

LHC vs. Precision Experiments

A Comparison of LFV D6 Operators QQLL

Michael A. Schmidt

5 July 2016 @ SUSY

The University of Sydney

based on

Yi Cai, MS JHEP02(2016)176



THE UNIVERSITY OF
SYDNEY



COEPP

ARC Centre of Excellence for
Particle Physics at the Terascale

Motivation

- Standard Model very successful

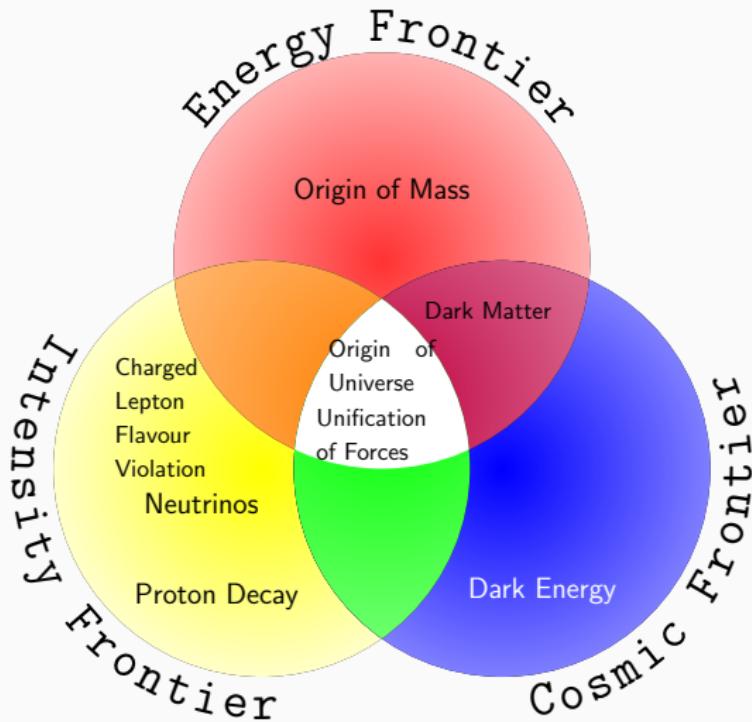
Motivation

- Standard Model very successful
- but incomplete

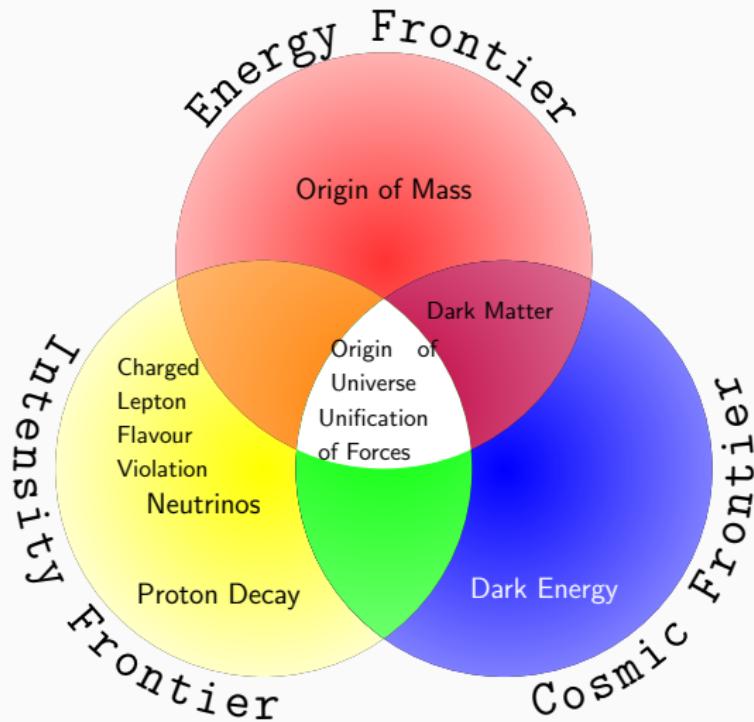
Motivation

- Standard Model very successful
- but incomplete
- Flavour changing processes are a sensitive probe

Motivation



Motivation



Can the LHC compete with precision experiments?

