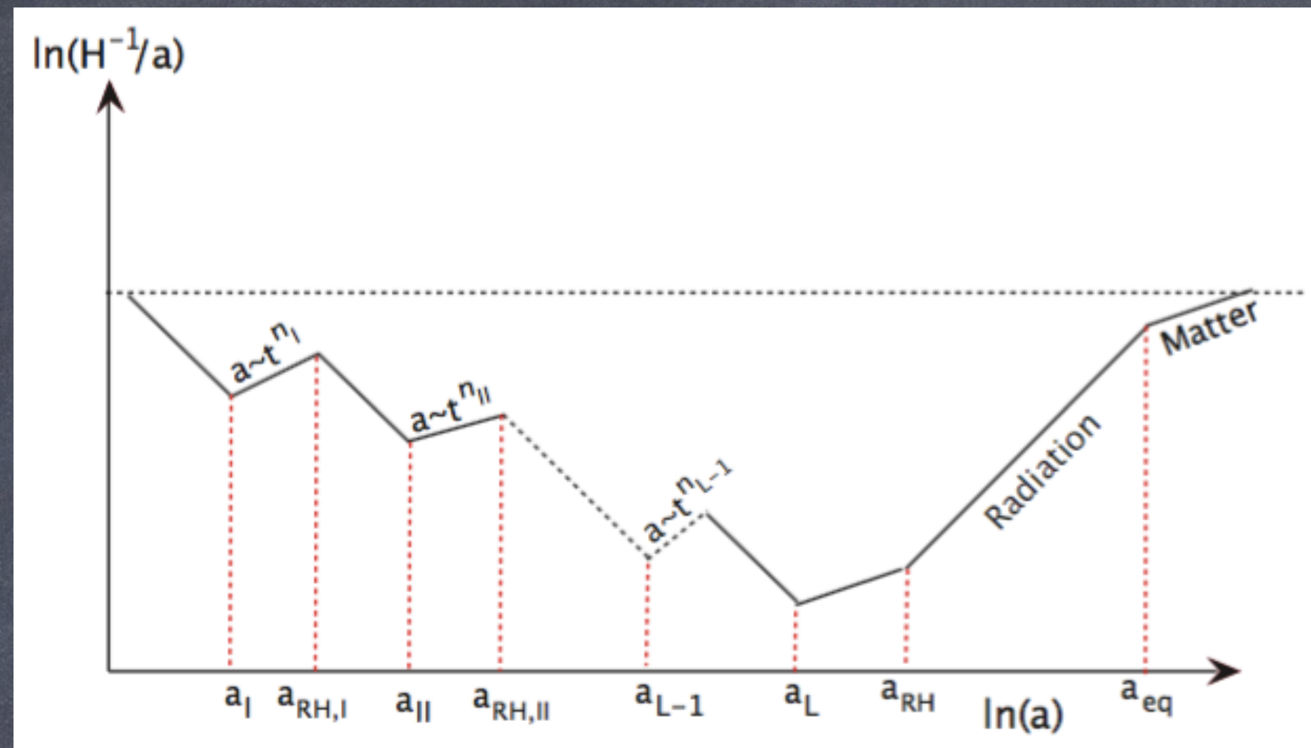
inflation	radiation or matter era
accelerated expansion	decelerated expansion
$\ddot{a} = \frac{d}{dt}(aH) > 0$	$\ddot{a} = \frac{d}{dt}(aH) < 0$

$$N(k) = 62 - \ln\frac{k = a_k H_k}{a_0 H_0} - \ln\frac{10^{16}\mathrm{GeV}}{V_k^{1/4}} + \ln\frac{V_k^{1/4}}{V_{end}^{1/4}} - \frac{1}{3}\ln\frac{V_{end}^{1/4}}{\rho_{rh}^{1/4}}$$

50 – 60 e – foldings are required, depending on thermal history

Lowest e – foldings will be 25 for successful BBN

Number of e-foldings of Inflation



$$\frac{k}{a_0H_0} = \frac{a_kH_k}{a_0H_0} = \left(\frac{a_k}{a_I}\right)\left(\frac{a_I}{a_{RH,I}}\right)\left(\frac{a_{RH,I}}{a_{II}}\right)\left(\frac{a_{II}}{a_{RH,II}}\right)\ldots\left(\frac{a_{RH,L-1}}{a_L}\right)\left(\frac{a_L}{a_{RH}}\right)\left(\frac{a_{RH}}{a_{eq}}\right)\left(\frac{H_k}{H_{eq}}\right)\left(\frac{a_{eq}H_{eq}}{a_0H_0}\right)$$

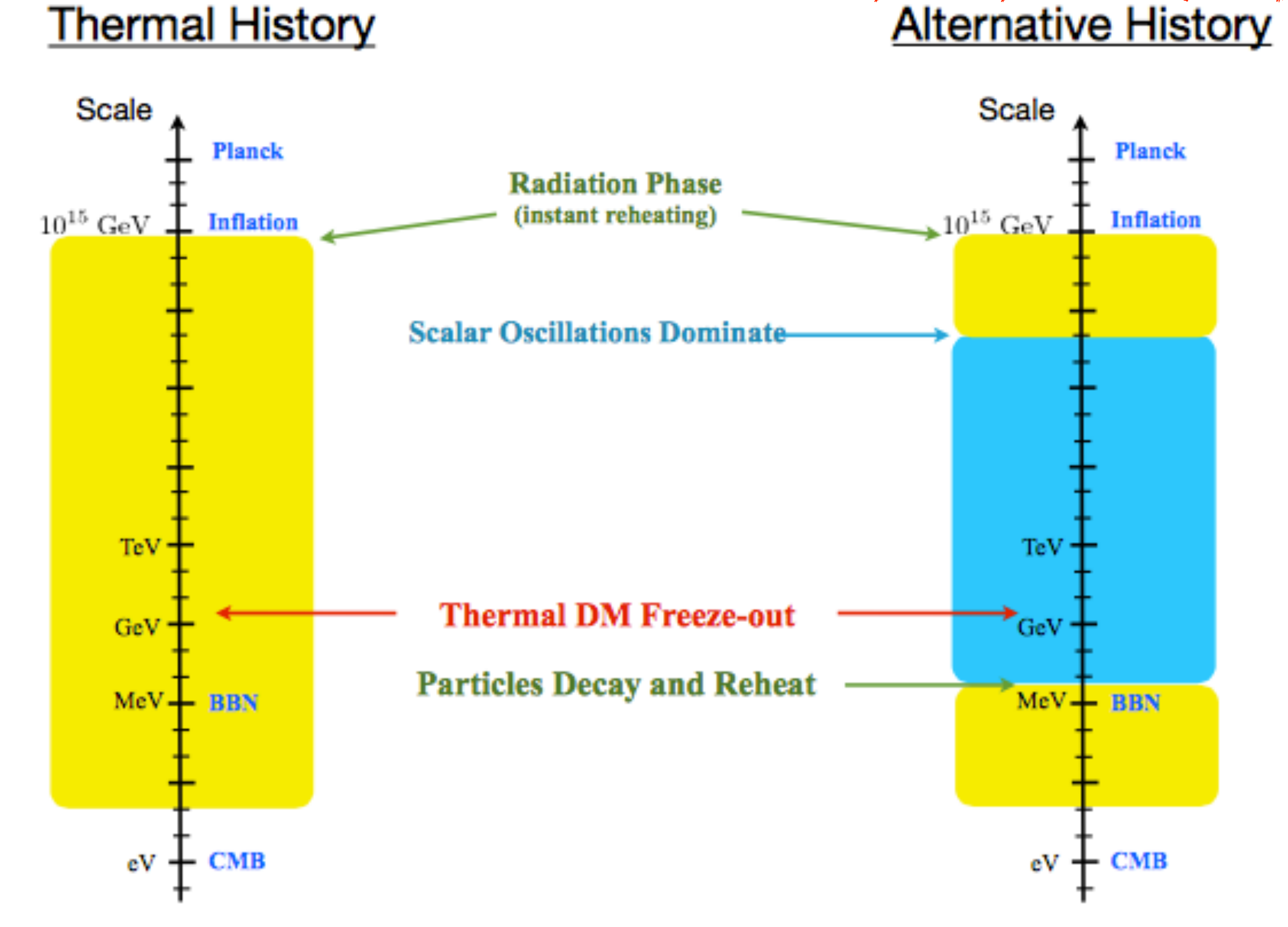
$$= \mathrm{e}^{-N_I(k)}\left(\frac{a_I}{a_{RH,I}}\right)\mathrm{e}^{-N_{II}}\left(\frac{a_{II}}{a_{RH,II}}\right)\ldots e^{-N_L}\left(\frac{\rho_L}{\rho_{RH}}\right)^{-1/3}\left(\frac{\rho_{RH,II}}{\rho_{eq}}\right)^{-1/4}\left(\frac{H_k}{H_{eq}}\right)\left(\frac{a_{eq}H_{eq}}{a_0H_0}\right)$$

Since during the period between i and $i+1$ phases of inflation, the Universe expands as $a \sim t_i^n$. Hence during these periods, $\ln(1/(aH)) \sim (1-n_i)/n_i \ln(a)$, and therefore

$$N_I(k) + \sum_{i=2}^{L} N_i = -\ln\left(\frac{k}{a_0H_0}\right) + \frac{1}{3}\ln\left(\frac{\rho_{RH,L}}{\rho_L}\right) + \frac{1}{4}\ln\left(\frac{\rho_{eq}}{\rho_{RH,L}}\right)$$

