Theoretical uncertainty
of the supersymmetric dark matter relic density
from scale and scheme variations

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I. Motivation & Introduction
dark matter relic density allows for constraining the MSSM parameter space strongly

precise determination of the relic density by PLANCK to

$$\Omega_{\text{CDM}} h^2 = 0.1198 \pm 0.0015$$


• need of precise theoretical prediction to meet experimental precision
• measure of theoretical uncertainty needed for a realistic estimation

Minimal Supersymmetric Standard Model (MSSM)

assume lightest neutralino $\tilde{\chi}_1^0$ being the LSP and thus the DM candidate

$\Omega_{\text{CDMtheor.}}$
Theoretical Prediction of the Relic Density

- Number density of DM in the early universe can be described by the Boltzmann equation:

\[ \dot{n} + 3Hn = -\langle \sigma_{\text{eff}}v \rangle (n^2 - n_{eq}^2) \]

1) Thermal equilibrium regime (T >> m)
   - Annihilation and production of DM in thermal equilibrium

2) Annihilation regime (T ~ m/10)
   - SM particles not energetic enough to create DM particles

3) Freeze-out (T ~ m/30)
   - Annihilation rate falls behind expansion rate → DM abundance

- Relic density proportional to cross section

\[ \Omega_\chi h^2 = \frac{n_\chi m_\chi}{\rho_{\text{crit}}} \propto \frac{1}{\langle \sigma_{\text{eff}}v \rangle} \]

Jungman, Kamionkowski, Griest, Phys. Reports 267 (1995)
number density of DM in the early universe can be described by the Boltzmann equation

\[ \dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2) \]

\[ \langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{i}^\text{eq}}{n_{\text{eq}}} \frac{n_{j}^\text{eq}}{n_{\text{eq}}} \]

with

\[ \frac{n_{i}^\text{eq}}{n_{\text{eq}}} \propto \exp\left(-\frac{(m_{i} - m_{\chi})}{T}\right) = \exp\left(-\frac{(m_{i} - m_{\chi})}{x m_{\chi}}\right) \]

- sizeable contributions of AX \( \rightarrow \) SM when particle A is almost degenerate in mass with particle X (DM candidate)
- various different processes can be important in the MSSM parameter space:
  \( \rightarrow \) pair annihilation of neutralinos
  \( \rightarrow \) coannihilation with other neutralinos, light stops, taus, etc…

we define

\[ \Delta M = \frac{m_{\tilde{t}_1} - m_{\tilde{\chi}_1}}{m_{\tilde{\chi}_1}} \]
Theoretical Uncertainties

Arising from Cosmology

- choice of cosmological model

- variation in hubble expansion rate

\[ H^2 = \frac{8\pi G}{2}(\rho_{\text{rad}} + \rho_D) \]

\[ \kappa_D = \frac{\rho_D(T_0)}{\rho_{\text{rad}}(T_0)} \]

\[ \Omega_{\text{LSP}} h^2 < 0.135 \]

- effective degrees of freedom of the Universe

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>Data set</th>
<th>( \Omega_{\text{LSP}} h^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>WMAP+SDSS</td>
<td>0.092 → 0.136</td>
</tr>
<tr>
<td>B</td>
<td>WMAP+SDSS+SNLS+BAO</td>
<td>0.100 → 0.123</td>
</tr>
<tr>
<td>C</td>
<td>WMAP+SDSS+SNLS+BAO</td>
<td>0.100 → 0.123</td>
</tr>
<tr>
<td>A</td>
<td>WMAP+SDSS+SNLS+BAO</td>
<td>0.094 → 0.136</td>
</tr>
</tbody>
</table>

Precision data from CMB measurements
PLANCK: ~ 1.5% uncertainty
Theoretical Uncertainties

Arising from Particle Physics

• determination of particle mass spectrum

• accuracy of effective cross section calculation
  MicrOMEGAs
  Belanger, Boudjema, et al., CPC (2002)
  DarkSUSY
  SuperIso Relic
  Arbey, Mamoudi, et al., CPC (2010)
  MadDM

For enhancing the precision and getting a first quantitative estimate of the theoretical predicted relic density
→ calculate SUSY-QCD NLO corrections to $\sigma_{\text{eff}}$
II. Processes & Technicalities
Classes of Diagrams

- Calculating SUSY-QCD corrections to all relevant processes for the relic density calculation
- providing a tool which extends public tools like micrOMEGAs (and DarkSUSY)
aim: Renormalisation scheme which is valid over a wide parameter space for all (co)annihilation processes

- relevant parameters: $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_t, m_b, A_t, A_b, \theta_{\tilde{t}}, \theta_{\tilde{b}}$

relevant as incoming known masses crucial for coannihilating scenarios; as dependent parameter problem of large effects for large tanbeta

- hybrid on-shell / \overline{\text{DR}} renormalisation scheme

\[
\begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{b}_2}^2 \end{pmatrix} = U^q \begin{pmatrix} M_Q^2 + (I_{\tilde{t}_1}^{3L} - e_q s_W^2) \cos 2\beta m_Z^2 + m_q^2 \\ m_q (A_q - \mu (\tan \beta)^{-2} I_{\tilde{t}_1}^{3L}) \end{pmatrix} (U^q)^\dagger
\]

- stable choice important

\[
\delta \theta_{\tilde{t}} \propto \frac{1}{(U_{21}^q U_{12}^q + U_{11}^q U_{22}^q)}
\]
→ What is the theoretical uncertainty of the calculation?

