Component versus Superspace Approaches to D=4, N=1 Conformal Supergravity

Ryo Yokokura
Keio univ.

7 July 2016
SUSY2016 @ The university of Melbourne

with Taichiro Kugo and Koichi Yoshioka

to be published in PTEP [arXiv:1602.04441]

Conformal SUGRA puzzle

restricted ?

component

practical ◎
symmetry △

superspace

practical △
symmetry ◎

What we did

not restricted!

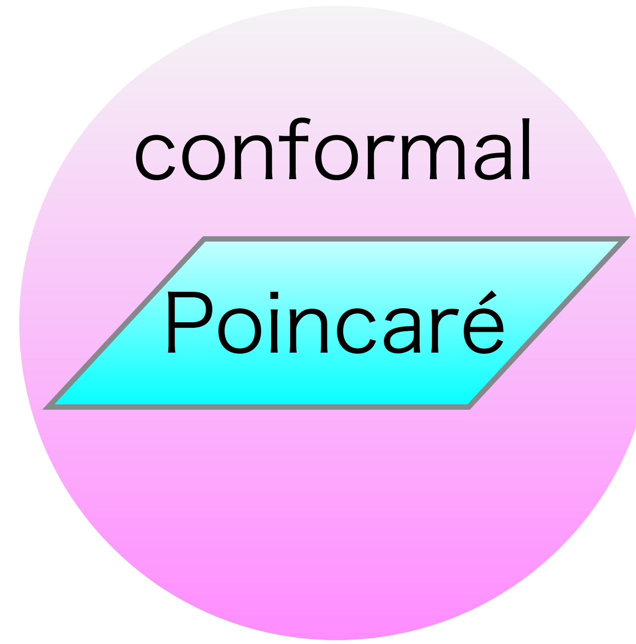
component	superspace
practical ◎	practical △
symmetry △	symmetry ◎

4D N=1 SUGRA

- A candidate beyond SM and Einstein gravity
- Low-energy string effective actions
- Construction:
  Gauge theory of super-Poincaré symmetry

Conformal SUGRA

Gauge theory of superconformal (SC) sym.



- Super-Poincaré extended to SC
- Having more gauge freedoms
- Poincaré SUGRA obtained by gauge-fixing

What is the most different point?

M. Kaku and P. K. Townsend(1978)

S-transformation in SC

$\overset{Q}{\underset{S}{\rightleftharpoons}}$ rule: ladder operators

$$0 \underset{S}{\leftharpoondown} B \overset{Q}{\underset{S}{\rightleftharpoons}} F \overset{Q}{\underset{S}{\rightleftharpoons}} \cdots$$

Q (SUSY generator) : raising scale weight

S (SUSY of inversion) : lowering scale weight

$$\{Q, S^T\} = -\frac{1}{2}C^{-1}D + \frac{1}{2}\sigma^{ab}C^{-1}M_{ab} + i\gamma_5 C^{-1}A$$

scale Lorentz chiral

Component approach

T. Kugo and S. Uehara (1985)

- Conformal multiplet defined by Q transf. laws

$$\mathcal{V}_A = [\mathcal{C}_A, \mathcal{Z}_{\alpha A}, \ldots] = [\![(\mathcal{C}_A)]\!]$$

$$0 \underset{S}{\leftharpoondown} \mathcal{C}_A \underset{S}{\overset{Q}{\rightleftharpoons}} \mathcal{Z}_{\alpha A} \underset{S}{\overset{Q}{\rightleftharpoons}} \cdots$$

A: general spinor indices

$$\delta_Q(\varepsilon)\mathcal{C}_A = \frac{1}{2}\bar{\varepsilon} i\gamma_5 \mathcal{Z}_A$$

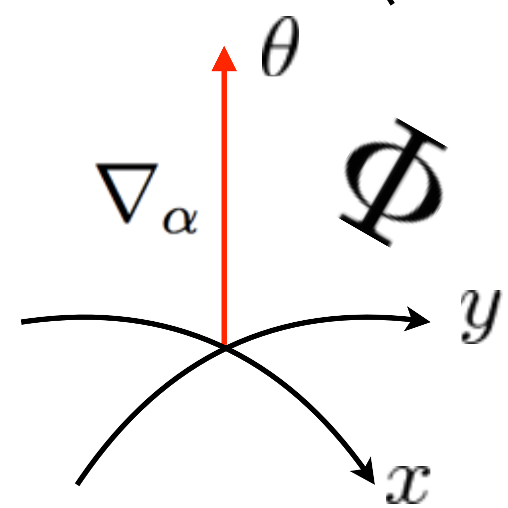
- Transf. laws determined by SC transf. laws of $\mathcal{C}_A$

$$\delta_S \mathcal{Z} \sim \delta_S \delta_Q \mathcal{C} = (\delta_M + \delta_D + \delta_A)\mathcal{C}$$

Lorentz scale chiral

Superspace approach

D. Butter (2010)



- Q = SC covariant spinor deriv. ∇_α

$$Q_\alpha \Phi_A = \nabla_\alpha \Phi_A$$

- Primary superfield vanishing by S transf.

$$S_\alpha \Phi_A = 0$$

- Component obtained by setting θ =0

$$\mathcal{C}_A = \Phi_A|_{\theta=0} \qquad \mathcal{Z}_{\alpha A} = (\nabla_\alpha \Phi_A)|_{\theta=0}$$

Additional restriction?

T. Kugo and S. Uehara (1985)

$$0 \xleftarrow[S]{?} \mathcal{Z}_{\alpha A} \underset{S}{\overset{Q}{\rightleftharpoons}} \cdots$$

- Introducing spinor derivative $\mathcal{D}_\alpha [\![\mathcal{C}_A]\!] := [\![\mathcal{Z}_{\alpha A}]\!]$

- Additional restriction occurs？

$$\delta_S \mathcal{Z}_{\alpha A} \sim (\delta_M + \delta_D + \delta_A)\mathcal{C}_A = 0$$

Lorentz scale chiral

- Previous superspace approach restricted?

