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An Introduction to Particle Dark Matter

Pre-SUSY Summer School
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✓ PhD **Theoretical Particle Physics** (2004)  
  *International School for Advanced Studies (SISSA-ISAS), Trieste, Italy*

✓ Postdoc, FSU and California Institute of Technology (2005-2007)  
  *Theoretical Astrophysics and Particle Physics*

✓ Joined **UCSC Physics** Faculty (Assistant Professor, 2007-2011,  
  Associate Professor, July 2011-2015  
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✓ Director of UCSC Physics **Graduate Studies** (2012-)

✓ **SCIPP Deputy Director** for **Theory** (July 2011-)
1. What is the origin of the tiny excess of matter over anti-matter?
2. What is the fundamental particle physics nature of Dark Matter?
Please come *introduce yourselves*!

[to myself, other Instructors, to each other...]

If you are ever on the **US West Coast** please let me know!

Never *underestimate* the importance of *networking* in science!
a new elementary particle

...as such it is of interest to particle physicists!
What this mini-lecture series is not:

✧ Review of “evidences” for dark matter

✧ Review of “models” for dark matter

✧ Review of possible claimed “signals” from dark matter
What this mini-lecture series will be

✓ Gross features of dark matter as a particle (lecture 1)

✓ Paradigms for dark matter in the early universe (lecture 1/2)

✓ Schematics of dark matter searches (lecture 2/3)

✓ Selected lessons from old and new particle dark matter models (lecture 3)
One thing we do **know well** about dark matter

Global amount of dark matter in the universe

**Reason:** very good handles on total energy density, total *matter* density, total *baryonic* matter

CMB data indicate the universe is nearly flat
→ energy density is close to critical...

What is the **critical density**? (very good number to have in mind!)

\[
\rho_{\text{crit}} \equiv \frac{3H_0^2}{8\pi G_N} \approx 10^{-29} \text{ g/cm}^3
\]

...since 1 GeV \(\sim 10^{-24}\) g, **10 protons per cubic meter** (=tincy!)
Various ways to "weigh" matter versus dark energy (CMB+SN+BAO)

...and ordinary (baryonic) matter versus non-baryonic (BBN, CMB)

Global amount of dark matter in the universe from simple subtractions!

\[ \bar{\rho}_{DM} = \Omega_{DM}\rho_{crit} \approx 0.3\rho_{crit}. \]
in "astro" units...
\[ \bar{\rho}_{\text{DM}} \sim 10^{10} \frac{M_\odot}{\text{Mpc}^3} \]

clusters... 10^5 denser!

in "particle physics" units...
\[ \bar{\rho}_{\text{DM}} \sim 10^{-6} \frac{\text{GeV}}{\text{cm}^3} \]

galaxies... 10^6 denser!

\[ \frac{\delta \rho}{\rho} \gg 1 \]

the Universe is highly non-linear!

...which is one of the reasons why modified gravity does not work!
CMB sky is very **boring** – $T$ fluctuations very **small**!

$T$ fluctuations proportional to (baryonic) **density** fluctuations,

$$\delta \rho / \rho \lesssim 10^{-4}$$

Good news! Matter **over-densities** in linear regime grow **linearly** with scale factor

But the scale factor since CMB decoupling grew by $z_{rec} \sim 1,100$

Not enough time for structures to go **non-linear**!
We need a species that has decoupled from photons much earlier (Dark Matter) so that its density perturbations are much larger at recombination!

\[
(\delta \rho / \rho)_{\text{DM}} \gg 10^{-4}
\]

Dark matter seeds timely structure formation!
Recombination

Dark Matter

Baryon-photon fluid oscillations
Things go **badly wrong without DM** for structure formation!

Even with best (covariant) incarnation of modified gravity (TeVeS), structure goes non-linear, but the **power spectrum** of matter density fluctuation is **entirely wrong**...
Don’t get fooled by the “Volcano” versus “Neptune” analogy

[Volcano: No new planet between Mercury and the Sun, but GR
Neptune: New planet]

Modified Gravity [MOND, TeVeS] actually does not work at all!!
Knowledge of the dark matter average \textit{density} is a powerful \textit{model-building} tool

Models that \textbf{predict} the “right” \textit{amount} of dark matter get kudos

Dark Matter “\textit{cosmogony}” well-motivated guideline to model building

prototypical example: dark matter as a \textit{thermal relic}... more on this shortly
What else do we know about the microscopic nature of dark matter from its macroscopic features?