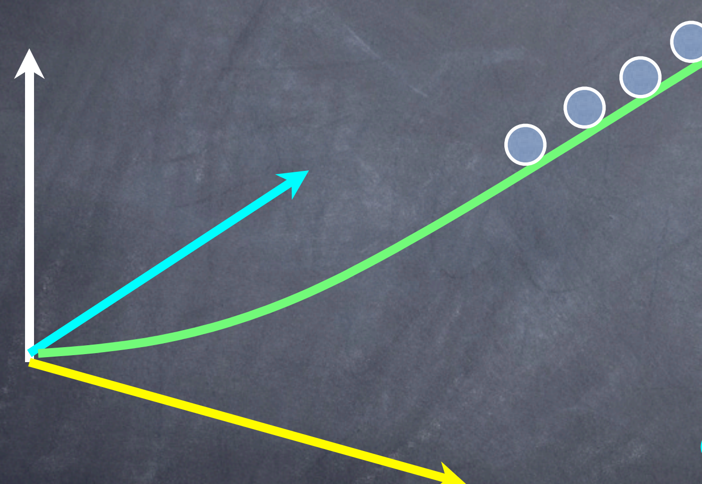
$$+\ln\left(\sqrt{\frac{8\pi V_k}{3m_{Pl}^2}}\frac{1}{H_{eq}}\right) + \ln(219\,\Omega_0 h) + \sum_{i=1}^{L-1}\frac{n_i}{2}\ln\left(\frac{V_{i+1}}{V_i}\right)$$

hep-th/0501125



Moduli Domination after Inflation

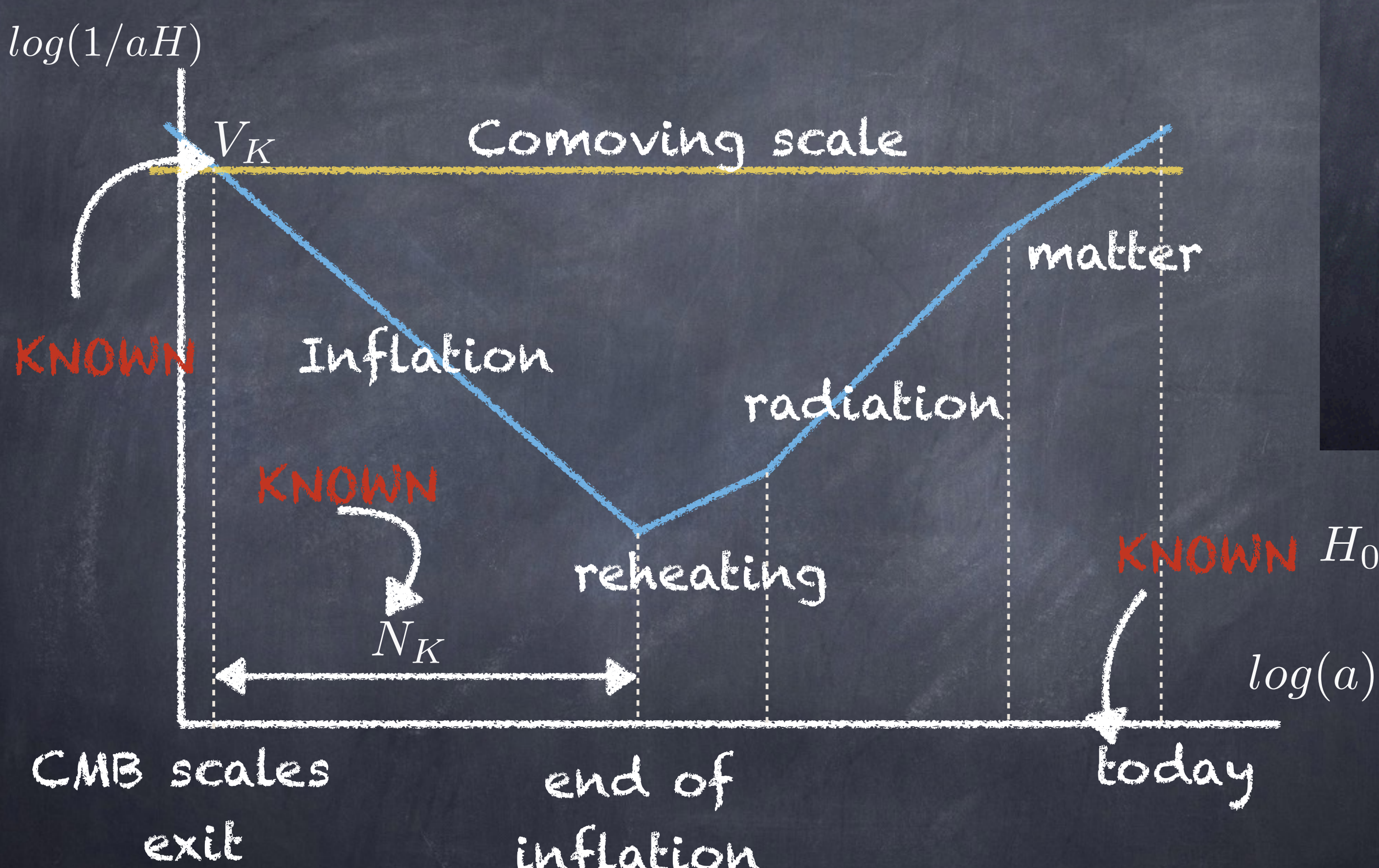
What happens after inflation ?



The flat direction rolls slowly due to Hubble friction

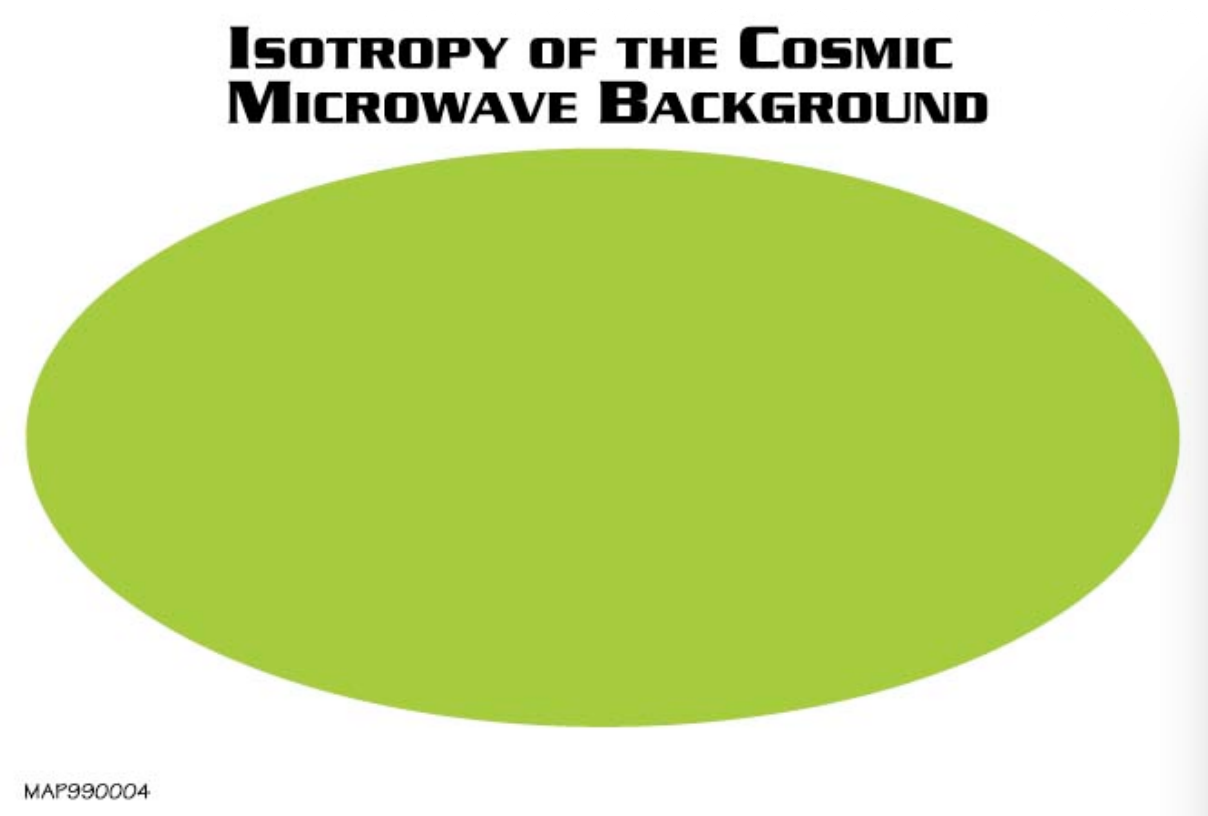
Eventually oscillates (even after inflaton decayed completely) when

