GUT SCALE THRESHOLD EFFECTS ON PROTON DECAY

Takumi KUWAHARA

Nagoya University

Based on
with J.Hisano and Y.Omura (Nagoya U.)
with B.Bajc (J.Stefan inst.), J.Hisano and Y.Omura (Nagoya U.)
Introduction

Discovery of the 126GeV Higgs boson (in 2012)

- Success of the Standard Model (SM)
- Approach to Beyond SM (BSM) more realistically,
  treat all SM parameters as known values

We can prepare theoretical predictions of BSM more precisely.

Intensity Frontier

Flavor Physics
CP violation
etc..

=> Indirect measurement,
but accessible to high-energy
The promising extension of the SM

**Supersymmetric Grand Unified Theories (SUSY GUTs)**

Unified description of

* strong and electro-weak interactions
* quarks and leptons

=> Predicting **Baryon-Number Violating Processes** (Proton Decay etc..)

* B# is accidentally preserved in SM
* Signature of BSM if we find

Precise prediction towards discovery
- Introduction
- SUSY SU(5) GUTs
- Proton Decay
- Procedure & Results
- Summary
Matter and Gauge sectors are almost universal in the SUSY SU(5) GUTs

**Matter Sector**: completely embedded in $5^* (\Phi)$ and $10 (\Psi)$

$$D^C, L \in \Phi, \quad U^C, Q, E^C \in \Psi$$

**Gauge Sector**

$$V(24) = \begin{pmatrix} G & X^+ \\ X & W \end{pmatrix} - \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 \\ -3 \end{pmatrix} B$$

**Higgs Sector**

MSSM Higgs doublets are embedded in fields in (anti-)fundamental reps.

$$\overline{H(5)} = \begin{pmatrix} H^C \\ H_d \end{pmatrix}, \quad H(5) = \begin{pmatrix} H^C \\ H_u \end{pmatrix}$$

+ GUT breaking Higgs, and etc..

So, Higgs sector depends on models
Higgs Sector (besides $5+5^*$ Higgs containing MSSM Higgses)

<table>
<thead>
<tr>
<th>Minimal SU(5)</th>
<th>Adjoint (24-dimensional) Higgs</th>
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<tbody>
<tr>
<td>Missing Partner Model</td>
<td>50+50$^*$</td>
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<tr>
<th>Models for Yukawa Realization</th>
<th>additional 45+45$^*$</th>
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<tbody>
<tr>
<td>Georgi, Jarlskog (1979) et al.</td>
<td></td>
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<tr>
<td>etc..</td>
<td>Blue Higgses: GUT breaking one</td>
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Higgs Sector (besides 5+5* Higgs containing MSSM Higgses)

In this talk, I focus on

**Minimal SU(5)**

- Simple
- Still valid in high-scale SUSY Scenario (D=5 decay)
- with fine-tuning in doublet-triplet splitting

**Missing Partner Model**

- Solving doublet-triplet splitting without fine-tuning
- Models requiring huge number of fields
  => prospect for large quantum correction to proton decay prediction
- Free from D=5 proton decay (if imposing Peccei-Quinn symmetry)
Proton Decay

X bosons give rise to baryon-number violating process!

\[ V(24) = \left( \begin{array}{ccc} G & X^+ \\ X & W \end{array} \right) - \frac{1}{2\sqrt{15}} \left( \begin{array}{cc} 2 \\ -3 \end{array} \right) B \]

Main decay mode: \( p \rightarrow \pi^0 + e^+ \)

Proton Decay induced by gauge-interaction: in general, model (= Higgs sector) independent decay

Current lower bound (future sensitivity) on proton decay.

<table>
<thead>
<tr>
<th>Current Decay</th>
<th>CURRENT</th>
<th>FUTURE</th>
</tr>
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<tbody>
<tr>
<td>( p \rightarrow \pi^0 + e^+ )</td>
<td>( 1.67 \times 10^{34} ) yrs</td>
<td>( 1.0 \times 10^{35} ) yrs</td>
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Super-K Result

2016 Moriond

Hyper-K Prospect

10-years exposure
Theoretical progress (Higher-order corrections to Wilson coeff. of D=6 operators)

- **QCD correction (2-loop)**
  - Arafune, Nihei (1994)

- **RGE in SM (2-loop)**

- **RGE in SUSY SM (2-loop)**
  - Hisano, Kobayashi, Nagata, Muramatsu (2013)

**Threshold Corrections**

RGE effects are computed @ 2-loop order (Gauge interaction)

1-loop threshold corrections are also expected as the same order

In addition,

Hadron Matrix Elements @ 2GeV are calculated by lattice simulation

with 30% errors

Aoki, Shintani, Soni (2013)
Results

Analytic formula for Threshold Corrections

In the Effective theory (~ MSSM),

\[ \mathcal{L}_{\text{dim.6}} = \int d^4 \theta \sum_{I=1,2} (1 - \lambda^{(I)}) C^{(0)}_I \mathcal{O}^{(0)}_I + \text{h.c.} \]

