SCIPP
SANTA CRUZ INSTITUTE FOR PARTICLE PHYSICS

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An Introduction to Particle Dark Matter

Pre-SUSY Summer School

Melbourne, June 29-July 1, 2016

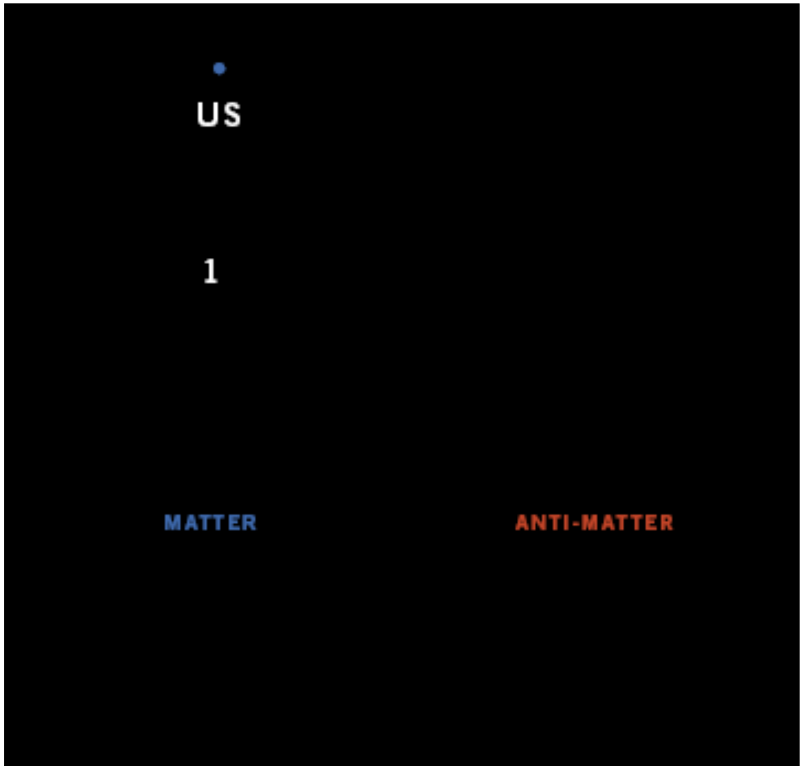
- ✓ PhD **Theoretical Particle Physics** (2004)
  International School for Advanced Studies (SISSA-ISAS), Trieste, Italy

- ✓ Postdoc, FSU and California Institute of Technology (2005-2007)
  Theoretical Astrophysics and Particle Physics

- ✓ Joined **UCSC Physics** Faculty (Assistant Professor, 2007-2011,
  Associate Professor, July 2011-2015
  Full Professor, July 2015-)

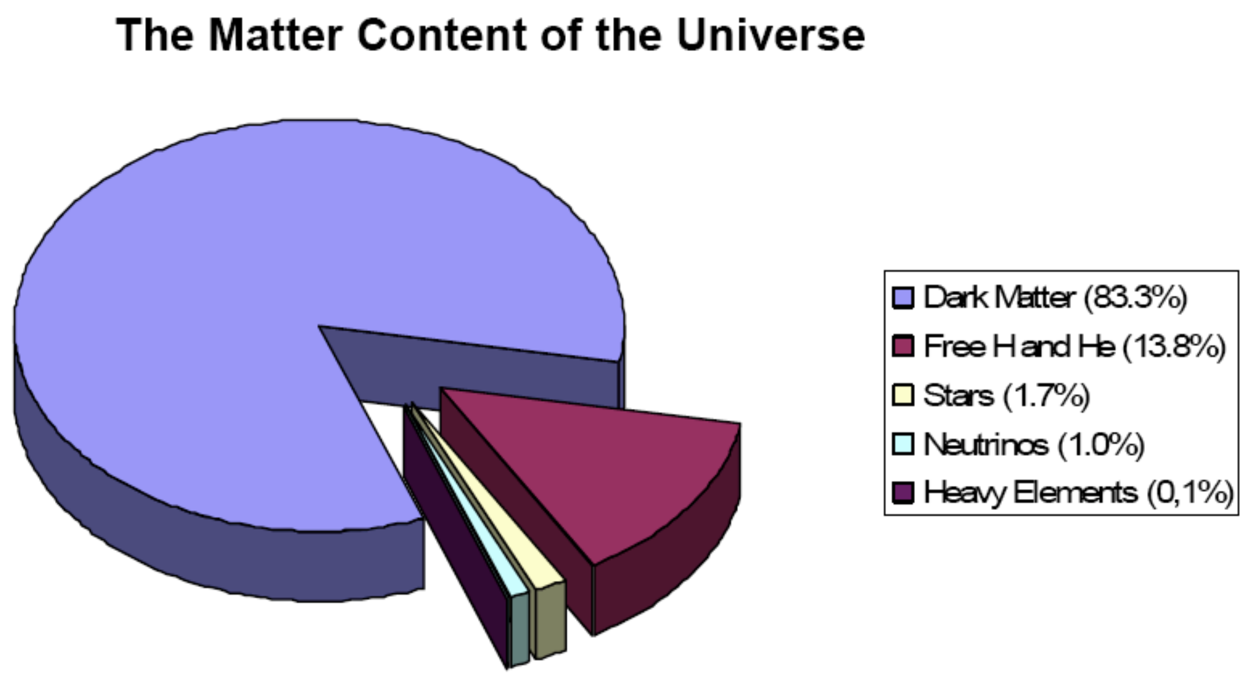
- ✓ Director of UCSC Physics **Graduate Studies** (2012-)

- ✓ **SCIPP Deputy Director** for **Theory** (July 2011-)



1. What is the origin of the tiny excess of matter over anti-matter?

2. What is the fundamental particle physics nature of Dark Matter?

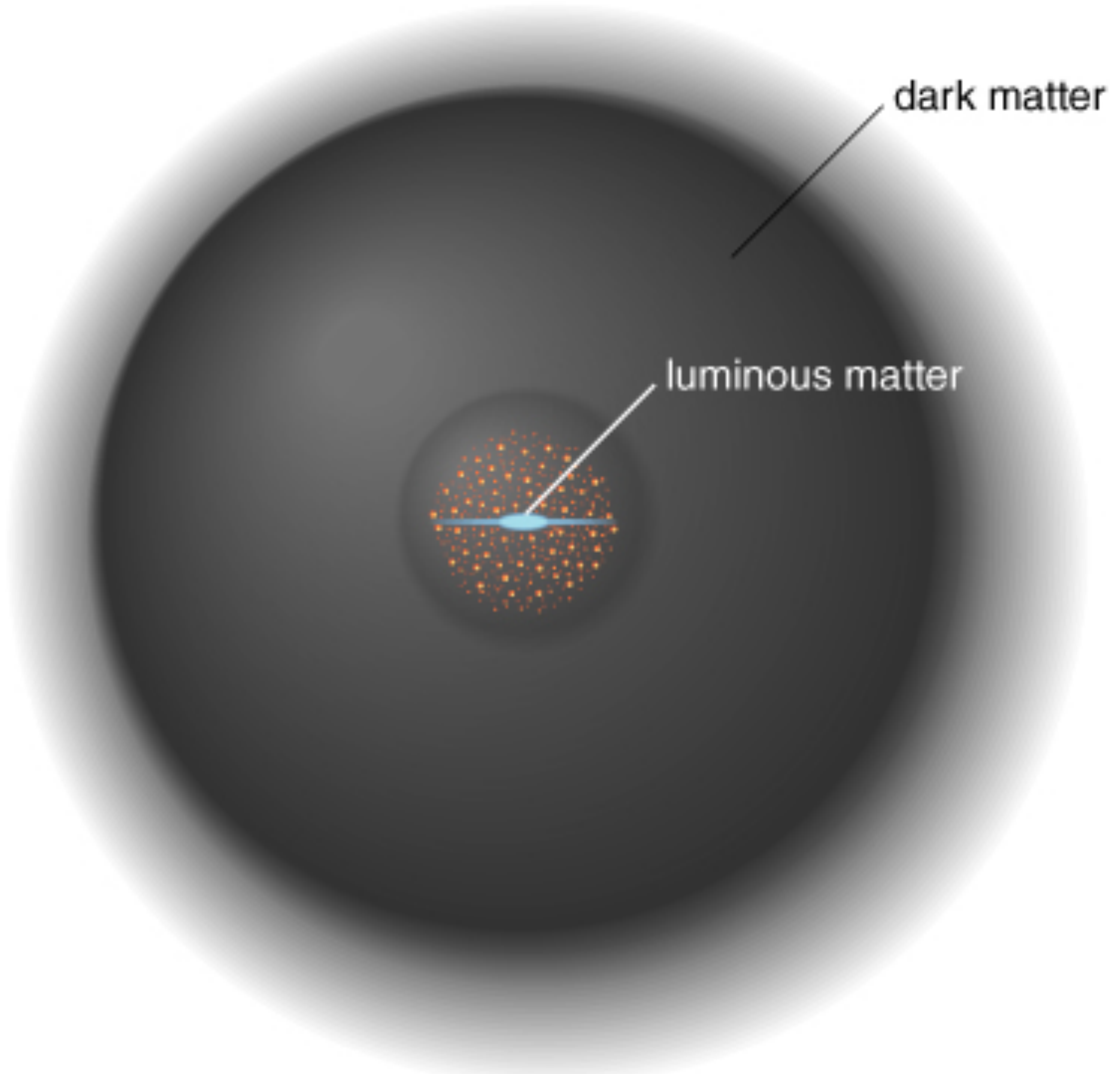


Please come **introduce yourselves**!