Conformal SUGRA puzzle

restricted ?

component		superspace
practical ◎	⇔	practical △
symmetry △		symmetry ◎

Results

Restriction does not exist.

$$0 \underset{S}{\leftharpoondown} \boxed{\Phi_A \underset{S}{\overset{Q}{\rightleftharpoons}} \nabla_\alpha \Phi_A} \underset{S}{\overset{Q}{\rightleftharpoons}} \cdots$$

- $\underset{S}{\overset{Q}{\rightleftharpoons}}$ rule: S transforming $\nabla_\alpha \Phi_A$ to Φ_A not 0
- Consistent with component $0 \underset{S}{\leftharpoondown} \mathcal{C}_A \underset{S}{\overset{Q}{\rightleftharpoons}} \mathcal{Z}_{\alpha A} \underset{S}{\overset{Q}{\rightleftharpoons}} \cdots$
- $\boxed{0 \underset{S}{\leftharpoondown} \mathcal{Z}_{\alpha A}}$: not required by $\underset{S}{\overset{Q}{\rightleftharpoons}}$ rule

Correspondence

not restricted!

Conformal multiplet

component	superspace
practical ◎	practical △
symmetry △	symmetry ◎

Correspondence of conformal multiplet

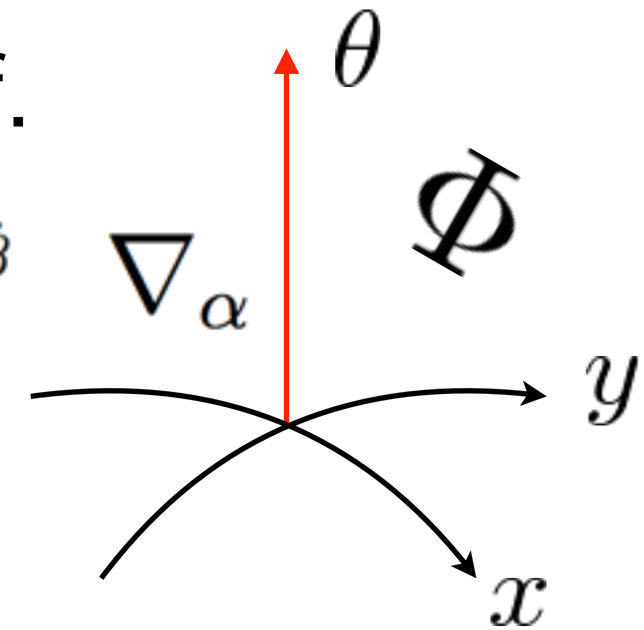
component	superspace
$\mathcal{C}_A$	$\Phi_A\|$
$\mathcal{Z}_A$	$\begin{pmatrix} -i\nabla_\alpha \Phi_A \\ +i\bar{\nabla}^{\dot{\alpha}}\Phi_A \end{pmatrix}\|$
$\mathcal{H}_A$	$\frac{1}{4}(\nabla^2\Phi_A + \bar{\nabla}^2\Phi_A)\|$
$\mathcal{K}_A$	$-\frac{1}{4}i(\nabla^2\Phi_A - \bar{\nabla}^2\Phi_A)\|$
$\mathcal{B}_{cA}$	$\frac{1}{2}\left(-\frac{1}{2}(\bar{\sigma}_c)^{\dot{\gamma}\gamma}[\nabla_\gamma, \bar{\nabla}_{\dot{\gamma}}]\Phi_A\right)\|$
Λ_A	$-\frac{i}{4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \bar{\nabla}^2\nabla_\alpha\Phi_A \\ \nabla^2\bar{\nabla}^{\dot{\alpha}}\Phi_A \end{pmatrix}\| + 2i\begin{pmatrix} \mathcal{W}_\alpha \\ \bar{\mathcal{W}}^{\dot{\alpha}} \end{pmatrix}\Phi_A\|$
$\mathcal{D}_A$	$\frac{1}{4}\left(\frac{1}{2}\bar{\nabla}_{\dot{\beta}}\nabla^2\bar{\nabla}^{\dot{\beta}}\Phi_A + (-1)(-2i)^2\bar{\mathcal{W}}_{\dot{\beta}}\bar{\nabla}^{\dot{\beta}}\Phi_A\right)\|$ $= \frac{1}{4}\left(\frac{1}{2}\nabla^\beta\bar{\nabla}^2\nabla_\beta\Phi_A + (-2i)^2\mathcal{W}^\beta\nabla_\beta\Phi_A\right)\|$

T. Kugo, RY and K. Yoshioka (2016)

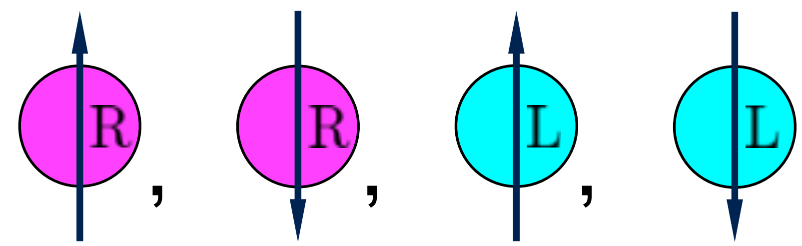
How to obtain

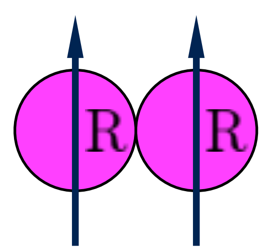
∇_α combinations reproducing Q transf.

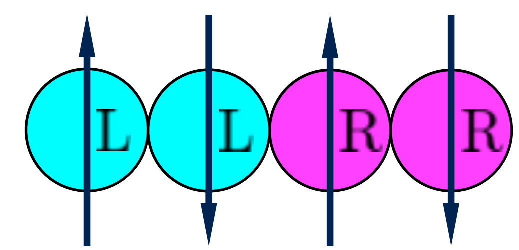
$$\{\nabla_\alpha, \nabla_\beta\} = 0, \quad \{\nabla_\alpha, \bar{\nabla}_{\dot{\beta}}\} = -2i\nabla_{\alpha\dot{\beta}}$$

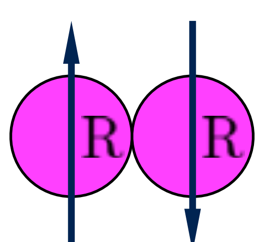


1. Four freedoms

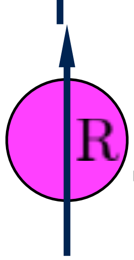
$\nabla_\alpha =$ 

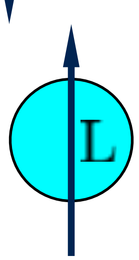
2. Nilpotent: 

$= 0$ ▶ Maximum: 

3. 