- "Dark": ...for the reason above! But detailed constraints on electric charge of dark matter are model-dependent... Milli-charge allowed... Phenomenologically: DM is nearly dissipationless (maybe not entirely though, see dark photons, dark disks...)

- Collisionless... really? Let's calculate the relevant constraints!
mean free path $\lambda$ larger than cluster size, $\sim 1$ Mpc

cluster density: $\rho \sim 1$ GeV/cm$^3$, thus...

$\lambda = 1/(\sigma (\rho/m)) > 1$ Mpc $\Rightarrow \sigma /m < 1$ Mpc / 1 GeV/cm$^3$

$\Rightarrow \sigma /m < 1$ cm$^2$/g, or 1 barn/GeV
1 barn/GeV... which is strong interaction-size...

is this small?

Also, if cross section is slightly smaller, no visible effect...
if cross section slightly larger, disaster...

Begs the question: is “collisional” self-interacting dark matter a “natural” possibility??
Classical: needs to be confined (gravitationally bound) on scales at least as large as dSph... if de Broglie wavelength is larger, disaster strikes!
little exercise: consider $\nu \sim 100 \text{ km/s}$, show that $\lambda = h/p$ is

$$\lambda \sim 3 \text{ mm} \left( \frac{1 \text{ eV}}{m} \right)$$

which means that to have $\lambda \ll \text{kpc} \sim 3 \times 10^{21} \text{ cm}$, $m > 10^{-22} \text{ eV}$
Much, much **better constraints** if the DM is a fermion – we know that the **phase space** density is bounded (Pauli blocking): \( f = gh^{-3} \)

Using **observed** density and velocity dispersion of dSph, **Tremaine-Gunn** limit (1979): observed phase space density cannot exceed upper bound! (Liouville theorem?) Exercise!

\[
\sigma \sim 150 \text{ km/s} \quad \quad \rho \gtrsim 1 \text{ GeV/cm}^3
\]

\[
m^4 > \frac{\rho h^3}{[g(2\pi\sigma^2)^{3/2}]} \sim (25 \text{ eV})^4.
\]
Fluid: don't want to disrupt pretty (and old!) clusters of stars

Neat exercise to estimate the energy exchanged by encounters of GC and BH, in the impulse approximation, demand that that energy be smaller than binding energy, get maximal mass for BH

Also constraints on disk disruption ("heating")

Bottom line: \( m > 10^3 \) solar masses \( \sim 10^{70} \) eV
...here's the name of the game:

(i) **Mass**: >90 orders of magnitude for *bosons*, 70 for *fermions*

(ii) **Interactions**: \(\sim\) *dark, self-interacting* at most \(\sim\) strong interactions

(iii) **Abundance**
Think left and think right and think low and think high.

Oh the things you can think up, if only you try!

Dr. Seuss
A successful framework for the origin of species in the early universe: thermal decoupling
A successful framework for the **origin of species** in the early universe: **thermal decoupling**

A successful **synergy** of **statistical mechanics**, **general relativity**, and of **nuclear and particle physics** making **predictions** testable to exquisite accuracy with **astronomical** observations!
Key idea of thermal decoupling:
if the reaction keeping a species in equilibrium is faster than the expansion rate of the universe, the reaction is in statistical equilibrium;
if it’s slower, the species decouples ("freeze-out")

\[ \Gamma \ll H(T) \quad \Gamma(T_{t.o.}) \sim H(T_{t.o.}) \]

the reaction rate (from definition of cross section!)

\[ \Gamma = n \cdot \sigma \cdot v \]
(1) borrow \textbf{equilibrium number densities} from stat mech

\[ n_{\text{rel}} \sim T^3 \quad \text{for } m \ll T, \]
\[ n_{\text{non-rel}} \sim (mT)^{3/2} \exp \left( -\frac{m}{T} \right) \quad \text{for } m \gg T. \]

(2) borrow \textbf{Hubble rate} from general relativity

(FRW solution to Einstein's eq.)

