The Spectra of Type IIB Flux Compactifications at Large Complex Structure

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Contents

Introduction

- Type IIB string theory is 10d
  $\Rightarrow$ (Usually) say 6d are 'compactified'
  Compactification has parameters ('moduli')
- Low energy limit is supergravity
  $\Rightarrow$ Compactification affects content of supergravity
  $\Rightarrow$ Different compactifications $\leftrightarrow$ different low energy theories
- Flux associates energy with parameters of compactification
  $\Rightarrow$ Important for constructing metastable vacua
  $\Rightarrow$ Flux compactification useful to get to particle physics
  (But: we *will not* be talking about moduli stabilisation)

Motivation: Flux landscape, computational difficulty

Many possible flux compactification choices ('flux landscape')
($\Rightarrow$ Many low-energy theories)
And too difficult to compute in generic compactifications
Computations possible only when there are few moduli

$\Rightarrow$ Large literature on statistical approach, e.g. random matrix theory (RMT)

But: difficult to verify applicability of RMT techniques, as used precisely where explicit compactifications not available

Here we present a way to test these statistical methods

Will focus on statistical models of two important matrices:
the Hessian $\mathcal{H}$ and the matrix $\mathcal{M}$

Type IIB flux compactifications, I

Low-energy degrees of freedom include axio-dilaton τ and complex structure moduli u^i ('large complex structure' $\leftrightarrow u^i \gtrsim 1$)

Integrally quantised flux wraps cycles of the compactification
$\Rightarrow$ Gives superpotential W (Gukov-Vafa-Witten),

$$W = \vec{N} \cdot \vec{\Pi} = (\vec{f} - \tau \vec{h}) \cdot (1, u^i, 2F - u^j F_j, F_i)$$

$\vec{N}$ is the 'flux vector', $\vec{\Pi}$ is the 'period vector', F is cubic in u^i

Kähler potential (in LCS expansion):

$$K = -\ln\left(\tfrac{i}{6}\varkappa_{ijk}(u^i - \overline{u}^i)(u^j - \overline{u}^j)(u^k - \overline{u}^k) - 2\mathrm{Im}\varkappa_0\right) - \ln\left(-i(\tau - \overline{\tau})\right)$$

F-term scalar potential:

$$V = e^K \left(K^{a\bar{b}} D_a W \overline{D}_{\bar{b}} \overline{W} - 3|W|^2\right) ,$$

K: Kähler potential, $K_{a\bar{b}}$: Kähler metric, D_a: Kähler covariant derivative ($D_a W = \partial_a W + \partial_a K\, W$), $a, b \in \{\tau, u^i\}$

Type IIB flux compactifications, II

Hessian matrix $\mathcal{H}$,

$$\mathcal{H} = \begin{pmatrix} \partial_a \overline{\partial}_{\overline{b}} V & \partial_a \partial_b V \\ \overline{\partial}_{\overline{a}} \overline{\partial}_{\overline{b}} V & \overline{\partial}_{\overline{a}} \partial_b V \end{pmatrix}$$

$\mathcal{H}$ important: must be positive definite for metastable vacuum

Matrix $\mathcal{M}$,

$$\mathcal{M} = \begin{pmatrix} 0 & Z_{ab} e^{-i\vartheta} \\ \overline{Z}_{\overline{a}\overline{b}} e^{i\vartheta} & 0 \end{pmatrix}$$

$\mathcal{M}$ important: $\partial_a V = 0 \Leftrightarrow$ eig'val equation for $\mathcal{M}$ (eig'val $2|W|$)

$$Z_{ab} \equiv D_a D_b W \,, \quad \vartheta \equiv \arg(W)$$

RMT spectra for $\mathcal{H}$ and $\mathcal{M}$ proposed in literature

Results: Numeric spectra at large complex structure

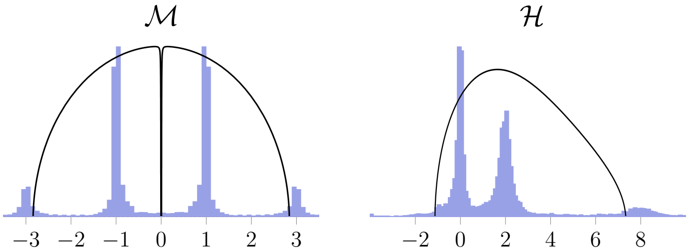


Figure 1: In blue: Sampled numerical spectra of $\mathcal{M}$ (on left) and $\mathcal{H}$ (on right) in explicit compactification, in units of $|W|$ and $m_{3/2}^2$. RMT expectations shown in black. Here $\tau \sim 5i$ and $u^i \sim 10i$.