[to myself, other Instructors, to each other...]

If you are ever on the **US West Coast** please let me know!

Never **underestimate** the importance of **networking** in science!


dark matter
luminous matter

4/5

a new

elementary particle

...as such it is of interest

to particle physicists!

What this mini-lecture series **is not**:

- ✧ Review of "**evidences**" for dark matter
- ✧ Review of "**models**" for dark matter
- ✧ Review of possible claimed "**signals**" from dark matter

What this mini-lecture series **will be**

- ✓ Gross **features** of dark matter *as a* ***particle*** (lecture 1)
- ✓ **Paradigms** for dark matter in the **early universe** (lecture 1/2)
- ✓ **Schematics** of dark matter **searches** (lecture 2/3)
- ✓ Selected **lessons** from **old and new** particle dark matter **models** (lecture 3)

One thing we do **know well** about dark matter

Global amount of dark matter in the **universe**

Reason: very good handles on **total energy density**, total **matter** density, total **baryonic** matter

CMB data indicate the **universe** is nearly **flat**

→ energy density is close to **critical**...

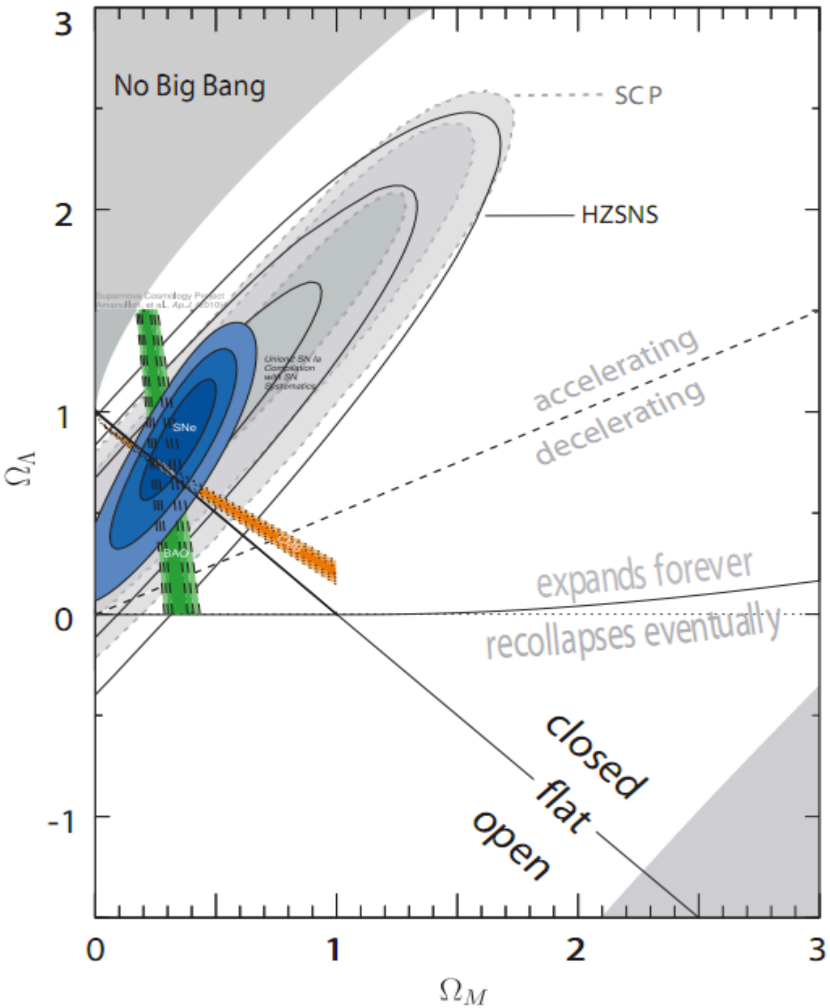
What is the **critical density**? (very good number to have in mind!)

$$\rho_{\rm crit} \equiv \frac{3H_0^2}{8\pi G_N} \simeq 10^{-29}\ {\rm g/cm^3}$$

...since 1 GeV ~10^{-24} g, **10 protons** per **cubic meter** (=tiny!)

Various ways to "**weigh**" **matter** versus dark energy (CMB+SN+BAO)

...and **ordinary** (baryonic) **matter** versus **non-baryonic** (BBN, CMB)



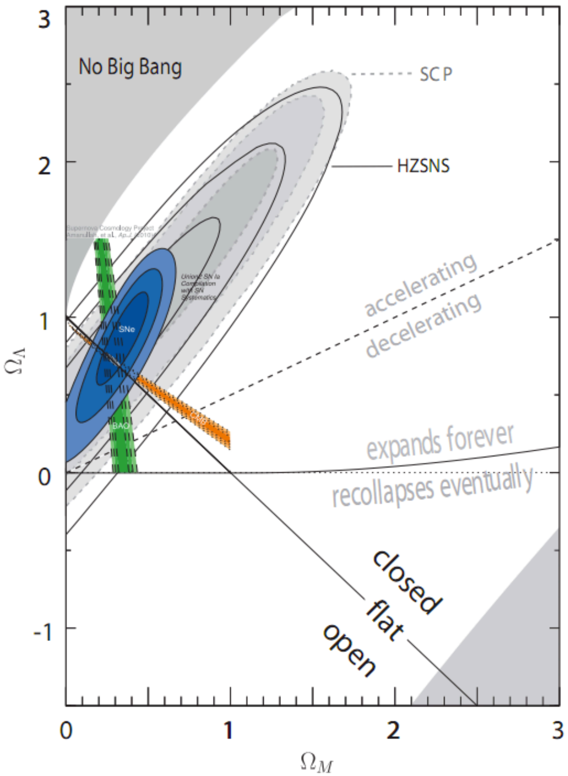
Global amount of **dark matter**
in the universe
from simple **subtractions**!

$$\bar{\rho}_{\rm DM} = \Omega_{\rm DM}\rho_{\rm crit} \simeq 0.3\rho_{\rm crit}.$$

in "**astro**" units...

$$\bar{\rho}_{\rm DM} \sim 10^{10}\ \frac{M_\odot}{\rm Mpc^3}$$

clusters... 10^5 denser!



in "**particle physics**" units...

$$\bar{\rho}_{\rm DM} \sim 10^{-6}\ \frac{\rm GeV}{\rm cm^3}$$

galaxies... 10^6 denser!

$$\delta\rho/\rho \gg 1$$

the Universe is highly **non-linear**!

...which is one of the reasons why **modified gravity** does not work!

CMB sky is very **boring** – *T* fluctuations very **small**!

T fluctuations proportional to (baryonic) **density** fluctuations,

$$\delta\rho/\rho \lesssim 10^{-4}$$

Good news! Matter **over-densities** in linear regime grow **linearly** with scale factor

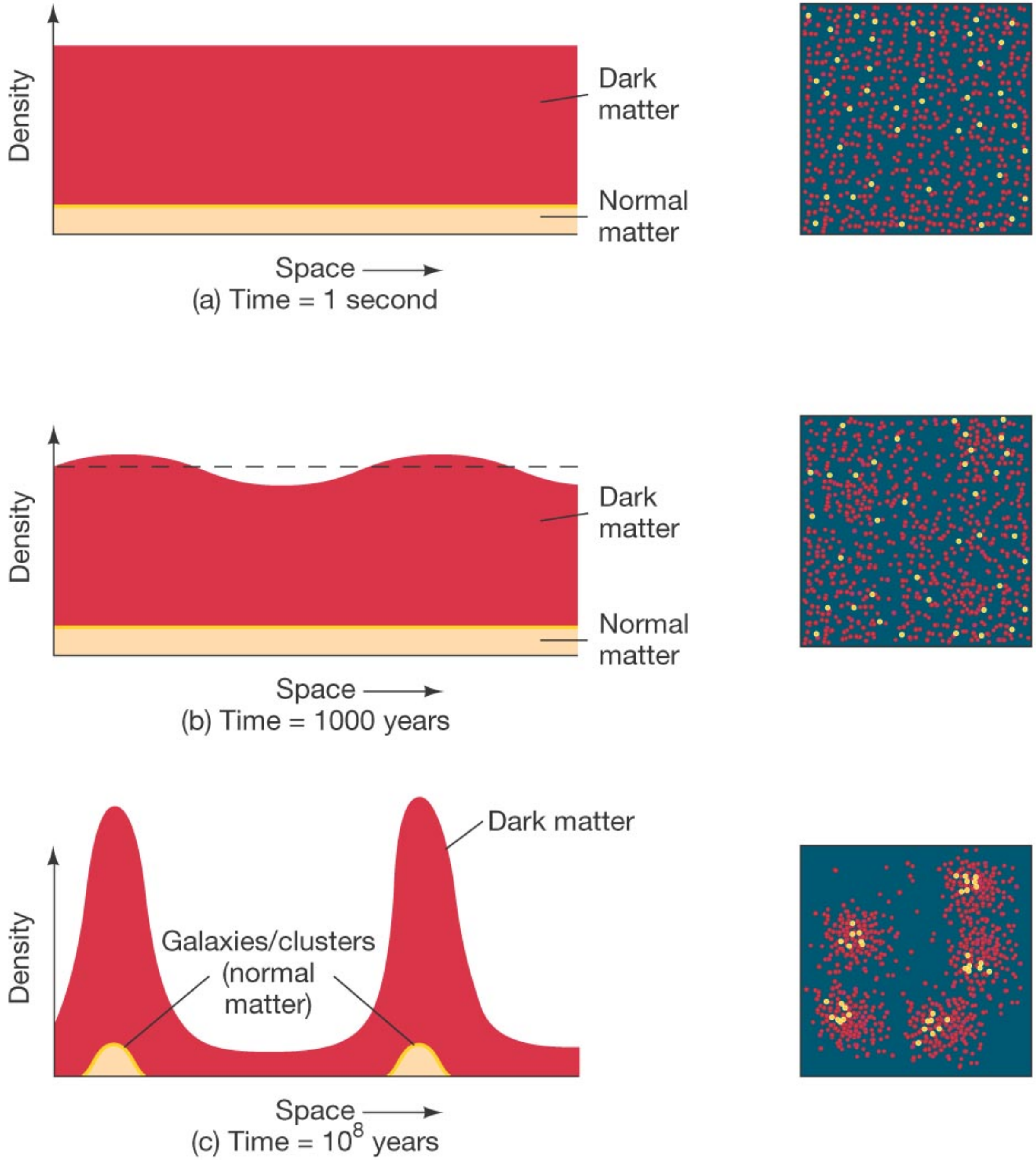
But the scale factor since CMB decoupling grew by z_{rec}**~1,100**