Operators

D6 Operators with 2 Quarks and 2 Leptons

Buchmüller, Wyler NPB268(1986)621; Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

Scalar

$$\mathcal{Q}_{ledq} = (\bar{L}^\alpha \ell)(\bar{d} Q^\alpha) \quad \mathcal{Q}_{lequ}^{(1)} = (\bar{L}^\alpha \ell)\epsilon_{\alpha\beta}(\bar{Q}^\beta u)$$

Vector

$$\mathcal{Q}_{lq}^{(1)} = (\bar{L}\gamma_\mu L)(\bar{Q}\gamma^\mu Q) \quad \mathcal{Q}_{lq}^{(3)} = (\bar{L}\gamma_\mu \tau^I L)(\bar{Q}\gamma^\mu \tau^I Q)$$

$$\mathcal{Q}_{eu} = (\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma^\mu u) \quad \mathcal{Q}_{ed} = (\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma^\mu d)$$

$$\mathcal{Q}_{lu} = (\bar{L}\gamma_\mu L)(\bar{u}\gamma^\mu u) \quad \mathcal{Q}_{ld} = (\bar{L}\gamma_\mu L)(\bar{d}\gamma^\mu d)$$

$$\mathcal{Q}_{qe} = (\bar{Q}\gamma_\mu Q)(\bar{\ell}\gamma^\mu \ell)$$

Tensor

$$\mathcal{Q}_{lequ}^{(3)} = (\bar{L}^\alpha \sigma_{\mu\nu} \ell)\epsilon_{\alpha\beta}(\bar{Q}^\beta \sigma^{\mu\nu} u)$$

D6 Operators with 2 Quarks and 2 Leptons

Buchmüller, Wyler NPB268(1986)621; Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884

Scalar with same-flavour quark

$$\mathcal{Q}_{ledq} = (\bar{L}^\alpha \ell)(\bar{d} Q^\alpha) \quad \mathcal{Q}_{lequ}^{(1)} = (\bar{L}^\alpha \ell)\epsilon_{\alpha\beta}(\bar{Q}^\beta u)$$

Vector Carpentier, Davidson 1008.0280; Petrov,Zhuridov 1308.6561

$$\begin{array}{ll} \mathcal{Q}_{lq}^{(1)} = (\bar{L}\gamma_\mu L)(\bar{Q}\gamma^\mu Q) & \mathcal{Q}_{lq}^{(3)} = (\bar{L}\gamma_\mu \tau^I L)(\bar{Q}\gamma^\mu \tau^I Q) \\ \mathcal{Q}_{eu} = (\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma^\mu u) & \mathcal{Q}_{ed} = (\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma^\mu d) \\ \mathcal{Q}_{lu} = (\bar{L}\gamma_\mu L)(\bar{u}\gamma^\mu u) & \mathcal{Q}_{ld} = (\bar{L}\gamma_\mu L)(\bar{d}\gamma^\mu d) \\ \mathcal{Q}_{qe} = (\bar{Q}\gamma_\mu Q)(\bar{\ell}\gamma^\mu \ell) & \end{array}$$

Tensor

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Scalar Operators

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Wilson coefficients $\Xi^{u,d}$ in Lagrangian [unbroken phase]

$$-\mathcal{L} = \Xi_{ij,kk}^d (\mathcal{Q}_{ledq})_{ij,kk} + \Xi_{ij,kk}^u (\mathcal{Q}_{lequ}^{(1)})_{ij,kk} + \text{h.c. .}$$

Effective four fermion Lagrangian [broken phase]

$$\begin{aligned} \mathcal{L}_{4f} = & \Xi_{ij,kl}^{Cd} (\bar{\nu}_{Li} \ell_{Rj})(\bar{d}_{Rk} u_{LI}) + \Xi_{ij,kl}^{Nd} (\bar{\ell}_{Li} \ell_{Rj})(\bar{d}_{Rk} d_{LI}) \\ & + \Xi_{ij,kl}^{Cu} (\bar{\nu}_{Li} \ell_{Rj})(\bar{d}_{Lk} u_{RI}) + \Xi_{ij,kl}^{Nu} (\bar{\ell}_{Li} \ell_{Rj})(\bar{u}_{Lk} u_{RI}) . \end{aligned}$$

Thus the most general four fermion coefficients are

$$\begin{aligned} \Xi_{ij,kl}^{Nd} &= U_{ii'}^{\ell*} V_{lk}^d \Xi_{ij,kk}^d & \Xi_{ij,kl}^{Cd} &= U_{ii'}^{\nu*} V_{lk}^u \Xi_{i'j,kk}^d \\ \Xi_{ij,kl}^{Nu} &= -U_{ii'}^{\ell*} V_{kl}^{u*} \Xi_{ij,II}^u & \Xi_{ij,kl}^{Cu} &= U_{ii'}^{\nu*} V_{kl}^{d*} \Xi_{i'j,II}^u \end{aligned}$$

⇒ In general there is quark flavour violation.

We do not consider top-quark, because phenomenology is different.