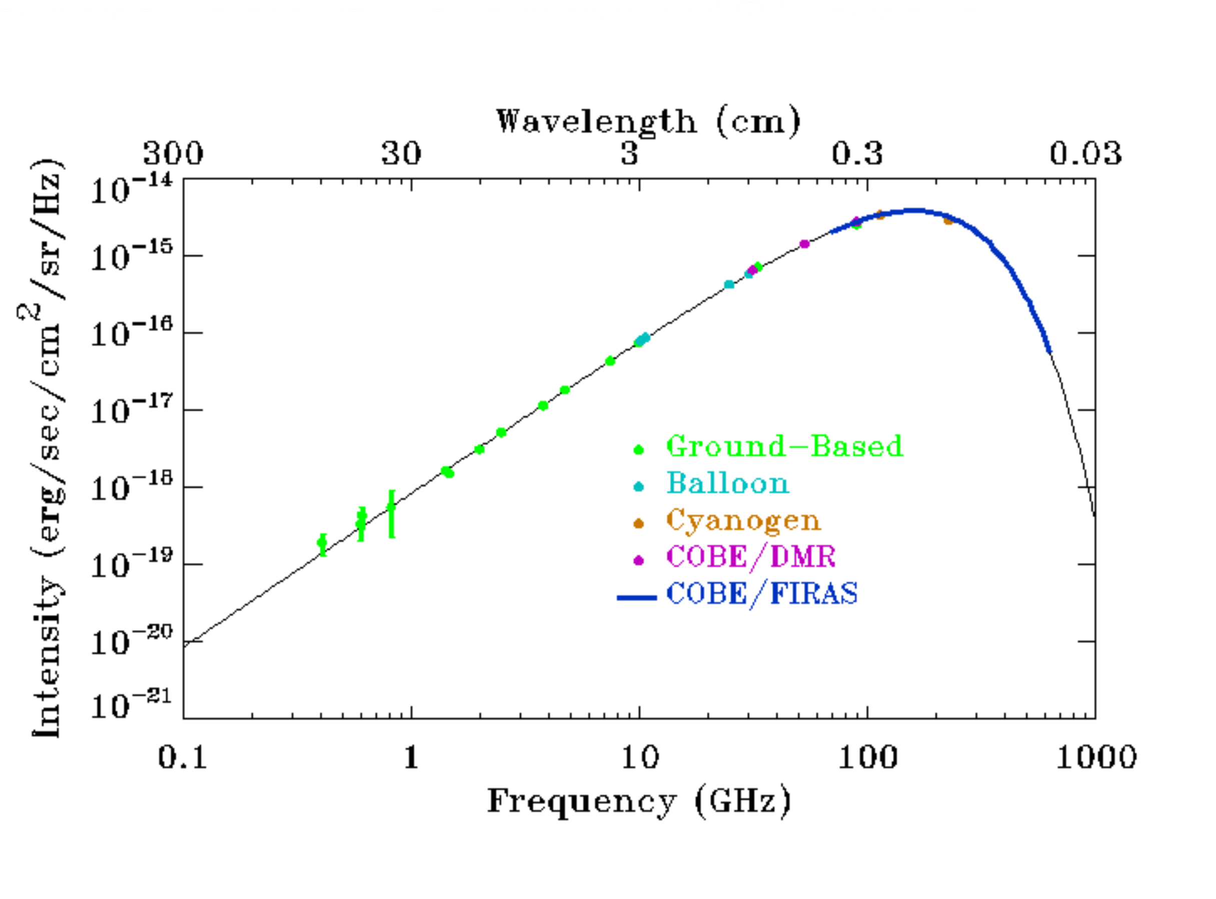
$$m_0(\sim 1\ TeV) \geq H(t)$$



Observations

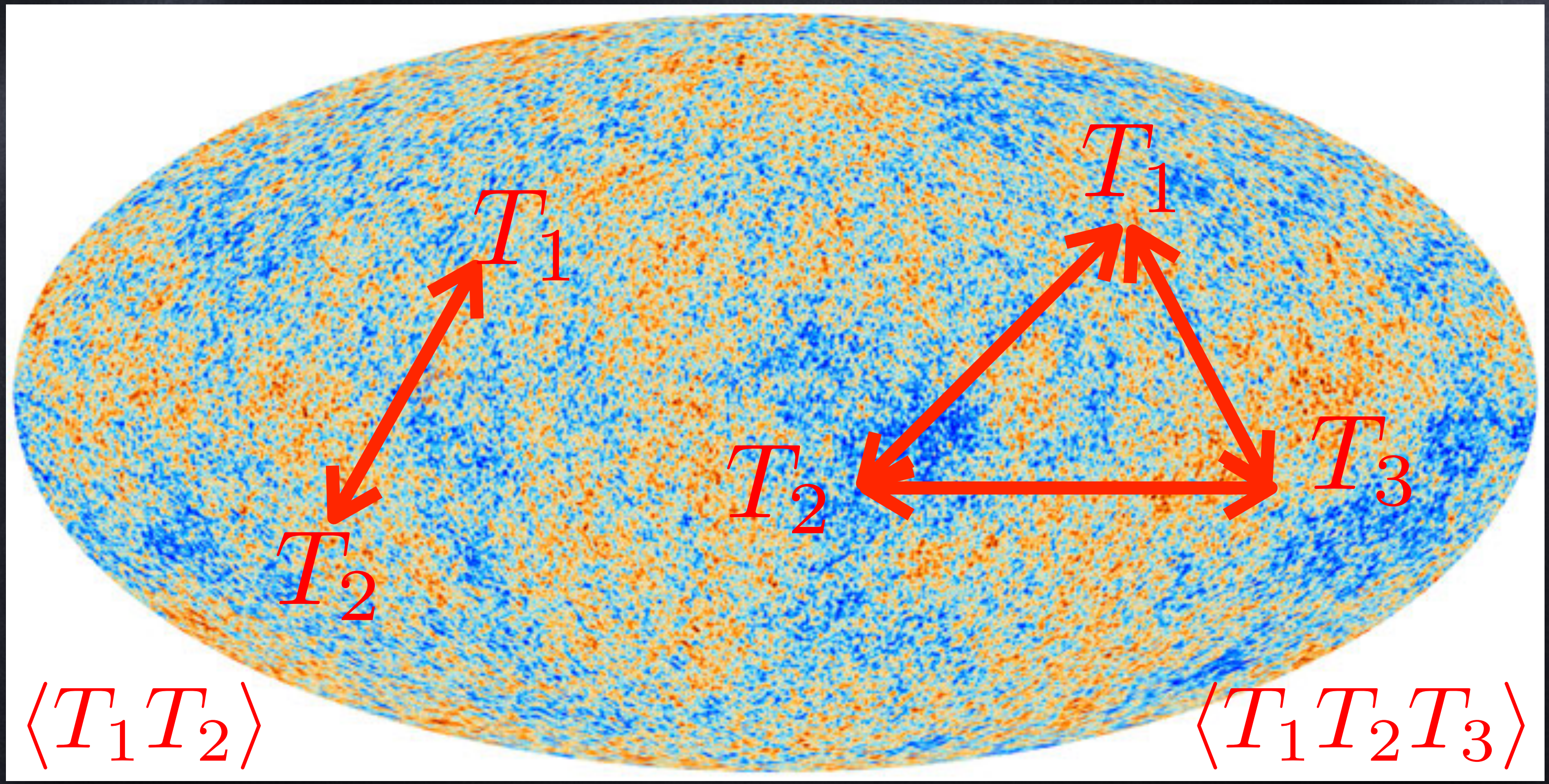
What Observations are telling us?