Not enough time for structures to go **non-linear**!

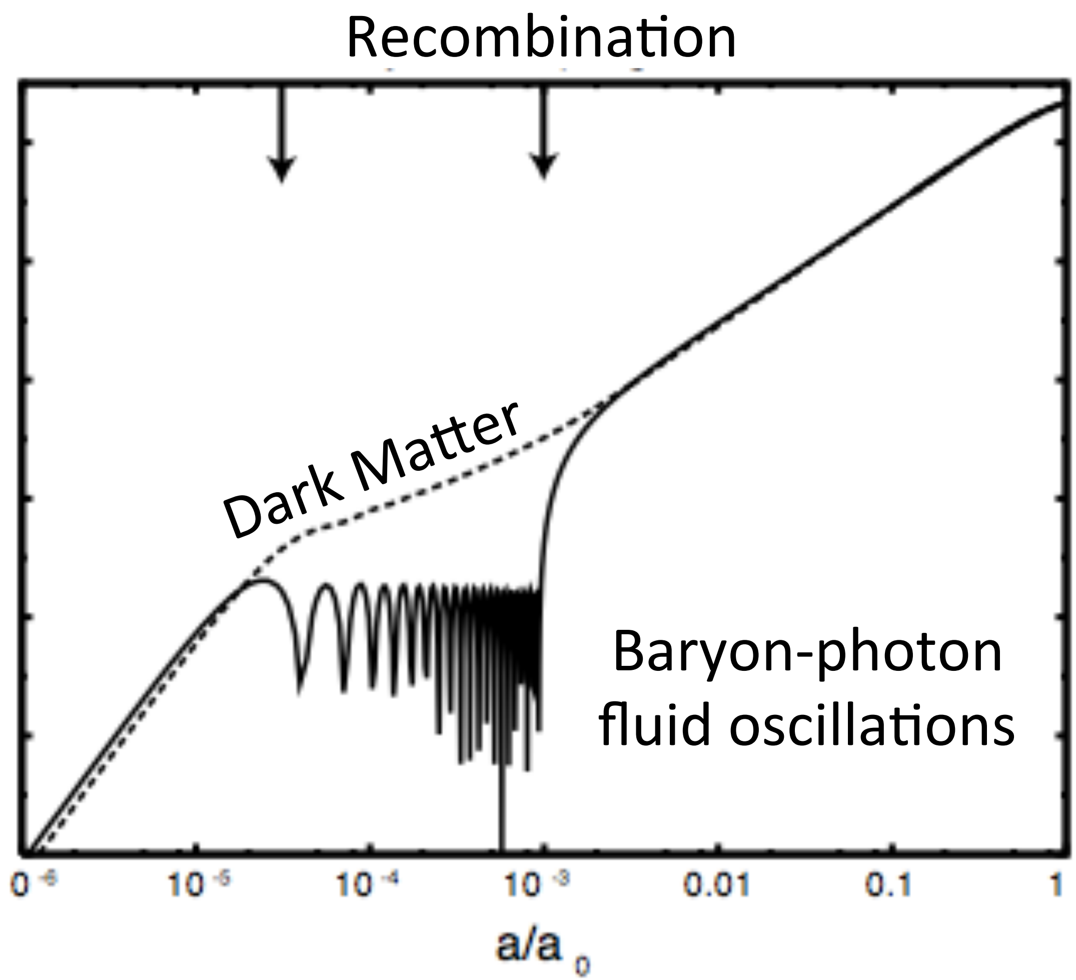
We need a **species** that has **decoupled** from photons much earlier (**Dark Matter**) so that its density perturbations are much larger at recombination!

$$(\delta\rho/\rho)_{\rm DM} \gg 10^{-4}$$

Dark matter **seeds** timely structure formation!



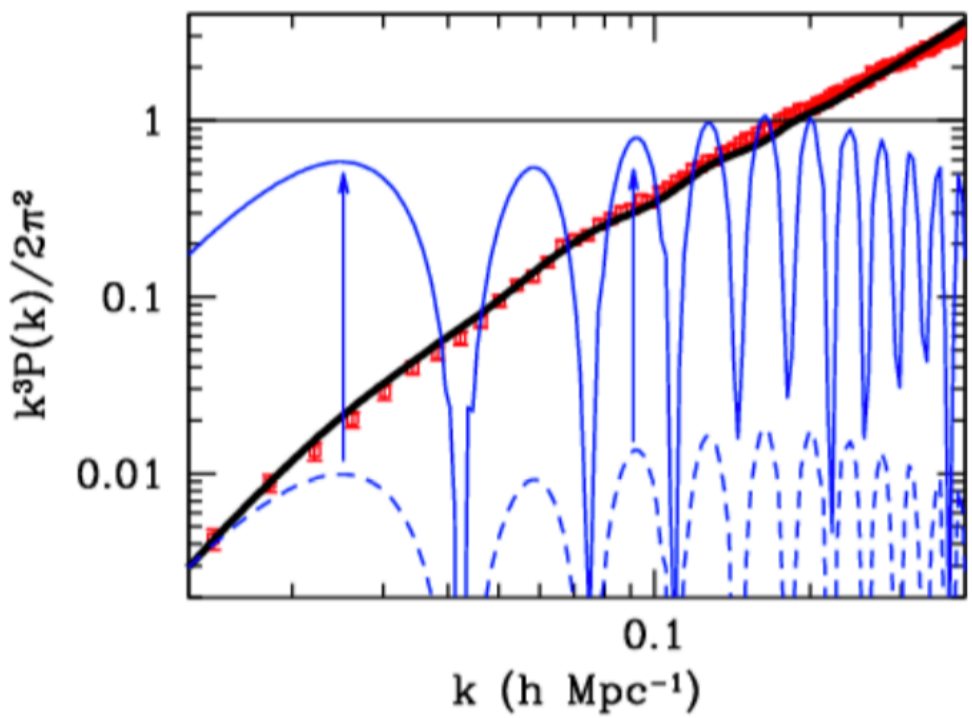
Density
Dark matter
Normal matter
Space
(a) Time = 1 second
Density
Dark matter
Normal matter
Space
(b) Time = 1000 years
Dark matter
Density
Galaxies/clusters (normal matter)
Space
(c) Time = 10^8 years



Recombination
Dark Matter
Baryon-photon
fluid oscillations
a/a 0

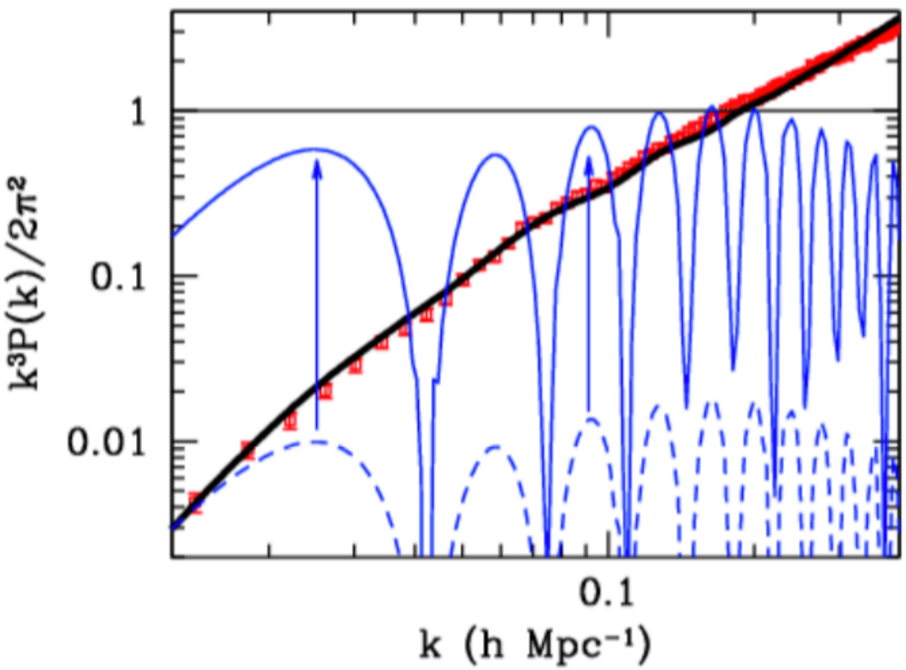
Things go **badly wrong without DM** for structure formation!

