# Gravitationally Bound Boson Bubbles in (1-cosine (A ) ) Scalar field theory

With Eby, Suranyi, Vaz

Ongoing work with Leembruggen and Leeney

# Based On

- Eby,Suranyi,Vaz, RW, JHEP03(2015)1-15
- Eby,Kouvaris,Nillson,RW , JHEP02(2016)28-46
- Eby,Suranyi,RW, Mod.Phys.Lett. A31 (2016) no.15, 1650090
- Ongoing work with Suranyi, Eby,Leembruggen,Leeney

# Relevant Papers


- Barranco,Bernal, Phys.Rev. D83 (2011) 043525
- Chavanis, P.R.D. Phys.Rev. D84 (2011) 043531
- Guth, Hertzberg,Prescod-Weinstein P.R.D. **Phys.Rev. D92 (2015) no.10, 103513**
- S.Davidson,T.Schwetz**,** Phys.Rev. D93 (2016) no.12, 123509
- Braaten, Mohapatra,Zhang,**arXiv:1604.00669** **arXiv:1512.00108**

# Summary

$$\text{Potential } V(A) = f^2 m^2 \left(1 - \text{Cos}\left(\frac{A}{f}\right)\right)$$

- Limiting Mass $\# \frac{M_p f}{m}$
- Size $\# \frac{M_p}{f\ m}$

For $f = 6 \times 10^{11} \text{GeV}$ $m = 10^{-5}\ \text{ev}$

Maximum Mass M= $10^{19} kg$

Size = 500 km

# Summary

- We only analyze weekly bound condensates
- A is a real field. Therefore particle number is not conserved and the condensate can decay
- Weak binding ensures long life
- If the number of particles exceeds a critical value the condensate would collapse. More about this at the end

# Gravitationally bound fermion bubbles

- Quantum degeneracy pressure balances gravity

$$\text{Maximum mass} = \text{Chandrasekhar limit} = \# \frac{M^3{}_P}{m^2}$$

# Gravitationally Bound Boson States

- Wheeler 1956,"Geons" Phys.Rev.97(2)511

A wave confined to a region by the gravitational effects of its own energy.

Electro-magnetic or a gravity wave.

Classsical model for stable elementary particles

Such solutions exist(Anderson, Brill 1996). They are unstable.

# Klein-Gordon Geons by Kaup(68)

- Kaup considered spherically symmetric , time independent configurations of a free massive complex scalar field, satisfying K.G. and Einstein's eqns. Phys.Rev.172,1331(1968).
- Numerically solved the eqns.
- Configurations stable under small radial perturbations.
- Maximum allowed mass. Kaup Bound.

# Kaup 1968

- Kaup Bound for free bosons $.633\frac{M_P{}^2}{m}$
- Noticed that perfect fluid model does not work. Radial stress is not the same as stresses in theta and phi directions.
- No equation of state in the conventional sense.
- For 1 GeV mass Kaup Bound 10^11 Kg

# Introduce Self Interactions

- Mielke, Scherzer PRD24,2011(1981)
- Colpi,Shapiro,Wasserman, P.R.L.57,2485(1986)

$$M_{crit} = 0.236\sqrt{\Lambda}\,\frac{M_P{}^2}{m}, \textit{with } \Lambda = \frac{\lambda M_P{}^2}{4\pi m^2}, \text{for large } \Lambda$$

$\lambda$ is the 4-scalar self coupling

Comparable to the Chandrasekhar limit when $\lambda \approx 1$ and $\mathrm{m} = 1\mathrm{GeV}$

Size $\sqrt{\lambda}\,\frac{M_p}{m^2}$

# Ruffini and Bonazzola(1969)

- Phys.Rev.187,1767(1969)
- Free massive Hermitian scalar field
- Second quantize the scalar field. Classical gravity
- Obtained  the same results as Kaup

# Ruffini-Bonazzola Method

- Spherical symmetry and time independence

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Asymptotically Flat Boundary Conditions
- Regularity at the origin

# Ruffini-Bonazzola Method

- Einstein's equations of motion

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- Energy Momentum Tensor

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu} L$$

- Expand the field in creation annihilation operators.
- Evaluate the expectation of the normal ordered energy momentum tensor in the N particle ground state

# Ruffini-Bonazzola Method

- Expand the field in creation annihilation operators

$$\Phi(\bar{r},t) = \sum_{n,l,m} R_{n,l,m}(r) Y_{l,m}(\theta,\phi)\ \exp(\text{-iE}_{\text{n,l}}t)\, a_{n,l,m} + \text{h.c}$$

$$[a_{n,l,m}, a^{+}{}_{n',l',m'}] = \delta_{n,n'}\delta_{l,l'}\delta_{m,m'}$$

# Ruffini-Bonazzola method

- Construct the N particle state

$$| N,000000....>=\prod_{1}^{N} a^{+} | 0 >$$

Evaluate the expectation value

$$< N,0000....|: T_{\mu\nu} :| N,00000....>$$

and feed it to the R.H.S. of Einstein's eqns.

LHS classical expression.

<N,000…| $\frac{\partial^2}{\partial t^2}-\nabla^2+V^{'}(A)$ |N-1,000…>=0

Re-derived the Kaup Bound

# (1-Cosine(A/f))potential

- Used by Barranco and Bernal
- We use an expansion method to truncate the potential
- Solutions are expressed as a function of a parameter $\lambda$

The parameter space we explore is different from that of Barranco and Bernal

# Ruffini-Bonazzola with (1-Cos(A)) potential

- Add a potential energy term

$$V(\phi) = f^2 m^2 \left[ 1 - \cos\left( \frac{\phi}{f} \right) \right] = \frac{m^2}{2}\phi^2 - \frac{1}{24}\left( \frac{m}{f} \right)^2 \phi^4 + \ldots\ldots\ldots$$

$$\text{Evaluate } < \mathrm{N} |: \cos\left( \frac{\phi}{\mathrm{f}} \right) :| \mathrm{N} > \approx \mathrm{J}_0\left( \frac{2\sqrt{N}R}{f} \right) \text{in the large N limit}$$

$$\phi = a \exp(-\mathrm{iEt})\, \mathrm{R(r)} + \mathrm{h.c.} \text{ is the mode expansion}$$

# Equations of Motion

$$\frac{A'}{A^2 r} + \frac{A-1}{Ar^2} = \frac{f^2}{M^2{}_P}\left[\frac{E^2 N R^2}{B f^2} + \frac{N R'^2}{A f^2} + m^2 (1 - J_0(X)\right]$$

$$X = \frac{2R\sqrt{N}}{f}$$

$$\frac{B'}{ABr} - \frac{A-1}{Ar^2} = -\frac{f^2}{M^2{}_P}\left[\frac{E^2 N R^2}{B f^2} + \frac{N R'^2}{A f^2} - m^2 (1 - J_0(X))\right]$$

$$\sqrt{N} R'' + \sqrt{N}\left(\frac{2}{r} + \frac{B'}{2B} - \frac{A'}{2A}\right) R' + A\left[\frac{E^2 R\sqrt{N}}{B} - f m^2 J_1(X)\right] = 0$$