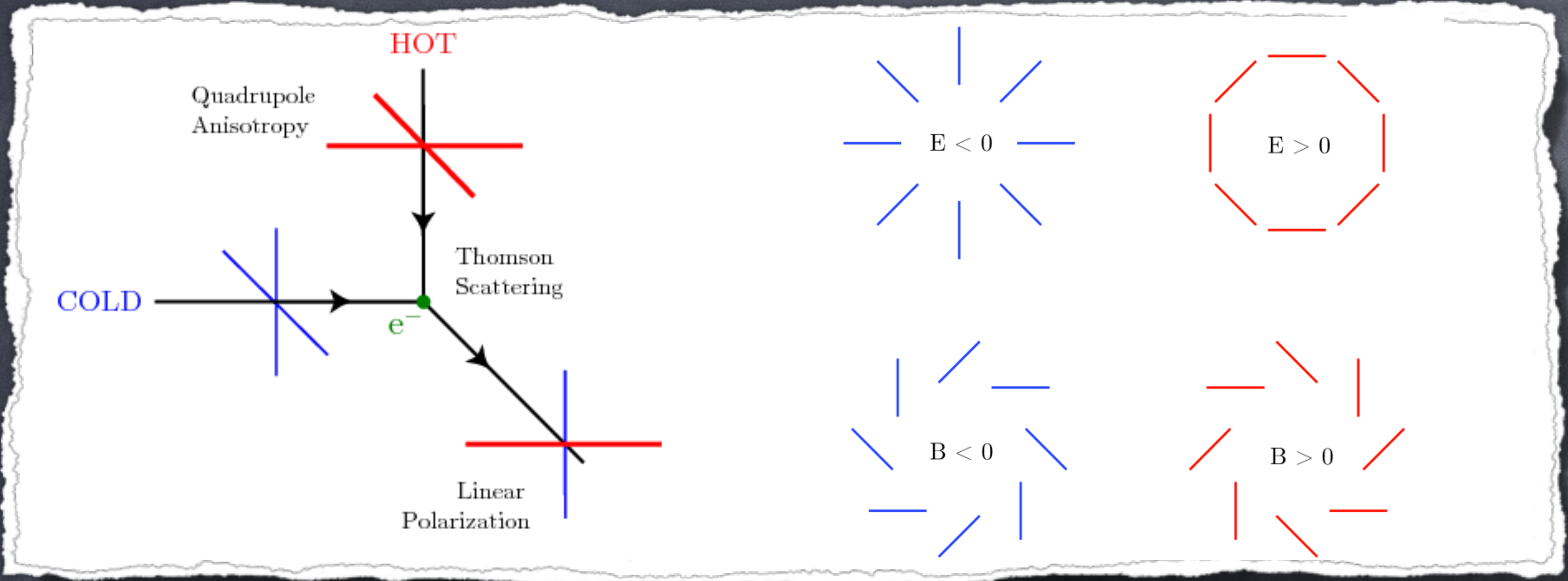


$$\frac{\Delta T}{T} = 4.6 \times 10^{-5}$$

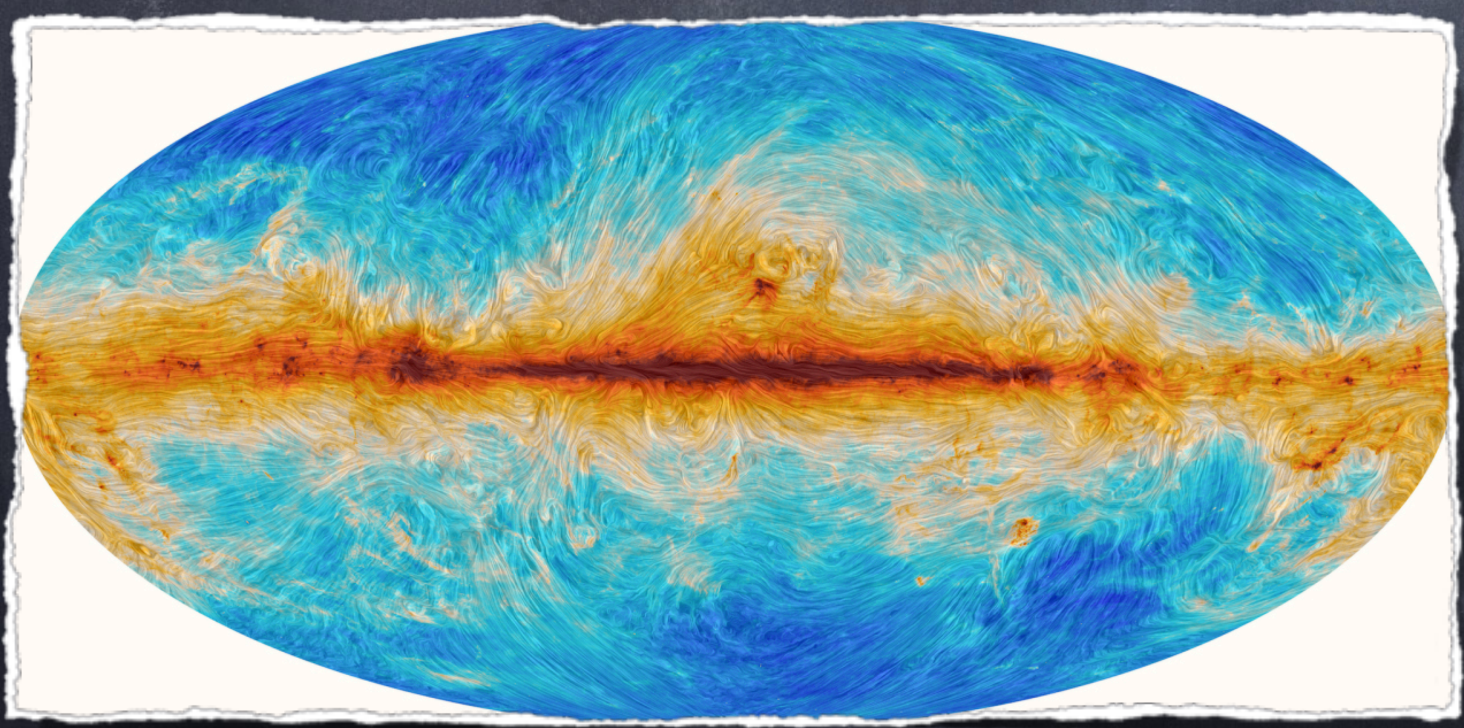
Perturbations are Gaussian

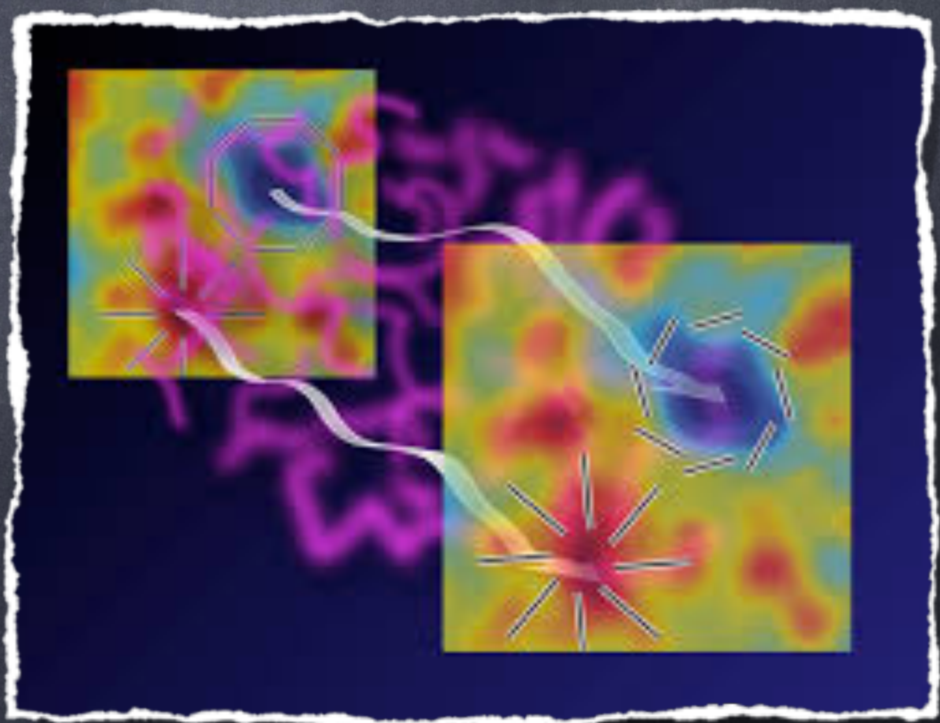


E-mode & B-mode Polarisations



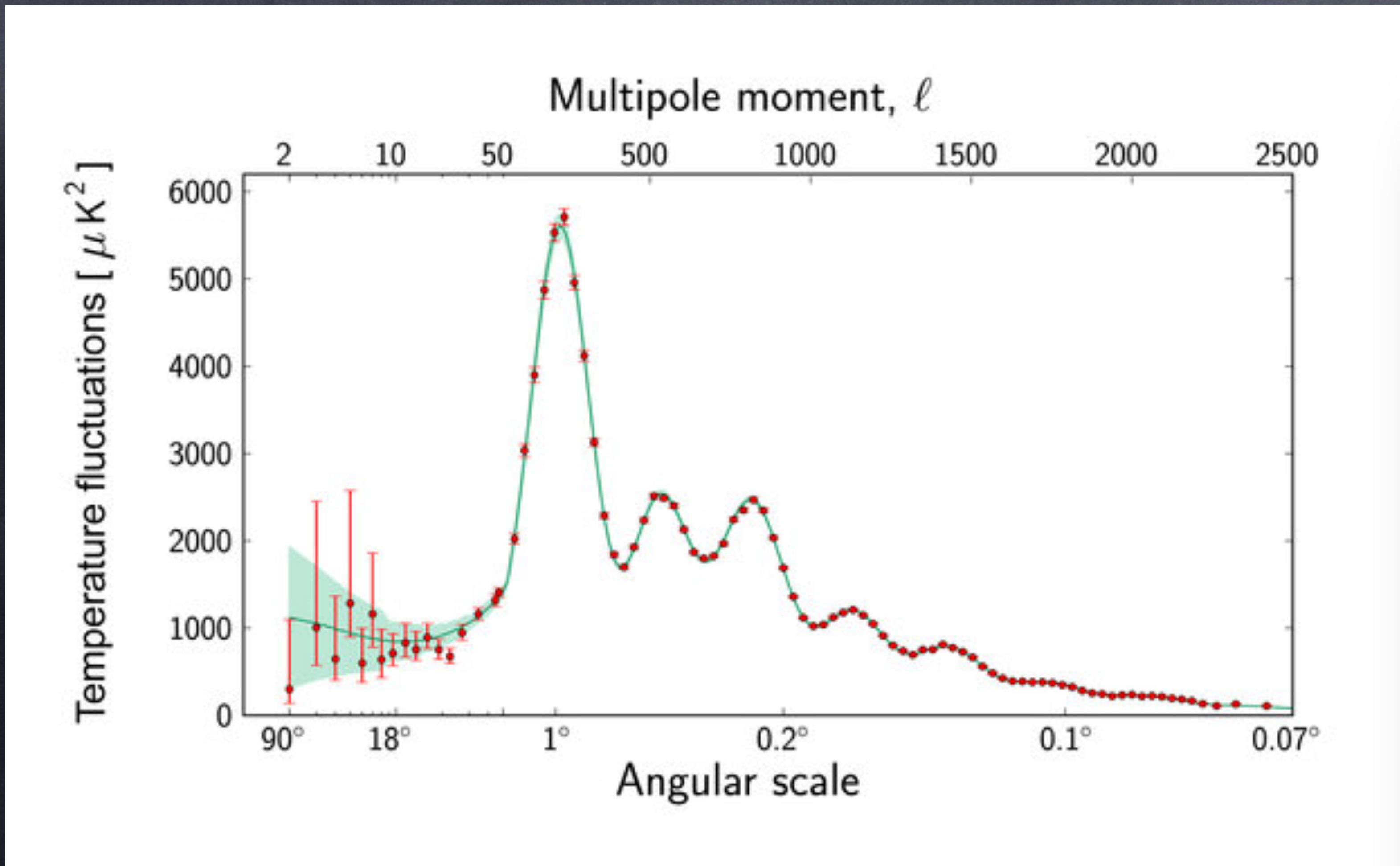
A pure E mode turns into B mode if we turn all polarisation vector by 45 degrees