: only singlet (triplet vanishing)

4. { 

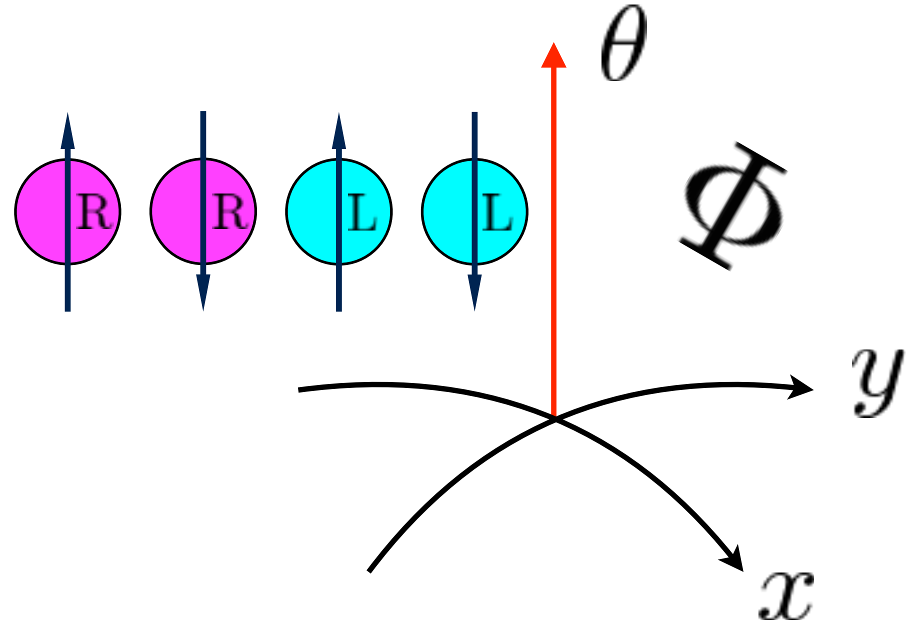
, 

}: spacetime derivative

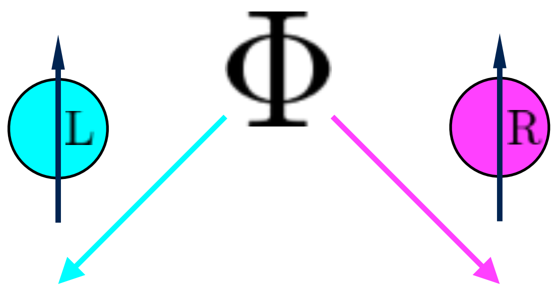
D. Butter (2010)