# Rescale

Dimension free radial cdt $z = rm$,

Rescaled wave function $X(z) = 2\sqrt{N}\, R(r)/f$ ,

$\varepsilon = E/m$ , $A = 1 + \delta\, a(z)$, and $B = 1 + \delta\, b(z)$ where $\delta = f^2 / M^2{}_P$

$$a' = -\frac{a}{z} + z\left[\frac{1}{4}\varepsilon^2 X^2 + \frac{1}{4}X'^2 + 1 - J_0(X)\right]$$

$$b' = \frac{a}{z} + z\left[\frac{1}{4}\varepsilon^2 X^2 + \frac{1}{4}X'^2 - 1 + J_0(X)\right]$$

$$X'' = \left[-\frac{2}{z} + \frac{\delta}{2}(a' - b')\right]X' - \varepsilon^2(1 + \delta\, a - \delta\, b)X + 2(1 + \delta\, a)J_1(x)$$

$a(z), b(z)$ and $X(z)$ are regular at $z = 0$ and vanish at $z = \infty$. $X(z)$ and $b(z)$ are finite, and $a(z) = 0$ at $z = 0$

# Expansion method of solution

- Double expansion
- Binding energy is the difference between the mass and the energy eigenvalue E

Define $\varepsilon$ to be $\frac{\mathrm{E}}{\mathrm{m}}$

Let $\delta = \left(\frac{\mathrm{f}}{\mathrm{M_P}}\right)^2 << 1$, and $\Delta = \sqrt{1 - \varepsilon^2} \approx \sqrt{2(m-E)/m} << 1$

the equations can be reduced to a system of equations depending

on $\varepsilon$ and $\delta$ through $\lambda = \frac{\sqrt{\delta}}{\Delta} = \frac{\mathrm{f}}{\mathrm{M_P}\Delta}$ only.

# Expansion Method

- Expansion parameters

$$\text{Let } \delta = \left(\frac{\mathrm{f}}{\mathrm{M_P}}\right)^2 << 1, \text{ and } \Delta = \sqrt{1\text{-}\varepsilon^2} \approx \sqrt{2(m-E)/m} << 1$$

The equations can be reduced to

a system of equations depending

$$\text{on } \varepsilon \text{ and } \delta \text{ through } \lambda = \frac{\sqrt{\delta}}{\Delta} = \frac{\mathrm{f}}{\mathrm{M_P}\Delta} \text{ only}.$$

Scale X and z such that $\mathrm{X} = \Delta\,\mathrm{Y}$ and $\mathrm{x} = \Delta \mathrm{z}$

# Equations of Motion

- **Equations of Motion After Scaling**

Scale the wave function X and distance z such that

$$X = \Delta Y \text{ and } x = \Delta z$$

Then the equations reduce to

$$a'(x) = \frac{x}{2} Y(x)^2 - \frac{a(x)}{x}$$

$$b'(x) = \frac{a(x)}{x}$$

$$Y''(x) = Y(x) - \frac{2}{x} Y'(x) - \frac{1}{8} Y(x)^3 + \lambda^2 b(x) Y(x)$$

$$\lambda = \frac{f}{m\Delta}$$

# Scaling Method

- Leading Corrections

Leading order corrections are of $O(\delta)$ and $O(\delta\,\lambda^2)$ and they should be $<<1$. We want $\Delta = \dfrac{f}{M_P \lambda} << 1$, and $\delta\,\lambda^2 = \left(\dfrac{\lambda\, f}{M_P}\right)^2 << 1$.

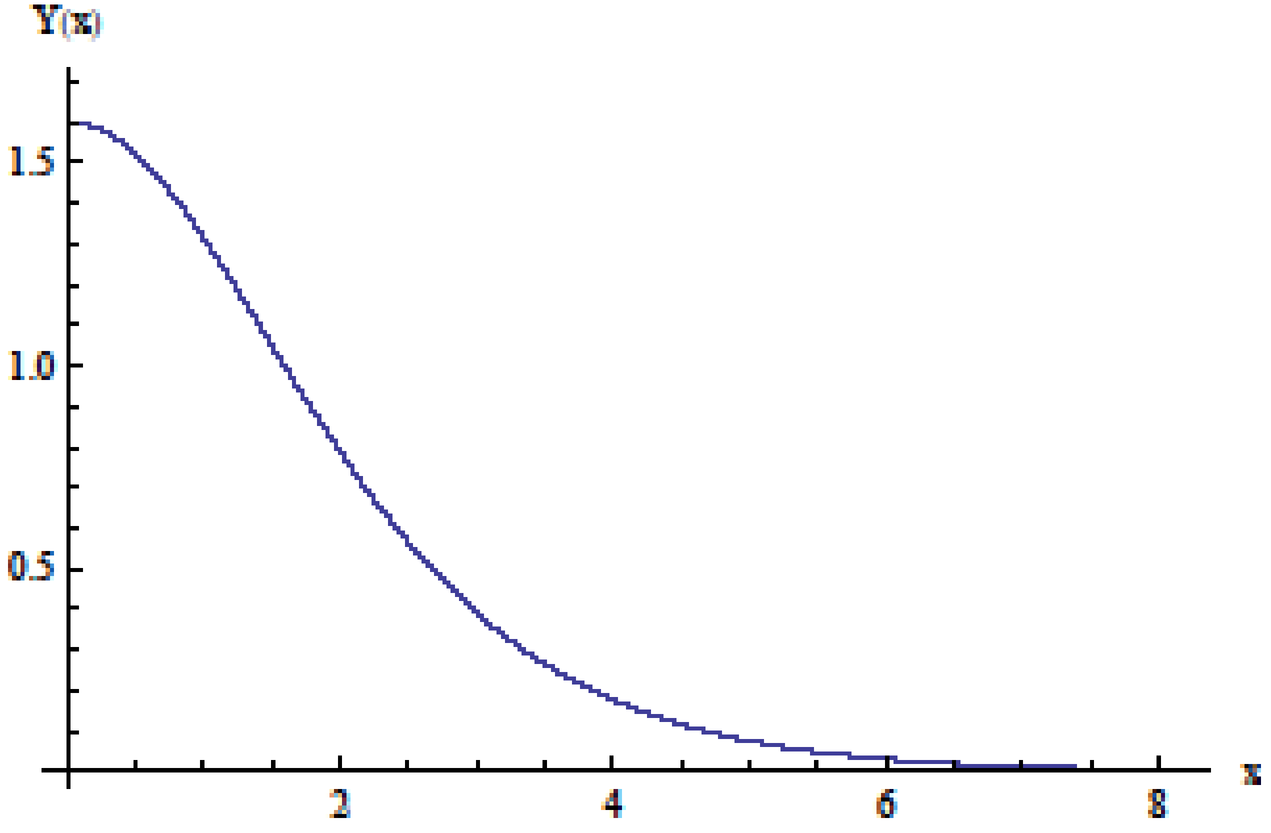
This implies $\dfrac{f}{M_P} << \lambda << \dfrac{M_P}{f}$. This enables us to cover a wide range of parameter space.