Scalar Operators

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Choose basis in which charged lepton mass matrix is diagonal as well as $\Xi_{ij,kk}^N$

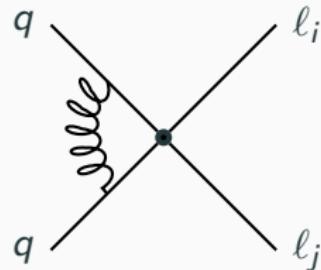
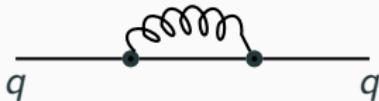
$$\begin{aligned} \Xi_{ij,kl}^{Nd} &= \delta_{kl} \Xi_{ij,kk}^d & \Xi_{ij,kl}^{Cd} &= U_{ii'}^* V_{kl}^* \Xi_{i'j,kk}^d \\ \Xi_{ij,kl}^{Nu} &= -\delta_{kl} \Xi_{ij,kk}^u & \Xi_{ij,kl}^{Cu} &= U_{ii'}^* V_{kl}^* \Xi_{i'j,II}^u \end{aligned}$$

⇒ No new flavour changing neutral current processes (FCNCs).

We do not consider top-quark, because phenomenology is different.

Renormalization Group Corrections

- Main effect are QCD corrections



- Following the standard discussion at NLO

Buchalla, Buras, Lautenbacher hep-ph/9512380

$$\Xi(\mu) = \Xi(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_0}{2\beta_0}}$$

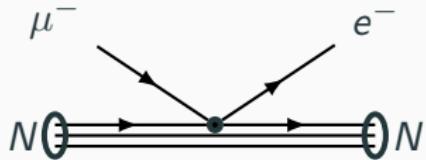
with coefficients

$$\beta_0 = 11 - 2n_F/3 \quad \text{and} \quad \gamma_0 = 6C_2(3) = 8$$

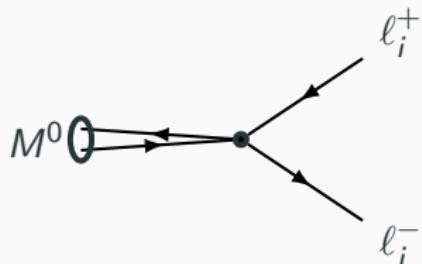
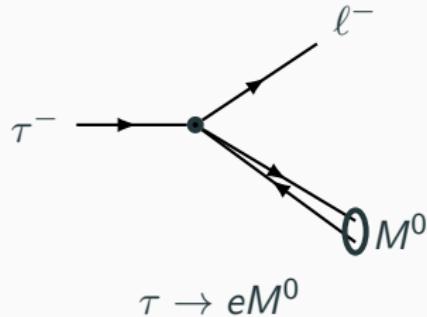
- Wilson coefficients become larger at smaller scales.
⇒ Increases reach of precision experiments

Precision Experiments

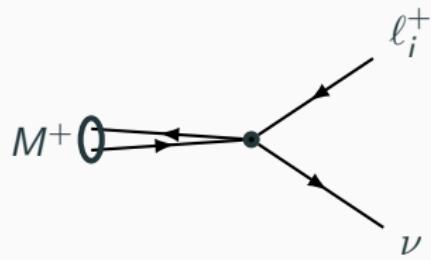
Precision Experiments



$\mu - e$ conversion in nuclei



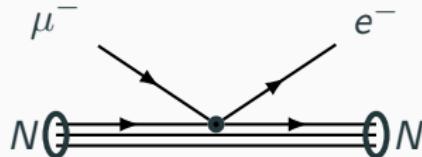
$$M^0 \rightarrow \ell_i^+ \ell_j^-$$



$$M^+ \rightarrow \ell_i^+ \nu$$

$\mu - e$ Conversion

- Agnostic about mediation mechanism
- Following discussion in [Gonzalez et al. 1303.0596](#)

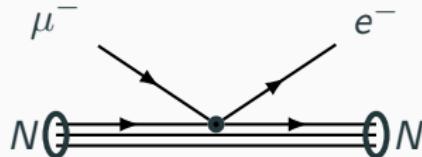


	^{48}Ti	^{197}Au	^{208}Pb
$R_{\mu e}^{\max}$	4.3×10^{-11}	7.0×10^{-13}	4.6×10^{-11}
$\bar{u}u$	1100 [870]	2100 [1700]	760 [610]
$\bar{d}d$	1100 [930]	2200 [1900]	780 [680]
$\bar{s}s$	480 [-]	950 [-]	340 [-]
$\bar{c}c$	150 [-]	290 [-]	110 [-]
$\bar{b}b$	84 [-]	170 [-]	61 [-]

Direct nuclear mediation [Meson mediation]

$\mu - e$ Conversion

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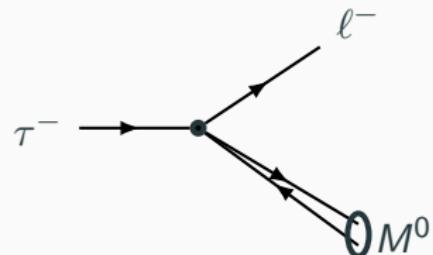
Direct nuclear mediation [Meson mediation]

⇒ Strongest limit for first generation quarks,
and non-negligible for other quarks if pure direct nuclear mediation

