Bayesian naturalness of Next-to-Minimal and Minimal Supersymmetric Models

A. Fowlie, D. Kim, P. Athron, C. Balazs, B. Farmer, and D. Harries, in preparation, A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph], D. Kim, P. Athron, C. Balazs, B. Farmer, and E. Hutchison, Phys. Rev. D90, 055008 (2014), arXiv:1312.4150 [hep-ph]

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Results from my previous work[1], Csaba et al.'s previous work[2], and our forthcoming joint paper on this topic.[3]

[1] A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph].

[2] D. Kim et al., Phys. Rev. D90, 055008 (2014), arXiv:1312.4150 [hep-ph].

[3] A. Fowlie et al., in preparation.

Minimal and next-to-minimal SUSY: problems & solutions

Minimal SUSY

Prior to the LHC, minimal SUSY solved many problems:

- Hierarchy problem
- Gauge coupling unification
- Dark matter

But not all.

The μ-problem

The superpotential includes

$$W \supset \mu H_u H_d$$

leading to

$$\frac{1}{2}M_Z^2 \approx -\mu^2 - m_{H_u}^2$$

Anticipated that M_{SUSY} governs the weak scale (radiative EWSB), $M_{\text{SUSY}} \sim M_{\text{WEAK}}$.

But why is $\mu \sim M_{\text{SUSY}}$?

Next-to-Minimal SUSY: μ-solution

Add a singlet superfield with a $\mathbb{Z}_3$ symmetry. Original μ-term forbidden. Effective μ-term generated by $\langle S \rangle$:

$$W \supset \mu H_u H_d \to \lambda S H_u H_d \to \lambda \langle S \rangle H_u H_d$$

Advantage: model now has a single scale, M_{SUSY}. The μ-term, $\lambda\langle S \rangle$, is a function of the soft-breaking parameters.

That $M_{\text{SUSY}} \sim \mu \sim M_{\text{WEAK}}$ is not a coincidence, as M_{SUSY} governs all scales, $M_{\text{SUSY}} \to \mu(M_{\text{SUSY}}) \to M_{\text{WEAK}}(M_{\text{SUSY}})$.

SUSY problem: Higgs mass $m_h^{\text{Tree}} \lesssim M_Z$

At tree-level the Higgs mass prediction in SUSY is

$$m_h \leq M_Z$$

We've known for a while that

$$m_h \geq M_Z$$

Require substantial loop corrections from sparticle masses, $\Delta m_h^2(M_{\text{SUSY}})$. For that, $M_{\text{SUSY}} \gg M_{\text{WEAK}}$.

But $M_{\text{SUSY}} \gg M_{\text{WEAK}}$ requires fine-tuning as weak scale is a function of μ and M_{SUSY}.

Next-to-minimal SUSY: Higgs mass $m_h^{\text{Tree}} \gtrsim M_Z$?

Higgs mass boosted at tree-level:

$$m_h^2 \approx M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta$$

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Though could be scotched by mixing which repels eigenvalues.

Furthermore, trade-off between $\cos^2$ and $\sin^2$ terms.

Nevertheless, perhaps $\Delta m_h^2(M_{\text{SUSY}})$ and thus M_{SUSY} needn't be so big.

Minimal and next-to-minimal SUSY: which is *better*?

Two problems. Two solutions. Next-to-minimal
more natural/better than minimal

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Recieved wisdom.

Two problems. Two solutions. Next-to-minimal *more natural/better than* minimal

Recieved wisdom.

Are we sure it's true? Can we quantify it?

Next-to-minimal *more natural/plausible than* minimal?

Old-fashioned answers

No — no justification for such claims. Minimal vs. next-to-minimal settled by experiments, not theorists

Yes ...

- ...but can't quantify it/make it rigorous
- ...calculate Barbieri-Giudice-Ellis style fine-tuning measures, e.g.

$$\Delta \equiv \sum \frac{\partial \ln M_Z}{\partial \ln p_i}$$

Identify parameter points in minimal and next-to-minimal models with the smallest Δ.

Next-to-minimal *more natural/better than* minimal?

Modern answer: a synthesis of the two

Maybe ...