# Numerical Solution

- Numerically integrate the equations for a series of values of lambda and calculate Y(x).
- Y, a, b tend to zero at infinity. a is zero at the origin. Y is a function with no nodes.
- Y as a function of x for lambda =1 is plotted

# Plot of the wave function

- Wave function for $\lambda = 1$



# Solutions

$$V(y) = 2\pi \int_0^y Y(x)^2 x^2 dx \text{ and } \mathrm{V(x_{99}) \le\le / V(\infty) = 0.99}$$

$$R_{99} = \frac{1}{m\Delta} x_{99} = \frac{\mathrm{M_P}}{\mathrm{mf}} \lambda \mathrm{x}_{99}$$

$$\mathrm{Mass\,M} == \frac{f^2}{m\Delta} V(\infty)$$

$$R_{99} = \frac{\mathrm{M}}{\mathrm{f}^2} \frac{\mathrm{x_{99}}}{\mathrm{V}(\infty)}$$

$\mathrm{V}(\infty)$ and $\mathrm{x_{99}}$ are computed for a series of values $.01 \le \lambda \le 10$

The ratio $\frac{\mathrm{x_{99}}}{\mathrm{V}(\infty)}$ is larger than 0.1 throughout the range of $\lambda$

Therefore $\mathrm{R_{99}} \ge 0.1 \frac{\mathrm{M}}{\mathrm{f}^2} = 0.1 \frac{\mathrm{M}}{\mathrm{M_P^2}\delta} = \frac{0.05}{\delta} R_s$

Safe from collapse for small $\delta$.

# Solutions

For $\lambda << 1$, a linear fit geives an excellent approxiamtion of $R_{99}$

$$R_{99} = \frac{M_P}{f\ m} 2.735\lambda = \frac{1}{\Delta m} 2.735$$

For $\lambda \geq 0.5$, $R_{99}(\lambda) = \frac{M_P}{f\ m}(0.456 + 5.75\lambda)$

$R_{99}$ increases throughout the range of $\lambda$ considered

Barranco and Bernal investigated the range $\lambda \approx 10^{-5} - 10^{-6}$

For $\lambda << 1$, $M(\lambda) \approx \frac{M_P f}{m} 50.26\lambda = 50.26 \frac{f^2}{m\Delta}$

*For* $\lambda > 1$, i.e. weak binding $V(\infty) \approx \frac{15}{\lambda^2}$ and

$$M = \frac{f^2}{m\Delta} V(\infty) = 15 \frac{M_P^2}{m} \Delta$$

# Solutions

- Mass has a maximum as a function of lambda

Number of particles in the condensate

$$N \approx \frac{M(\Delta)}{m\sqrt{1-\Delta^2}}$$

$M_{max}$ is attained at $\lambda_{max} = .58$

For $N < N_{max}$ there are two masses $M(\Delta_1)$ and $M(\Delta_2)$

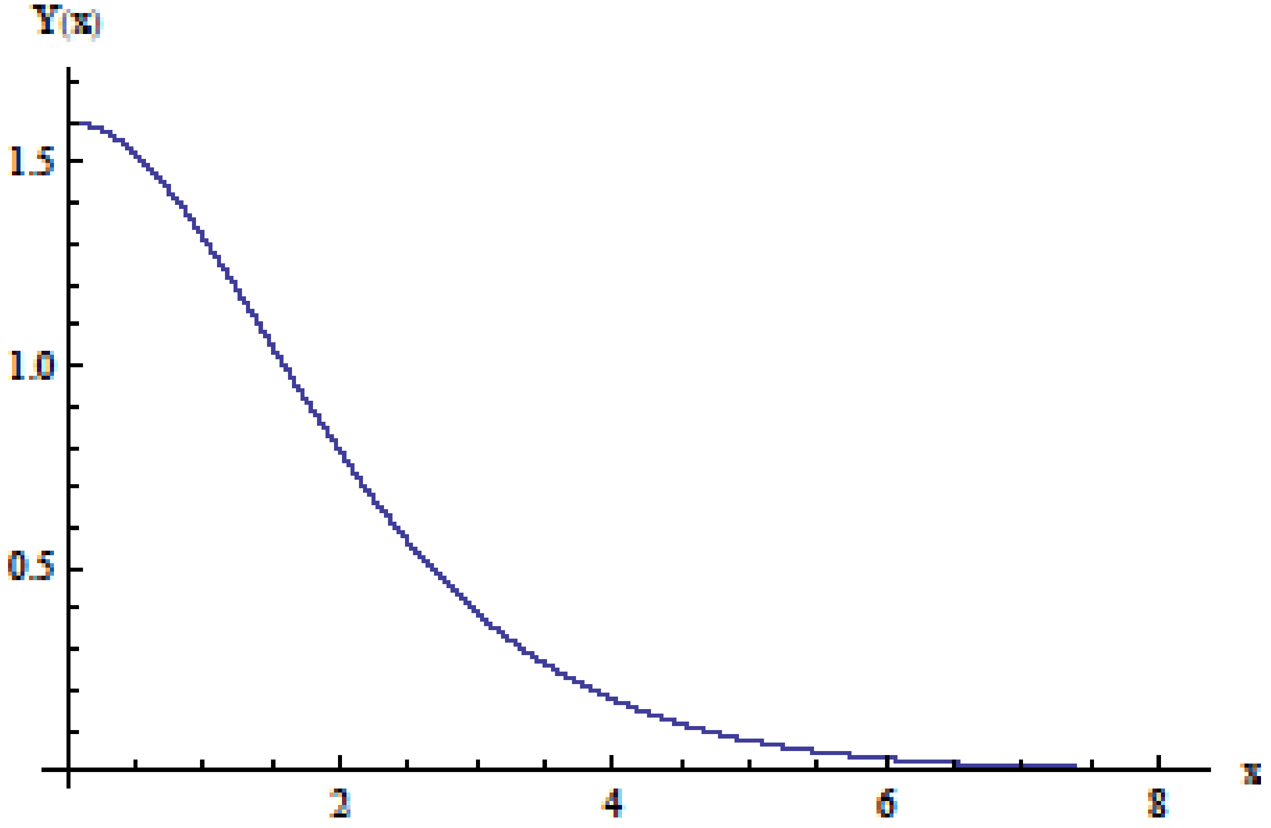
giving the same particle number. Assume $\Delta_1 > \Delta_2$, or equivalently

$\lambda_1 < \lambda_{max} < \lambda_2$. Then $M(\Delta_1) < M(\Delta_2)$.