LFV τ Decays

- Only light quarks u,d,s
- Weak dependence on phase
- f_0 : φ_m parameterizes quark content
- Quark FCNC parameterized by λ

$$\Xi_{ij,kl}^u = \lambda \Xi_{ij,II}^u V_{kl} \quad \Xi_{ij,kl}^d = \lambda \Xi_{ij,kk}^d V_{kl}$$

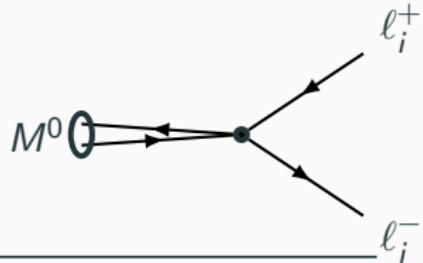


decay	Br_i^{\max}	cutoff scale Λ [TeV]		
		$\Xi_{ij,uu}^u$	$\Xi_{ij,dd}^d$	$\Xi_{ij,ss}^d$
$\tau^- \rightarrow e^- \pi^0$	8.0×10^{-8}	10	10	-
$\tau^- \rightarrow e^- \eta$	9.2×10^{-8}	34	34	7.9
$\tau^- \rightarrow e^- \eta'$	1.6×10^{-7}	42	42	12
$\tau^- \rightarrow e^- K_S^0$	2.6×10^{-8}	-	$7.8 \sqrt{\lambda}$	$7.8 \sqrt{\lambda}$
$\tau^- \rightarrow e^- (f_0(980) \rightarrow \pi^+ \pi^-)$	3.2×10^{-8}	$13 \sqrt{\sin \varphi_m}$	$13 \sqrt{\sin \varphi_m}$	$16 \sqrt{\cos \varphi_m}$
$\tau^- \rightarrow \mu^- \pi^0$	1.1×10^{-7}	9.0 – 9.6	9.0 – 9.6	-
$\tau^- \rightarrow \mu^- \eta$	6.5×10^{-8}	36 – 38	36 – 38	8.4 – 8.9
$\tau^- \rightarrow \mu^- \eta'$	1.3×10^{-7}	42 – 46	42 – 46	12 – 13
$\tau^- \rightarrow \mu^- K_S^0$	2.3×10^{-8}	-	$(7.8 - 8.3) \sqrt{\lambda}$	$(7.8 - 8.3) \sqrt{\lambda}$
$\tau^- \rightarrow \mu^- (f_0(980) \rightarrow \pi^+ \pi^-)$	3.4×10^{-8}	$(12 - 14) \sqrt{\sin \varphi_m}$	$(12 - 14) \sqrt{\sin \varphi_m}$	$(15 - 16) \sqrt{\cos \varphi_m}$

Leptonic Neutral Meson Decays $M^0 \rightarrow \ell_i^+ \ell_j^-$

- Assumption: no quark FCNC processes
- What if assumption does not hold?

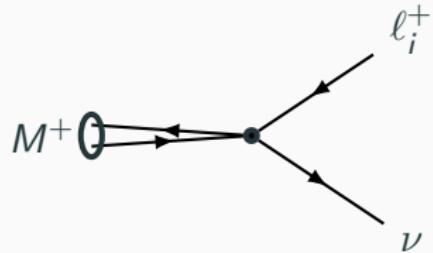
$$\Xi_{ij,kl}^u = \lambda \Xi_{ij,II}^u V_{kl} \quad \Xi_{ij,kl}^d = \lambda \Xi_{ij,kk}^d V_{kl}$$



decay	Br_i^{max}	cutoff scale Λ [TeV]				
		$\Xi_{ij,uu}^u$	$\Xi_{ij,dd}^d$	$\Xi_{ij,ss}^d$	$\Xi_{ij,cc}^u$	$\Xi_{ij,bb}^d$
$\pi^0 \rightarrow \mu^+ e^-$	3.8×10^{-10}	2.2	2.2	-	-	-
$\pi^0 \rightarrow \mu^- e^+$	3.4×10^{-9}	1.2	1.2	-	-	-
$\pi^0 \rightarrow \mu^+ e^- + \mu^- e^+$	3.6×10^{-10}	2.6	2.6	-	-	-
$\eta \rightarrow \mu^+ e^- + \mu^- e^+$	6×10^{-6}	0.52	0.52	0.12	-	-
$\eta' \rightarrow e\mu$	4.7×10^{-4}	0.091	0.091	0.026	-	-
<hr/>						
$K_L^0 \rightarrow e^\pm \mu^\mp$	4.7×10^{-12}	-	$86\sqrt{\lambda}$	$86\sqrt{\lambda}$	-	-
$D^0 \rightarrow e^\pm \mu^\mp$	2.6×10^{-7}	$6.4\sqrt{\lambda}$	-	-	$6.4\sqrt{\lambda}$	-
$B^0 \rightarrow e^\pm \mu^\mp$	2.8×10^{-9}	-	$10\sqrt{\lambda}$	-	-	$6.6\sqrt{\lambda}$
$B^0 \rightarrow e^\pm \tau^\mp$	2.8×10^{-5}	-	$0.97\sqrt{\lambda}$	-	-	$0.62\sqrt{\lambda}$
$B^0 \rightarrow \mu^\pm \tau^\mp$	2.2×10^{-2}	-	$0.18\sqrt{\lambda}$	-	-	$0.12\sqrt{\lambda}$