- Need a logical framework for quantifying plausibility/belief in a model
- Should condition belief on experimental data only, not theorists' prejudices or arbitrary functions Δ
- Make the calculations and discover which model is best

Bayesian statistics is a *unique* logical framework for quantifying plausibility

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$$p(M \mid D)$$

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Condition belief only on experimental data,

$$p(M \,|\, D)$$

No weird inventions, Δ_{EWSB} etc. No cheating.

What do you calculate?

Calculate the Bayesian evidence for each model under consideration

$$p(D \mid M) = \int p(D \mid M, \boldsymbol{p}) \cdot p(\boldsymbol{p} \mid M) \prod \mathrm{d}\boldsymbol{p}$$

Probability of data given point in model (likelihood).
Probability of point given model (prior). *Somewhat* subjective, though should reflect knowledge or ignorance about parameters.

Compare the evidences in a so-called Bayes-factor:

$$p(D \mid M_b) / p(D \mid M_a) \propto p(M_b \mid D) / p(M_a \mid D)$$

which is proportional to the posterior odds. May not agree with frequentist methods, even with *informative* data.[4]

[4] D. V. Lindley, Biometrika 44, 187–192 (1957), M. S. Bartlett, Biometrika 44, 533–534 (1957).

Why Bayesian evidence captures fine-tuning in one slide[5]

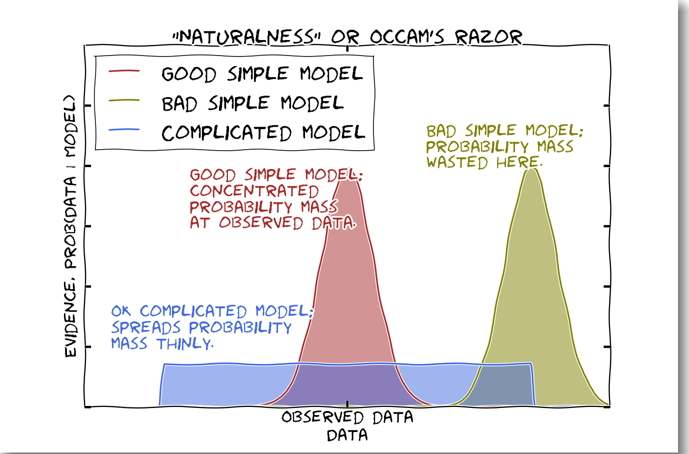

Figure 1: Bayesian evidence captures old-fashioned ideas about FT. SM is a bad simple model: concentrates probability at $M_Z \sim M_P$.

[5] D. Mackay, (1992).

Trick for speeding up the integral reveals a connection

Re-parameterize integral such that M_Z is an input parameter, $\boldsymbol{p} \to \boldsymbol{p}', M_Z$:

$$p(D \mid M) = \int p(\hat{M}_Z \mid M_Z) \cdot p(D' \mid M, \boldsymbol{p}) \cdot p(\boldsymbol{p}', M_Z \mid M) \prod \mathrm{d}\boldsymbol{p}' \, \mathrm{d}M_Z$$

where there is a Jacobian: $p(\boldsymbol{p}', M_Z \mid M) = \sum \mathcal{J} p(\boldsymbol{p} \mid M)$.

We approximate measurement of the Z-mass with a Dirac distribution, $p(\hat{M}_Z \mid M_Z) \to \delta(\hat{M}_Z - M_Z)$ as we don't expect rest of integrand to change much over the width of the Gaussian. Integrating the Dirac,

$$p(D \mid M) = \int p(D' \mid M, \boldsymbol{p}', \hat{M}_Z) \cdot \left[\mathcal{J} p(\boldsymbol{p} \mid M)\right]_{M_Z = \hat{M}_Z} \prod \mathrm{d}\boldsymbol{p}'$$

Trick for speeding up the integral reveals a connection

In the integral for the evidence, a factor

$$\left[\mathcal{J}\, p(\boldsymbol{p} \mid M)\right]_{M_Z = \hat{M}_Z}$$

appeared in the integrand. What is it?