Every Bubble belonging to the $\lambda > \lambda_{max}$ Branch is unstable

# Plot of the wave function

- Wave function for $\lambda = 1$

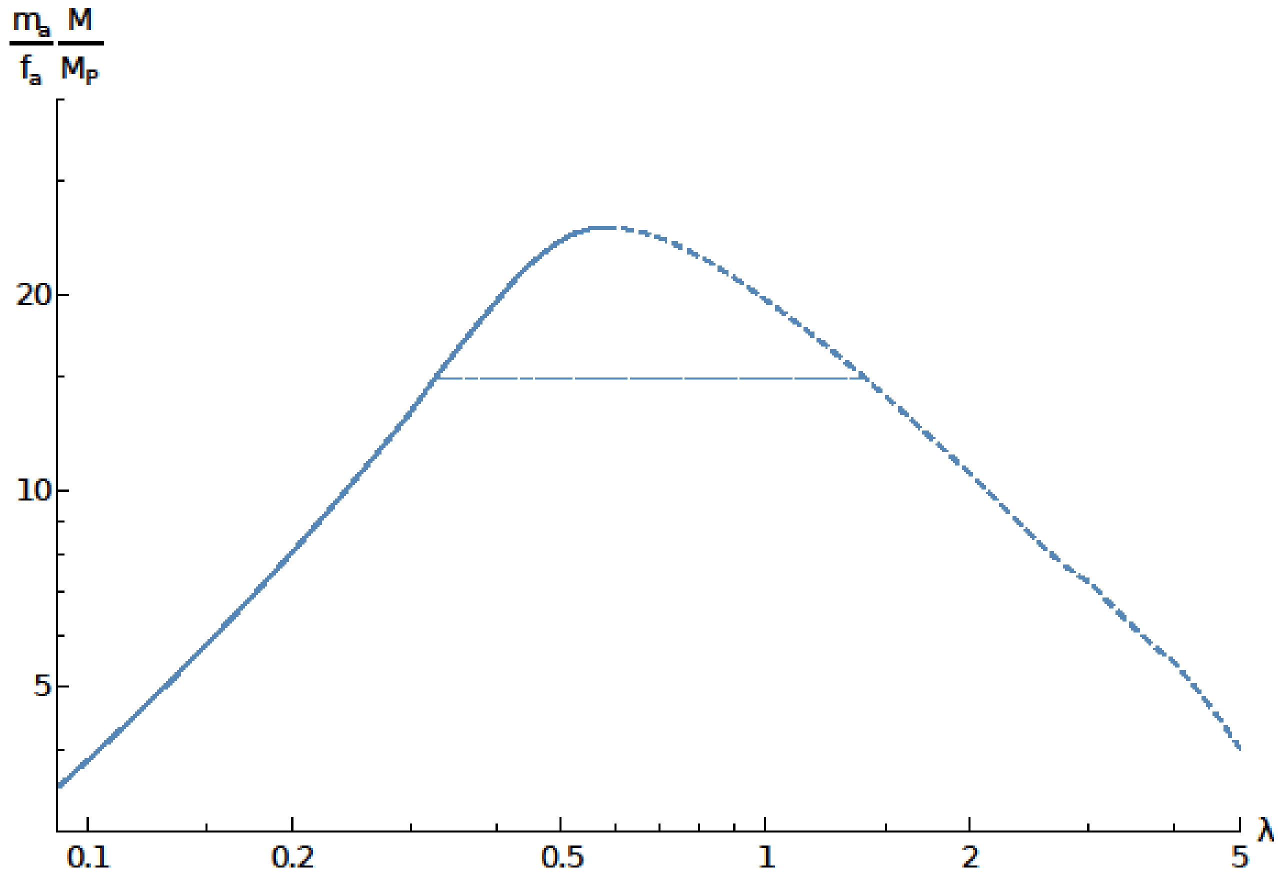


# Table

$$\frac{f}{M_P} < \lambda < \frac{M_P}{f}$$

$$m = 10^{-5}\, ev \text{ and } f = 6x10^{11}\text{GeV}$$

| $\lambda = f_a/(M_P\Delta)$ | M/10^18 kg | $R_{99}$ (km) | $\rho$ (kg/m$^3$) | $\delta M$ (kg) |
|---|---|---|---|---|
| 0.1 | 1.00 | 115 | 155 | 125,000 |
| 0.3 | 3.45 | 386 | 14.3 | 46,300 |
| 0.4 | 5.08 | 593 | 5.80 | 33,500 |
| 0.5 | 6.33 | 854 | 2.43 | 16,300 |
| 0.54 | 6.56 | 972 | 1.70 | 8,350 |
| 0.58 | 6.63 | 1076 | 1.27 | 1,170 |
| 0.62 | 6.61 | 1183 | .95 | -5,370 |
| 0.8 | 5.98 | 1652 | .32 | --23,200 |
| 1.0 | 5.14 | 2145 | 0.12 | -33,100 |
| 2.0 | 2.78 | 4499 | .0070 | -53,400 |
| 4.0 | 1.42 | 9062 | .00050 | -88,700 |
| 10.0 | 5.74 | 22849 | .000012 | -267,000 |



$\frac{m_a}{f_a}\frac{M}{M_P}$
20
10
5
0.1
0.2
0.5
1
2
5
λ

# Conclusions

- Solved the field equation for a spherically

Symmetric configuration when the decay constant is much less than the Planck scale.

Considered only weakly bound configurations.

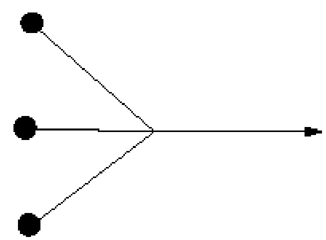
For QCD axions mass of the bubble is of the order of 10^19 kg.

Radius 100-1000 km.

No galaxy sized bubbles?

# Decay Process

- Bound states combine and emit a plane wave state



- Three bound states produce one relativistic state which escapes from the condensate
- Momentum is absorbed by the remaining N-3 particle state