Leptonic Charged Meson Decays $M^+ \rightarrow \ell_i^+ \nu$

- $R_M = \frac{\text{Br}(M^+ \rightarrow e^+ \nu)}{\text{Br}(M^+ \rightarrow \mu^+ \nu)}$
- Theoretical error for R_π (R_K) about 5%
- Improvement by factor 20 (2) possible
- ✓ indicates constraints
- Second index corresponds to charged lepton

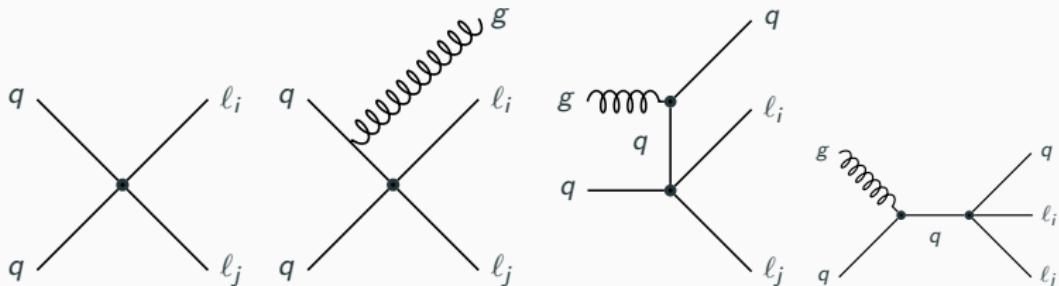


decay	constraint	cutoff scale Λ [TeV]		Wilson coefficients				
		$\Lambda_{\mu e, e\mu, e\tau}$	$\Lambda_{\tau e, \tau\mu, \mu\tau}$	$\Xi_{ij,uu}^u$	$\Xi_{ij,dd}^d$	$\Xi_{ij,ss}^d$	$\Xi_{ij,cc}^u$	$\Xi_{ij,bb}^d$
R_π	$R_\pi^{\exp} \pm 5\%$	25 – 280	25 – 260	✗	✗	-	-	-
R_K	$R_K^{\exp} \pm 5\%$	24 – 160	24 – 150	✓	-	✓	-	-
$\text{Br}(D^+ \rightarrow e^+ \nu)$	$< 8.8 \times 10^{-6}$	2.8 – 2.9	2.9	-	✓	-	✓	-
$\text{Br}(D_s^+ \rightarrow e^+ \nu)$	$< 8.3 \times 10^{-5}$	3.2 – 3.3	3.2 – 3.3	-	-	✓	✓	-
$\text{Br}(B^+ \rightarrow e^+ \nu)$	$< 9.8 \times 10^{-7}$	2.0	2.0	✓	-	-	-	✓
$\text{Br}(\pi^+ \rightarrow \mu^+ \nu)$	$\text{Br}^{\exp} \pm 5\%$	1.9 – 7.4	1.9 – 9.4	✗	✗	-	-	-
$\text{Br}(K^+ \rightarrow \mu^+ \nu)$	$\text{Br}^{\exp} \pm 5\%$	1.7 – 5.8	1.7 – 7.4	✓	-	✓	-	-
$\text{Br}(D^+ \rightarrow \mu^+ \nu)$	$(3.82 \pm 0.33) \times 10^{-4}$	1.1 – 2.7	1.1 – 3.4	-	✓	-	✓	-
$\text{Br}(D_s^+ \rightarrow \mu^+ \nu)$	$(5.56 \pm 0.25) \times 10^{-3}$	1.3 – 4.3	1.3 – 5.3	-	-	✓	✓	-
$\text{Br}(B^+ \rightarrow \mu^+ \nu)$	$< 1.0 \times 10^{-6}$	1.9 – 2.7	1.7 – 3.0	✓	-	-	-	✓
$\text{Br}(D^+ \rightarrow \tau^+ \nu)$	$< 1.2 \times 10^{-3}$	0.21 – 0.78	0.23 – 0.73	-	✗	-	✓	-
$\text{Br}(D_s^+ \rightarrow \tau^+ \nu)$	$(5.54 \pm 0.24) \times 10^{-2}$	0.33 – 1.2	0.33 – 1.1	-	-	✓	✓	-
$\text{Br}(B^+ \rightarrow \tau^+ \nu)$	$(1.14 \pm 0.27) \times 10^{-4}$	0.49 – 1.3	0.49 – 1.2	✗	-	-	-	✓

