Intro to SUSY III: MSSM

Archil Kobakhidze

PRE-SUSY 2016 SCHOOL
27 JUNE - 1 JULY 2016, MELBOURNE
Recap from the second lecture:

- Chiral and anti-chiral superfields

\[ \bar{D}_{\dot{\alpha}} S(x, \theta, \bar{\theta}) = 0, \quad D_\alpha S(x, \theta, \bar{\theta}) = 0 \]

\[
\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \\
= \phi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) \\
+ i\theta\sigma^\mu \bar{\theta} \partial_\mu \phi(x) + \frac{i}{\sqrt{2}} \theta^2 \partial_\mu \psi(x) \sigma^\mu \bar{\theta} - \frac{1}{4} \theta^2 \bar{\theta}^2 \partial_\mu \partial^\mu \phi(x)
\]
Recap from the second lecture:

- Vector (real) superfield

\[ S^+(x, \theta, \bar{\theta}) = S(x, \theta, \bar{\theta}) \]

- In the Wess-Zumino gauge

\[ V_{WZ}(x, \theta, \bar{\theta}) = \theta \sigma^\mu \bar{\theta} A_\mu(x) + \theta^2 \bar{\theta} \lambda + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta}^2 D(x) \]

- Super-Yang-Mills

\[ V \rightarrow V^a T^a \]
Recap from the second lecture:

- Strength tensor superfields

\[ W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V, \quad \bar{W}_\dot{\alpha} = -\frac{1}{4} D^2 \bar{D}_{\dot{\alpha}} V \]

- In the Wess-Zumino gauge

\[ W_\alpha = \lambda_\alpha + \theta_\alpha D + \frac{i}{2} (\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha F_{\mu \nu} + i \theta^2 (\sigma^\mu \partial_\mu \bar{\lambda})_\alpha \]

- SUSY invariant Lagrangians – F and D terms for chiral and vector superfields, respectively.
Nonrenormalisation theorems


\[ \mathcal{L} = K \left[ \Phi^+ e^{gV} \Phi \right] \big|_{\theta^2 \bar{\theta}^2} + W(\Phi) \big|_{\theta^2} + h.c. + f(\Phi) W^\alpha W_\alpha \big|_{\theta^2} + h.c. \]

- Kahler potential \( K \left[ \Phi^+ e^{gV} \Phi \right] \) receives corrections order by order in perturbation theory

- Only 1-loop corrections for \( f(\Phi) \)

- \( W(\Phi) \) is not renormalised in the perturbation theory!
Nonrenormalisation theorems


- Consider just Wess-Zumino model:

\[ W(\Phi) = \frac{m}{2} \Phi^2 + \frac{\lambda}{3} \Phi^3 \]

- R-symmetry and U(1) charges:

<table>
<thead>
<tr>
<th>'field'</th>
<th>( \Phi )</th>
<th>( m )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U(1)</td>
<td>1</td>
<td>−2</td>
<td>−3</td>
</tr>
<tr>
<td>U(1)_R</td>
<td>1</td>
<td>0</td>
<td>−1</td>
</tr>
</tbody>
</table>
Nonrenormalisation theorems


- Quantum corrected superpotential:

\[ W_{eff}(\Phi) = m\Phi^2 f \left( \frac{\lambda\Phi}{m} \right) = \sum_{n \geq 0} c_n \lambda^n m^{1-n} \Phi^{n+2} \]

- Consider \( \lambda \to 0 \to n \geq 0 \)
- Consider \( m \to 0 \to n \leq 1 \)

- Hence, \( W_{eff}(\Phi) = W(\Phi) \)
Outline of part III: MSSM

- Standard Model. Great success and some problems
- Building MSSM
- Soft supersymmetry breaking. Spontaneous supersymmetry breaking
- Sparticle spectra
- Current data and future prospects
Standard Model

- Standard Model of particle physics is theoretically consistent model of known elementary particles and fundamental interactions which successfully describes (almost) all observed phenomena in particle physics.
Standard Model

The SM has been tested with very high precision (one part in a thousand)
Standard Model

- Theoretical foundation of the Standard Model is the relativistic local quantum field theory (QFT) with local gauge invariance. QFT is the unique theory that consistently merges quantum mechanics and relativity, while local gauge invariance is the only known framework which consistently describes force carrier spin 1 particles.

