# DBI action of real linear superfield in 4D N = 1 conformal supergravity

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# Outline

- DBI action of real linear multiplet in global SUSY
- short review of conformal SUGRA
- DBI action of real linear multiplet in conformal SUGRA
- Summary

# Introduction

▪ DBI Lagrangian

$$1 - \sqrt{\det\left(\eta_{ab} + \partial_a\phi\partial_b\phi + B_{ab} + F_{ab}\right)}$$

$\phi$ : scalar  $B_{ab}$ : anti-symmetric tensor  $F_{ab}$ : Field strength of vector

It describes effective action of D-brane.

# Introduction

・DBI Lagrangian

$$1-\sqrt{\det\left(\eta_{ab}+\partial_a\phi\partial_b\phi+B_{ab}+F_{ab}\right)}$$

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It describes effective action of D-brane.

・What I want to do

=extend it to SUSY version based on superfield

- SUSY is manifest
- easy to generalize
- …

※We focus on four dimension.

# Introduction

Each part is constructed separately.

target

$$1-\sqrt{\det\left(\eta_{ab}+\partial_a\phi\partial_b\phi+B_{ab}+F_{ab}\right)}$$

M. Rocek, A. A. Tseytlin (1999)

…

N. Ambrosetti, et al. (2010)

…

S. Cecotti, S. Ferrara (1987)

J. Bagger, A. Galperin (1997)

S. M. Kuzenko and S. A. McCarthy (2003) …

This part can be constructed in 4D N=1 SUSY using linear superfield.

What is linear superfield?

How do we construct it ?

# DBI of real linear superfield in global SUSY (Review)

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + \mathrm{i}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m,$$
$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - \mathrm{i}\theta^\alpha \sigma^m_{\alpha\dot{\alpha}}\partial_m,$$

X:chiral $\bar{D}_{\dot{\alpha}} X = 0,$

$X$ : complex scalar $\psi$ : Weyl spinor $F$ : auxiliary field

L:linear $D^2 L = \bar{D}^2 L = 0$

$C$ : real scalar $\chi$ : Majorana spinor $b_{ab}$ : anti-symmetric tensor

$$B_a = \frac{1}{\sqrt{2}} \varepsilon_{abcd} \partial^b b^{cd}$$

※ There exists duality relation between chiral and linear (linear-chiral duality)

# DBI of real linear superfield in global SUSY (Review)

constraint

J. Bagger, A. Galperin (1997)

$$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$

(comes from N=2 partial breaking)

$$X = X(L)$$

$$\mathcal{L} = \int d^2\theta X(L) + \text{h.c.}$$
$$= 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2} + \ldots$$

We call DBI of linear superfield

linear-chiral dual

$$1 - \sqrt{\det\left(\eta_{ab} + \frac{1}{2}\partial_a \phi \partial_b \bar{\phi}\right)}$$

# Outline

✔ ▪ DBI action of real linear multiplet in global SUSY

Constraint is important !!

▪ short review of conformal SUGRA

▪ DBI action of real linear multiplet in conformal SUGRA

▪ Summary

# Short review of Conformal SUGRA

Conformal SUGRA is “good” tool for constructing SUGRA action.

| $P$ | $Q$ | $M$ | $D$ | $A$ | $S$ | $K$ |
|---|---|---|---|---|---|---|
| translation | SUSY | Lorentz | dilation | u(1) | S-SUSY | conformal boost |

Poincare SUGRA

conformal SUGRA

# Short review of Conformal SUGRA

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Poincare SUGRA ($P$, $Q$, $M$)

conformal SUGRA ($P$ through $K$)

Why are they needed?

- Weyl rescaling is not required due to larger symmetries.
- different formulations of SUGRA can be treated in a unified manner.

(Old minimal or New minimal)

Compensator
(unphysical multiplet)

- chiral compensator S_0 → Old minimal
- linear compensator L_0 → New minimal

# Short review of Conformal SUGRA

Conformal SUGRA is "good" tool for constructing SUGRA action.

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Poincare SUGRA ($P$, $Q$, $M$)

conformal SUGRA ($P$ through $K$)

The multiplet of conformal SUGRA $\Phi$ is characterized by weights of dilation and u(1) called Weyl $w$ and chiral weight $n$ .

$$D\Phi = w\Phi \qquad A\Phi = n\Phi$$

e.g. chiral multiplet w=n

Linear multiplet w=2, n=0

# Outline

- ✔ ▪DBI action of real linear multiplet in global SUSY

  Constraint is important !!