Large Hadron Collider

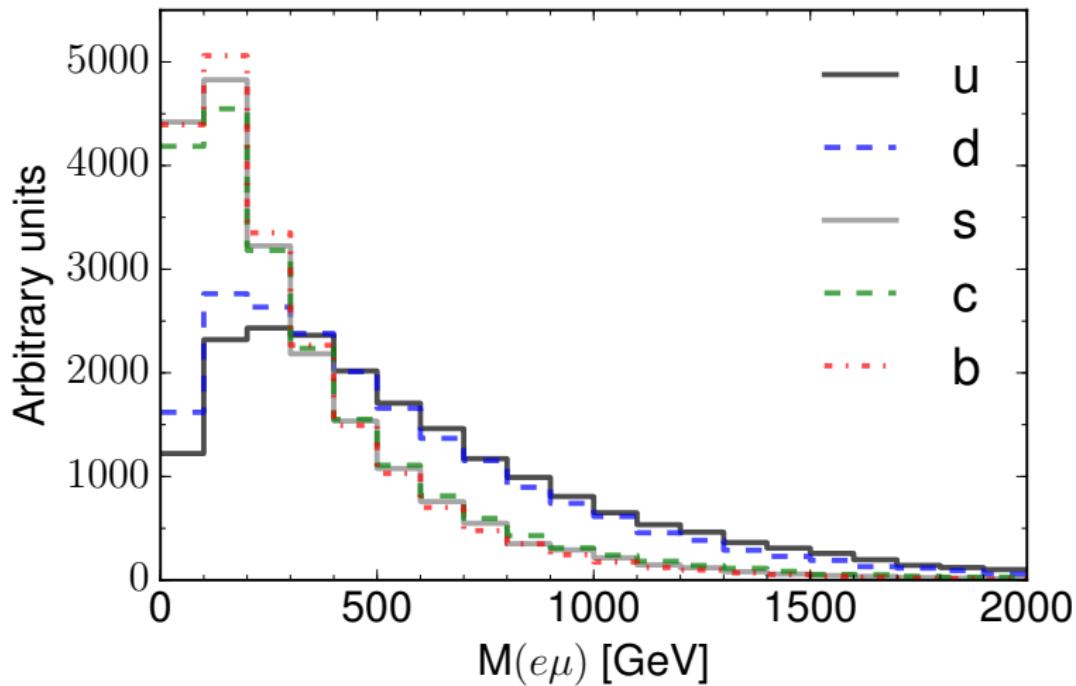
LFV at the Large Hadron Collider (LHC)

- Processes at LHC: $pp \rightarrow \ell_i \ell_j + \text{jets}$



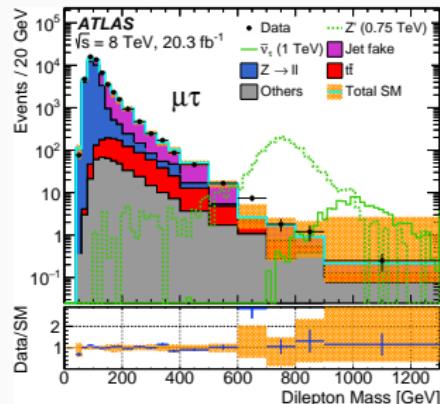
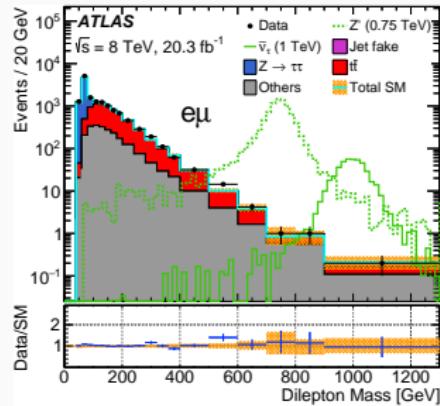
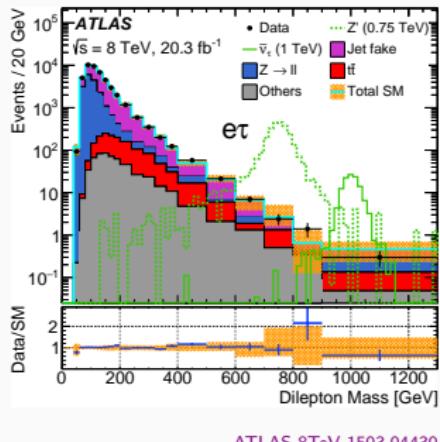
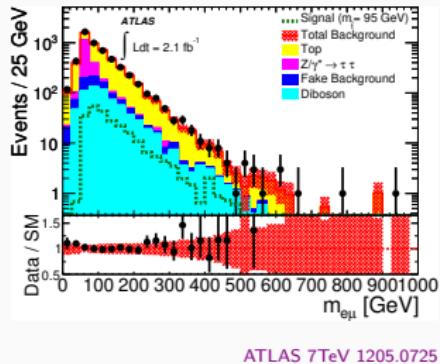
- Signal: opposite-sign **different flavour** pair of leptons and possibly jets
- ATLAS 7 TeV: LFV in $e\mu$ continuum [\tilde{t} and R] ATLAS 1205.0725
- ATLAS 8 TeV: LFV heavy neutral particle decay ATLAS 1503.04430
- Projection to 14 TeV with 300 fb^{-1}
- Study invariant mass distribution of $e\mu$ pair