Threshold corrections to Wilson coeff. \( \lambda^{(I)} \)

For each threshold corrections, we obtain; Hisano, TK, Omura (2015)

\[ \lambda^{(1)} = \frac{\Sigma(0)}{M_X^2 + \Sigma(0)} + \frac{g_5^2}{16\pi^2} \frac{16}{5} \left( 1 - \ln \frac{M_X^2}{\mu^2} \right), \]

\[ \lambda^{(2)} = \frac{\Sigma(0)}{M_X^2 + \Sigma(0)} + \frac{g_5^2}{16\pi^2} \frac{18}{5} \left( 1 - \ln \frac{M_X^2}{\mu^2} \right). \]

Vacuum polarization Vertex + Box

Vacuum polarization strongly depends on GUT mass spectrum
Comparing with the previous study

\[
(Ratio) \equiv \frac{\Gamma(p \rightarrow \pi^0 + e^+)}{\Gamma(p \rightarrow \pi^0 + e^+)} \bigg|_w \quad \bigg| \frac{\Gamma(p \rightarrow \pi^0 + e^+)}{\Gamma(p \rightarrow \pi^0 + e^+)} \bigg|_{w/o}
\]

Ratio of decay rate with and without threshold corrections

Numerical Results: among the GUT models \((M_x = 2\times10^{16}\text{GeV})\)

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<th>Ratio</th>
<th>Minimal SU(5)</th>
<th>Missing-Partner</th>
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<tr>
<td>(\Gamma(p \rightarrow e^+ \pi^0))</td>
<td>0.994</td>
<td>0.394</td>
</tr>
<tr>
<td>(\tau(p \rightarrow e^+ \pi^0))</td>
<td>(2.23\times10^{36}\text{yrs})</td>
<td>(7.09\times10^{35}\text{yrs})</td>
</tr>
</tbody>
</table>

Bajc, Hisano, TK, Omura (2015)

- Suppressed rate <= thanks to threshold effects
- Short lifetime <= Large unified coupling @GUT scale due to many fields

Determination of GUT Mass Spectrum

\[
\alpha_i^{-1}(\mu) = \alpha_G^{-1}(\mu) + \lambda_i(\mu)
\]

\(\alpha_i^{-1}(\mu)\): MSSM couplings
\(\lambda_i(\mu)\): Unified coupling

Constraining on
* Color-triplet Mass
* \(M_x^2 M_\Sigma\)

Depends on GUT Scale Masses
Summary

- We have derived 1-loop threshold correction to Wilson coefficients of Dim.-6 operators at GUT scale.

- Proton lifetime becomes longer about a few % due to threshold corrections in the minimal SUSY SU(5).

- Large suppression of decay rate in the missing-partner SU(5) model (due to many fields and mass splitting)
Backups
SUSY SU(5) GUTs and Its Spectrum

Minimal SUSY SU(5) GUT

Matter Sector
\[ \Phi_A(\overline{5}) = \begin{pmatrix} D_a^C \\ \epsilon_{rs} L^s \end{pmatrix}, \quad \Psi^{AB}(10) = \begin{pmatrix} \epsilon^{abc} U^C_c \\ -Q^{br} \quad Q^{as} \\ \epsilon^{rs} E^C \end{pmatrix} \]

Gauge Sector
\[ V(24) = \begin{pmatrix} G \\ X \\ W \end{pmatrix} - \frac{1}{2 \sqrt{15}} \begin{pmatrix} 2 \\ -3 \end{pmatrix} B \]

Higgs Sector
\[ \overline{H}(\overline{5}) = \begin{pmatrix} H^C_c \\ H_d \end{pmatrix}, \quad H(5) = \begin{pmatrix} H^C_c \\ H_u \end{pmatrix} \]
\[ \Sigma_{24} = \begin{pmatrix} \Sigma_8 \\ \Sigma_{(3^*,2)} \quad \Sigma_{(3,2)} \\ \Sigma_3 \end{pmatrix} + \frac{1}{2 \sqrt{15}} \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} \Sigma_S \]
Minimal SUSY SU(5) GUT

\[ \langle (\Sigma_{24})^r_s \rangle = -3v_{24}\delta^r_s, \quad \langle (\Sigma_{24})^\alpha_\beta \rangle = 2v_{24}\delta^\alpha_\beta \]

\[ K_{24} = (\Sigma_{24}^\dagger)^A_B (e^{2g_5V})^B_C (e^{-2g_5V})^D_A (\Sigma_{24})^C_D, \quad \rightarrow \quad M_X = 5g_5v_{24} \]

\[ W = \lambda \overline{H}(\Sigma_{24} + 3v_{24})H, \quad \rightarrow \quad M_{HC} = 5\lambda v_{24} \]

\[ W = \frac{f}{3} \text{Tr}(\Sigma_{24})^3 + \frac{m_{24}}{2} \text{Tr}(\Sigma_{24})^2 \]