Power spectrum
of density
perturbations



δρ/ρ~1

Even with best (covariant) incarnation of modified gravity (TeVeS), structure goes non-linear, but the **power spectrum** of matter density fluctuation is **entirely wrong**...



Don’t get fooled by the “**Volcano**” versus “**Neptune**” analogy

[Volcano: No new planet between Mercury and the Sun, but GR
Neptune: New planet]

Modified Gravity [MOND,TeVeS] actually **does not work** at all!!

Knowledge of the dark matter average **density**
is a powerful **model-building** tool

Models that **predict** the "right" **amount** of dark matter get kudos

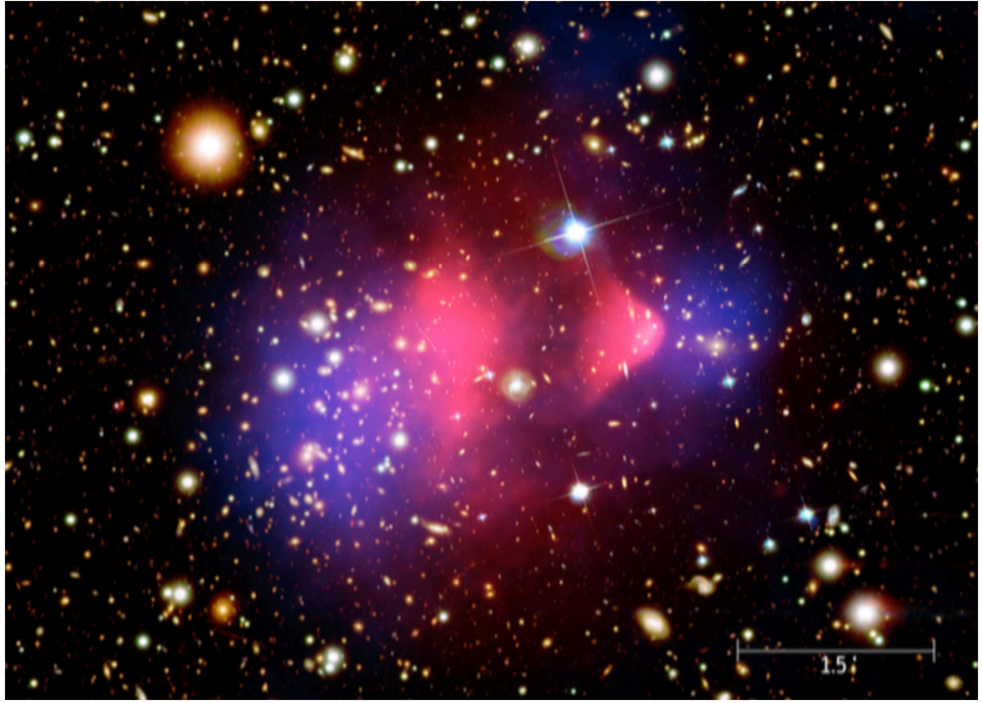
Dark Matter "**cosmogony**" well-motivated guideline to model building

prototypical example: dark matter
as a ***thermal relic***… more on this shortly

What else do we know about the **microscopic** nature of dark matter from its **macroscopic** features?

- "**Dark**": ...for the **reason above**! But detailed constraints on electric charge of dark matter are model-dependent... Milli-charge allowed... Phenomenologically: DM is nearly **dissipationless** (maybe not entirely though, see dark photons, dark disks...)

- **Collisionless**... really? Let's calculate the relevant constraints!

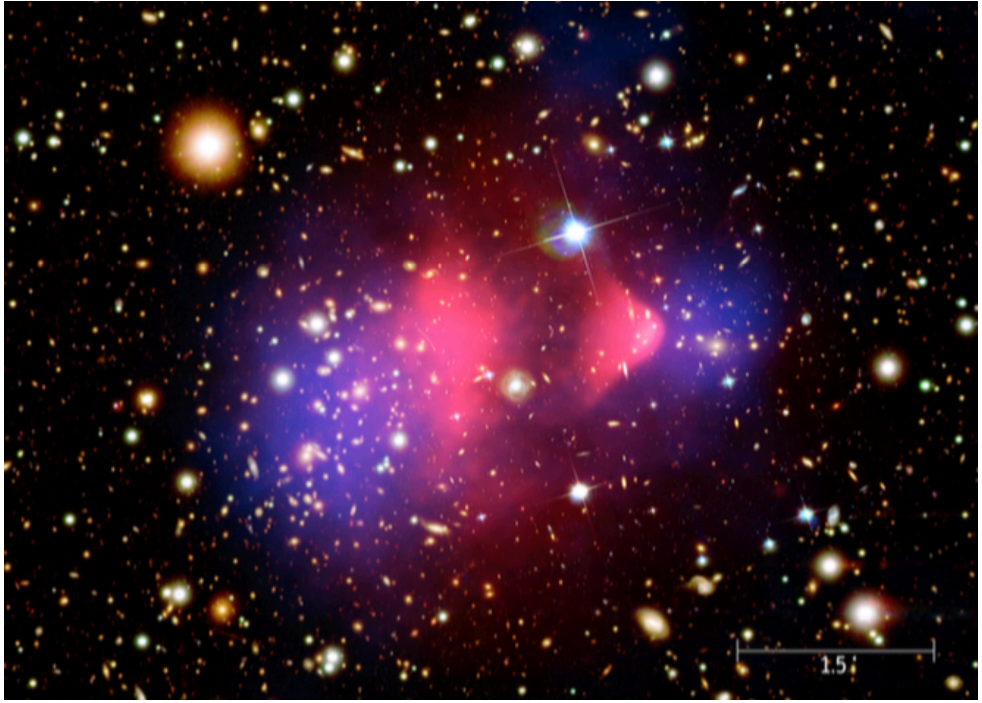


mean free path λ larger than cluster size, ~ 1 Mpc

cluster **density**: ρ ~ 1 GeV/cm^3, thus…

$\lambda = 1/(\sigma\,(\rho/m)) > 1$ Mpc → $\sigma/m < 1$ Mpc / 1 GeV/cm^3

→ ***$\sigma/m < 1$ cm^2/g***, or ***1 barn/GeV***



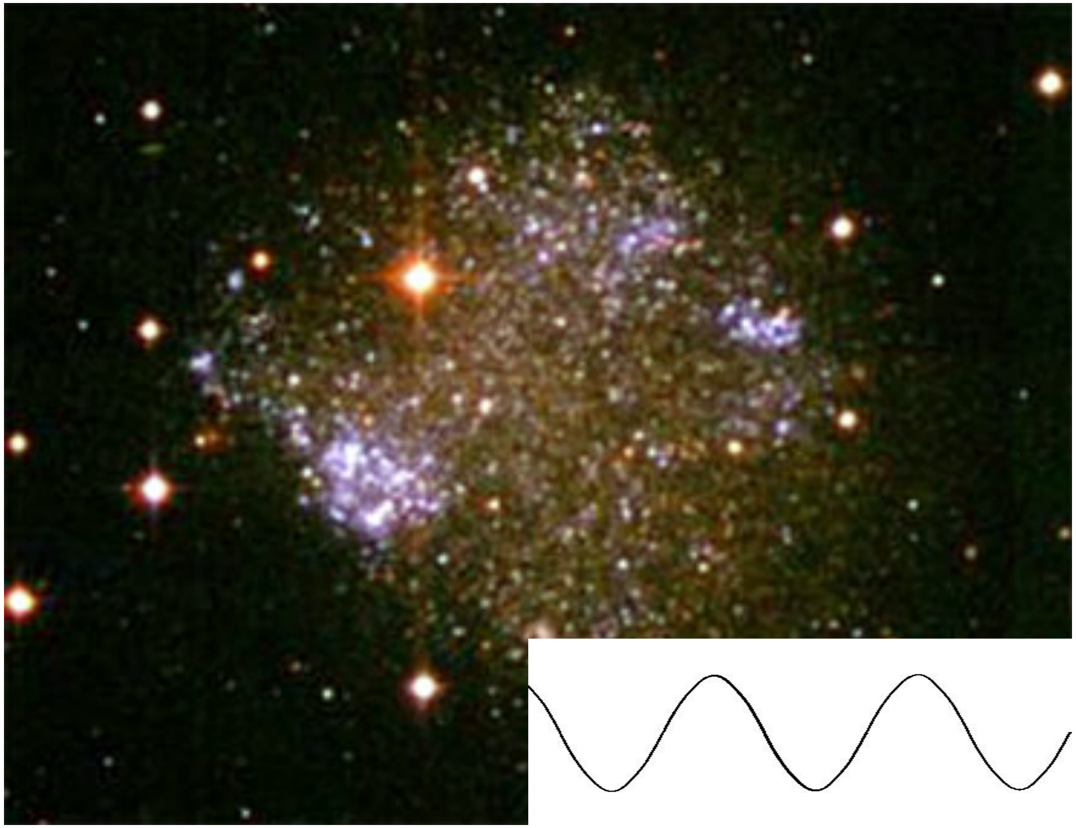
1 barn/GeV... which is **strong interaction**-size...