\[ H^2 = \frac{8\pi G_N}{3} \rho. \]
\[ H^2 = \frac{8\pi G_N}{3} \rho. \]

GR+SM: energy density in radiation

\[ \rho \sim \rho_{\text{rad}} = \frac{\pi^2}{30} \cdot g \cdot T^4 \quad \Rightarrow \quad H \sim T^2/\tilde{M}_P \]
first application: hot thermal relic

language definition: hot = relativistic at $T_{f.o}$
   cold = $v < c = 1$. (actually not by much, typically!)

simple application: relic SM neutrinos (cosmo $\nu$ background)

$$\nu + \bar{\nu} \leftrightarrow f + \bar{f},$$
\[ \nu + \bar{\nu} \leftrightarrow f + \bar{f}, \]

\[ n(T_{\nu}) \cdot \sigma(T_{\nu}) = H(T_{\nu}) \]

\[ \sigma \sim G_F^2 T_{\nu}^2 \]

suppose this is a hot relic... \( n^\sim T_{\nu}^3 \)

\[ T_{\nu}^3 G_F^2 T_{\nu}^2 = T_{\nu}^2 / M_P \]

\[ T_{\nu} = (G_F^2 M_P)^{-1/3} \approx (10^{-10} \times 10^{18})^{-1/3} \text{ GeV} \approx 1 \text{ MeV} \]
happy about two things in particular:

1. hot relic assumption works! \( T_\nu \gg m_\nu \)

2. Fermi effective theory OK! \( T_\nu \ll m_W \)

\[
T_\nu = (G_F^2 M_P)^{-1/3} \approx (10^{-10} \times 10^{18})^{-1/3} \text{ GeV} \sim 1 \text{ MeV}
\]
now, how do we calculate the **relic** thermal abundance of this prototypical hot relic?

Introduce $Y = n/s$ (number and entropy density, $V = a^3$)

If universe is iso-entropic, $s \times a^3 = S$ is conserved

$Y \sim n \cdot a^3$ is thus $\sim$ comoving number density, and (without entropy injection)

$$Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu)$$

$$Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)}$$
\[ Y_{\text{today}} = Y_{\text{freeze-out}} = Y(T_\nu) \]

\[ Y_{\text{freeze-out}} = \frac{n(T_\nu)}{s(T_\nu)} = \frac{\rho_\nu(T_\nu)}{m_\nu \cdot s(T_\nu)} \]

\[ n_{\text{today}} = s_{\text{today}} \times Y_{\text{today}} = s_{\text{today}} \times Y_{\text{freeze-out}} \]

\[ \rho_{\nu,\text{today}} = m_\nu \times Y_{\text{freeze-out}} \times s_{\text{today}} \]

\[ \Omega_\nu h^2 = \frac{\rho_\nu}{\rho_{\text{crit}}} h^2 \simeq \frac{m_\nu}{91.5 \text{ eV}} \]

Cowsik-Mc-Clelland limit
That was **fun**! Let's see if it works for something else...

Try **proton-antiproton** freeze-out: what’s the **relic** matter **abundance** in a baryon-symmetric Universe?

\[
\sigma \sim \Lambda_{QCD}^{-2}
\]

\[
n \sigma = H \Rightarrow T^3 \Lambda^{-2} = T^2/M_p \Rightarrow T = \Lambda^2/M_p
\]

doesn't quite work, we're way **outside** the regime of validity for **hot relics**, since \(T << m_p\)...

Need to work out the case of **cold relics**, which looks nastier by eye

\[
n \sim (m_\chi T)^{3/2} \exp\left(-\frac{m_\chi}{T}\right)
\]
Here's the trick: **freeze-out** condition gives

\[ n_{\text{f.o.}} \sim \frac{T_{\text{f.o.}}^2}{M_P \cdot \sigma} . \]

now define \( m_\chi / T \equiv x \)  
(cold relic: \( x \gg 1 \))

**Freeze-out** condition \((x)\) now reads

\[ \frac{m_\chi^3}{x^{3/2}} e^{-x} = \frac{m_\chi^2}{x^2 \cdot M_P \cdot \sigma} . \]