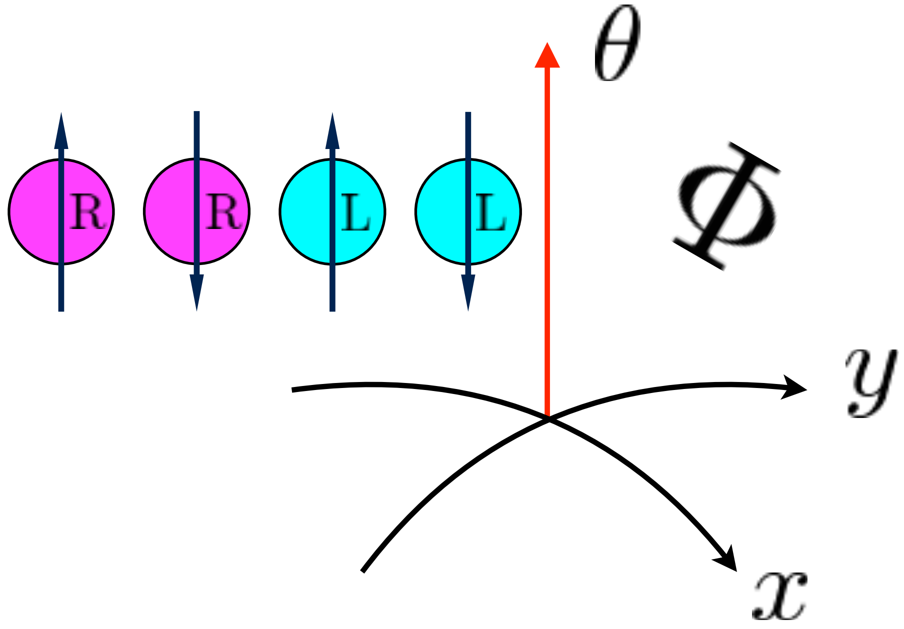
The 1st, 2nd order

Φ

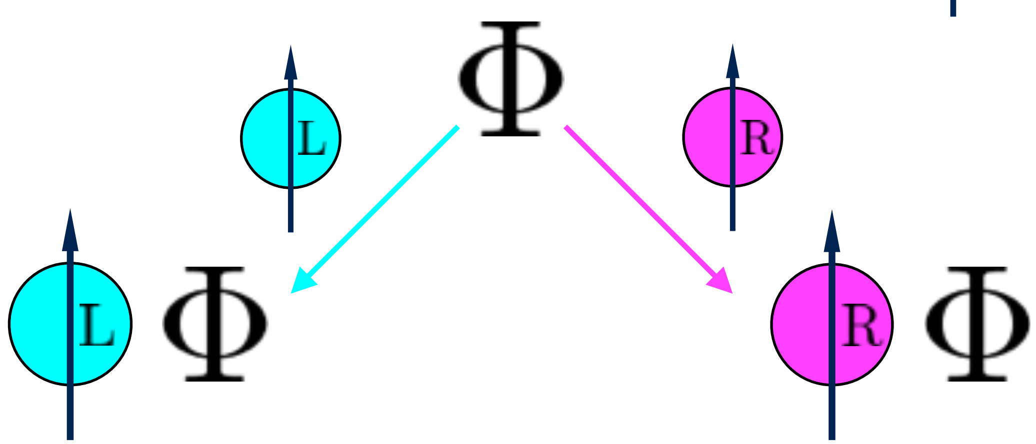


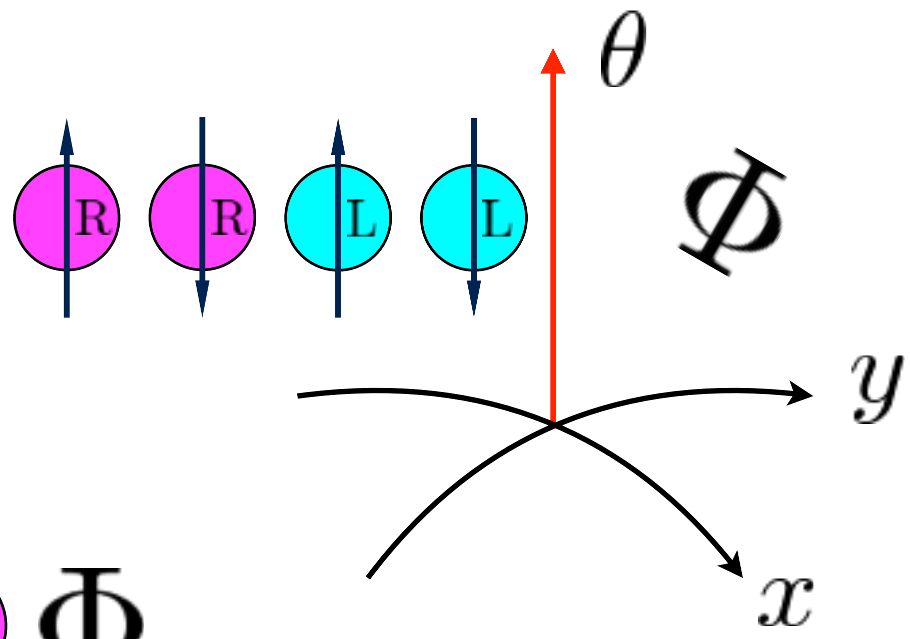
The 1st, 2nd order



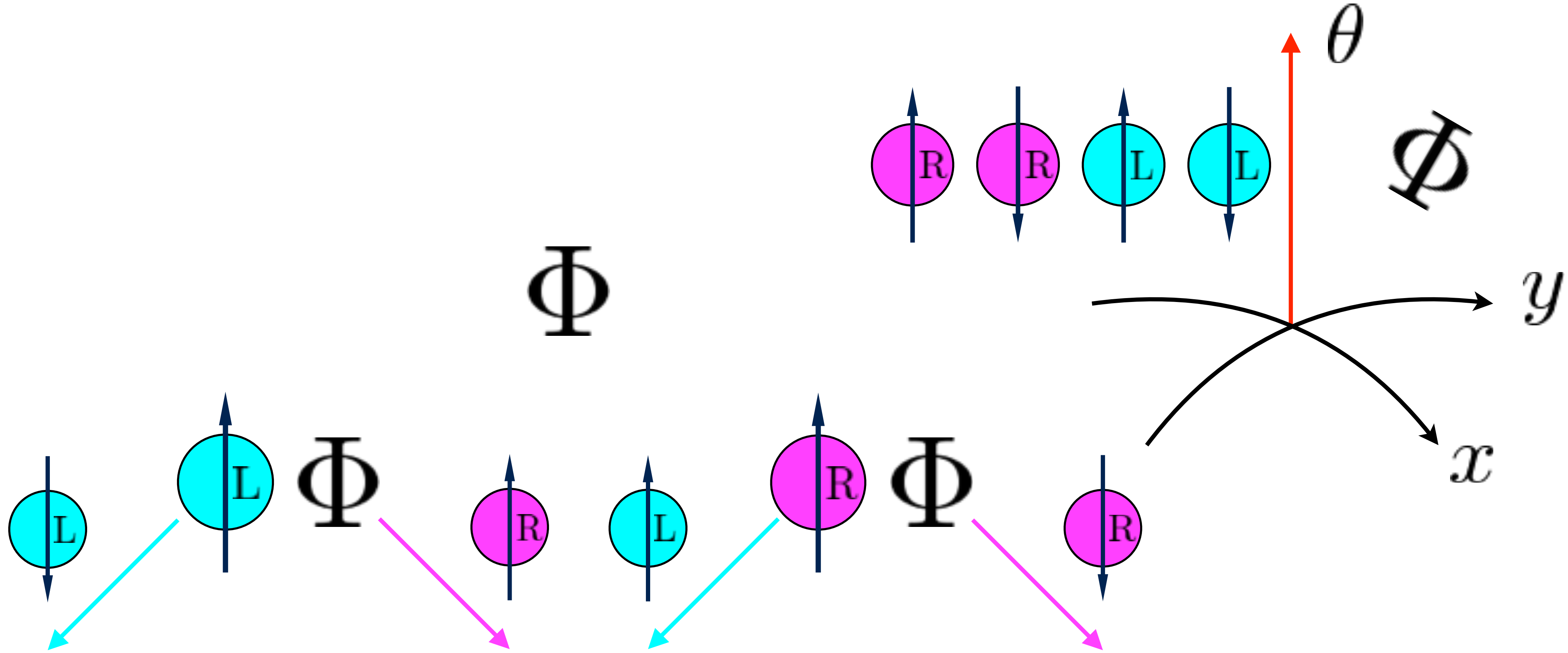


The 1st, 2nd order

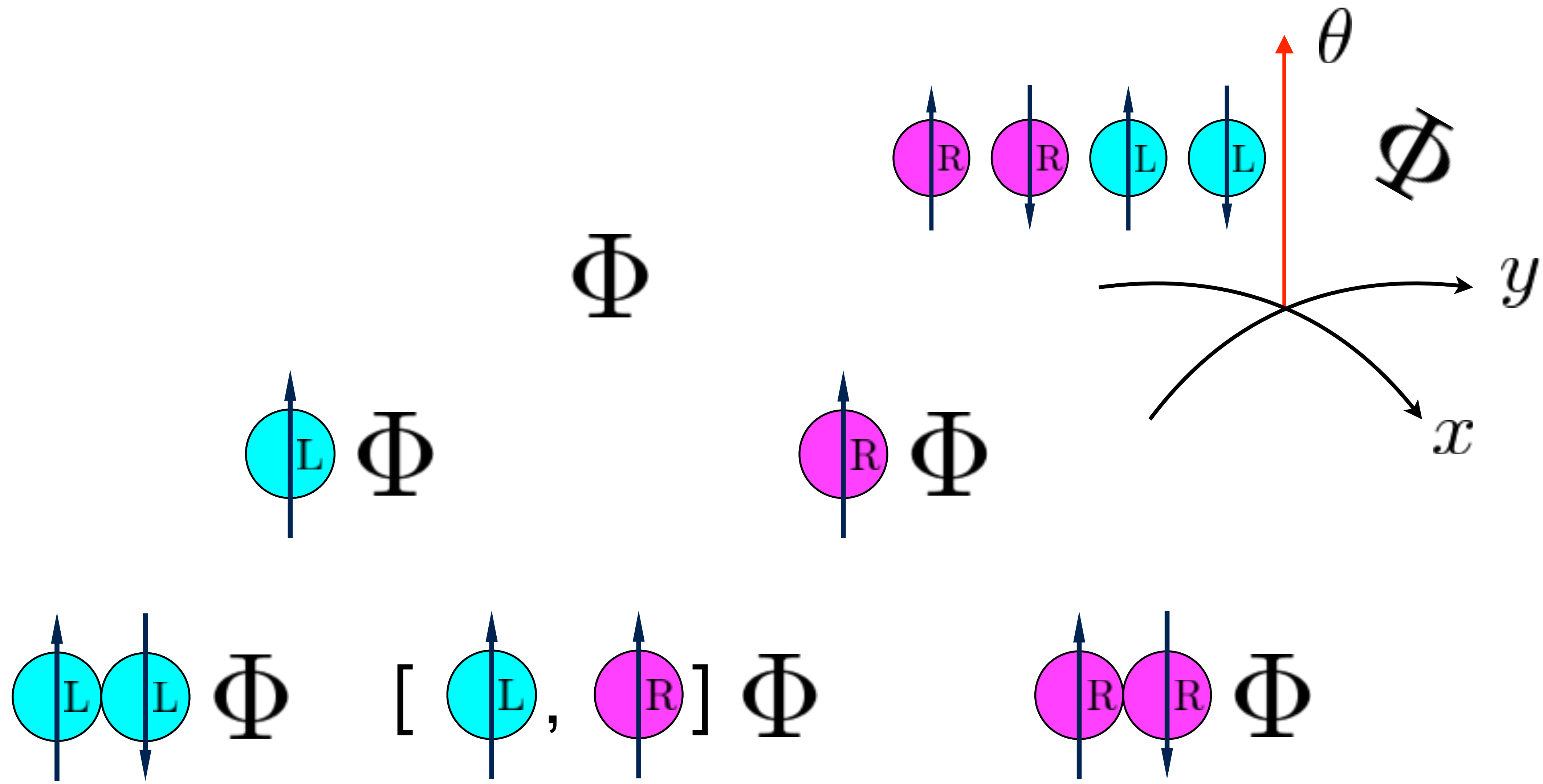




The 1st, 2nd order



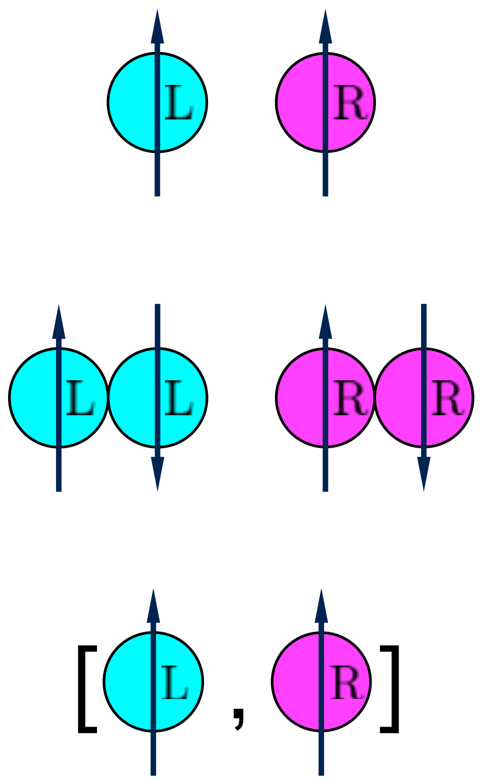
The 1st, 2nd order



anti-commutator:
spacetime derivative

Correspondences of the 1st, 2nd order

T. Kugo, RY and K. Yoshioka (2016)