- Measurement of the Z-mass. Enforces $M_Z = 91.$ GeV.
- Prior for Lagrangian parameters. Perhaps logarithmic priors if we are ignorant of scale.

$$p(\boldsymbol{p} \mid M) \propto \prod \frac{1}{p_i} = \frac{1}{\mu^2} \cdots$$

- Interesting! Jacobian from re-parameterisation. In the CMSSM, $(b, \mu^2) \to (M_Z, \tan\beta)$

$$\mathcal{J} = \frac{\partial \mu^2}{\partial M_Z} \frac{\partial b}{\partial \tan\beta}$$

In the integrand of the Bayesian evidence we have a factor

$$p(D \,|\, M) \supset \int \frac{1}{\mu^2} \frac{\partial \mu^2}{\partial M_Z} \cdots = \int \frac{1}{M_Z} \frac{1}{\Delta_{\mu^2}} \cdots$$

In the integrand of the Bayesian evidence we have a factor

$$p(D \mid M) \supset \int \frac{1}{\mu^2} \frac{\partial \mu^2}{\partial M_Z} \cdots = \int \frac{1}{M_Z} \frac{1}{\Delta_{\mu^2}} \cdots$$

Reciprocal of Barbieri-Giudice-Ellis style fine-tuning measure!

This means

The *intuition* behind BG-style measures was correct. There is a connection between Δ and plausibility

Does *not* mean that you should just keep on calculating Δ with renewed justification and vigour! You need all the factors in the integrand and to perform the integral

You then need to compare more than one model

What we calculated: Bayes-factor CNMSSM vs. CMSSM[6], priors/fine-tuning maps[7], and, forthcoming, full comparison of Bayesian vs. Δ methods[8]

[6] A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph].

[7] D. Kim et al., Phys. Rev. D90, 055008 (2014), arXiv:1312.4150 [hep-ph].

[8] A. Fowlie et al., in preparation.

Models/priors: CNMSSM vs. CMSSM

Well-known constrained models for simplicity (speed is still an issue with many CPUs) — CMSSM and CNMSSM

Uninformative *honest* priors for Lagrangian parameters (logarithmic if we're ignorant of scale).

CMSSM		CNMSSM	
m_0	Log, $0.3, 20$ TeV	λ	Log, $0.001, 4\pi$
$m_{1/2}$	Log, $0.3, 10$ TeV	m_S	Log, $0.3, 20$ TeV
A_0	Flat, $-20, 20$ TeV	κ	Log, $0.001, 4\pi$
μ	Log, 1 GeV, M_P		
b	Log, $0.3, 20$ TeV		

Table 1: Priors for the CMSSM and CNMSSM model parameters.

Models/priors: CNMSSM vs. CMSSM

Make a change of variables from Lagrangian parameters to $M_Z, \tan\beta$. This introduces Jacobian.

μ extends to Planck scale, as unrelated to soft-breaking masses. This captures μ-problem.

Experimental data

Two approaches: if the goal is somewhat pedagogical, keep it simple and consider only measurements of M_Z and m_H. Alternatively, include more relevant data for full inference.

Quantity	Experimental data, $\mu \pm \sigma$	Theory error, τ
M_Z	$91.1876\,\text{GeV}$	
δa_μ	$(28.8 \pm 8.0) \times 10^{-10}$	1.0×10^{-10}
$\text{BR}(B_s \to \mu\mu)$	$(3.2 \pm 1.5) \times 10^{-9}$	14%
$\text{BR}(B_s \to X_s\gamma)$	$(3.43 \pm 0.22) \times 10^{-4}$	0.21×10^{-4}
$\text{BR}(B_u \to \tau\nu)$	$(1.14 \pm 0.22) \times 10^{-4}$	0.38×10^{-4}
Search for SUSY in $\sim 20/\text{fb}$ at $\sqrt{s} = 8\,\text{TeV}$.		
LHC, Tevatron and LEP Higgs searches.		

Table 2: Experimental data included in our likelihood function.