- The basic lesson one can draw from the success the Standard Model is that symmetry principle plays a defining role in our understanding of microworld.
Problems of the Standard Model

Empirical evidence for BSM physics:

- SM can’t explain massive neutrinos
- No candidate for dark matter particle
- Current measurements of the Higgs and top quark masses indicate that the Higgs vacuum is unstable


A. Kobakhidze (U. of Sydney)
Problems of the Standard Model

Theoretical evidence for BSM physics:

- The very existence of 125 GeV elementary Higgs boson is somewhat puzzling. Scalar masses do receive quantum correction from UV physics, which is proportional to the UV scale:

  \[ \delta m_h^2 \sim \Lambda^2 \]

  Quadratic sensitivity of the Higgs mass to high energy mass scale is known as the hierarchy problem.

- However, from Part II we know that quadratic divergences cancel out in supersymmetric theories. Thus, supersymmetric extension of the Standard Model provides natural framework for the solution of the hierarchy problem.
Problems of the Standard Model

Theoretical evidence for BSM physics:

- Other hierarchies: strong CP problem, CC problem
- Too many free parameters...more symmetries, e.g. GUTs?

Supersymmetry also provides more friendly framework (gauge coupling unification) for GUTs and incorporates dark matter candidate.
Minimal Supersymmetric Standard Model (MSSM)

- Recall from Part I and II that superalgebra implies equal number of bosonic and fermionic degrees of freedom.

- The superalgebra also implies that particle and sparticle are degenerate in mass. We do not observe this. SUSY must be broken symmetry!
MSSM

- Matter fields and their superpartners are residing in chiral superfields:

\[
q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow Q \sim (3, 2, 1/6)
\]

\[
l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \rightarrow L \sim (1, 2, -1/2)
\]

\[
u^c_L \rightarrow U \sim (3^*, 1, -2/3), \quad d^c_L \rightarrow D \sim (3^*, 1, 1/3), \quad e^c_L \rightarrow E \sim (1, 1, 1)
\]
MSSM

- SU(3), SU(2) and U(1) gauge fields and their superpartners are residing in vector superfields:

\[ V_{SU(3)} \sim (8, 1, 1), \ V_{SU(2)} \sim (1, 3, 1), \ V_{U(1)} \sim (1, 1, 0) \]
MSSM

- The electroweak Higgs doublet can be placed in chiral superfield:

\[ H_d \sim (1, 2, -1/2) \]

- However, the above superfield contains also a fermionic partner (Higgsino) with quantum numbers of the lepton doublet. Gauge anomaly cancellation than requires to introduce another Higgs superfield:

\[ H_u \sim (1, 2, 1/2) \]
MSSM

- The Lagrangian of the supersymmetric Standard Model:

\[ L_{SUSY\;SM} = \int d\theta^2 d\bar{\theta}^2 \sum_{i=\text{all chiral superfields}} \Phi_i^+ e^{\sum_a g_a V_a} \Phi_i + \int d\theta^2 \text{Tr} \left( \sum_a \frac{1}{4g_a^2} W_\alpha^a W_{a\alpha} \right) + h.c. \]

\[ + \int d\theta^2 \mu H_u H_d + \hat{y}_u QU H_u + \hat{y}_d QD H_d + \hat{y}_l LE H_d + h.c. \]

- The power of SUSY: no new parameter has been introduced! Moreover, Higgs self-coupling is defined by electroweak gauge couplings!

A. Kobakhidze (U. of Sydney)
MSSM

- Higgs potential (neutral components):

\[ V = |\mu|^2 \left( |H_u^0|^2 + |H_d^0|^2 \right) + \frac{1}{8} (g^2 + g'^2) \left( |H_u^0|^2 - |H_d^0|^2 \right)^2 \]

- No EWSB without SUSY breaking

\[ \langle H_u^0 \rangle = \langle H_d^0 \rangle = 0 \]

- Since self-interaction \( \sim g \), we expect a light Higgs,

\[ m_h \sim m_Z \sim 100 \text{ GeV} \]
MSSM

- Gauge invariance alone does not forbid lepton and baryon number violating interactions:

\[ \tilde{\chi}' L L E + \tilde{\chi}'' L Q D + \tilde{\chi}''' U D D . \]