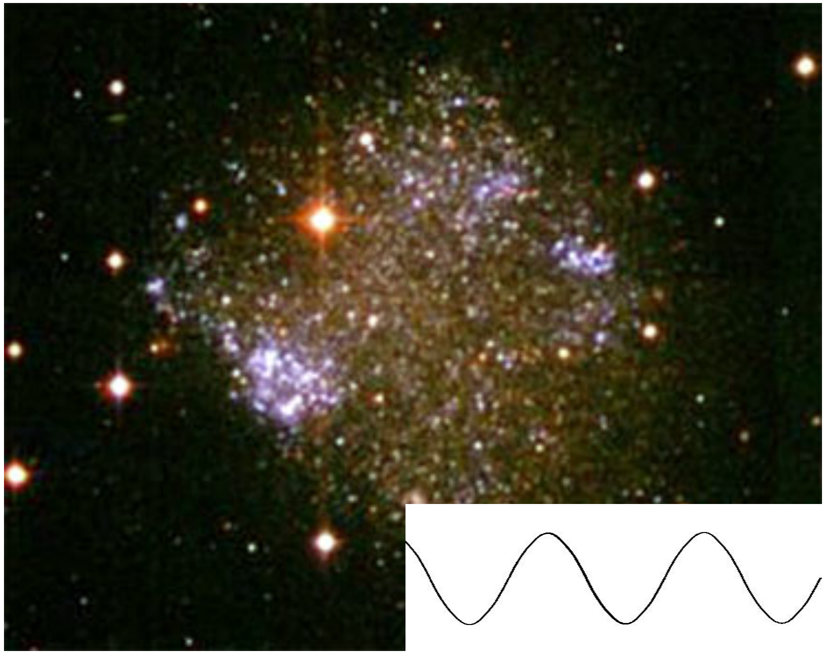
is this **small**?

Also, if cross section is **slightly smaller**, no **visible effect**...
if cross section **slightly larger**, **disaster**...

Begs the question: is "collisional" **self-interacting** dark matter a "**natural**" possibility??

- **Classical**: needs to be confined (gravitationally bound) on scales at least as large as dSph... if de Broglie wavelength is larger, disaster strikes!





little exercise: consider *v ~ 100 km/s*, show that ***λ = h/p*** is

$$\lambda \sim 3\ \mathrm{mm}\ \left(\frac{1\ \mathrm{eV}}{m}\right)$$

which means that to have λ << kpc ~ 3×10^{21} cm, **m>10^{-22} eV**

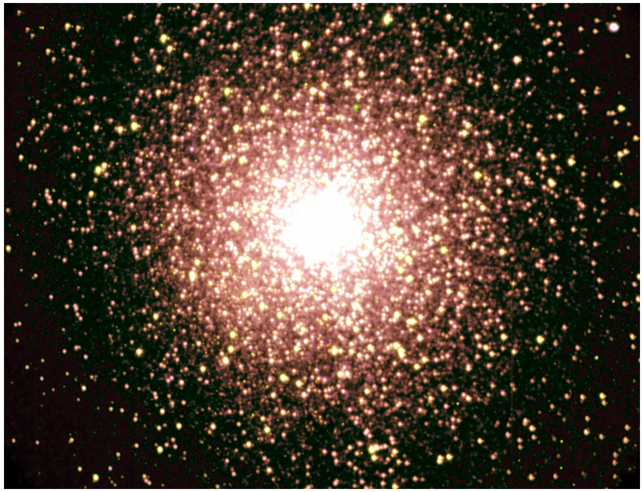
Much, much **better constraints** if the DM is a fermion – we know that the **phase space** density is bounded (Pauli blocking): $f = gh^{-3}$

Using **observed** density and velocity dispersion of dSph, **Tremaine-Gunn** limit (1979): observed phase space density cannot exceed upper bound! (Liouville theorem?) Exercise!

$$\sigma \sim 150 \text{ km/s} \qquad \rho \gtrsim 1 \text{ GeV/cm}^3$$

$$m^4 > \frac{\rho h^3}{[g(2\pi\sigma^2)^{3/2}]} \sim (25 \text{ eV})^4.$$

- **Fluid**: don't want to **disrupt** pretty (and old!) **clusters** of stars



Neat exercise to estimate the **energy exchanged** by encounters of GC and BH, in the impulse approximation, demand that that energy be smaller than binding energy, get maximal mass for BH

Also constraints on **disk disruption** ("heating")

Bottom line: $m > 10^3$ solar masses $\sim 10^{70}$ eV

…here's the **name of the game**:

(i) **Mass**: >**90** orders of magnitude for **bosons**, **70** for **fermions**

(ii) **Interactions**: ~**dark**, **self-interacting** at most ~ strong interactions

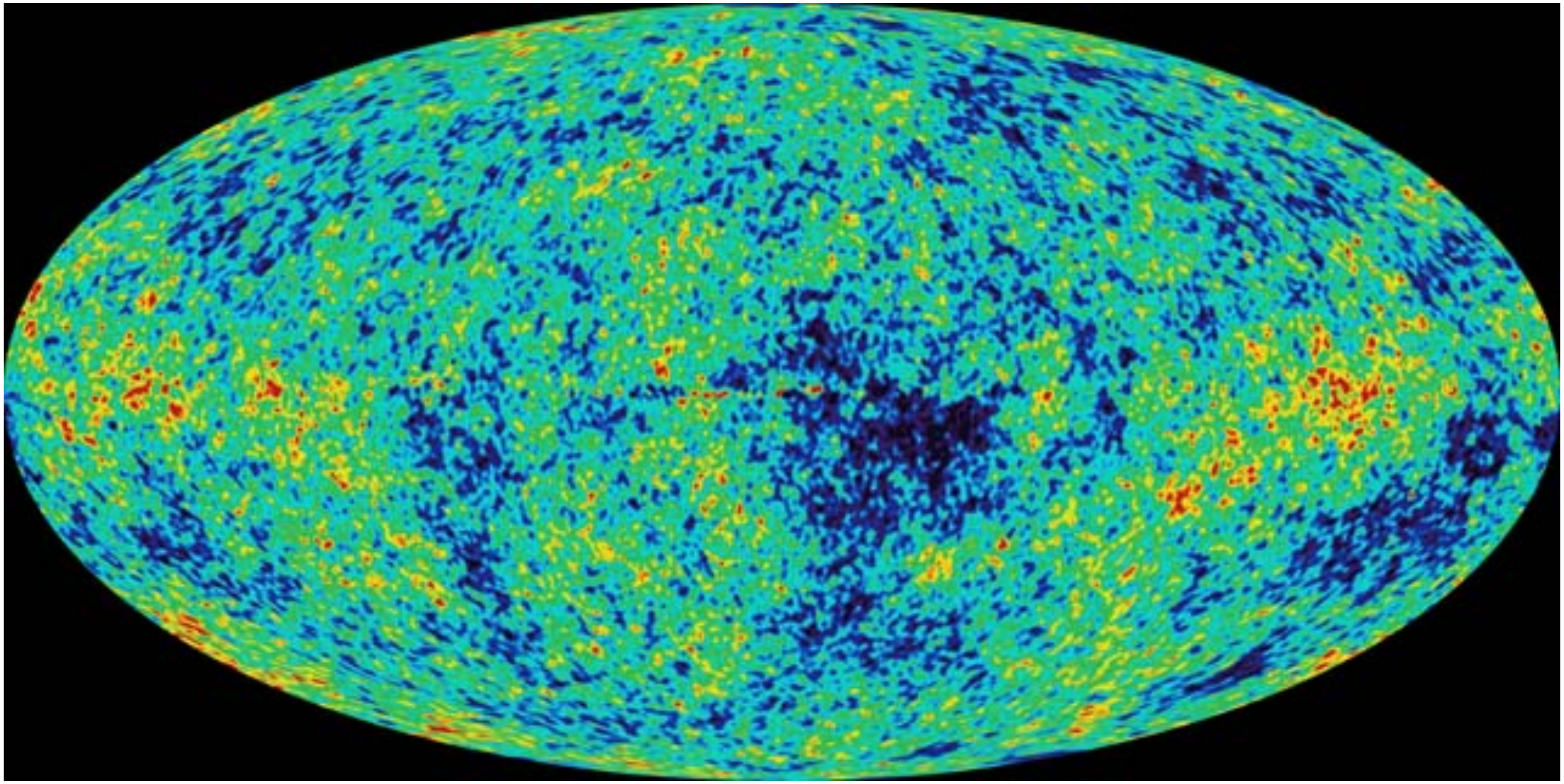
(iii) **Abundance**

Think left and think right and think low and think high.