...so we gotta **solve**

\[ \sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} . \]
\[ \sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \]
Take e.g. a "weakly interacting massive particle"

\[ \sqrt{x} \cdot e^{-x} = \frac{1}{m_\chi \cdot M_P \cdot \sigma} \]

\[ \sigma \sim G_F^2 m_\chi^2 \]

\[ m_\chi \sim 10^2 \text{ GeV.} \]

\[ \sqrt{x} \cdot e^{-x} \sim \frac{1}{10^2 \cdot 10^{18} \cdot 10^{-6}} \sim 10^{-14}. \]

Thus \( x = m_\chi / T \sim 35 \)
Off to calculating the **thermal relic density**

\[
\Omega_\chi = \frac{m_\chi \cdot n_\chi(T = T_0)}{\rho_c} = \frac{m_\chi T_0^3}{\rho_c} \frac{n_0}{T_0^3}
\]

 Iso-entropic universe \(aT \sim \text{const}\)

\[
\frac{n_0}{T_0^3} \simeq \frac{n_{f.o.}}{T_{f.o.}^3}
\]

\[
\Omega_\chi = \frac{m_\chi T_0^3}{\rho_c} \frac{n_{f.o.}}{T_{f.o.}^3} = \frac{T_0^3}{\rho_c} x_{f.o.} \left( \frac{n_{f.o.}}{T_{f.o.}^2} \right) = \left( \frac{T_0^3}{\rho_c M_P} \right) \frac{x_{f.o.}}{\sigma}
\]

\[
\left( \frac{\Omega_\chi}{0.2} \right) \simeq \frac{x_{f.o.}}{20} \left( \frac{10^{-8} \text{ GeV}^{-2}}{\sigma} \right)
\]
Notice we neglected relative velocity... What is the velocity of a cold relic at freeze-out?

\[ \frac{3}{2} T = \frac{1}{2} m v^2 \]

...just use equipartition theorem...

Now, back to relic density:

\[ \left( \frac{\Omega_X}{0.2} \right) \approx \frac{x_{f.o.}}{20} \left( \frac{10^{-8} \text{ GeV}^{-2}}{\sigma} \right) \]

\[ \sigma_{\text{EW}} \sim G_F^2 T_{f.o.}^2 \sim G_F^2 \left( \frac{E_{\text{EW}}}{20} \right)^2 \sim 10^{-8} \text{ GeV}^{-2}, \]
Is this unique to WIMPs? No.

\[
\left( \frac{\Omega_\chi}{0.2} \right) \approx \frac{x_{\text{f.o.}}}{20} \left( \frac{10^{-8} \text{ GeV}^{-2}}{\sigma} \right)
\]

\[\sigma \sim \frac{g^4}{m_\chi^2}\]

"WIMPless" miracle... what did we use?

\[m_\chi \cdot \sigma \cdot M_P \gg 1\]

\[\sigma \sim 10^{-8} \text{ GeV}^{-2}\]

Substitute and find that \(m_\chi >> 0.1 \text{ eV}\)!

In practice various constraints on light thermal relics from structure formation, relativistic degrees of freedom at BBN, CMB... \(m_\chi > \text{MeV}\)
Put everything together: suppose you have a mediator $Z'$, mass $m_{Z'}$.
What is the **range** of **masses** expected for cold relics?

Cross section cannot be arbitrarily large: **unitarity** limit

\[ \sigma \lesssim \frac{4\pi}{m_X^2} \]

\[ \frac{\Omega_X}{0.2} \gtrsim 10^{-8} \text{ GeV}^{-2} \cdot \frac{m_X^2}{4\pi} \]

\[ \left( \frac{m_X}{120 \text{ TeV}} \right)^2 \lesssim 1 \]
What is the **range** of **masses** expected for cold relics?