- Same peaks seen in many compactifications, so universal
- Very different from expectations of random matrix theory
- Peaks are sharper for larger moduli (not shown)
- Peaks visible only for canonically normalised fields and in units of $|W|$ and $m_{3/2}^2 = e^{K/2}\,W$

Results: Analytic spectra at large complex structure, I

Restrict to LCS [1]: $(u^i)^3$ dominates W, $(u^i - \overline{u}^i)^3$ dominates K
$\Rightarrow$ Can derive analytic spectra:

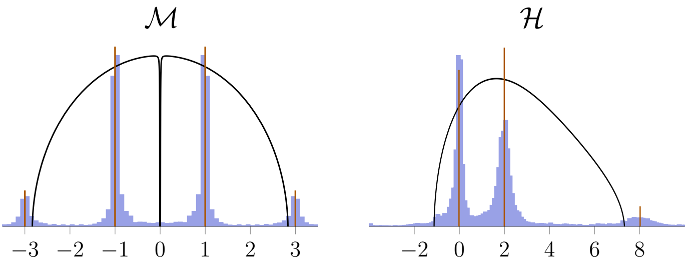
$$\mathrm{Spectrum}(\mathcal{M}) = \begin{cases} |W| & \text{multiplicity } h_-^{1,2}\,, \\ 3|W| & \text{multiplicity } 1\,, \\ -|W| & \text{multiplicity } h_-^{1,2}\,, \\ -3|W| & \text{multiplicity } 1\,. \end{cases}$$

$$\mathrm{Spectrum}(\mathcal{H}) = \begin{cases} 0 & \text{multiplicity } h_-^{1,2}\,, \\ 2m_{3/2}^2 & \text{multiplicity } h_-^{1,2}+1\,, \\ 8m_{3/2}^2 & \text{multiplicity } 1\,. \end{cases}$$

These are 'universal': true for any non-vanishing choice of flux, any not-all-vanishing triple intersections $\varkappa_{ijk}$, and any number of complex structure moduli u^i

[1] See [Marsh, Sousa, JHEP 1603 (2016) 064] for extension to this full subspace

Results: Analytic spectra at large complex structure, II



Blue: Randomly sampled numerical spectra of $\mathcal{M}$ and $\mathcal{H}$ in explicit compactification, in units of $|W|$ and $m^2_{3/2}$ respectively

Black: Random matrix theory expectations

Brown: Analytic spectra, with schematic degeneracies

Things to note

- Spectra remain highly peaked at not-so-large complex structure in explicit compactifications
- Can show $F^2 = 4|W|^2 \Rightarrow$ no supersymmetric points here
- No eig'val of $\mathcal{M}$ at $2|W| \Rightarrow$ no critical points here
- Slow-roll parameters take universal large values: $\epsilon = 4$, $\eta_{||} = 8$
- Analytic results can also be extended to F-theory: see [Marsh, Sousa, JHEP 1603 (2016) 064]
- Popular continuous flux approximation breaks down quickly at large complex structure (as a result of the dominance of one term in the superpotential)
- Strong linear correlation between $|W|$ and the supersymmetric mass at large complex structure

Summary

- Many flux compactifications, and difficult to compute
  $\Rightarrow$ Statistical perspective, including RMT
- In subspace of moduli space, can attempt to verify RMT
- Explicit compactifications: spectra show peaks, not well-described by RMT
- At LCS, can find analytic spectra of Hessian matrix $\mathcal{H}$, and matrix $\mathcal{M}$ that governs the critical point equation - find highly degenerate eigenvalues - these results are 'universal'
- $\Rightarrow$ Statistical models not as applicable as one might have believed (or hoped)
- But we can find other generic or universal predictions in some regions of parameter space, via analytic results