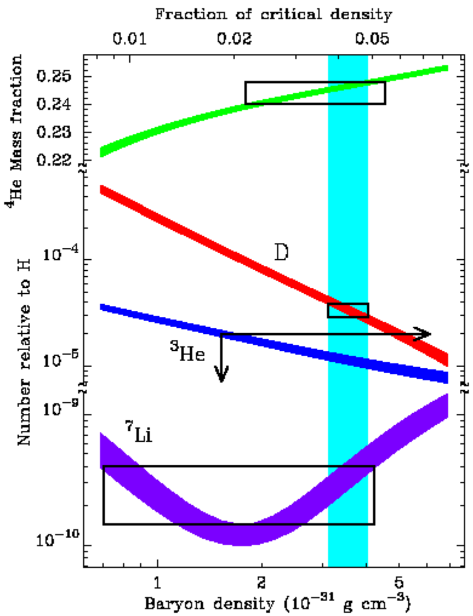
Oh the things you can think up, if only you try!

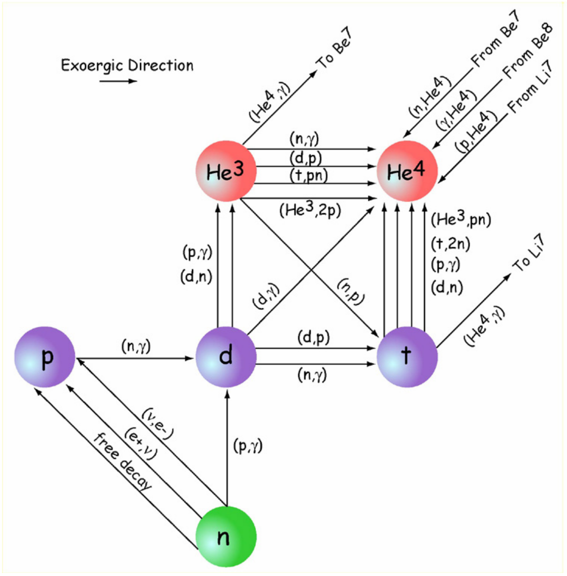
Dr. Seuss

A successful framework for the **origin of species** in the early universe: **thermal decoupling**



A successful framework for the **origin of species** in the early universe: **thermal decoupling**





A successful **synergy** of ***statistical mechanics***, ***general relativity***, and of ***nuclear and particle physics*** making **predictions** testable to exquisite accuracy with ***astronomical*** observations!

Key **idea** of thermal decoupling:

if the **reaction** keeping a species in equilibrium

is **faster** than the **expansion rate** of the universe,

the reaction is in **statistical equilibrium**;

if it's **slower**, the species **decouples** ("freeze-out")

$$\Gamma \ll H(T) \qquad \Gamma(T_{\rm t.o.}) \sim H(T_{\rm t.o.})$$

the **reaction rate** (from definition of cross section!)

$$\Gamma = n \cdot \sigma \cdot v$$

(1) borrow **equilibrium number densities** from stat mech

$$n_{\rm rel} \sim T^3 \quad \text{for} \;\; m \ll T,$$
$$n_{\rm non-rel} \sim (mT)^{3/2} \exp\left(-\frac{m}{T}\right) \quad \text{for} \;\; m \gg T.$$

(2) borrow **Hubble rate** from general relativity
(FRW **solution** to Einstein's eq.)

$$H^2 = \frac{8\pi G_N}{3}\rho.$$

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GR+SM: **energy density** in radiation

$$\rho \simeq \rho_{\text{rad}} = \frac{\pi^2}{30} \cdot g \cdot T^4 \longrightarrow H \simeq T^2/M_P$$

first application: **hot** thermal relic

language definition: **hot** = relativistic at $T_{f.o}$
cold = $v<c=1$. (actually not by much, typically!)

simple **application**: **relic** SM **neutrinos** (cosmo ν background)

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$

$$n(T_\nu) \cdot \sigma(T_\nu) = H(T_\nu)$$

$$\sigma \sim G_F^2 T_\nu^2$$

suppose this is a hot relic… $n \sim T_\nu^3$

$$T_\nu^3 G_F^2 T_\nu^2 = T_\nu^2 / M_P$$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \mathrm{GeV} \sim 1\ \mathrm{MeV}$$

happy about **two** things in particular:

1. **hot** relic assumption works! $T_\nu \gg m_\nu$

2. **Fermi** effective theory OK! $T_\nu \ll m_W$

$$T_\nu = (G_F^2 M_P)^{-1/3} \simeq (10^{-10} \times 10^{18})^{-1/3}\ \mathrm{GeV} \sim 1\ \mathrm{MeV}$$

now, how do we calculate the **relic** thermal **abundance** of this prototypical hot relic?

Introduce ***Y=n/s*** (number and entropy **density**, *V*=*a*[3])

If universe is iso-entropic, ***s x a*[3]*=S*** is conserved

***Y ~ n a*[3]** is thus ~ **comoving number density**, and (without entropy injection)

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$Y_{\rm today} = Y_{\rm freeze-out} = Y(T_\nu)$$

$$Y_{\rm freeze-out} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$

$$n_{\rm today} = s_{\rm today} \times Y_{\rm today} = s_{\rm today} \times Y_{\rm freeze-out}$$

$$\rho_{\nu,{\rm today}} = m_\nu \times Y_{\rm freeze-out} \times s_{\rm today}$$

$$\Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\rm crit}} h^2 \simeq \frac{m_\nu}{91.5\ {\rm eV}}$$

Cowsik-Mc-Clelland limit

That was **fun**! Let's see if it works for something else...

Try **proton-antiproton** freeze-out:
what's the **relic** matter **abundance** in a baryon-symmetric Universe?

$$\sigma \sim \Lambda_{\mathrm{QCD}}^{-2}$$

$$n\,\sigma = H \rightarrow T^3\,\Lambda^{-2} = T^2/M_P \rightarrow T = \Lambda^2/M_P$$

doesn't quite work, we're way **outside**
the regime of validity for **hot relics**, since $T<<<<<<<m_p$ ...

Need to work out the case of **cold relics**, which looks nastier by eye

$$n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)$$

Here's the trick: **freeze-out** condition gives

$$n_{\mathrm{f.o.}} \sim \frac{T_{\mathrm{f.o.}}^2}{M_P \cdot \sigma}$$

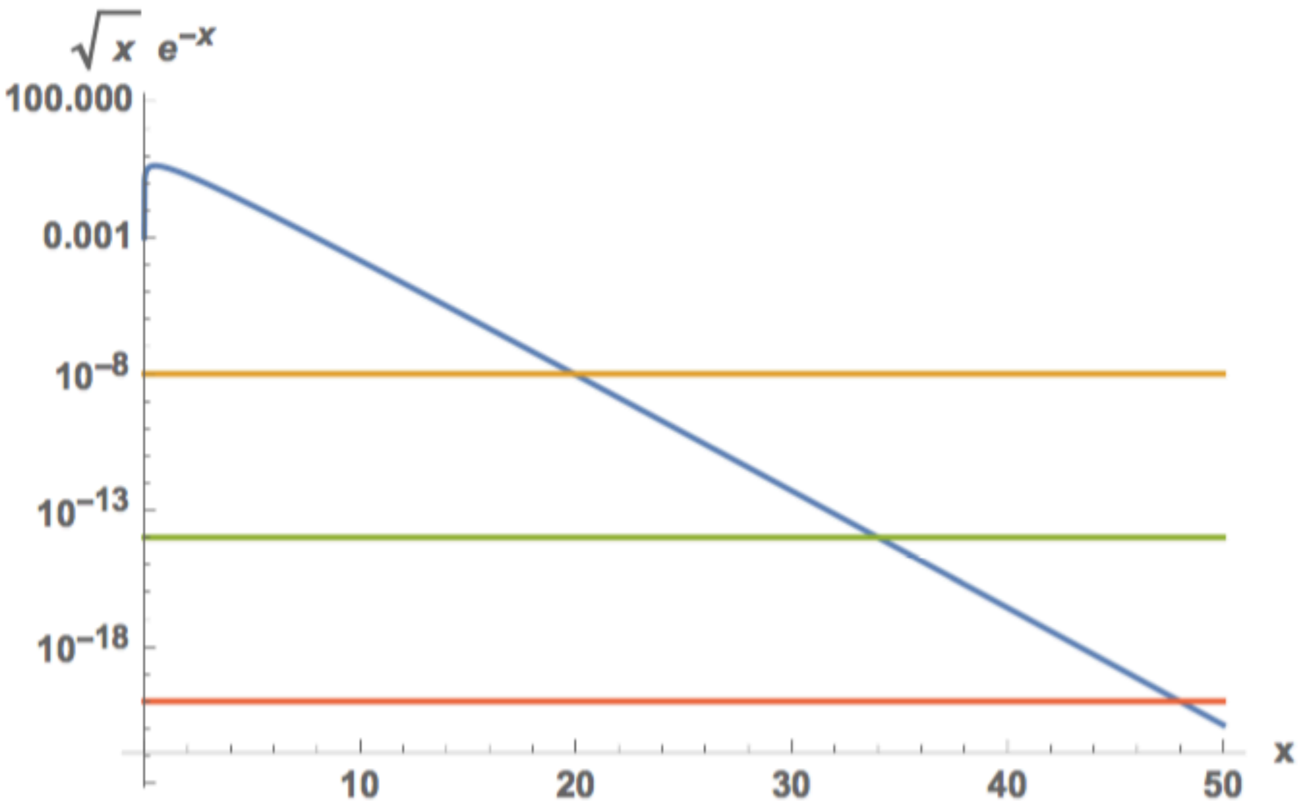
now define $m_\chi / T \equiv x$ (cold relic: **x>>1**)