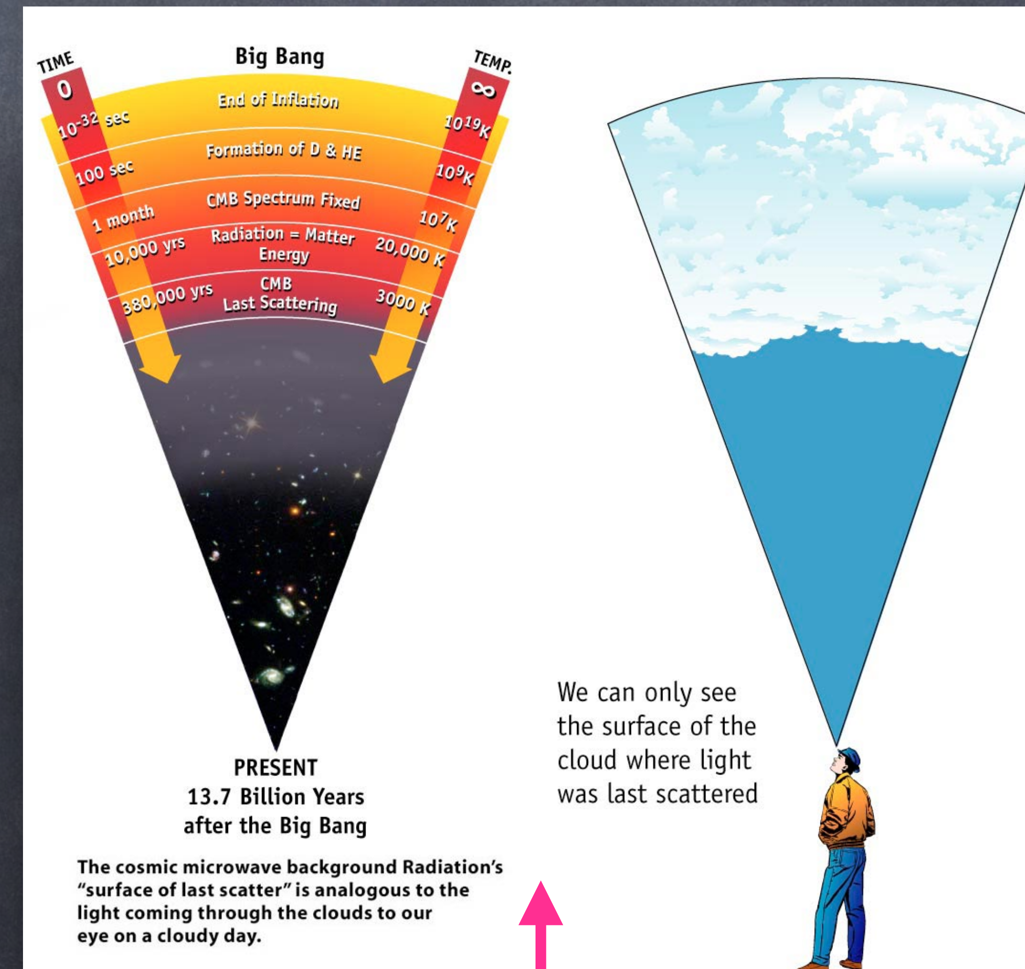




No primordial B-modes detected so far

Angular Power Spectrum





(1) Baryon density

(2) Dark Matter density

(3) Dark Energy density

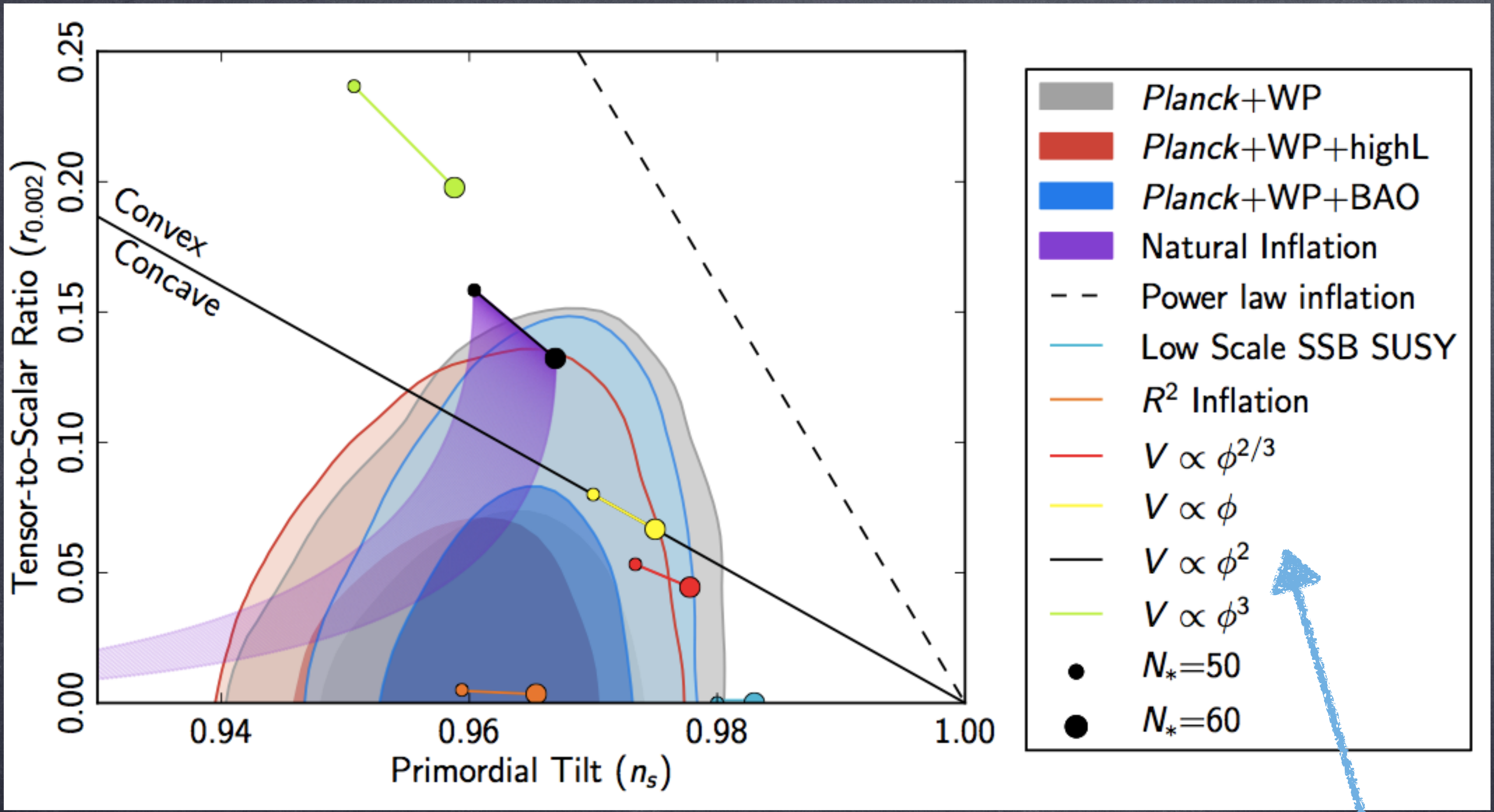
(4) Amplitude

4.6×10^{-5}

(5) Tilt: $n_s = 0.96$

(6) Reionization Optical depth

Summary Plot for Theorists from Planck



$$n_s = 0.959 \pm 0.007 \quad r_{0.02} < 0.11 \ (95\% \ CL)$$

$$\frac{dn_s}{d\ln k} = -0.015 \pm 0.009$$

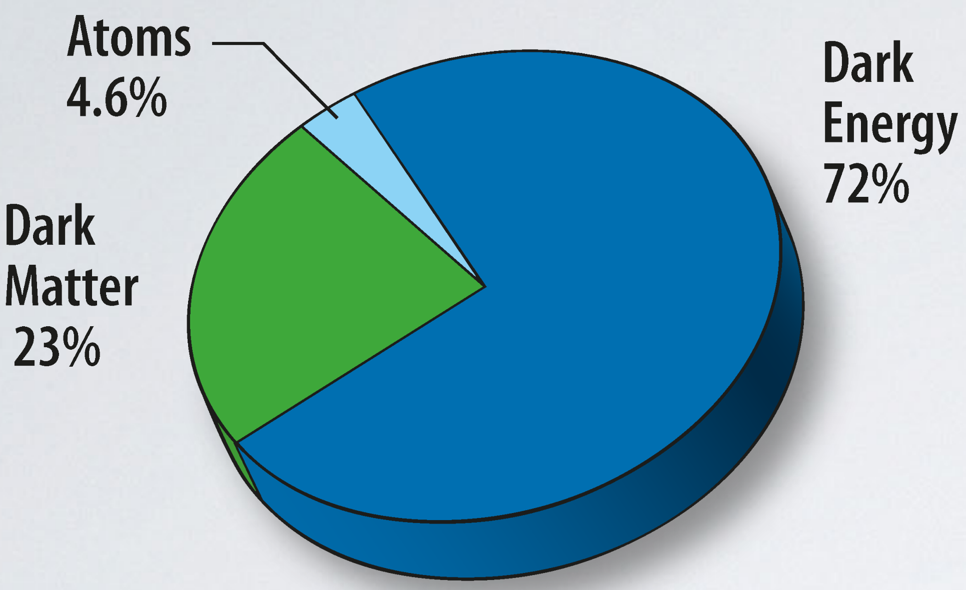
Bench mark points - No real physics

6 Model Parameters

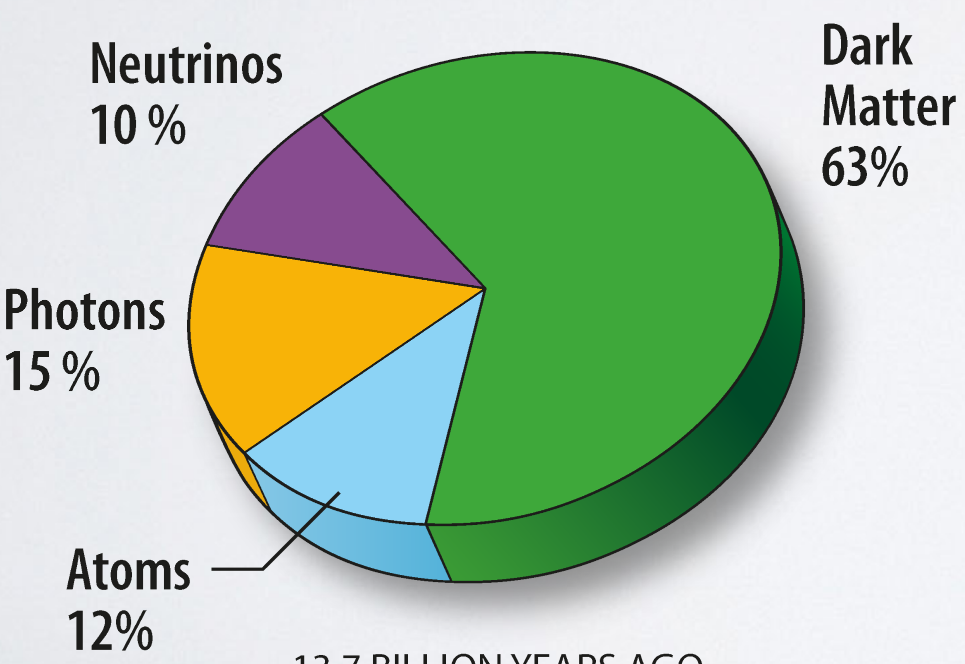
	Planck (CMB+lensing)		*Planck*+WP+highL+BAO	
Parameter	Best fit	68 % limits	Best fit	68 % limits
$\Omega_b h^2$	0.022242	0.02217 ± 0.00033	0.022161	0.02214 ± 0.00024
$\Omega_c h^2$	0.11805	0.1186 ± 0.0031	0.11889	0.1187 ± 0.0017
$100\theta_{MC}$	1.04150	1.04141 ± 0.00067	1.04148	$\boxed{1.04147 \pm 0.00056}$
τ	0.0949	0.089 ± 0.032	0.0952	0.092 ± 0.013
n_s	0.9675	0.9635 ± 0.0094	0.9611	$\boxed{0.9608 \pm 0.0054}$
$\ln(10^{10} A_s)$	3.098	3.085 ± 0.057	3.0973	3.091 ± 0.025

$$P_s \sim 3 \times 10^{-10} \left(\frac{k}{k_0}\right)^{n_s - 1} \qquad k_0 = 0.04\ \mathbf{Mpc}^{-1}$$