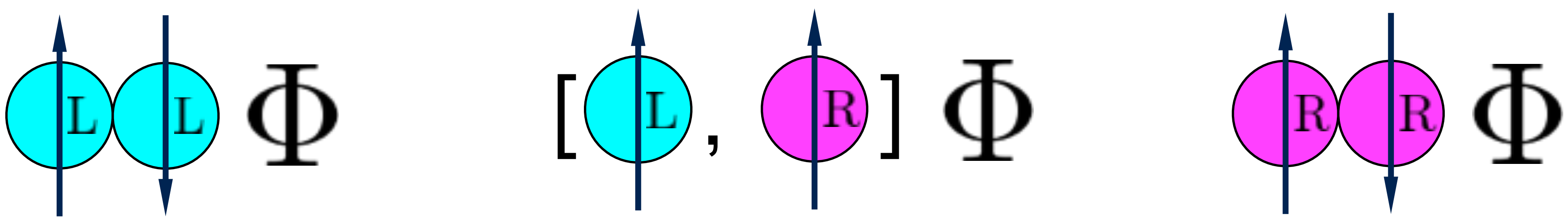
component	superspace
$\mathcal{C}_A$	$\Phi_A\|$
$\mathcal{Z}_A$	$\begin{pmatrix} -i\nabla_\alpha \Phi_A \\ +i\bar{\nabla}^{\dot{\alpha}}\Phi_A \end{pmatrix}\|$
$\mathcal{H}_A$	$\frac{1}{4}(\nabla^2\Phi_A + \bar{\nabla}^2\Phi_A)\|$
$\mathcal{K}_A$	$-\frac{1}{4}i(\nabla^2\Phi_A - \bar{\nabla}^2\Phi_A)\|$
$\mathcal{B}_{cA}$	$\frac{1}{2}\left(-\frac{1}{2}(\bar{\sigma}_c)^{\dot{\gamma}\gamma}[\nabla_\gamma, \bar{\nabla}_{\dot{\gamma}}]\Phi_A\right)\|$