CMSSM vs CNMSSM: Lagrangian parameters

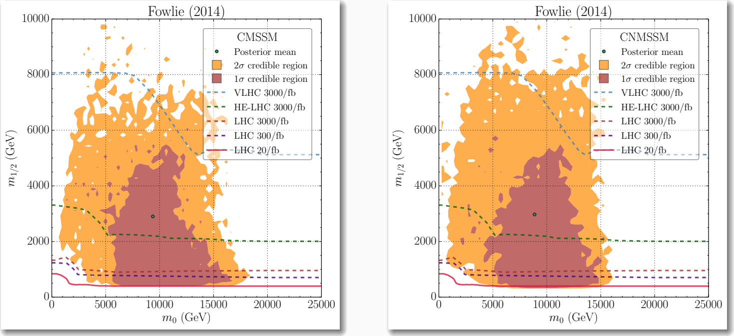

Figure 2: Similar credible regions on $(m_0, m_{1/2})$ planes. LEFT = CMSSM. RIGHT = CNMSSM.[9]

[9] A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph].

CMSSM vs CNMSSM: Mass spectra

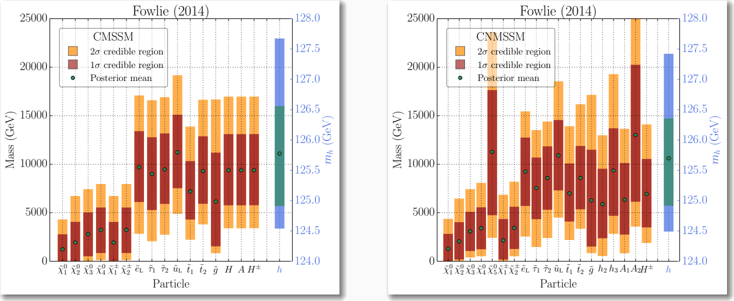

Figure 3: Consequently, similar mass spectra too, with $m_{\tilde{q},\tilde{g}} \gtrsim 5\,\text{TeV}$. LEFT = CMSSM. RIGHT = CNMSSM.[10]

[10] A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph].

Comparison of priors vs. FT measures

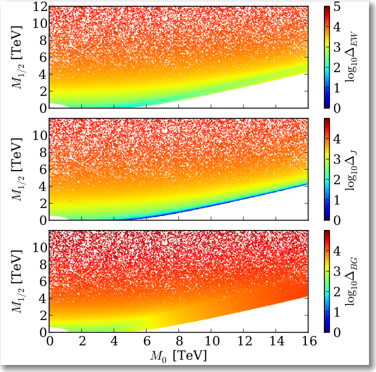

Figure 4: Slice of CNMSSM parameter space. The old-fashioned FT measures don't entirely match the Jacobian from the effective prior (center), but are similar.[11]

[11] D. Kim et al., Phys. Rev. D90, 055008 (2014), arXiv:1312.4150 [hep-ph].

Bayesian evidences

CNMSSM about ~ 10 times more plausible than CMSSM

In light of experimental data, the CNMSSM was favored versus the CMSSM by a factor of $\sim 10^{+100}_{-5}$.[12]

Of which a factor of ~ 5 resulted from solving the μ-problem.

These numbers are not particularly impressive. The case for the CNMSSM is possibly overstated in the literature.

Unfortunately large uncertainties (though estimate of uncertainty is extremely conservative). Drawback is that precise calculations are moderately CPU intensive.

We are working on this.[13]

[12] A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph].

[13] A. Fowlie et al., in preparation.

Conclusions

CNMSSM slightly favoured versus CMSSM

- Bayes-factors provide relative plausibility/correct "measure" of fine-tuning between two models
- Traditional fine-tuning measures are *at best* a poor numerical approximation to an effective prior[14]
- We find that the CNMSSM is mildly favoured[15] versus the CMSSM by a Bayes-factor of ~ 10
- *Some* support for claims about μ-problem and Higgs mass in CNMSSM
- Many more results and a pedagogical comparison of FT measures vs. Bayesian analyses forthcoming[16]

[14] D. Kim et al., Phys. Rev. D90, 055008 (2014), arXiv:1312.4150 [hep-ph].

[15] A. Fowlie, Eur. Phys. J. C74, 3105 (2014), arXiv:1407.7534 [hep-ph].

[16] A. Fowlie et al., in preparation.

Questions?

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