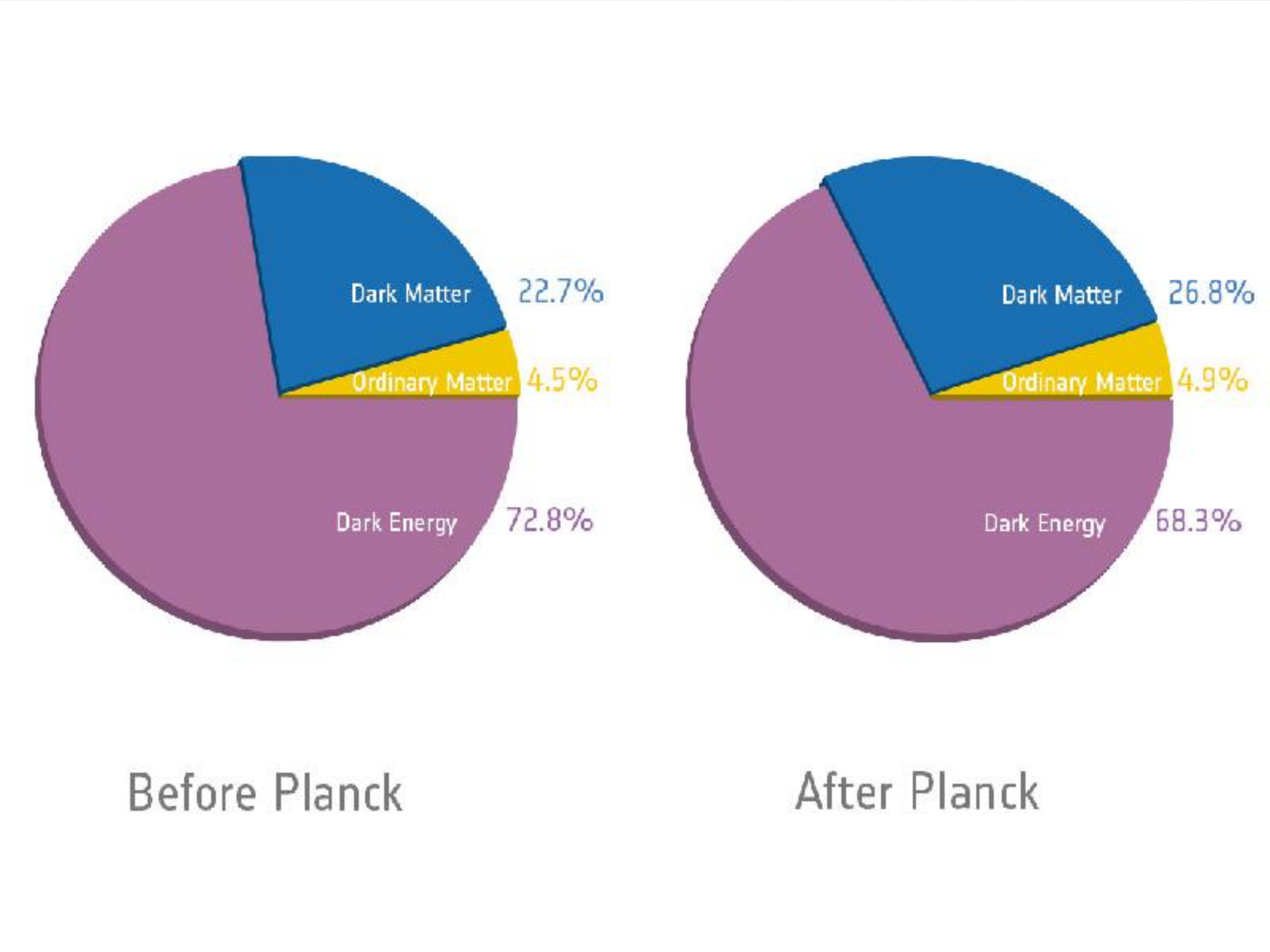
Energy Budget



TODAY



13.7 BILLION YEARS AGO
(Universe 380,000 years old)



Curvature of the Universe

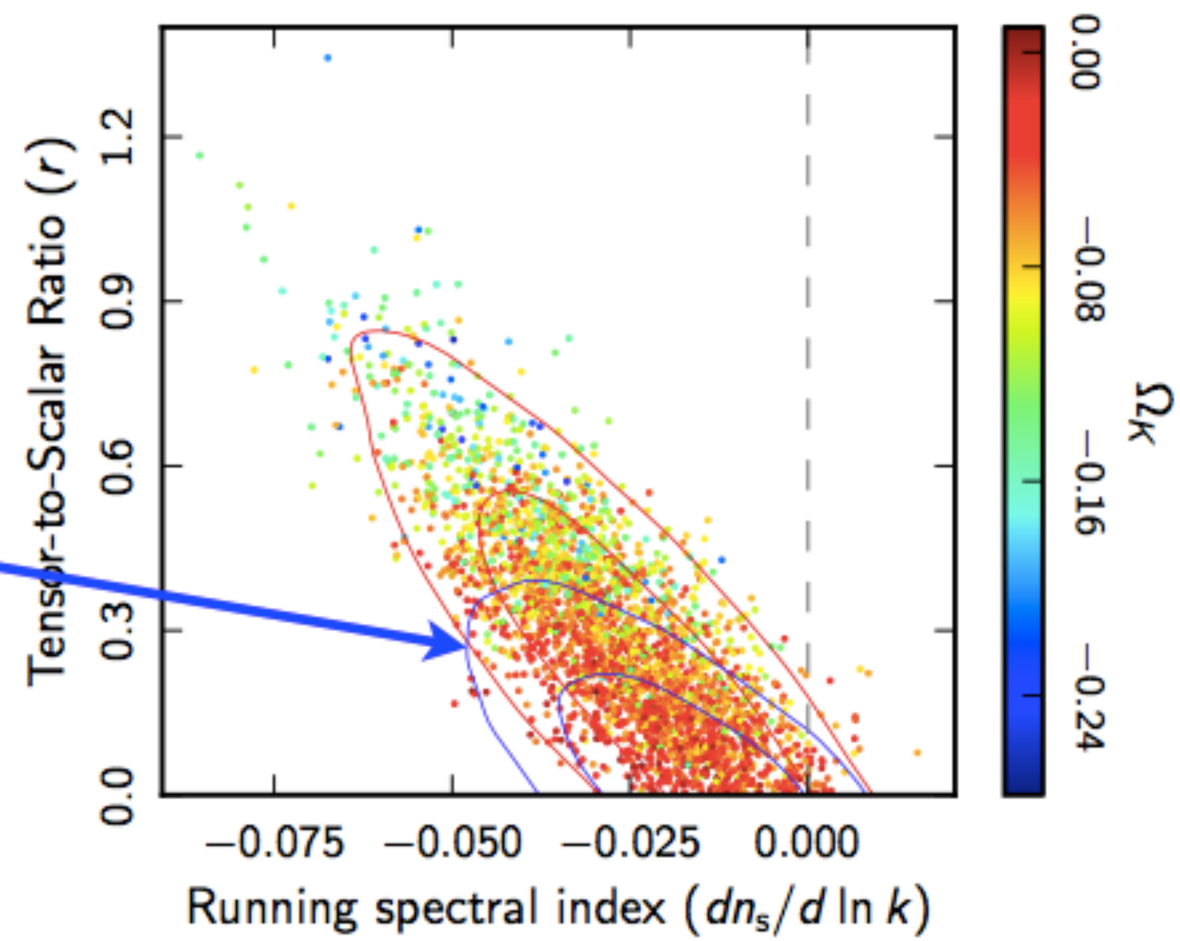
Simplest inflationary models predict $|\Omega_K| < 10^{-5}$

Open inflation (e.g. bubble nucleation, landscape) can predict larger negative spatial curvature, $O(10^{-4})$;

positive curvature (closed universe) much harder to get in inflationary paradigm.

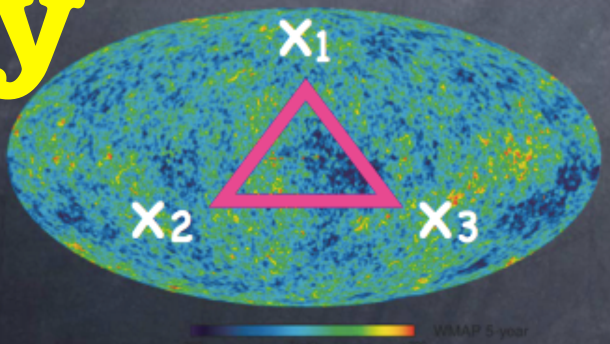
$$\Omega_K = -0.0004 \pm 0.0036$$

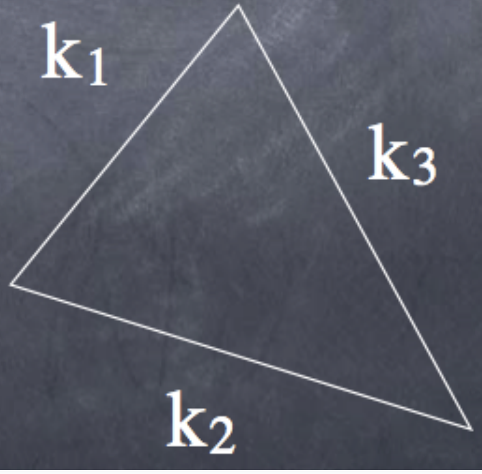
(Planck+WP+BAO)