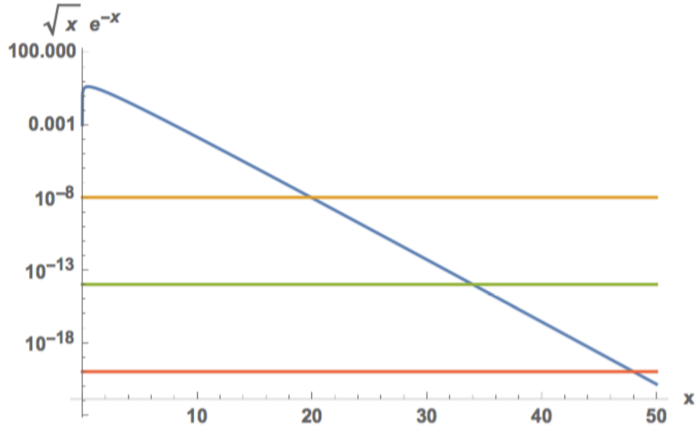
Freeze-out condition (x) now reads

$$\frac{m_\chi^3}{x^{3/2}} e^{-x} = \frac{m_\chi^2}{x^2 \cdot M_P \cdot \sigma}$$

...so we gotta **solve** $\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$





$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma}$$

$$\sigma \sim G_F^2 m_\chi^2$$

Take e.g. a "**weakly interacting massive particle**"

$$m_\chi \sim 10^2 \text{ GeV.}$$

$$\sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \sim \frac{1}{10^2 \cdot 10^{18} \cdot 10^{-6}} \sim 10^{-14}.$$

thus ***$x = m_\chi / T \sim 35$***

Off to calculating the **thermal relic density**

$$\Omega_\chi = \frac{m_\chi \cdot n_\chi(T = T_0)}{\rho_c} = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_0}{T_0^3}$$

iso-entropic universe aT ~const $\quad \frac{n_0}{T_0^3} \simeq \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3}$

$$\Omega_\chi = \frac{m_\chi\, T_0^3}{\rho_c} \frac{n_{\rm f.o.}}{T_{\rm f.o.}^3} = \frac{T_0^3}{\rho_c} x_{\rm f.o.} \left(\frac{n_{\rm f.o.}}{T_{\rm f.o.}^2}\right) = \left(\frac{T_0^3}{\rho_c\, M_P}\right) \frac{x_{\rm f.o.}}{\sigma}$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\rm f.o.}}{20} \left(\frac{10^{-8}\ {\rm GeV}^{-2}}{\sigma}\right)$$

Notice we neglected relative **velocity**...
What is the velocity of a cold relic at freeze-out?

$$\frac{3}{2}T = \frac{1}{2}mv^2$$

...just use **equipartition** theorem...

Now, back to **relic density**:

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\mathrm{f.o.}}}{20}\left(\frac{10^{-8}\ \mathrm{GeV}^{-2}}{\sigma}\right)$$

$$\sigma_{\mathrm{EW}} \sim G_F^2 T_{\mathrm{f.o.}}^2 \sim G_F^2\left(\frac{E_{\mathrm{EW}}}{20}\right)^2 \sim 10^{-8}\ \mathrm{GeV}^{-2},$$

$$\left(\frac{\Omega_\chi}{0.2}\right) \simeq \frac{x_{\mathrm{f.o.}}}{20}\left(\frac{10^{-8}\ \mathrm{GeV}^{-2}}{\sigma}\right)$$

Is this **unique** to **WIMPs**? **No**.

$$\sigma \sim \frac{g^4}{m_\chi^2}$$

"**WIMPless**" miracle... what did we use?

$$m_\chi \cdot \sigma \cdot M_P \gg 1$$

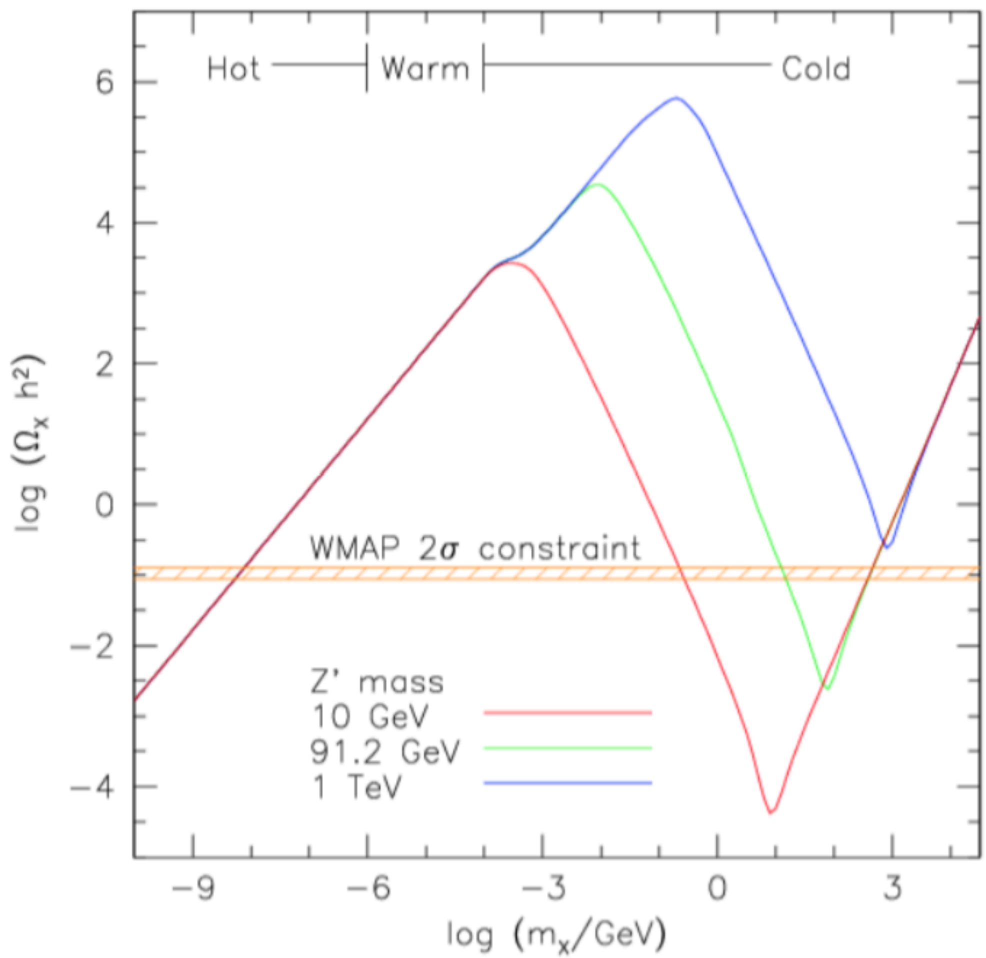
$$\sigma \sim 10^{-8}\ \mathrm{GeV}^{-2}$$

Substitute and find that ***m_χ >> 0.1 eV*** !

In practice various **constraints** on light thermal relics from structure formation, relativistic degrees of freedom at BBN, CMB... **m_χ > MeV**

Put everything together: suppose you have a **mediator** Z', mass ***$m_{Z'}$***

$$\sigma \sim \frac{m_\chi^2}{(s - m_{Z'}^2)^2 + m_{Z'}^4}$$



What is the **range** of **masses** expected for cold relics?

Cross section cannot be arbitrarily large: **unitarity** limit

$$\sigma \lesssim \frac{4\pi}{m_\chi^2}$$

$$\frac{\Omega_\chi}{0.2} \gtrsim 10^{-8}\ \mathrm{GeV}^{-2} \cdot \frac{m_\chi^2}{4\pi}$$

$$\left(\frac{m_\chi}{120\ \mathrm{TeV}}\right)^2 \lesssim 1$$

What is the **range** of **masses** expected for cold relics?