# Decay Calculation

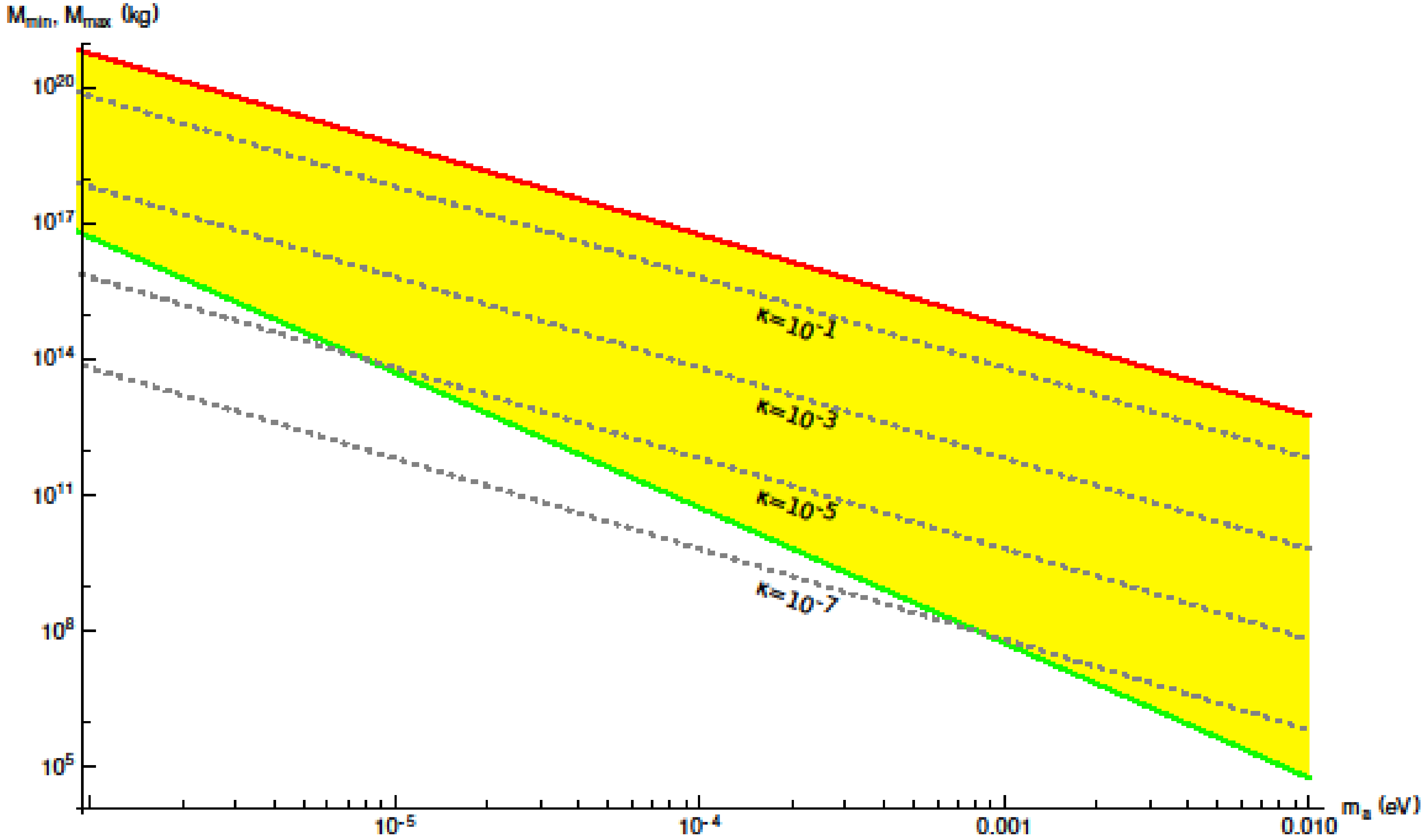
- Calculate <N-3, p |V(A) |N>

$$A = R(r)\exp(-iEt)\,a + \int \frac{d^3p}{\sqrt{2E_p}} a_p \exp(ip.r - iEt) + h.c.$$

- Bound state wave function is localized at a distance scale R= $1/m\Delta$
- Momentum uncertainty $m\Delta$ . The small width of the momentum distribution for weak binding makes this process rare.

# Condensate Decay

- Emitted particle is relativistic. It has energy 3E, which is close to 3m for small binding energy.
- This particle will escape from the bound state and given enough time from the local neighborhood.



$M_{min}$, $M_{max}$ (kg)
$10^{20}$
$10^{17}$
$10^{14}$
$10^{11}$
$10^{8}$
$10^{5}$
$\kappa=10^{-1}$
$\kappa=10^{-3}$
$\kappa=10^{-5}$
$\kappa=10^{-7}$
$10^{-5}$
$10^{-4}$
0.001
0.010
$m_a$ (eV)

# Life Time

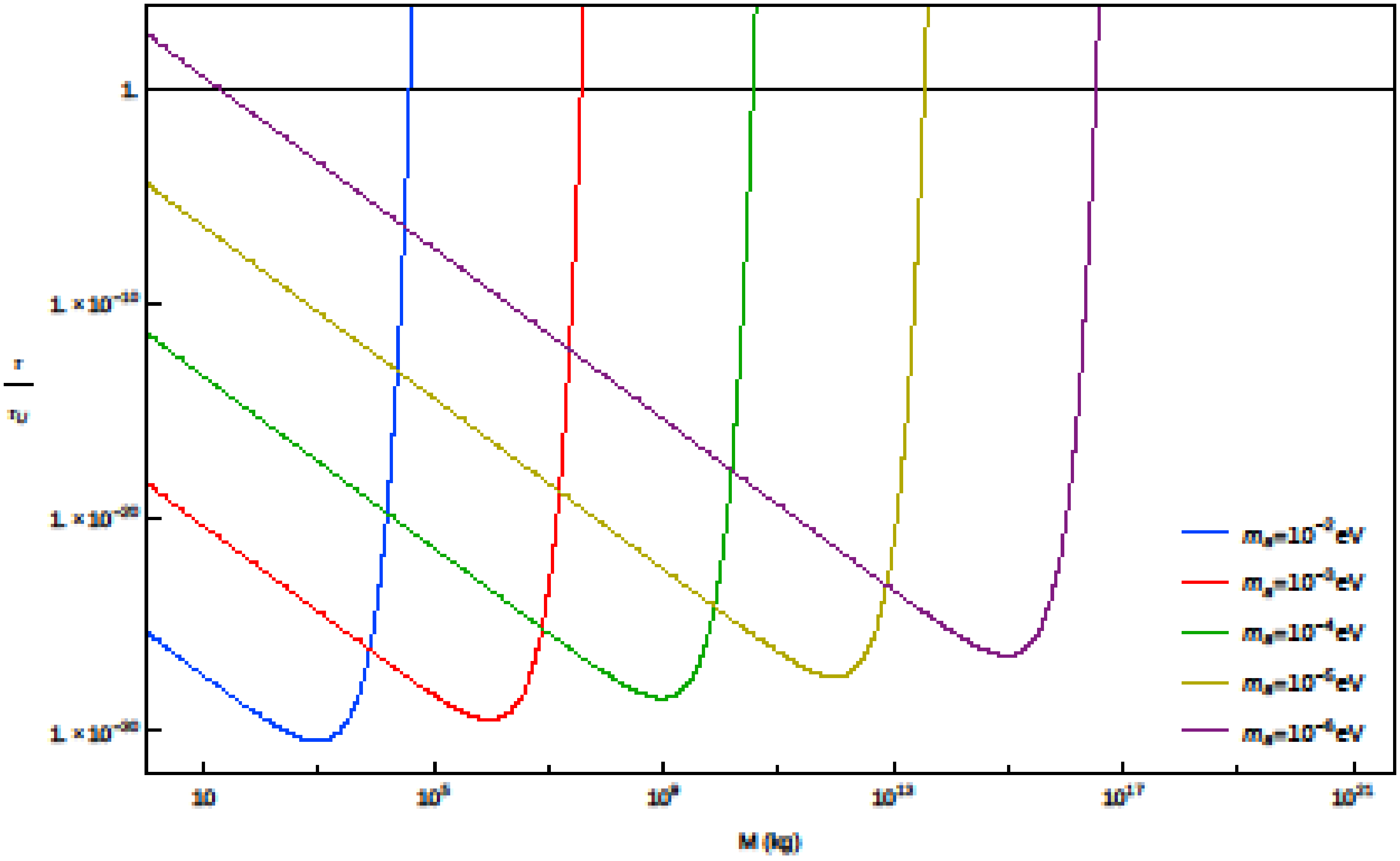
- Life time formula

$$\tau = a \frac{\mathrm{f}^4}{\mathrm{m}^3 M^2} \exp(b \frac{mM}{f^2})$$

$$a = 59.23, and\ b = .13$$

- M is the mass of the condensate

# Ratio of Lifetime to the Age of the Universe



# Collapse Beyond Criticality

- Recent paper by Chavanis -Print: **arXiv:1604.05904 predicts collapse to a BH**
- **Ongoing work.  Higher order will stop it**
- **Will form a bound state at 1 meter (Madelyn Leembruggen)**
- **Corresponds to large binding. Will decay before the collapse**
- **No BH formation**

# Backup Slides

# Self gravitating condensates with attractive interactions

- Quantum pressure keeps the condensate from collapse up to a certain limiting mass
- Use the variational method to analyze it
- Analized by CM physicists(Stoof, Journal of Statistical Physics 97,Freire,Arovas, P.R.A.59,1461
- Simple variational ansatz is to assume the density to be constant up to a certain radius and zero beyond it.