Non-Gaussianity

$$\Phi(x) = \Phi_G(x) - f_{NL}\Phi_G(x)^2$$





Slow-roll single-field inflation: $f_{NL} < 1$

Some interesting inflation models predict much higher f_{NL}

WMAP9: $f_{NL} = 37 \pm 20$

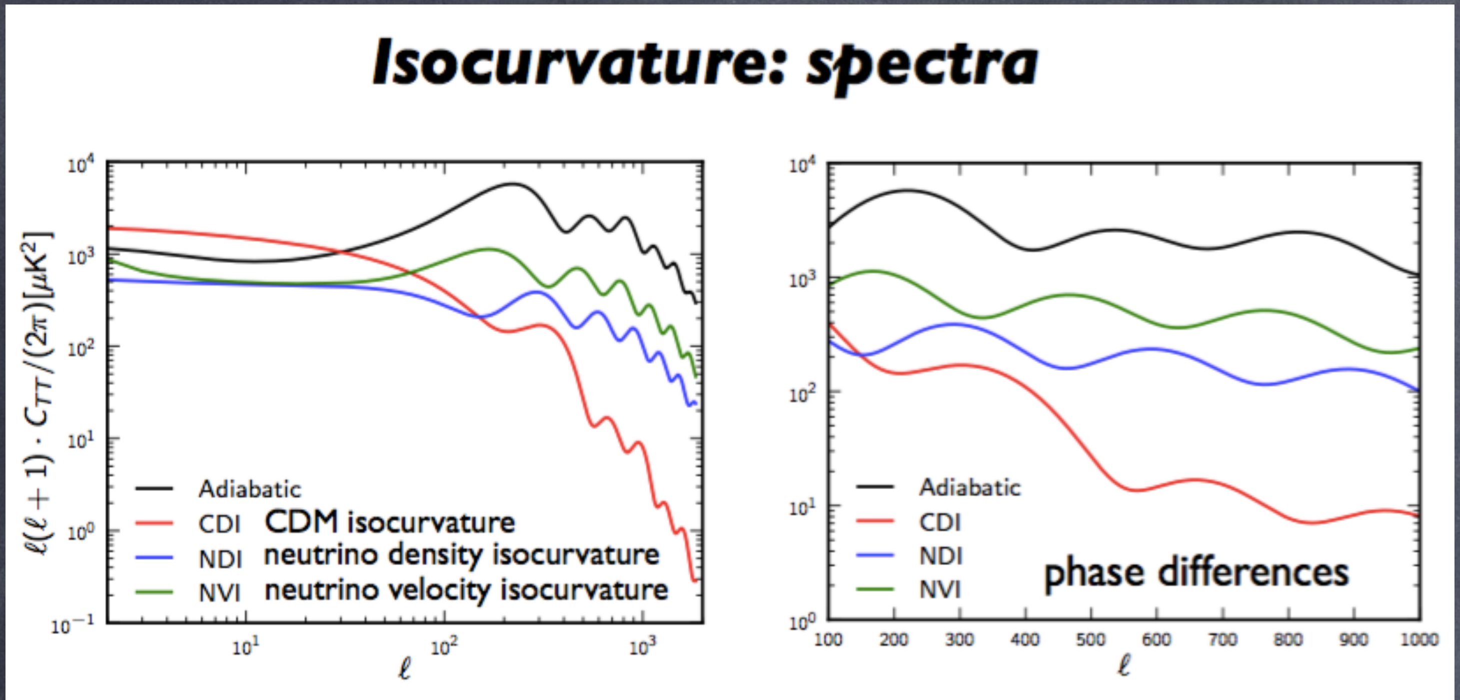
Nonlinear effects cause additional non-Gaussianity in the CMB: coupling between weak gravitational lensing and ISW from evolving gravitational potential

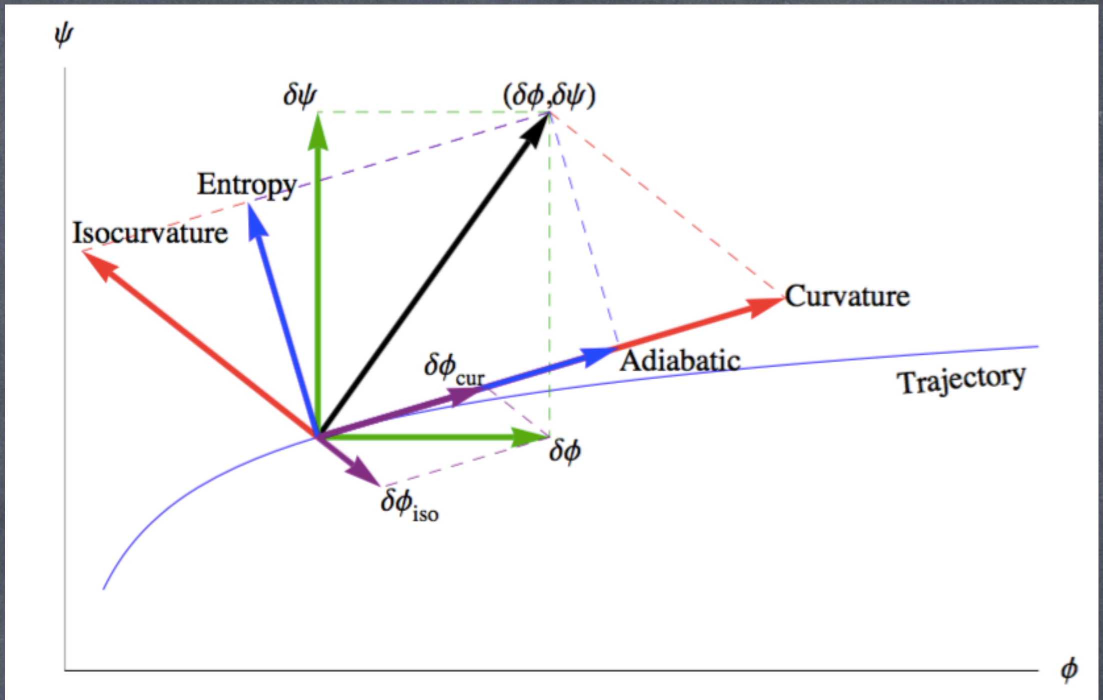
- This effect was clearly detected by Planck

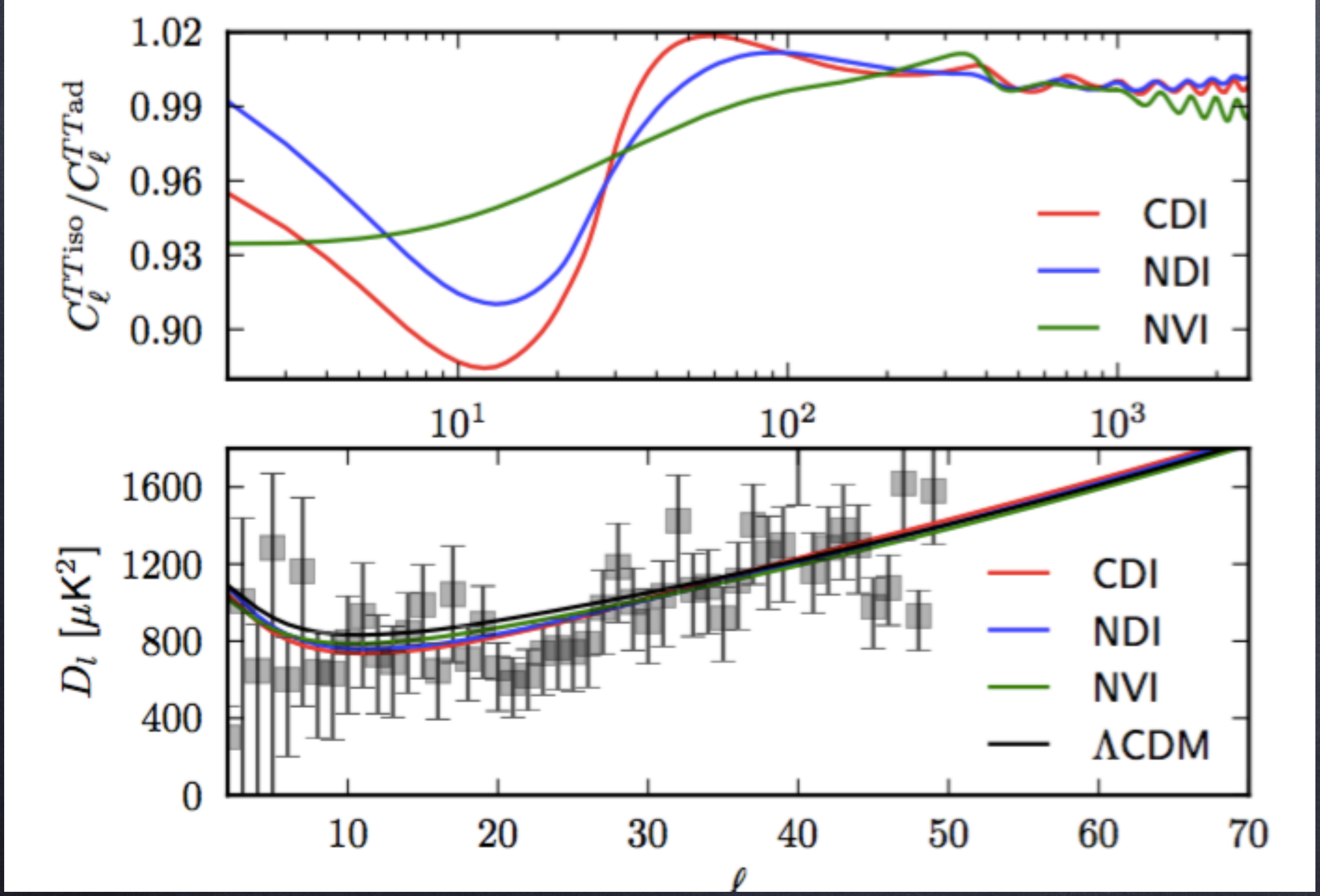
Planck:

- before correcting for ISW-lensing effect: $f_{NL} = 9.8 \pm 5.8$
- ISW-lensing subtracted: $f_{NL} = 2.7 \pm 5.8$