Comparison for Different Quarks

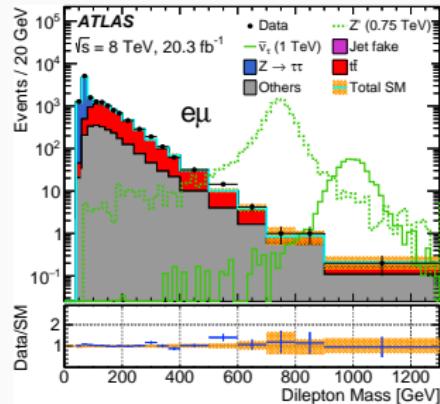
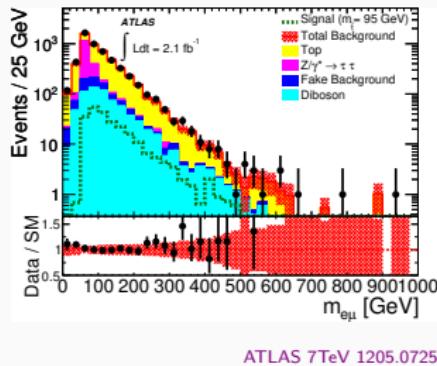


Production cross section normalised to same value for each quark.

ATLAS Searches



Backgrounds



- **Main backgrounds:** $t\bar{t}$, WW , $Z/\gamma^* \rightarrow \tau\tau$
also W/Z plus jets, WZ/ZZ , single top and $W/Z + \gamma$
- ⇒ Efficiently reduced in exclusive 7 TeV analysis by rejecting jets and $E_T^{miss} < 20 \text{ GeV}$
- **Limit setting:** Maximum likelihood estimator for 7 and 8 TeV and estimate significance for 14 TeV.

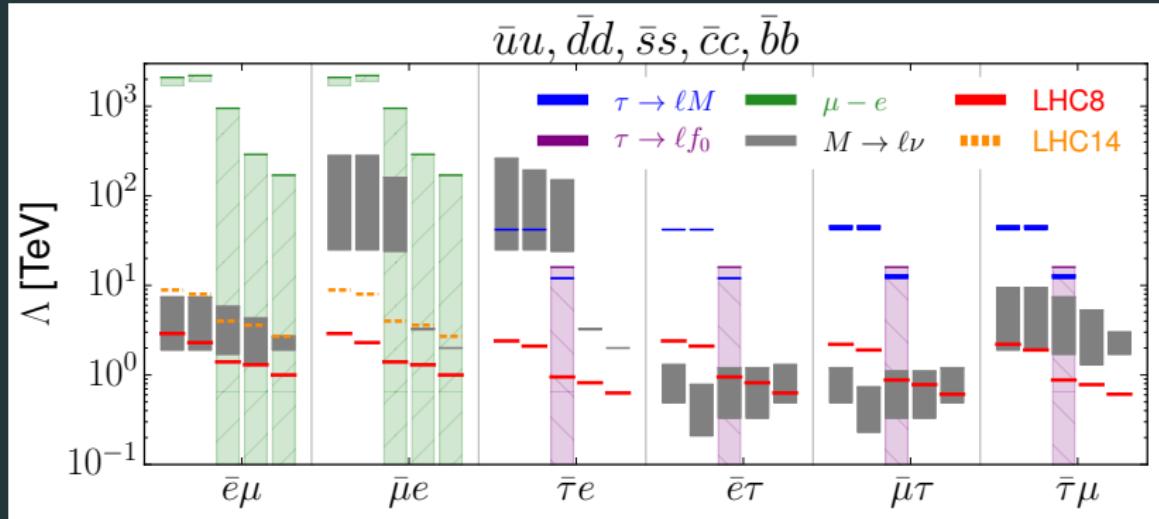
Limits from LHC on Cutoff Scale in TeV

$\bar{q}q$	$\bar{l}_i l_j$	$\bar{e}\mu$	$\bar{e}\tau$	$\bar{\mu}\tau$
	7 TeV	8 TeV	14 TeV	8 TeV
$\bar{u}u$	2.6	2.9	8.9	2.4
$\bar{d}d$	2.3	2.3	8.0	2.1
$\bar{s}s$	1.1	1.4	4.0	0.95
$\bar{c}c$	0.97	1.3	3.6	0.82
$\bar{b}b$	0.74	1.0	2.7	0.63