# Astrophysical sized BEC's using Gross-Pitaevskii method

- Employed by Bohmer and Harko and by Chavanis
- Employs the Hartree-Fock approximation and the pseudo potential Hamiltonian
- Valid when the inter-particle separation is greater than the particle particle scattering length.

# Gross-Pitaevskii formalism

$$H = \sum_i \left( \frac{p^2{}_i}{2m} + V(\vec{r}_i) \right) + g \sum_{i<j} \delta(\vec{r}_i - \vec{r}_j)$$

$$g = \frac{4\pi\hbar^2 a}{m}, a \text{ the scattering length}$$

$$\Psi(\vec{\mathrm{r}}_1, \vec{\mathrm{r}}_2, \quad, \vec{\mathrm{r}}_\mathrm{N}) = \prod_i \Phi(\vec{r}_i)$$

Expectation of H in $\Psi(\vec{\mathrm{r}}_1, \vec{\mathrm{r}}_2, \quad, \vec{\mathrm{r}}_\mathrm{N})$

$$\mathrm{E} = \int d^3\bar{r} \, (N \frac{\hbar^2}{2m} |\nabla\Phi|^2 + NV(\vec{r}) |\Phi(\vec{r})|^2 + \frac{N(N-1)}{2} g |\Phi(\vec{r})|^4)$$

With the rescaling $\Psi(\vec{\mathrm{r}}) = \sqrt{\mathrm{N}}\Phi(\vec{r})$

$$E = \int d^3\bar{r} (\frac{\hbar^2}{2m} |\nabla\Psi|^2 + V(\vec{r}) |\Psi(\vec{r})|^2 + \frac{g}{2} |\Psi(\vec{r})|^4)$$

# Gross–Pitaevskii equation

- Derivation of the GP equation
- Extremize E with N the particle number fixed.

$$\textit{Extremize}\ \mathrm{E} - \mu \mathrm{N}\ \text{with respect to}\ \Psi^*$$

$$\mu\Psi(\vec{\mathrm{r}}) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) + g\,|\,\Psi(\vec{r})\,|^2\ \Psi(\vec{r})$$

# Gross-Pitaevskii formalism

- Time dependent equation

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi(\vec{r}) + V(\vec{r})\Psi(\vec{r}) + g\,|\Psi(\vec{r})|^2\,\Psi(\vec{r})$$

# Variational Method
# Minimize the energy

$$\varepsilon = \frac{E}{N} = \frac{\hbar^2}{2mR^2} - \frac{3Gm^2N}{5R} + \frac{3\hbar^2 aN}{2mR^3}$$

*a is the scattering length.*

Solve the equation $\frac{\partial \mathrm{e}}{\partial \mathrm{R}} = 0$

Upperlimit for mass $= \mathrm{m\ N_c} \approx M_P\sqrt{\frac{1}{ma}} \approx M_P\sqrt{\frac{1}{\lambda}}$

Derived by Chavanis

# GP Poisson System

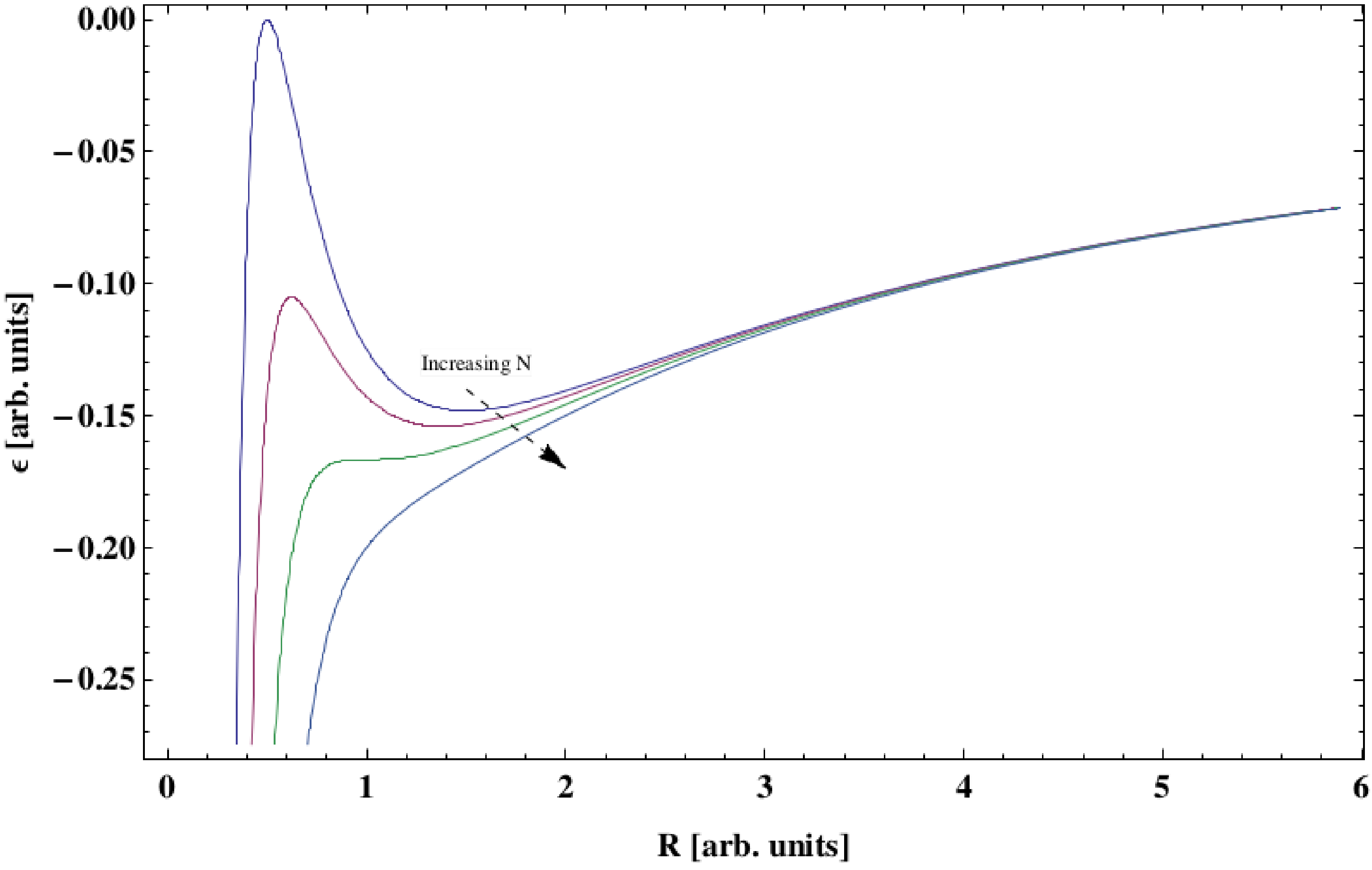
- Supplement the GP with the Poisson equation
- Follow the method of Bohmer, Harko Com.Ast.Part.Phy.06,025(2007). Recent work by Chavanis