No Evidence for Iso-curvature Perturbations

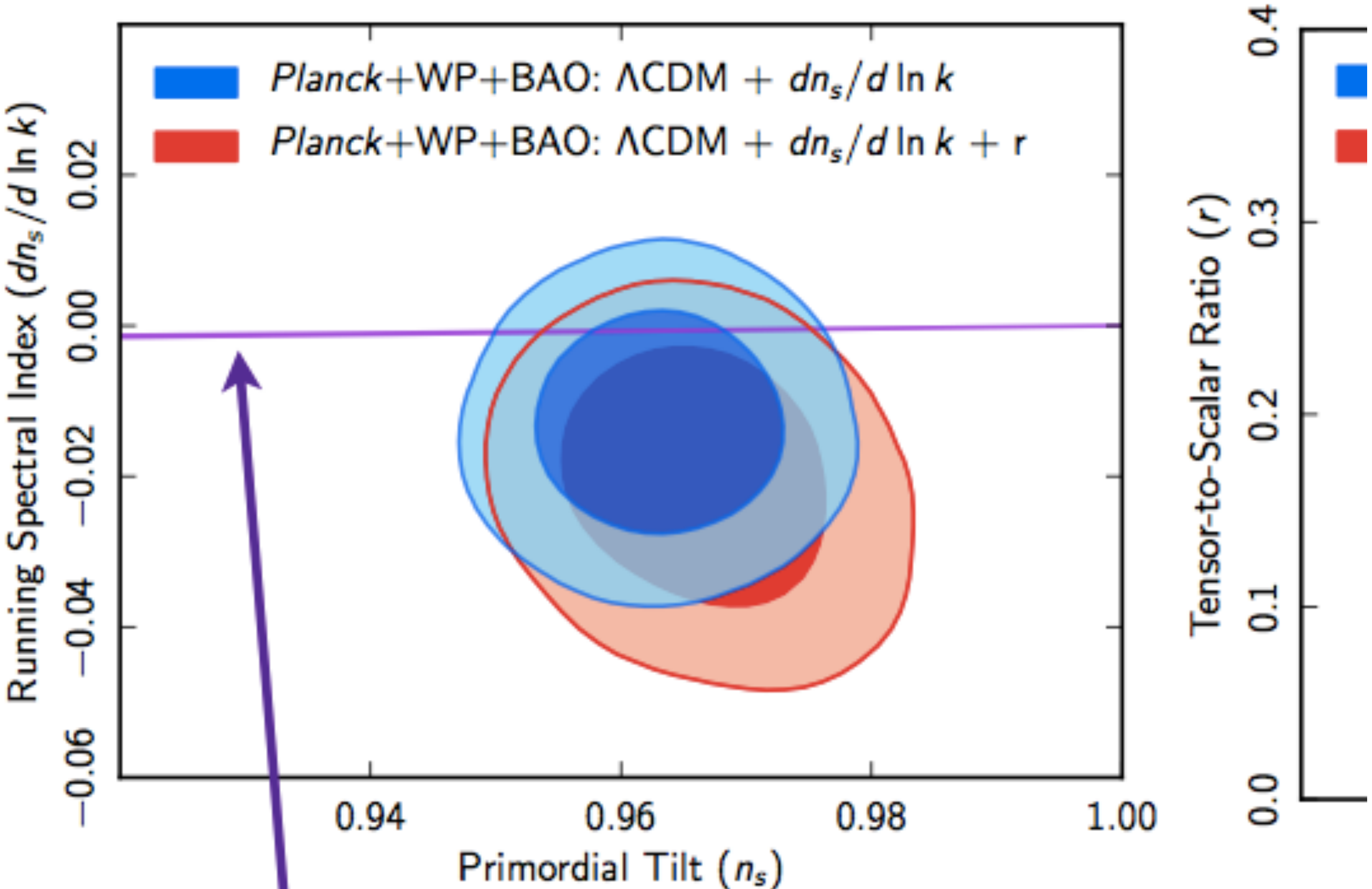


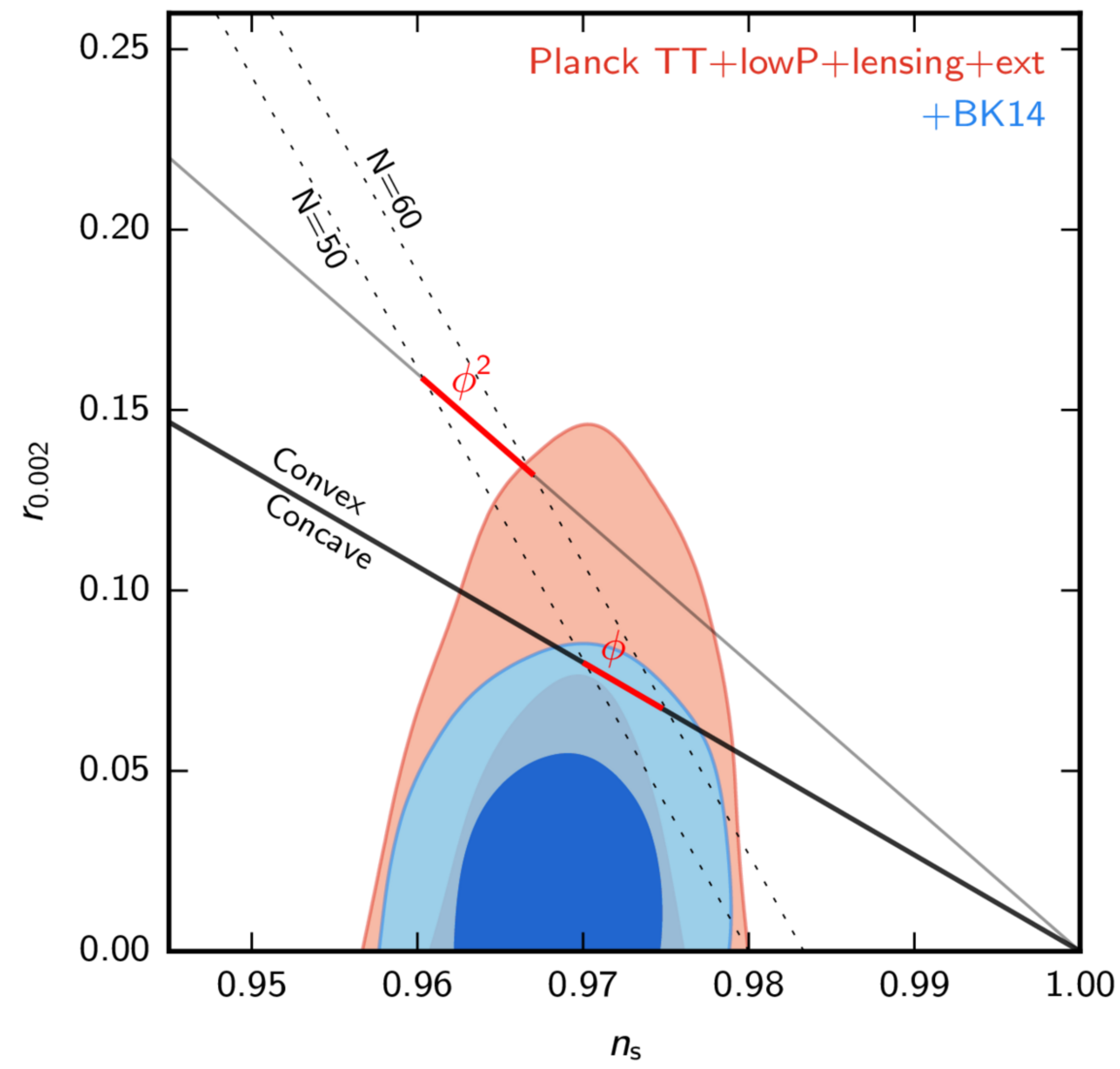




Inflationary perturbations are Adiabatic in nature

RUNNING OF THE SPECTRAL TILT



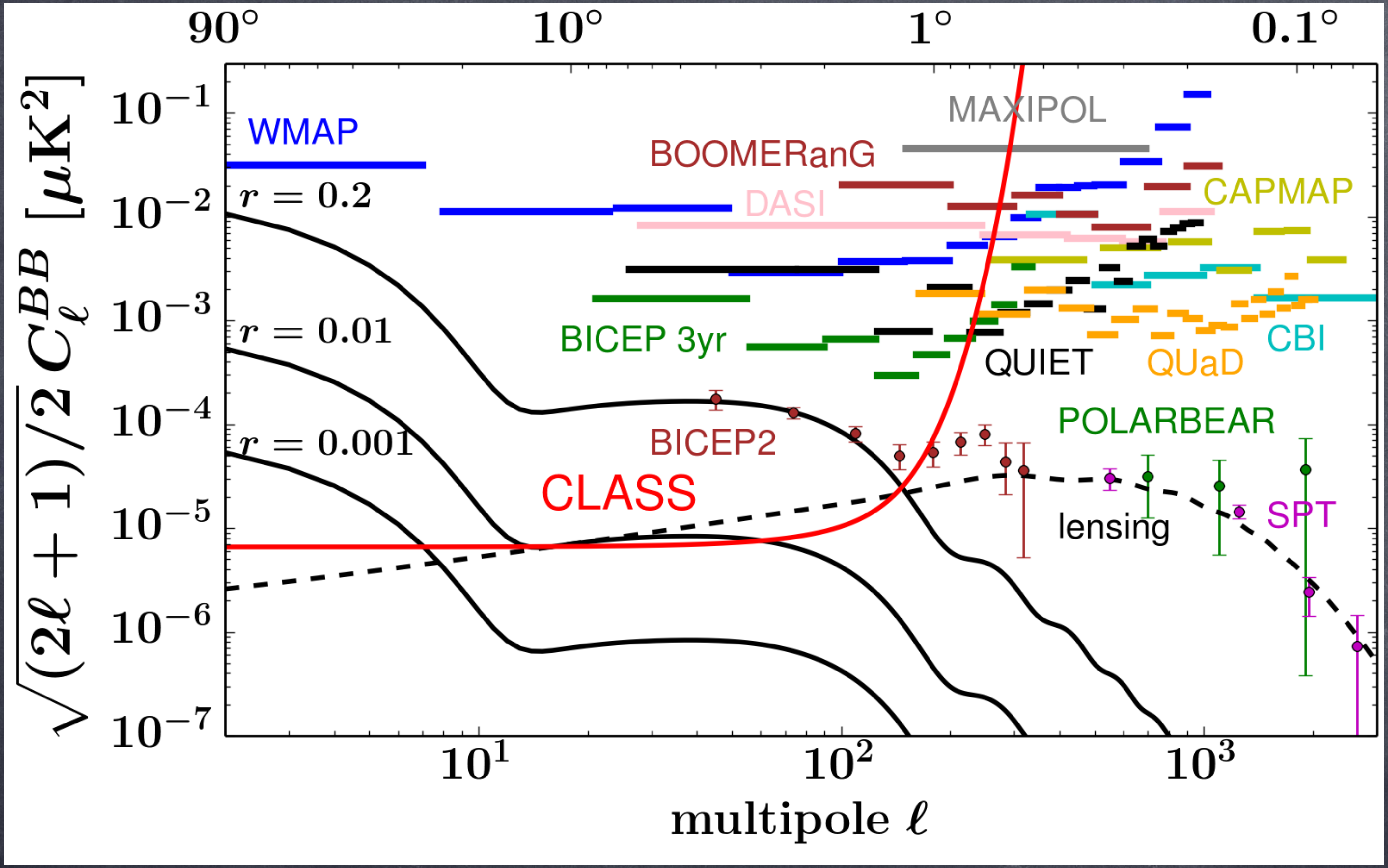


predictions of monomial chaotic models with $N_* \sim [50,60]$

$$\text{Planck+WP:}\ dn_s/d\ln k = -0.013 \pm 0.009$$

Bicep+Planck jont analysis: 1510.09217

Immediate Future for CMB: B modes



$$f_{\rm NL}^{h\zeta\zeta} \equiv \langle h\zeta\zeta\rangle/(P_\zeta^{3/2}P_h^{1/2})$$

Tensor to scalar ratio: $r \equiv P_h(k_*)/P_\zeta(k_*)$

Non-Gaussianity: $f_{\rm NL} \propto \langle\zeta\zeta\zeta\rangle/P_\zeta^2(k)$