- The above terms are forbidden due to R-parity:

\[ (-1)^{3(B-L)+2S} \]

Ordinary particles R-even (+), sparticles R-odd (-).
MSSM

- Conservations of R-parity implies:
  
  i. Sparticles produce in pairs;
  
  ii. The lightest supersymmetric particle, the LSP, is stable and may be a dark matter particle (usually neutralino);
  
  iii. Large missing energy signature at colliders.
Soft supersymmetry breaking

- We would like to break supersymmetry without introducing undesired quadratic divergences in scalar masses. There are three types of explicit soft-breaking terms:

  i. Mass terms for scalar components of chiral superfields
  ii. Mass terms for fermionic component of vector superfields
  iii. Trilinear couplings for scalar components of chiral superfields


A. Kobakhidze (U. of Sydney)
Soft supersymmetry breaking

\[ \mathcal{L}_{soft} = -\frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + \text{c.c.} \right) \]

\[ -\tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{u}^c m_u^2 \tilde{u}^c \dagger - \tilde{d}^c m_d^2 \tilde{d}^c \dagger - \tilde{L}^\dagger \tilde{L} - m_\tilde{e}^2 \tilde{\tilde{e}}^c \tilde{\tilde{e}}^c \dagger \]

\[ -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (B H_u H_d + \text{c.c.}) \]

- Sparticles must be within the reach of LHC, if SUSY is indeed responsible for the solution of the hierarchy problem!
**Spontaneous supersymmetry breaking**

- Explicit soft SUSY breaking is problematic nevertheless:
  
i. Introduces $\sim 100$ new a priori unknown parameters
  
ii. Unacceptably large contribution to flavour changing neutral processes (SUSY flavour problem)
  
iii. Unacceptably large CP-violating effects (SUSY CP problem)

- It is more desirable to have spontaneous SUSY breaking:
  
i. Fayet-Iliopoulos mechanism – D-term breaking
  
ii. O’Raifeartaigh mechanism – F-term breaking

Note that upon the spontaneous SUSY breaking, $\text{Str } M^2=0$ still holds!

A. Kobakhidze (U. of Sydney)
Supersymmetry mediation scenarios

- Standard approach to realistic SUSY breaking:
  i. Break SUSY spontaneously in the “hidden sector” at high energy scale
  ii. Find the interactions that mediate “hidden sector” breaking to the visible sector

  (a) Gravity mediation
  (b) Gauge mediation
  (c) Anomaly mediation
RG evolution and REWSB
‘Typical’ sparticle spectra

Generic features:
- Coloured particles are heavy
- Uncoloured particles are light

Overall SUSY breaking scale is a free parameter
<table>
<thead>
<tr>
<th>Model</th>
<th>$\ell, \mu, \tau, Y$ Jets</th>
<th>$E^\text{miss}_T$</th>
<th>$\mathcal{L} \int d\mathcal{L}$ (fb$^{-1}$)</th>
<th>Mass limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSUGRA/CMSSM</td>
<td>0-3, $\ell$, $\mu$-2 $\tau$</td>
<td>2-10 jets/3 $\ell$</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g} \rightarrow q\bar{q}$</td>
<td>0</td>
<td>2-6 jets</td>
<td>Yes</td>
<td>3.2</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g} \rightarrow \tilde{\chi}_1^0 (\text{compressed})$</td>
<td>0</td>
<td>mono-jet</td>
<td>Yes</td>
<td>3.2</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g} \rightarrow (\ell +c)^{\text{CMSSM}}$</td>
<td>2 $\ell$, $\mu$ ($\text{CMSSM}$)</td>
<td>2 jets</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g} \rightarrow (\ell + c)^{\text{CMSSM}}$</td>
<td>0</td>
<td>2-6 jets</td>
<td>Yes</td>
<td>3.2</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g} \rightarrow (\ell + c)$</td>
<td>1 $\ell$, $\mu$</td>
<td>2-6 jets</td>
<td>Yes</td>
<td>3.2</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g} \rightarrow (\ell + c)^{\text{CMSSM}}$ &amp;</td>
<td>0 $\ell$, $\mu$</td>
<td>2-6 jets</td>
<td>Yes</td>
<td>3.2</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>GMSB ($\tilde{\chi}_1^0$, NLS)</td>
<td>2 $\tau$</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
<td>1503.06943</td>
</tr>
<tr>
<td>GGM (bino NLS)</td>
<td>2 $\tau$</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
<td>1503.06943</td>
</tr>
<tr>
<td>GGM (higgsino-bino NLS)</td>
<td>2 $\tau$</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
<td>1503.06943</td>
</tr>
<tr>
<td>GGM (higgsino NLS)</td>
<td>2 $\tau$</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
<td>1503.06943</td>
</tr>
<tr>
<td>Granatino LSP</td>
<td>0 $\ell$, $\mu$</td>
<td>Mono-jet</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
</tbody>
</table>