If you have a WIMP, defined by a cross section

\[
\sigma \sim G_F^2 \frac{m^2}{m_\chi}
\]

\[
\Omega_\chi h^2 \sim 0.1 \frac{10^{-8} \text{ GeV}^{-2}}{G_F^2 \frac{m^2}{m_\chi}} \sim 0.1 \left( \frac{10 \text{ GeV}}{m_\chi} \right)^2
\]

"Lee-Weinberg" limit
Discussion so far OK for a **qualitative** assessment of relic density

State of the art much more sophisticated: Solve Boltzmann equation

\[
\hat{L}[f] = \hat{C}[f]
\]

\[
\hat{L}_{\text{NR}} = \frac{d}{dt} + \frac{d\vec{x}}{dt} \nabla_x + \frac{d\vec{v}}{dt} \nabla_v
\]

\[
\hat{L}_{\text{cov}} = p_\alpha \frac{\partial}{\partial x^\alpha} - \Gamma_\beta^\alpha_\gamma p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}
\]

Looks ugly, but for the **FRW** metric **phase-space** density simplifies...

\[
f(\vec{x}, \vec{p}, t) \rightarrow f(|\vec{p}|, t)
\]

\[
f(E, t)
\]

\[
\hat{L}[f] = E \frac{\partial f}{\partial t} - \frac{\dot{a}}{a} |\vec{p}|^2 \frac{\partial f}{\partial E}
\]
Now, what we are interested in are **number** densities, which in terms of **phase-space** densities are simply...

\[ n(t) = \sum_{\text{spin}} \int \frac{d^3p}{(2\pi)^3} f(E, t) \]

...**integrate** the Liouville operator over **momentum space** and get

\[ \int L[f] \cdot g \frac{d^3p}{(2\pi)^3} = \frac{dn}{dt} + 3H \cdot n. \]
Back to **Boltzmann** equation, suppose a 2-to-2 reaction, with 3, 4 in eq.

\[
1 + 2 \leftrightarrow 3 + 4
\]

Consider the **collision** factor, and again integrate over momenta...

\[
g_1 \int \hat{C}[f_1] \frac{d^3 p}{(2\pi)^3} = -\langle \sigma \cdot v_{\text{Møl}} \rangle (n_1 n_2 - n_1^{\text{eq}} n_2^{\text{eq}})
\]

...where the **cross section**

\[
\sigma = \sum_f \sigma_{12\rightarrow f}
\]
\[ g_1 \int \hat{C}[f_1] \frac{d^3p}{(2\pi)^3} = -\langle \sigma \cdot v_{M\phi} \rangle (n_1 n_2 - n_1^{eq} n_2^{eq}) \]

let’s understand the rest of the equation:

\[ v_{M\phi} \equiv \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2} \]

\[ \langle \sigma \cdot v_{M\phi} \rangle = \frac{\int \sigma \cdot v_{M\phi} e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2}{\int e^{-E_1/T} e^{-E_2/T} d^3p_1 d^3p_2} \]

Final version of **Boltzmann Eq.**

\[ \dot{n} + 3Hn = \langle \sigma v \rangle (n_{eq}^2 - n^2) \]
\[ \dot{n} + 3Hn = \langle \sigma v \rangle \left( n_{\text{eq}}^2 - n^2 \right) \]

\[ \frac{dY(x)}{dx} = -\frac{xs\langle \sigma v \rangle}{H(m)} \left( Y(x)^2 - Y_{\text{eq}}^2(x) \right) \]

\[ \langle \sigma v \rangle = \langle \sigma v \rangle_0 x^{-n} \]

\[ Y_{\text{today}} \simeq \frac{n + 1}{\lambda} x_{f.o.}^{n+1} \]

\[ \lambda = \frac{\langle \sigma v \rangle_0 s_0}{H(m)} \]
There exist important "exceptions" to this standard story:

1. Resonances

\[
\langle s \rangle \simeq 4m_\chi^2 + 6m_\chi T.
\]

2. Thresholds

3. Co-annihilation

\[
\langle \sigma v \rangle \rightarrow \langle \sigma_{\text{eff}} v \rangle = \frac{\sum_{i<j=1}^N \sigma_{ij} \exp\left(-\frac{\Delta m_i + \Delta m_j}{T}\right)}{\sum_{i=1}^N g_i \exp\left(-\frac{\Delta m_i}{T}\right)}.
\]

Affects what the pair-annihilation rate today is compared to what it was at freeze-out!

\[
\langle \sigma_{\text{eff}} v \rangle = \int_0^\infty dp_{\text{eff}} \frac{W_{\text{eff}}(p_{\text{eff}})}{4E_{\text{eff}}^2} \kappa(p_{\text{eff}}, T) \quad E_{\text{eff}}^2 = \sqrt{p_{\text{eff}}^2 + m^2}.
\]