These combinations reproducing the transf. laws (Kugo and Uehara '85)

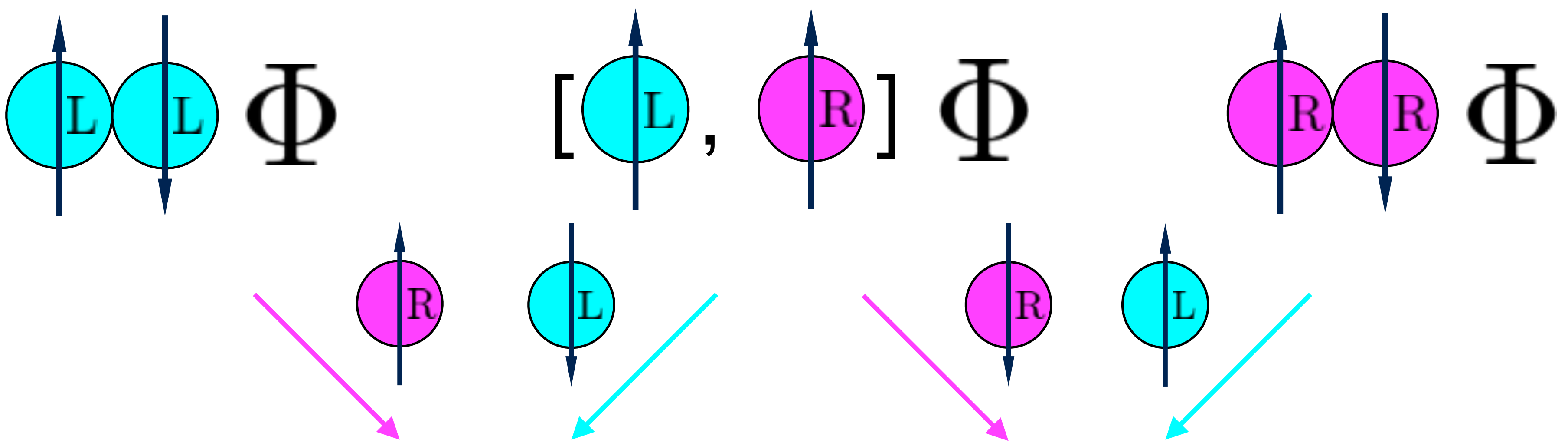
$$\delta_Q(\varepsilon)\mathcal{C}_A = \frac{1}{2}\bar{\varepsilon}i\gamma_5\mathcal{Z}_A,$$

$$\delta_Q(\varepsilon)\mathcal{Z}_A = (-)^A\frac{1}{2}(i\gamma_5\mathcal{H}_A - \mathcal{K}_A - \not{\mathcal{B}}_A + \not{D}\mathcal{C}_A i\gamma_5)\varepsilon$$