- ✔ ▪short review of conformal SUGRA
  - ▪compensator determines formulation of SUGRA.
  - ▪multiplet has specific weights.
- ▪ DBI action of real linear multiplet in conformal SUGRA
- ▪ Summary

How do we construct DBI action of liner multiplet in SUGRA?

In global $$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$

In conformal SUGRA ?

constraint $$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$

difficulty

global SUSY → conformal SUGRA

$\bar{D}^2 \rightarrow \Sigma$

$\bar{D}_{\dot{\alpha}} \rightarrow \bar{\mathcal{D}}_{\dot{\alpha}}$

※ we cannot apply these operators to arbitrary multiplet.

e.g., $\bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L \rightarrow \bar{\mathcal{D}} L \bar{\mathcal{D}} L$ ✗

Spinor derivative in conformal SUGRA cannot be applied unless weights of operand must satisfy w=n.

⇕

L has weights w=2, n=0.

U- associated derivative $\mathcal{D}^{(\mathbf{u})}L$

T. Kugo and S. Uehara,(1985)

U-multiplet $\mathbf{u} = \{\mathcal{C}_u, \mathcal{Z}_u, \mathcal{H}_u, \mathcal{K}_u, \mathcal{B}_{au}, \Lambda_u, \mathcal{D}_u\},$

It can be applied to any multiplet.

$\bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L$ ⇒ $\bar{\mathcal{D}}^{(\mathbf{u})} L \bar{\mathcal{D}}^{(\mathbf{u})} L$ ◎

How do we construct DBI action of liner multiplet in SUGRA?

In global

$$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$

In conformal SUGRA

$$X + \frac{1}{2} X \Sigma \left( \frac{1}{\mathbf{v}} \bar{X} \right) + \frac{1}{4\mathbf{s}} \bar{\mathcal{D}}^{(\mathbf{u})} L \bar{\mathcal{D}}^{(\mathbf{u})} L = 0$$

?

$\mathbf{u}, \mathbf{v}, \mathbf{s}$ : general multiplet

How do we construct DBI action of liner multiplet in SUGRA?

In global
$$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$

In conformal SUGRA
$$X + \frac{1}{2} X \Sigma \left( \frac{1}{\mathbf{v}} \bar{X} \right) + \frac{1}{4\mathbf{s}} \bar{\mathcal{D}}^{(\mathbf{u})} L \bar{\mathcal{D}}^{(\mathbf{u})} L = 0$$ ?

$\mathbf{u}, \mathbf{v}, \mathbf{s}$ : general multiplet

chiral

chiral

This term is not necessarily chiral
(we must impose chirality condition )

how three multiplets u,v,s are restricted by the chirality condition

※We choose compensator as u,v,s-multiplet

= gravitational correction

Remember

Chiral compensator S_0 (Old minimal)

Linear compensator L_0 (New minimal)

Old minimal formulation ☹ $\mathbf{u}, \mathbf{v}, \mathbf{s} = \text{the function of } S_0, \bar{S}_0$

⇒ chirality condition cannot be satisfied

no matter how we choose the form of u,v,s.

New minimal formulation ☺

If $\mathbf{u} = (L_0)^n, \mathbf{v} = L_0^2, \mathbf{s} = L_0$ ⇒ chirality condition is satisfied.

e.g. $\frac{1}{L_0}\bar{\mathcal{D}}^{(L_0)} L \bar{\mathcal{D}}^{(L_0)} L.$

# Main result

In global $$X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_{\dot{\alpha}} L \bar{D}^{\dot{\alpha}} L = 0,$$

In conformal SUGRA

$$X + \frac{1}{2} X \Sigma \left( \frac{1}{L_0^2} \bar{X} \right) + \frac{1}{4L_0} \bar{\mathcal{D}}^{(L_0)} L \bar{\mathcal{D}}^{(L_0)} L = 0,$$

can be realized in new minimal

$$S_B = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R + 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2} \right].$$

# Summary

- construct DBI action of real linear multiplet in SUGRA
- derive the condition under which DBI action can be realized in SUGRA (chirality condition)
- show that it can be constructed in new minimal SUGRA

future prospects

- relation to N=2 supergravity?
- application to cosmology

Thank you!!