$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r})+mV(\vec{r})\Psi(\vec{r})+g\,|\,\Psi(\vec{r})\,|^2\,\Psi(\vec{r})$$

$$\nabla^2 V=4\pi G\rho$$

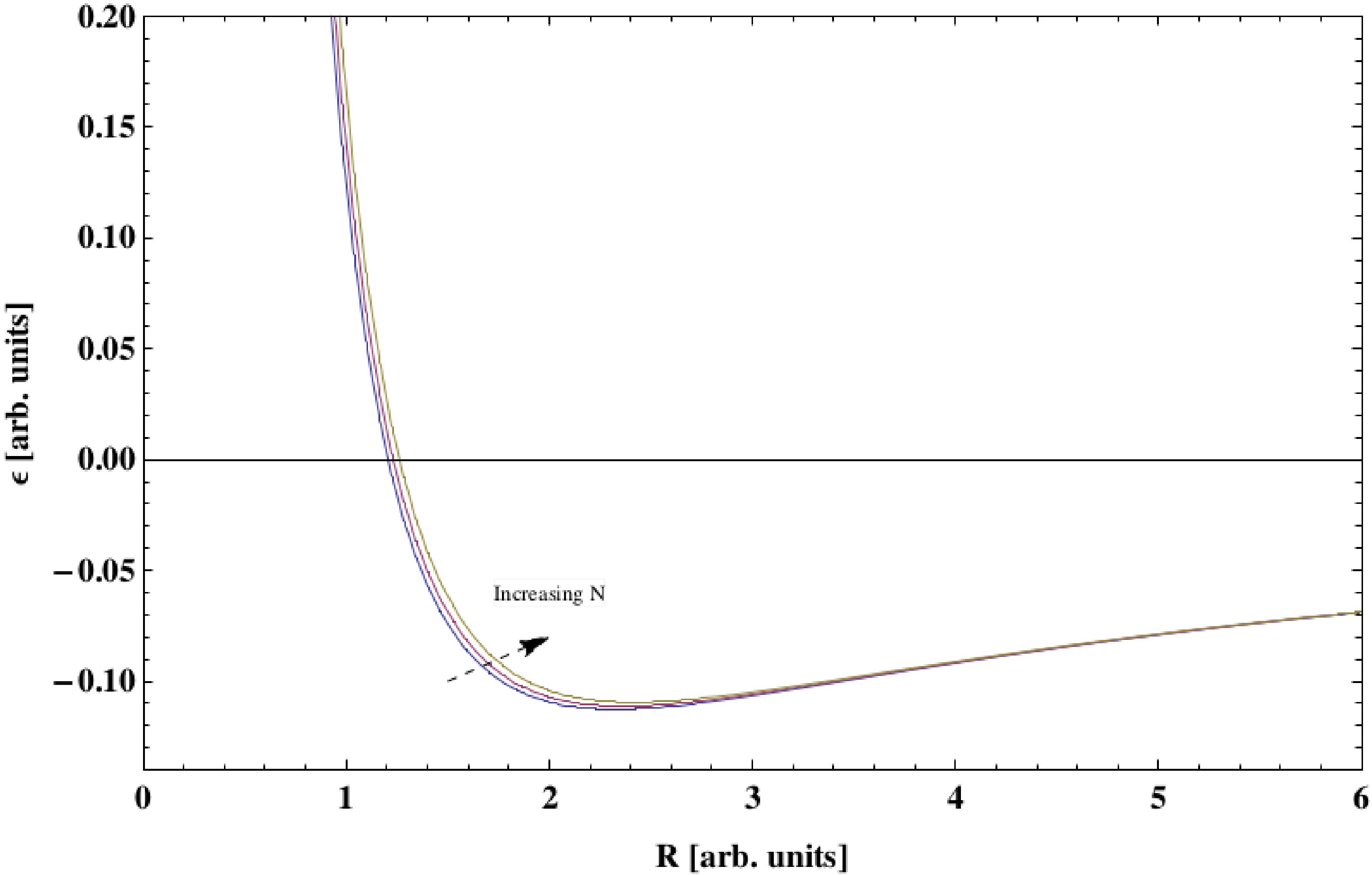
$$\rho(\vec{r},t)=m\Psi^*\Psi$$

# a<0



0.00
−0.05
−0.10
−0.15
−0.20
−0.25
ϵ [arb. units]
Increasing N
0
1
2
3
4
5
6
R [arb. units]