If you have a WIMP, defined by a cross section $\sigma \sim G_F^2 \, m_\chi^2$

$$\Omega_\chi h^2 \sim 0.1 \frac{10^{-8} \text{ GeV}^{-2}}{G_F^2 \, m_\chi^2} \sim 0.1 \left(\frac{10 \text{ GeV}}{m_\chi} \right)^2$$

"**Lee-Weinberg**" limit

Discussion so far OK for a **qualitative** assessment of **relic density**

State of the art much more sophisticated: Solve **Boltzmann equation**

$$\hat{L}[f] = \hat{C}[f]$$

$$\hat{L}_{\mathrm{NR}} = \frac{\mathrm{d}}{\mathrm{d}t} + \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}\vec{\nabla}_x + \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\vec{\nabla}_v$$

$$\hat{L}_{\mathrm{cov}} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma}\, p^\beta\, p^\gamma \frac{\partial}{\partial p^\alpha}$$

Looks ugly, but for the **FRW** metric **phase-space** density simplifies…

$$f(\vec{x}, \vec{p}, t) \rightarrow f(|\vec{p}|, t) \qquad f(E, t)$$

$$\hat{L}[f] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|\vec{p}|^2\, \frac{\partial f}{\partial E}$$

Now, what we are interested in are **number** densities, which in terms of **phase-space** densities are simply…

$$n(t) = \sum_{\text{spin}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f(E,t)$$

…**integrate** the Liouville operator over **momentum space** and get

$$\int L[f] \cdot g \frac{\mathrm{d}^3 p}{(2\pi)^3} = \frac{\mathrm{d}n}{\mathrm{d}t} + 3H \cdot n,$$

Back to **Boltzmann** equation, suppose a **2-to-2** reaction, with 3, 4 **in eq.**

$$1 + 2 \leftrightarrow 3 + 4$$

Consider the **collision** factor, and again integrate over **momenta**…

$$g_1 \int \hat{C}[f_1] \frac{\mathrm{d}^3 p}{(2\pi)^3} = -\langle \sigma \cdot v_{\mathrm{M\o l}} \rangle \left(n_1 n_2 - n_1^{\mathrm{eq}} n_2^{\mathrm{eq}} \right)$$

…where the **cross section**

$$\sigma = \sum_f \sigma_{12 \to f}$$

$$g_1 \int \hat{C}[f_1] \frac{\mathrm{d}^3 p}{(2\pi)^3} = -\langle \sigma \cdot v_{\mathrm{Møl}} \rangle \left(n_1 n_2 - n_1^{\mathrm{eq}} n_2^{\mathrm{eq}} \right)$$

let's understand the rest of the equation:

$$v_{\mathrm{Møl}} \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1\, E_2}$$

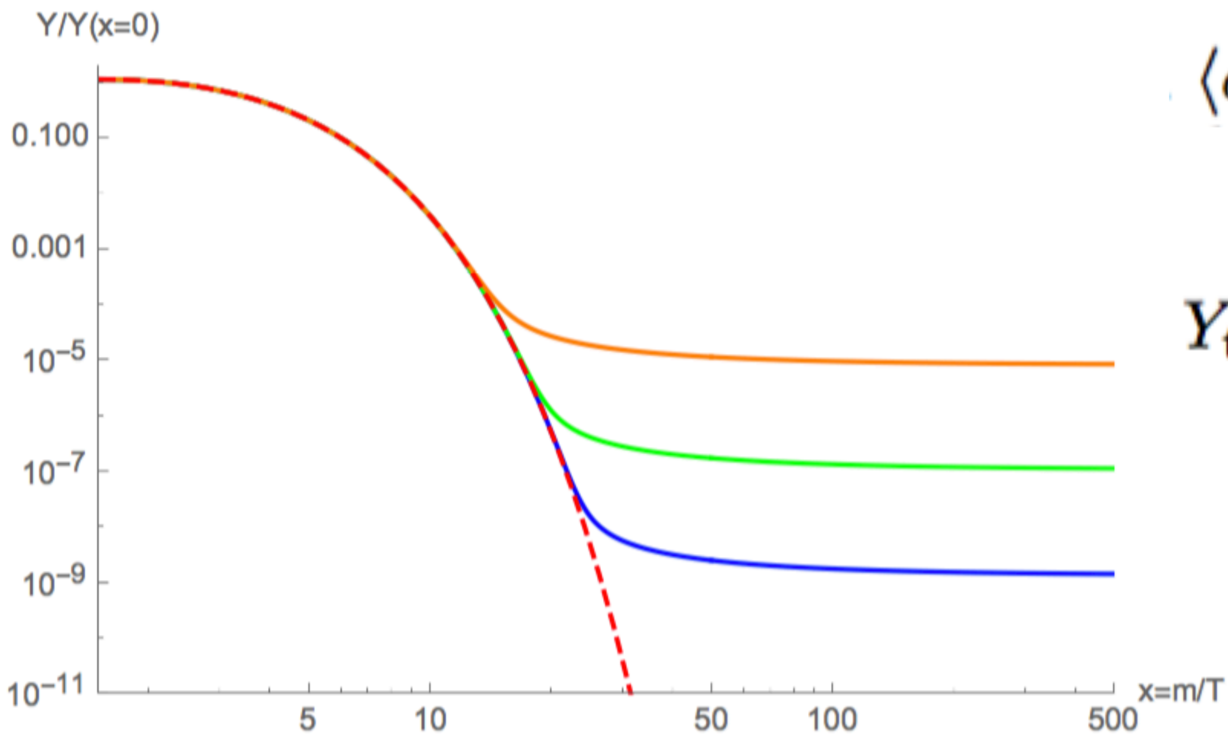
$$\langle \sigma \cdot v_{\mathrm{Møl}} \rangle = \frac{\int \sigma \cdot v_{\mathrm{Møl}}\, e^{-E_1/T} e^{-E_2/T}\, \mathrm{d}^3 p_1\, \mathrm{d}^3 p_2}{\int e^{-E_1/T} e^{-E_2/T}\, \mathrm{d}^3 p_1\, \mathrm{d}^3 p_2}$$

Final version of **Boltzmann Eq.**

$$\dot{n} + 3Hn = \langle \sigma v \rangle \left(n_{\mathrm{eq}}^2 - n^2 \right)$$

$$\dot{n} + 3Hn = \langle \sigma v \rangle \left(n_{\text{eq}}^2 - n^2 \right)$$

$$\frac{dY(x)}{dx} = -\frac{xs\langle \sigma v \rangle}{H(m)} \left(Y(x)^2 - Y_{\text{eq}}^2(x) \right)$$



$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 x^{-n}$$

$$Y_{\text{today}} \simeq \frac{n+1}{\lambda} x_{\text{f.o.}}^{n+1}$$

$$\lambda = \frac{\langle \sigma v \rangle_0 s_0}{H(m)}$$

There exist important "**exceptions**" to this standard story:

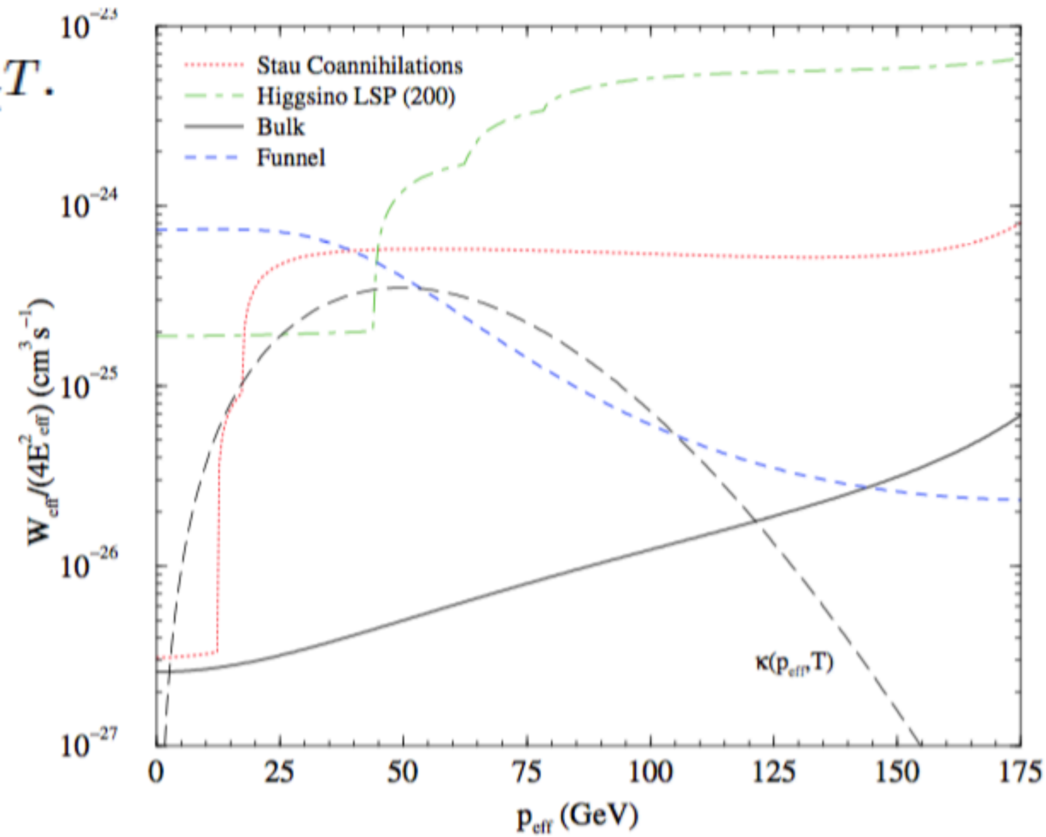
1. **Resonances**

$$\langle s \rangle \simeq 4m_\chi^2 + 6m_\chi T.$$

2. **Thresholds**

3. **Co-annihilation**

$$\langle \sigma v \rangle \;\to\; \langle \sigma_{\rm eff} v \rangle = \frac{\sum_{i<j=1}^{N} \sigma_{ij} \exp\left(-\frac{\Delta m_i + \Delta m_j}{T}\right)}{\sum_{i=1}^{N} g_i \exp\left(-\frac{\Delta m_i}{T}\right)}.$$



Affects what the **pair-annihilation** rate **today** is compared to what it was at **freeze-out**!

$$\langle \sigma_{\rm eff} v \rangle = \int_0^\infty dp_{\rm eff} \frac{W_{\rm eff}(p_{\rm eff})}{4E_{\rm eff}^2} \kappa(p_{\rm eff}, T) \qquad E_{\rm eff}^2 = \sqrt{p_{\rm eff}^2 + m^2}.$$