**Inclusive Searches**

<table>
<thead>
<tr>
<th>Model</th>
<th>$\ell, \mu, \tau, Y$ Jets</th>
<th>$E^\text{miss}_T$</th>
<th>$\mathcal{L} \int d\mathcal{L}$ (fb$^{-1}$)</th>
<th>Mass limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{b}_1 \rightarrow b \tilde{\chi}_1^0$</td>
<td>0</td>
<td>3 $\ell$</td>
<td>Yes</td>
<td>3.3</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{b}_1 \rightarrow b \tilde{\chi}_1^0$</td>
<td>0</td>
<td>3 $\ell$</td>
<td>Yes</td>
<td>3.3</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{b}_1 \rightarrow b \tilde{\chi}_1^0$</td>
<td>0</td>
<td>3 $\ell$</td>
<td>Yes</td>
<td>3.3</td>
<td>$m(\chi^0)$</td>
</tr>
</tbody>
</table>

**EW Direct**

<table>
<thead>
<tr>
<th>Model</th>
<th>$\ell, \mu, \tau, Y$ Jets</th>
<th>$E^\text{miss}_T$</th>
<th>$\mathcal{L} \int d\mathcal{L}$ (fb$^{-1}$)</th>
<th>Mass limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{g}_{1 \ell} \rightarrow t \tilde{\chi}_1^0$</td>
<td>2 $\ell$, $\mu$</td>
<td>0</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g}_{1 \ell} \rightarrow t \tilde{\chi}_1^0$</td>
<td>2 $\ell$, $\mu$</td>
<td>0</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
<tr>
<td>$\tilde{g}_{1 \ell} \rightarrow t \tilde{\chi}_1^0$</td>
<td>2 $\ell$, $\mu$</td>
<td>0</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
</tbody>
</table>

**Other**

<table>
<thead>
<tr>
<th>Model</th>
<th>$\ell, \mu, \tau, Y$ Jets</th>
<th>$E^\text{miss}_T$</th>
<th>$\mathcal{L} \int d\mathcal{L}$ (fb$^{-1}$)</th>
<th>Mass limit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar charm</td>
<td>2 $\ell$, $\mu$</td>
<td>0</td>
<td>Yes</td>
<td>20.3</td>
<td>$m(\chi^0)$</td>
</tr>
</tbody>
</table>

*Only a selection of the available mass limits on new states or phenomena is shown.*

---

A. Kobakhidze (U. of Sydney)
Is fine-tuning back?

A. Strumia JHEP 1104 (2011) 073
Beyond MSSM?

- Tensions: Higgs mass vs naturalness. To bring the tree level Higgs mass up to 125 GeV via radiative corrections we need massive superparticles (stop, gluino), but this results in increased fine-tuning

- Two ways to resolve the tension:
  i. Extensions of MSSM with larger tree level Higgs mass – e.g., NMSSM
  ii. Extra protection for the Higgs mass, e.g. Higgs as a PGB (e.g., Z. Berezhiani et al., `Double protection of the Higgs potential in a supersymmetric little Higgs model,’’ Phys. Rev. Lett. 96 (2006) 031801)

- Maybe weak scale fine-tuning (little hierarchy) is irrelevant, if an underlying UV theory is natural

A. Kobakhidze (U. of Sydney)
Summary

- SUSY is an attractive framework for beyond the Standard Model physics:
  - Low energy softly broken SUSY stabilizes the electroweak scale against radiative corrections.
  - In SUSY models with R-parity the LSP may play the role of dark matter.
  - Low-energy SUSY provides unification of gauge couplings and hence a framework for GUTs.
  - No empirical evidence of SUSY so far. The discovery of SUSY (if it there) undoubtedly will be the major breakthrough in fundamental physics.