# a>0



0.20
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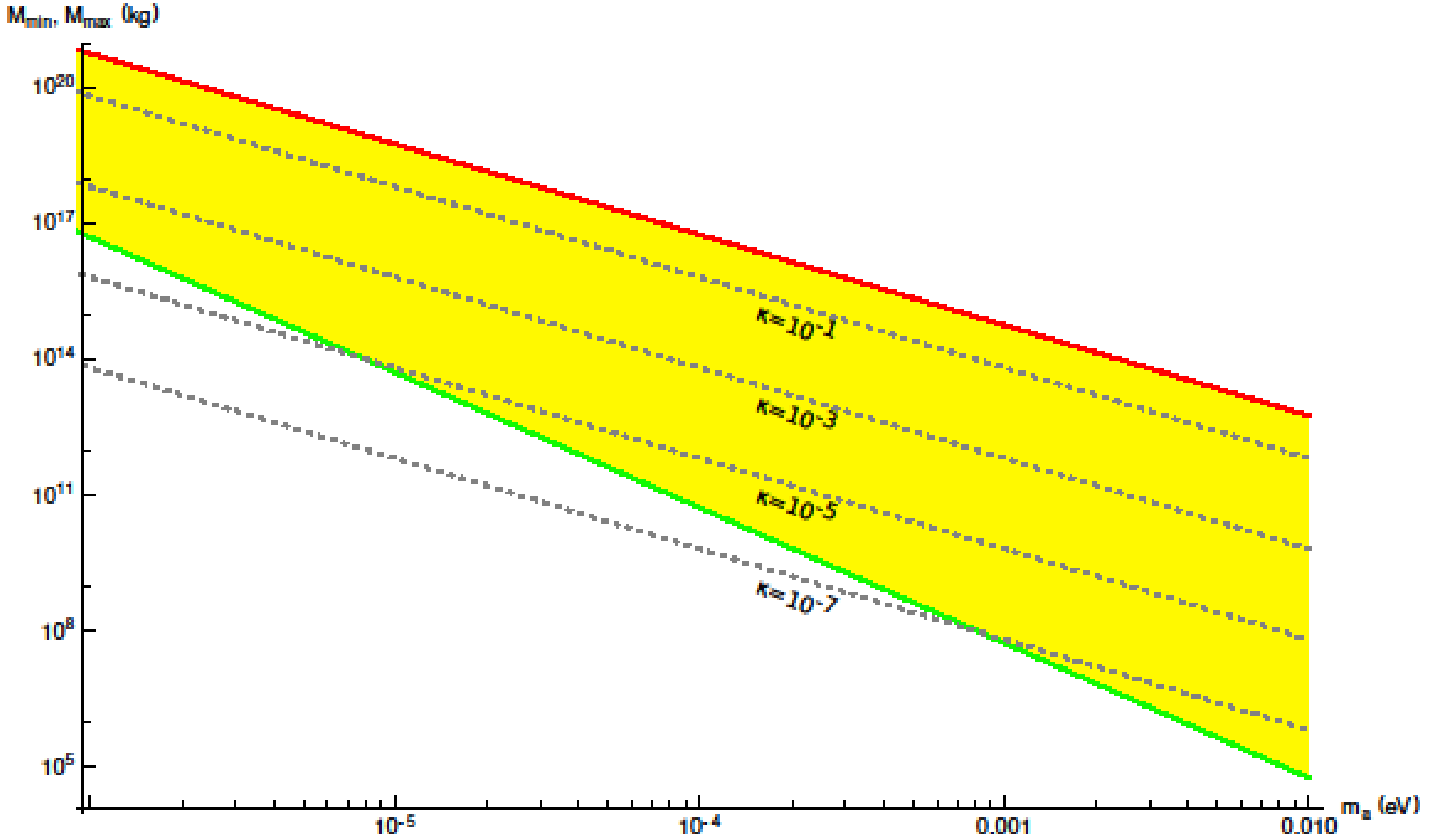
0
1
2
3
4
5
6
R [arb. units]

# Axions and GP Eqn

- Maximum Mass # $\frac{M_p f}{m}$
- Corresponding Radius #$\frac{M_p}{fm}$



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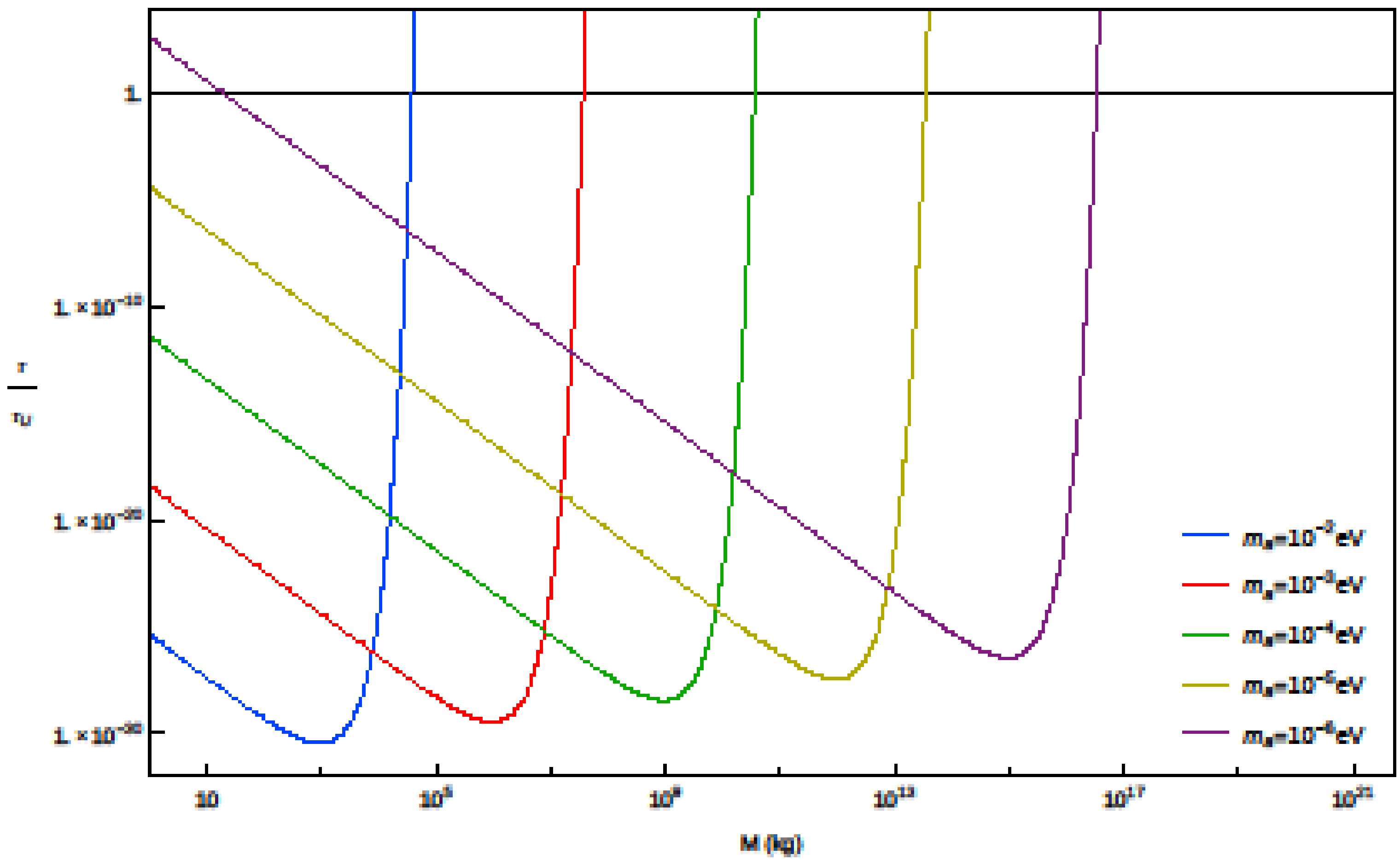
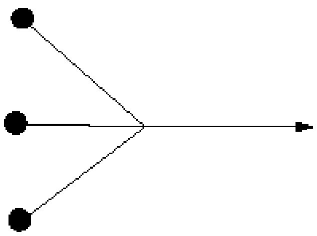


1.
1 × 10^-10
1 × 10^-20
1 × 10^-30
τ/τ_U
10
10^5
10^9
10^13
10^17
10^21
M (kg)
m_a=10^-2eV
m_a=10^-3eV
m_a=10^-4eV
m_a=10^-5eV
m_a=10^-6eV

# Decay Process

- Bound states combine and emit a plane wave state



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- Momentum is absorbed by the left over N-3 particle state

# Decay Calculation

- Calculate <N-3, p |V(A) |N>

$$A = R(r)\exp(-iEt)\,a + \int \frac{d^3p}{\sqrt{2E_p}} a_p \exp(ip.r - iEt) + h.c.$$

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