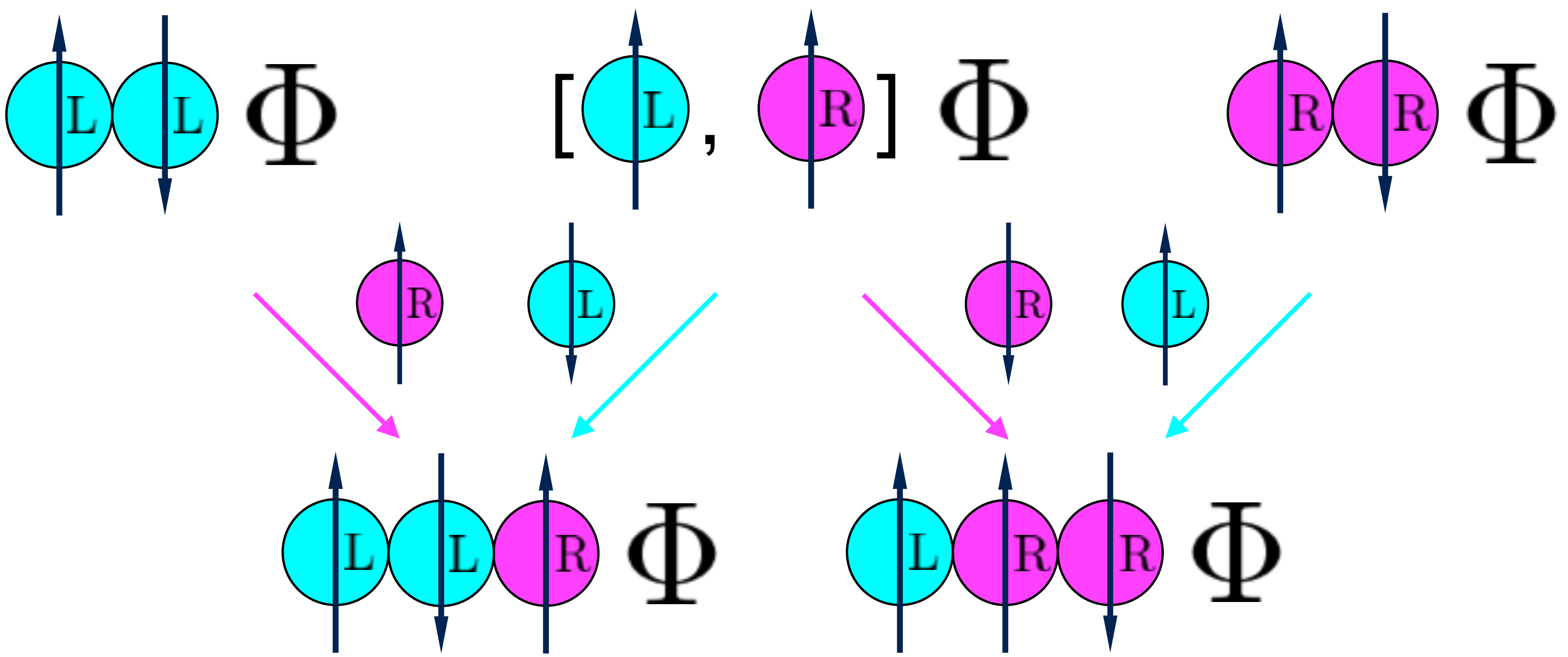
The 3rd, 4th order



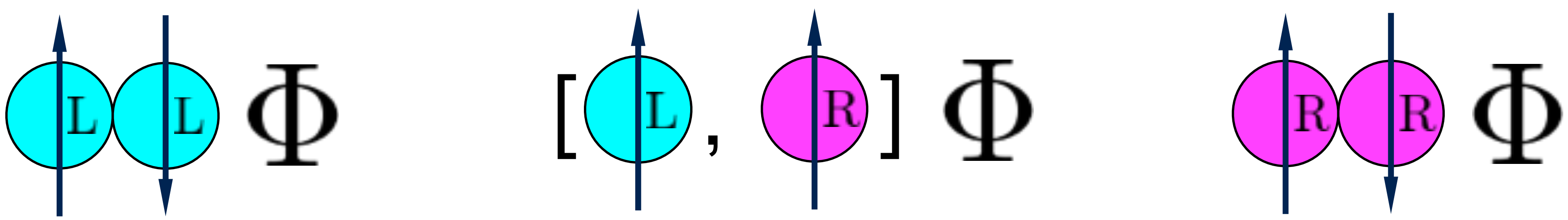
The 3rd, 4th order

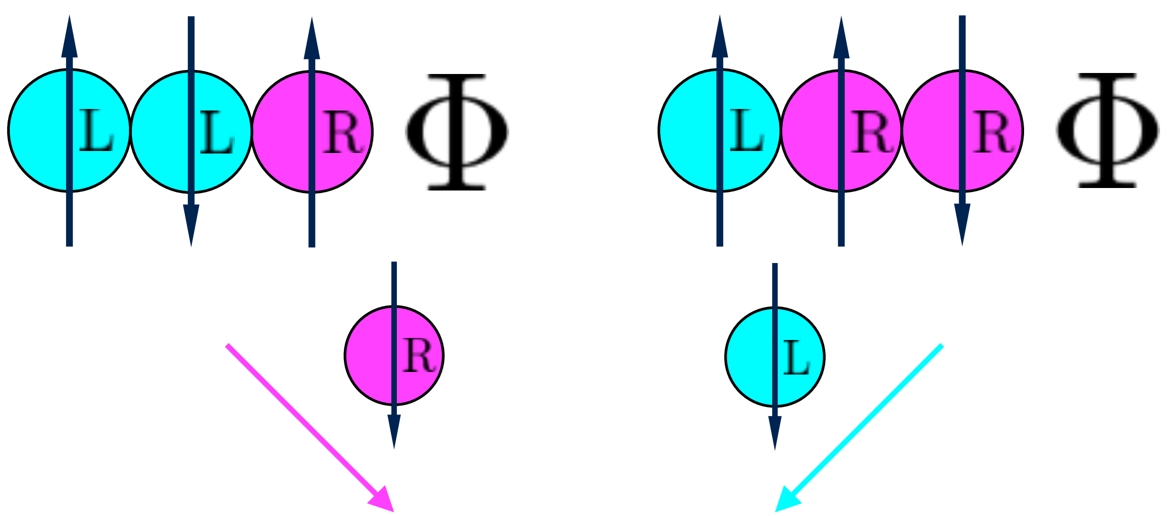


The 3rd, 4th order

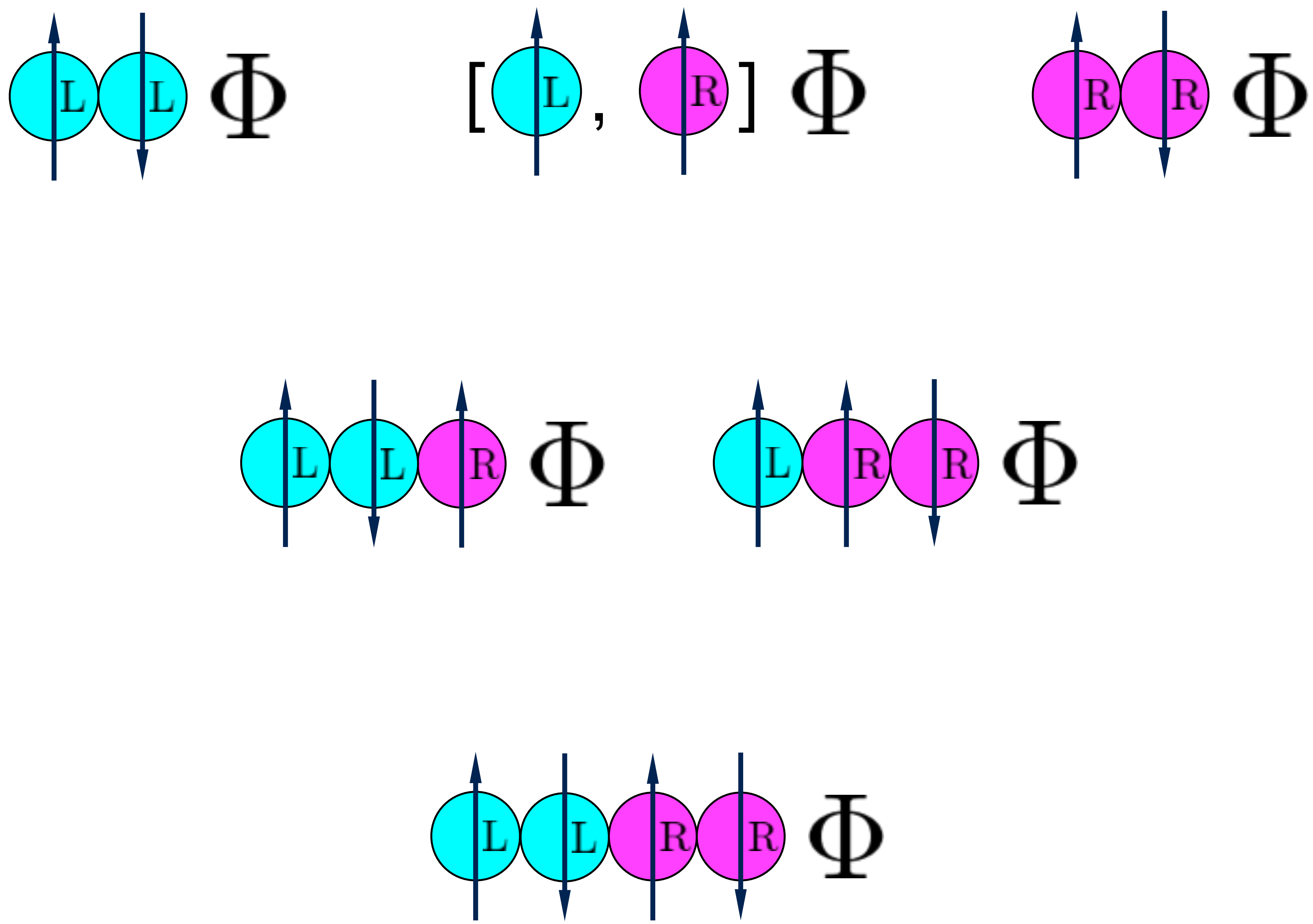


The 3rd, 4th order

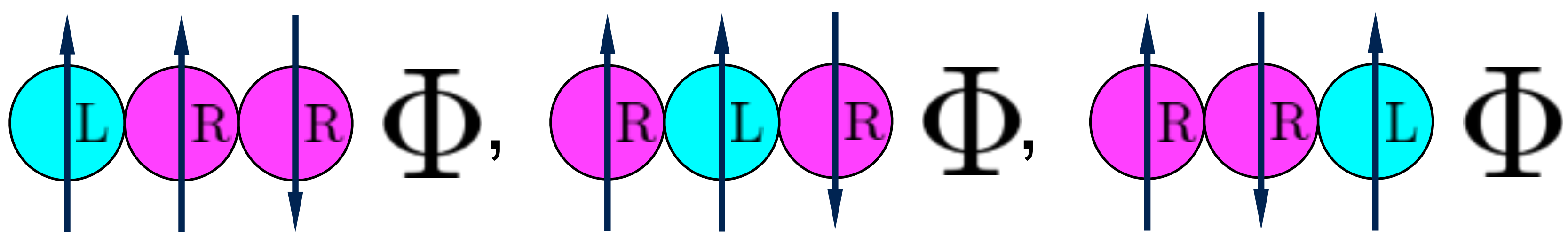


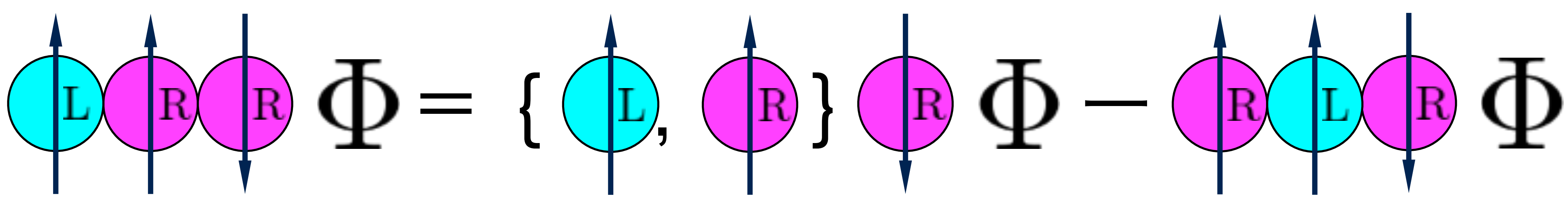


The 3rd, 4th order



Nontrivial combinations

-  different

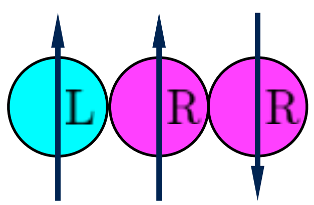


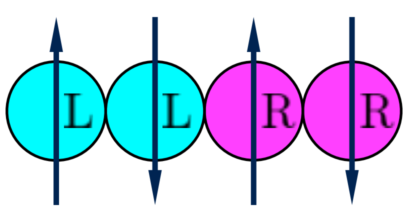
spacetime derivative

- Which combination reproducing Q transf. law?

Correspondences of the 3rd, 4th order

T. Kugo, RY and K. Yoshioka (2016)





component	superspace
Λ_A	$-\frac{i}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\bar\nabla^2\nabla_\alpha\Phi_A\\ \nabla^2\bar\nabla^{\dot\alpha}\Phi_A\end{pmatrix}\| + 2i\begin{pmatrix}\mathcal{W}_\alpha\\ \bar{\mathcal{W}}^{\dot\alpha}\end{pmatrix}\Phi_A\|$