- 8 TeV analysis gives only a slight improvement compared to 7 TeV despite 10 times more data because of large background
- $e\tau$ and $\mu\tau$ limits weaker than $e\mu$ because of low τ -tagging rate and higher fake background
- 14 TeV projection: same search strategy as 7 TeV exclusive search

Conclusions

Conclusions



Precision experiments win for light quarks

LHC competitive for heavy quarks and
right-handed τ -leptons

$\Lambda > 600 - 800$ GeV

Outlook

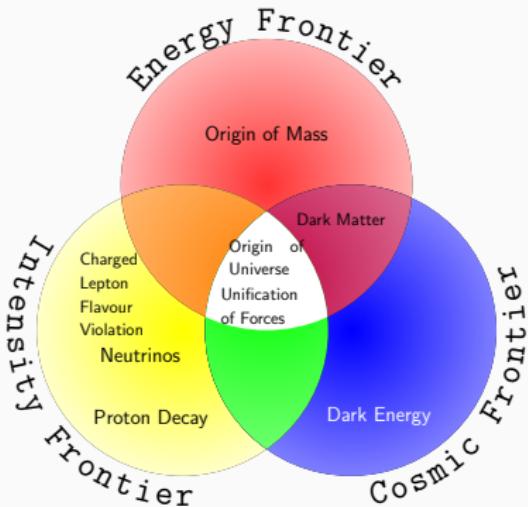
LHC more competitive for vector operators with right-handed quark currents

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$$\mathcal{Q}_{ed} = (\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma^\mu d)$$

$$\mathcal{Q}_{lu} = (\bar{L}\gamma_\mu L)(\bar{u}\gamma^\mu u)$$

$$\mathcal{Q}_{ld} = (\bar{L}\gamma_\mu L)(\bar{d}\gamma^\mu d)$$



Outlook

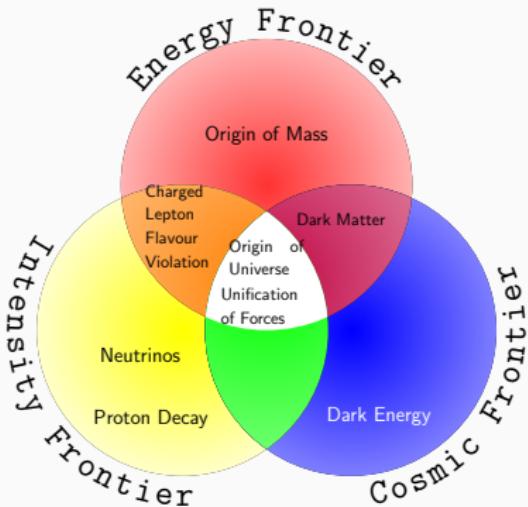
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Thank you!

Backup Slides

Selection Criteria 7 and 8 TeV

Same selection criteria as in ATLAS analysis.

- oppositely charged leptons
- Electrons:
 - $E_T > 25 \text{ GeV}$
 - tight identification criteria
 - $|\eta| < 1.37$ or $1.52 < |\eta| < 2.47$
- Muons: $p_T > 25 \text{ GeV}$, $|\eta| < 2.4$
- Tau: $E_T > 25 \text{ GeV}$, $0.03 < |\eta| < 2.47$
- Lepton isolation: scalar sum of lepton p_T within cone of $\Delta R = 0.2(0.4)$ is less than 10% (6%) of lepton p_T for 7 (8) TeV search
- Jets reconstructed anti- k_T algorithm with radius parameter 0.4
- 7 TeV analysis: jets rejected if $p_T > 30 \text{ GeV}$ or $E_T^{miss} < 25 \text{ GeV}$
- Invariant mass of lepton pair: $> 100(200) \text{ GeV}$ in 7(8) TeV analysis
- azimuthal angle difference $\Delta\phi > 3(2.7)$ in 7 (8) TeV analysis

Selection Criteria 14 TeV

Cuts follow 7 TeV inclusive analysis.

In addition

- $p_T(\ell) > 300 \text{ GeV}$
- $E_T^{miss} < 20 \text{ GeV}$

Limit setting

7 and 8 TeV

- Maximum likelihood estimator for 7 and 8 TeV

$$\mathcal{L}_i(\mu, \tilde{\theta}_i | n_i) = \mathcal{P}(n_i | \mu s_i + b_i) \mathcal{G}(\tilde{\theta}_i, 0, 1)$$

\mathcal{P} is Poisson function and \mathcal{G} Gaussian function

- SM background and observed events taken from ATLAS publications
- Total likelihood function is product

$$\mathcal{L} = \prod_i \mathcal{L}_i$$

14 TeV

- Estimate reach for 14 TeV using

$$\text{Significance} = \frac{S}{\sqrt{S + (\Delta S)^2 + (\Delta B)^2}}$$

with $\Delta S = 10\%S$ and $\Delta B = 10\%B$.