Gravitational Wave Instabilities in the Cosmic Neutrino Background

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Outline

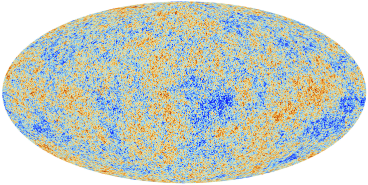
Cosmic Neutrino Background

- Decoupling of neutrinos results in the Cosmic Neutrino Background (CνB), $T \sim 1$ MeV

- The CνB temperature is related to that of the CMB:

$$\frac{T_\nu}{T_0} = \left(\frac{4}{11}\right)^{\frac{1}{3}}$$

where $T_0 = 2.725$ K is the temperature of the CMB today.



- The weakly interacting nature and low temperature of the CνB means it has yet to be observed

Neutrino Masses and Relic Asymmetry

- A very large lepton asymmetry can be stored in the CνB.
- Flavour dependent bound on the asymmetries is

$$L_\alpha^{C\nu B} = \frac{n_{\nu_\alpha} - \bar{n}_{\nu_\alpha}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)}\left(\xi_\alpha + \frac{\xi_\alpha^3}{\pi^2}\right)$$

  where the flavour independent bounds on ξ_α are $-0.07 < \xi < 0.22$.

- Majorana particles $\rightarrow$ the CνB will be parity violating.
- Possible indirect evidence of the CνB via induced parity violating radiative corrections to the graviton propagator.

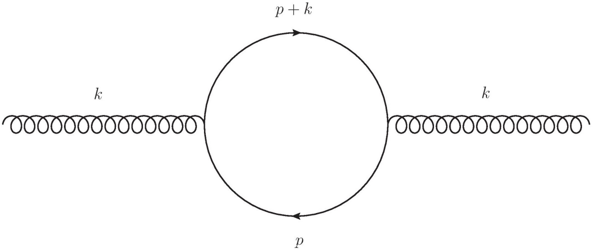
Fermion Propagator in CνB

- Homogeneous neutrino gas with a net lepton asymmetry Δn_ν.
- Leads to an addition to the fermion propagator.
- First order in Δn_ν parity violating contribution:

$$S(p) = S_0(p) + \underbrace{\frac{1}{p^2 - m^2}\left[\frac{i\sigma_{\alpha\beta}\gamma^5 p^\alpha \Delta f^\beta (\not{p} + m)}{p^2 - m^2} + \frac{1}{2}\gamma_\beta \gamma^5 \Delta f^\beta\right]}_{S_1(p)} + \ldots$$

- $\Delta f^0 = \sqrt{2} G_F \sum \Delta n_{\nu_\alpha}$ embodies the strength of the neutrino interactions and asymmetry density ($\lesssim 10^{-42}$ today).

Addition to the Graviton Propagator



Parity violating contribution in an homogeneous neutrino background:

$$\Pi_{\mu\nu\rho\sigma} = -\int \frac{d^4p}{(2\pi)^4}(2p+k)_\nu(2p+k)_\sigma\Big[Tr(\gamma_\mu S_0(p+k)\gamma_\rho S_1(p)) + Tr(\gamma_\rho S_0(p)\gamma_\mu S_1(p+k))\Big]$$

$$
\begin{aligned}
\Pi_{\mu\nu\rho\sigma} =& \frac{\Delta f^{\beta}}{8\pi^2} k^{\alpha} \varepsilon_{\mu\nu\alpha\beta} \int_0^1 dx \left[\frac{4\pi^2\lambda^2}{M^2}\right]^{\epsilon} \times \\
& \Bigg[8x^2(1-x)^2(1-2x)^2 \frac{k^2}{M^2} \Gamma(1+\epsilon) k_\nu k_\sigma \\
& + (24x^2 - 44x + 18)\Gamma(\epsilon - 1) M^2 \eta_{\nu\sigma} - 16x^2(1-x)^2 \Gamma(\epsilon) k^2 \eta_{\nu\sigma} \\
& - (80x^4 - 192x^3 + 156x^2 - 50x + 5)\Gamma(\epsilon) k_\nu k_\sigma \Bigg]
\end{aligned}
$$

where $M^2 = m^2 - x(1-x)k^2$.

We find a divergent quantity:

$$
\Pi^{(div)}_{\mu\nu\rho\sigma} = -\frac{1}{\epsilon} \frac{\Delta f^{\beta}}{2\pi^2} k^{\alpha} \varepsilon_{\mu\rho\alpha\beta} m^2 \eta_{\nu\sigma}
$$

Find that this term cannot contribute as it violates the gauge invariance ($h_{\mu\nu} \to h_{\mu\nu} + k_\mu \lambda_\nu + k_\nu \lambda_\mu$).
Transversality requires: $k^\mu \Pi_{\mu\nu\rho\sigma} = 0$ and $k^\nu \Pi_{\mu\nu\rho\sigma} = 0$.

Graviton Polarisation Tensor

We obtain the following simple form for the polarisation tensor,

$$\Pi_{\mu\nu\rho\sigma} = \varepsilon_{\mu\rho\alpha\beta} k^{\alpha} \Delta f^{\beta} [k_{\nu} k_{\sigma} - k^2 \eta_{\nu\sigma}] C(k^2)$$

where

$$C(k^2) = \frac{1}{192\pi^2} - \frac{m^2}{16\pi^2 (k^2)^{3/2}} \left[\sqrt{k^2} - \sqrt{4m^2 - k^2} \tan^{-1}\left(\frac{\sqrt{k^2}}{\sqrt{4m^2 - k^2}} \right) \right]$$

where

$$C(k^2) = \begin{cases} -\frac{1}{1920\pi^2} \frac{k^2}{m^2}, & \text{if } k^2/m^2 \ll 1 \\ \frac{1}{192\pi^2}, & \text{if } k^2/m^2 \gg 1 \end{cases}$$

From which we can find the addition to the propagator.