\[ \rightarrow \quad W = \frac{m_8}{2} \Sigma^A_8 \Sigma^A_8 + \frac{m_3}{2} \Sigma^A_3 \Sigma^A_3 + \frac{m_S}{2} \Sigma_S \Sigma_S + \cdots \]

\[ m_8 : m_3 : m_S = \frac{5m_{24}}{2} : \frac{5m_{24}}{2} : m_{24}/2 \]

\[ = 5 : 5 : 1 \]
Missing-Partner SU(5)

$$\langle (\Sigma_{75})^{[rs]}_{[tu]} \rangle = \frac{3}{2} v_{75} (\delta^r_t \delta^s_u - \delta^r_u \delta^s_t), \quad \langle (\Sigma_{75})^{[\alpha \beta]}_{[\gamma \delta]} \rangle = \frac{1}{2} v_{75} (\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma),$$

$$\langle (\Sigma_{75})^{[ar]}_{[bs]} \rangle = -\frac{1}{2} v_{75} \delta^a_\beta \delta^r_s.$$ 

$$\mathcal{K}_{75} = (\Sigma^\dagger_{75})^{[AB]}_{[CD]} (e^{2g_5 V})^C_E (e^{2g_5 V})^D_F (e^{-2g_5 V})^G_A (e^{-2g_5 V})^H_B (\Sigma_{75})^{[EF]}_{[GH]}.$$ 

after integrating out 50+50*

$$W = M_{H_C} H_C H'_C + M'_{H_C} H'_C H_C,$$

with

$$M_{H_C} \equiv \frac{48 v_{75}^2}{M_{Pl}} g_H g'_H, \quad M'_{H_C} \equiv \frac{48 v_{75}^2}{M_{Pl}} g'_H g_H.$$

typically, $\sim 10^{15}$ GeV
Missing-Partner SU(5)

\[
\langle (\Sigma_{75})^{[rs]}_{[tu]} \rangle = \frac{3}{2} v_{75} (\delta^r_\delta^s - \delta^r_\delta^s), \quad \langle (\Sigma_{75})^{[\alpha\beta]}_{[\gamma\delta]} \rangle = \frac{1}{2} v_{75} (\delta^\alpha_\delta^\beta - \delta^\alpha_\delta^\beta),
\]

\[
\langle (\Sigma_{75})^{[\alpha r]}_{[\beta s]} \rangle = -\frac{1}{2} v_{75} \delta^\alpha_\beta \delta^r_s,
\]

\[
W = m_{75} (\Sigma_{75})^{[CD]}_{[AB]} (\Sigma_{75})^{[AB]}_{[CD]} - \frac{1}{3} \lambda_{75} (\Sigma_{75})^{[AB]}_{[EF]} (\Sigma_{75})^{[CD]}_{[AB]} (\Sigma_{75})^{[EF]}_{[CD]}
\]

\[
75 = (1,1)_0 \oplus (3,1)_{-\frac{5}{3}} \oplus \overline{(3,1)}_{\frac{5}{3}} \oplus (\overline{3},2)_{\frac{5}{6}} \oplus (\overline{3},2)_{-\frac{5}{6}} \oplus (6,2)_{\frac{5}{6}} \oplus (6,2)_{-\frac{5}{6}} \oplus (8,1)_0 \oplus (8,3)_0
\]

\[
= 2 : 4 : \text{(NG Mode)} : 2 : 1 : 5
\]

with \( M_{(8,3)_0} = 5m_{75} \)
Constrained Mass Spectra

By using central values for couplings (& sparticles around 1 TeV)

**Minimal SU(5)**
\[
\begin{align*}
\frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} &= \frac{1}{8\pi^2} \frac{12}{5} \ln \frac{M_{H_C}}{\mu}, \\
\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} &= \frac{1}{8\pi^2} 12 \ln \frac{M_X^2 M_{\Sigma_{24}}}{\mu^3}.
\end{align*}
\]

\[M_{H_C} = 6.4 \times 10^{15} \text{ GeV}\]
\[(M_X^2 M_{\Sigma_{24}})^{1/3} = 1.5 \times 10^{16} \text{ GeV}\]

**MP SU(5)**
\[
\begin{align*}
\frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} - \frac{1}{g_1^2(\mu)} &= \frac{1}{8\pi^2} \left( \frac{12}{5} \ln \frac{M_{H_C} M_{\overline{H_C}}}{M_{H'_f} \mu} + 6 \ln \frac{2^6}{5^5} \right), \\
\frac{5}{g_1^2(\mu)} - \frac{3}{g_2^2(\mu)} - \frac{2}{g_3^2(\mu)} &= \frac{1}{8\pi^2} \left( 12 \ln \frac{M_X^2 M_{\Sigma_{75}}}{\mu^3} + 54 \ln \frac{5}{4} \right).
\end{align*}
\]

\[
\frac{M_{H_C} M_{\overline{H_C}}}{M_{H'_f}} = 1.1 \times 10^{20} \text{ GeV}
\]
\[(M_X^2 M_{\Sigma_{75}})^{1/3} = 5.4 \times 10^{15} \text{ GeV}\]