$\mathcal{D}_A$	$\frac{1}{4}\left(\frac{1}{2}\bar\nabla_{\dot\beta}\nabla^2\bar\nabla^{\dot\beta}\Phi_A + (-1)(-2i)^2\bar{\mathcal{W}}_{\dot\beta}\bar\nabla^{\dot\beta}\Phi_A\right)\|$
	$= \frac{1}{4}\left(\frac{1}{2}\nabla^\beta\bar\nabla^2\nabla_\beta\Phi_A + (-2i)^2\mathcal{W}^\beta\nabla_\beta\Phi_A\right)\|$

$$\mathcal{W}_\alpha = \frac{1}{8}[\bar\nabla_{\dot\alpha}, \{\bar\nabla^{\dot\alpha}, \nabla_\alpha\}]$$

These combinations reproducing the transf. laws (Kugo and Uehara '85)

$$\delta_Q(\varepsilon)\mathcal{H}_A = \frac{1}{2}\bar\varepsilon i\gamma_5(\not{\mathcal{D}}\mathcal{Z}_A + \Lambda_A),$$

$$\delta_Q(\varepsilon)\Lambda_A = (-)^A\frac{1}{2}(\sigma\cdot\mathcal{F}_A + i\gamma_5\mathcal{D}_A)\varepsilon$$
$$+\frac{1}{8}\left(\gamma_m\varepsilon R_{ab}(Q)\gamma_m(\Sigma^{ab}\mathcal{Z})_A + \gamma_5\gamma_m\varepsilon R_{ab}(Q)\gamma_5\gamma_m(\Sigma^{ab}\mathcal{Z})_A\right)$$

What we did

not restricted!

component	⟷	superspace
practical ◎		practical △
symmetry △		symmetry ◎