Two Possible Scenarios: $k^2/m_\nu^2 \ll 1$

Can write an effective term in the action,

$$S_{eff} \propto \frac{1}{m^2} \int d^4x \varepsilon_{\mu\rho\alpha\beta} \Delta f^\beta h^{\mu\nu} \partial^\alpha \Box(\Box h^{\rho\sigma} \eta_{\nu\sigma} - \partial_\nu \partial_\sigma h^{\rho\sigma})$$

In combination with the usual E-H action we find,

$$(\omega^2 - |\mathbf{k}|^2) \mp \frac{\Delta f^0}{1920\pi^2 m^2 M_p^2} |\mathbf{k}| (\omega^2 - |\mathbf{k}|^2)^2 = 0$$

This contains an instability, but only for very large $|\mathbf{k}|$; likely unphysical.

$$(\omega^2 - |\mathbf{k}|^2) = \pm \frac{1920\pi^2 m^2 M_p^2}{\Delta f^0 |\mathbf{k}|}$$

Two Possible Scenarios: $k^2/m_\nu^2 \gg 1$

In this limit,

$$S_{eff} = -\frac{1}{192\pi^2}\int d^4x \varepsilon_{\mu\rho\alpha\beta}\Delta f^\beta h^{\mu\nu}\partial^\alpha(\Box h^{\rho\sigma}\eta_{\nu\sigma} - \partial_\nu\partial_\sigma h^{\rho\sigma})$$
$$= \frac{1}{48\pi^2}\int d^4x\ \Delta f_\mu K^\mu$$

- The 4 dimensional Chern-Simons topological current:

$$K^\beta = \varepsilon^{\beta\alpha\mu\nu}(\Gamma^\sigma_{\alpha\rho}\partial_\mu\Gamma^\rho_{\nu\sigma} - \frac{2}{3}\Gamma^\sigma_{\alpha\rho}\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma}).$$

- Replicating Chern-Simons modified gravity.

$$S_{CS} = \int d^4x\ (\partial_\mu\theta)K^\mu = \int d^4x\ \theta({}^*RR)$$

Graviton Propagation Effects

- Chern-Simons modification induces a birefringence effect.
- eLISA can measure sources $z \lesssim 30$, and differentiates polarisations.
- Consider propagation in a FRW universe.

Consider $h_{ij} = \frac{\mathcal{A}_{ij}}{a(\eta)} \exp[-i(\phi(\eta) - \kappa n_k x^k)]$

- Decomposing into the two circularly polarised states: e^R_{ij} and e^L_{ij}
- Take the action $S = S_{EH} + S_{eff}$
- Find the accumulated phase over propagation.

Graviton Propagation Effects

- From the equations of motion:

$$(i\phi^{R,L}_{,\eta\eta} + (\phi^{R,L}_{,\eta})^2 + \mathcal{H}_{,\eta} + \mathcal{H}^2 - \kappa^2)\left(1 - \frac{\lambda_{R,L}\kappa\theta_{,\eta}}{a^2}\right) = \frac{i\lambda_{R,L}\kappa}{a^2}(\theta_{,\eta\eta} - 2\mathcal{H}\theta_{,\eta})(\phi^{R,L}_{,\eta} - i\mathcal{H})$$

- Solve in the matter dominated epoch, $a(\eta) = a_0\eta^2 = \frac{a_0}{1+z}$.
- Accumulated phase to first order in θ,

$$\Delta\phi^{R,L} = i\lambda_{R,L}kH_0\int_{\eta}^{1}\left[\frac{1}{4}\theta_{,\eta\eta} - \frac{1}{\eta}\theta_{,\eta}\right]\frac{d\eta}{\eta^4}$$

- Time dependence of Δf_0 is dilution $\Delta n \Rightarrow \theta_{,\eta} = \left(\frac{a(\eta_0)}{a(\eta)}\right)^4 \frac{\Delta f_0}{48\pi^2 M_p^2}$

Birefringence of Gravitational Waves

- For the CνB,

$$\Delta\phi^{R,L} = -i\frac{\lambda_{R,L}\Delta f_0 H_0}{288\pi^2 M_p^2}\left(\frac{k}{1\ \mathrm{GeV}}\right)(1+z)^6$$

- Ratio of the polarisations:

$$\frac{h_R}{h_L} \propto e^{-2|\Delta\phi|}$$

- From the current bounds on the CνB, $|i\Delta\phi^{R,L}| \lesssim 10^{-115}\left(\frac{k}{1\ \mathrm{GeV}}\right)$, for $z \sim 30$
- Significantly larger in the early universe, a higher asymmetry density.
- All sources could provide constraints.

Gravitational Wave Instabilities

- Chiral plasma in the early universe, composed of the neutrinos.
- Breakdown of the neutrino plasma, with a characteristic timescale.
- Production of gravitons through the gravitational anomaly coupled to the chiral charge, embodied by the Chern-Simons term.
- Potentially large instabilities before nucleosynthesis.
- Produce inhomogeneities, due to entropy production, the absence of which can provide bounds on Δf_0.

Conclusion and Future Work

- Parity asymmetric CνB.
- Chern-Simons like term induced in the graviton action.
- Negligible birefringent effect for small z.
- Bounds from early universe sources and instabilities in the neutrino plasma.

Future work

- Further exploration of the mechanism, and effects on GW propagation.
- Further investigation of early universe implications and bounds.
- Finite temperature field theory analysis.