DBI action of real linear superfield in 4D N = 1 conformal supergravity

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Outline

- DBI action of real linear multiplet in global SUSY
- short review of conformal SUGRA
- DBI action of real linear multiplet in conformal SUGRA
- Summary
Introduction

- DBI Lagrangian

\[ 1 - \sqrt{\det (\eta_{ab} + \partial_a \phi \partial_b \phi + B_{ab} + F_{ab})} \]

\( \phi \) : scalar \hspace{1em} B_{ab} : anti-symmetric tensor \hspace{1em} F_{ab} : Field strength of vector

It describes effective action of D-brane.
Introduction

- DBI Lagrangian

\[ 1 - \sqrt{\det (\eta_{ab} + \partial_a \phi \partial_b \phi + B_{ab} + F_{ab})} \]

\( \phi \) : scalar \quad B_{ab} \quad : \text{anti-symmetric tensor} \quad F_{ab} \quad : \text{Field strength of vector}

It describes effective action of D-brane.

- What I want to do
  = extend it to SUSY version based on superfield

  - SUSY is manifest
  - easy to generalize
  - ...

※ We focus on four dimension.
Each part is constructed separately.

This part can be constructed in 4D N=1 SUSY using \textit{linear superfield}.

What is linear superfield? How do we construct it?
DBI of real linear superfield in global SUSY (Review)

$X$: chiral

$\bar{D}_\dot{\alpha} X = 0,$

$X$: complex scalar  $\psi$: Weyl spinor  $F$: auxiliary field

$L$: linear

$D^2 L = \bar{D}^2 L = 0$

$C$: real scalar  $\chi$: Majorana spinor  $b_{ab}$: anti-symmetric tensor

$B_a = \frac{1}{\sqrt{2}} \varepsilon_{abcd} \partial^b b^{cd}$

※ There exists duality relation between chiral and linear (linear-chiral duality)
DBI of real linear superfield in global SUSY (Review)

Constraint

\[
X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_\alpha L \bar{D}^{\bar{\alpha}} L = 0,
\]

We call DBI of linear superfield

linear-chiral dual

\[
1 - \sqrt{\det \left( \eta_{ab} + \frac{1}{2} \partial_a \phi \partial_b \phi \right)}
\]

J. Bagger, A. Galperin (1997)

(comes from N=2 partial breaking)
Outline

- DBI action of real linear multiplet in global SUSY
  - Constraint is important !!
  - short review of conformal SUGRA

- DBI action of real linear multiplet in conformal SUGRA

- Summary
Short review of Conformal SUGRA

Conformal SUGRA is “good” tool for constructing SUGRA action.

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Poincare SUGRA \hspace{1cm} \text{conformal SUGRA}
Short review of Conformal SUGRA

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Why are they needed?

- Weyl rescaling is not required due to larger symmetries.
- Different formulations of SUGRA can be treated in a unified manner. (Old minimal or New minimal)

Compensator

- Chiral compensator $S_0$
- Linear compensator $L_0$

(unphysical multiplet)
Short review of Conformal SUGRA

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Poincare SUGRA  

Conformal SUGRA

The multiplet of conformal SUGRA $\Phi$ is characterized by weights of dilation and $u(1)$ called Weyl $w$ and chiral weight $n$.

$$D\Phi = w\Phi$$

$$A\Phi = n\Phi$$

e.g. chiral multiplet  $w=n$
Linear multiplet  $w=2$, $n=0$
Outline

- DBI action of real linear multiplet in global SUSY
  - Constraint is important !!
- short review of conformal SUGRA
  - compensator determines formulation of SUGRA.
  - multiplet has specific weights.
- DBI action of real linear multiplet in conformal SUGRA

- Summary
How do we construct DBI action of linear multiplet in SUGRA?

In global

\[ X - \frac{1}{4}X \bar{D}^2 \bar{X} - \bar{D}_\alpha L \bar{D}^\alpha L = 0, \]

In conformal SUGRA

?
difficulty

we cannot apply these operators to arbitrary multiplet.

e.g.,

Spinor derivative in conformal SUGRA cannot be applied unless weights of operand must satisfy $w=n$.

$L$ has weights $w=2$, $n=0$. 

$X - \frac{1}{4} X D^2 X - D_\alpha L \bar{D}^{\dot{\alpha}} L = 0,$
U- associated derivative

\[ D^{(u)} L \]

T. Kugo and S. Uehara, (1985)

U-multiplet

\[ u = \{ C_u, Z_u, H_u, K_u, B_{au}, \Lambda_u, D_u \}, \]

It can be applied to any multiplet.

\[ \bar{D}_\alpha L \bar{D}^{\alpha} L \] \rightarrow \[ D^{(u)} L \bar{D}^{(u)} L \]
How do we construct DBI action of linear multiplet in SUGRA?

In global

\[ X - \frac{1}{4} X \bar{D}^{2} \bar{X} - \bar{D}_{\alpha} L \bar{D}^{\alpha} L = 0, \]

In conformal SUGRA

\[ X + \frac{1}{2} X \Sigma \left( \frac{1}{v} \bar{X} \right) + \frac{1}{4s} \bar{D}^{(u)} L \bar{D}^{(u)} L = 0 \]

\( u, v, s : \) general multiplet
How do we construct DBI action of linear multiplet in SUGRA?

In global

\[ X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_\alpha L \bar{D}^\alpha L = 0, \]

In conformal SUGRA

\[ X + \frac{1}{2} X \Sigma \left( \frac{1}{\nu} \bar{X} \right) + \frac{1}{4s} \bar{D}^{(u)} L \bar{D}^{(u)} L = 0 \]

\( u, \nu, s \): general multiplet

This term is not necessarily chiral (we must impose chirality condition)
how three multiplets $u,v,s$ are restricted by the chirality condition

※We choose **compensator** as $u,v,s$-multiplet

= gravitational correction

Remember

Chiral compensator $S_0$ (Old minimal)
Linear compensator $L_0$ (New minimal)

Old minimal formulation 😞

- chirality condition cannot be satisfied no matter how we choose the form of $u,v,s$.

New minimal formulation 😊

If $u = (L_0)^n, v = L_0^2, s = L_0$

- chirality condition is satisfied.

e.g.

$$\frac{1}{L_0} D(L_0) L \bar{D}(L_0) L.$$
Main result

In global

\[ X - \frac{1}{4} X \bar{D}^2 \bar{X} - \bar{D}_\alpha L \bar{D}^\alpha L = 0, \]

In conformal SUGRA

\[ X + \frac{1}{2} X \Sigma \left( \frac{1}{L_0^2} \bar{X} \right) + \frac{1}{4L_0} \bar{D}^{(L_0)} L \bar{D}^{(L_0)} L = 0, \]

can be realized in new minimal

\[ S_B = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R + 1 - \sqrt{1 - B \cdot B + \partial C \cdot \partial C - (B \cdot \partial C)^2} \right]. \]
Summary

- construct DBI action of real linear multiplet in SUGRA

- derive the condition under which DBI action can be realized in SUGRA (chirality condition)

- show that it can be constructed in new minimal SUGRA

future prospects

- relation to N=2 supergravity?
- application to cosmology

Thank you!!