• loop calculation contains explicit uncanceled logs of the renormalisation scale as well as scale implicitly scale dependent parameter $\alpha_s, \theta_{\tilde{t}}, \theta_{\tilde{b}}, A_t, A_b, m_b, m_{\tilde{t}_2}$

• scale variation gives an estimate of the uncertainty of the calculation $\frac{1}{2} \mu_R < \mu < 2 \mu_R$

$\alpha_s$ less scale dependent as in TeV range

trilinear couplings more scale dependent

first quantitative study of theoretical uncertainty of Neutralino relic density calculation possible
III. Results & Phenomenology
Example Scenarios

For studying the uncertainty from scale & scheme variations we define 3 example scenarios:

**gaugino annihilation**

default case with interesting features depending on final state

<table>
<thead>
<tr>
<th>Annihilation Channel</th>
<th>Scenario A</th>
<th>Scenario B</th>
<th>Scenario C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\chi}_1^0 \chi_1^0 \rightarrow tt$</td>
<td>2%</td>
<td>9%</td>
<td>16%</td>
</tr>
<tr>
<td>$\tilde{\chi}_2^0 \chi_2^0 \rightarrow tt$</td>
<td>3%</td>
<td>23%</td>
<td></td>
</tr>
<tr>
<td>$\tilde{\chi}_1^0 \chi_1^\pm \rightarrow tb, tb$</td>
<td>43%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{\chi}_1^0 t_1 \rightarrow th^0$</td>
<td></td>
<td>1%</td>
<td>23%</td>
</tr>
<tr>
<td>$tg$</td>
<td></td>
<td>6%</td>
<td>23%</td>
</tr>
<tr>
<td>$tZ^0$</td>
<td></td>
<td>5%</td>
<td>11%</td>
</tr>
<tr>
<td>$bW^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_1 t_1^* \rightarrow h^0 h^0$</td>
<td></td>
<td>12%</td>
<td>5%</td>
</tr>
<tr>
<td>$h^0 H^0$</td>
<td></td>
<td>11%</td>
<td></td>
</tr>
<tr>
<td>$Z^0 A^0$</td>
<td></td>
<td>7%</td>
<td></td>
</tr>
<tr>
<td>$W^\pm H^\mp$</td>
<td></td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td>$Z^0 Z^0$</td>
<td></td>
<td>8%</td>
<td>2%</td>
</tr>
<tr>
<td>$W^+ W^-$</td>
<td></td>
<td>14%</td>
<td>3%</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>80%</td>
<td>72%</td>
</tr>
</tbody>
</table>

**Neutralino-stop Coannihilation**

study of scheme dependence on cross section and combination of all channels

**stop-antistop annihilation**

receives huge corrections due to Coulomb enhancement effects

| Scenario | $m_{\tilde{\chi}_1^0}$ | $m_{\tilde{\chi}_2^0}$ | $m_{\tilde{\chi}_1^\pm}$ | $m_{\tilde{\chi}_2^\pm}$ | $m_{\chi_1^0}$ | $m_{\chi_2^0}$ | $m_{\chi_1^\pm}$ | $m_{\chi_2^\pm}$ | $m_{t_1}$ | $m_{t_2}$ | $m_{h^0}$ | $\Omega_{\tilde{\chi}_1^0 h^0} h^2$ | $\text{BR}(b \to s\gamma)$ |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|--------------|--------------|----------------|------------------|------------------|------------------|
| A        | 738.1          | 802.5          | 802.4          | 1295.3         | 1032.1         | 1682.0         | 126.5          | 0.1248         | 3.0 \cdot 10^{-4} |
| B        | 1306.3         | 1827.0         | 1827.2         | 2640.0         | 1361.7         | 2157.3         | 123.7          | 0.1134         | 3.1 \cdot 10^{-4} |
| C        | 338.3          | 1996.6         | 1996.7         | 2909.0         | 376.3          | 1554.0         | 121.7          | 0.1193         | 3.49 \cdot 10^{-4} |
Gaugino Annihilation

Tree level

- $\chi_0^0/\pm$ with $\gamma, W^\pm, Z^0$ and $q, q'$
- $H^\pm/\tilde{H}_i^0$
- $\tilde{q}, \tilde{q}'$

Virtual one-loop corrections

Real gluon emission processes
NLO value does not lie within LO uncertainty band ← pure electroweak process
• Larger LO uncertainty band ← scale dependent $m_b^{DR}$
• Smaller NLO uncertainty band ← scale dependence decreased by virtual corrections to $A^0_{bb}$ coupling

NLO calculation shows 10-15% correction with respect to default MicrOMEGAs value

• Tiny scale dependence at LO and NLO
• **Smaller** LO uncertainty band ⤷ scale independent $m_{t}^{OS}$
• **Larger** NLO uncertainty band ⤷ first appearance of scale dependent $\alpha_{S}$

→ only at NLO it becomes possible to quantify the theoretical error for the first time

Gaugino Annihilation

- NLO calculation shows 5-10% relative correction with respect to default MicrOMEGAs value
- including theoretical error, still shift in the parameter space with respect to the MO value
- first quantitative estimation of theoretical uncertainty

Stop-Antistop Annihilation

• during freeze-out stop-antistop pair is moving slowly
• exchange of $n$ gluons lead to a correction factor proportional to $\left(\frac{\alpha_s}{\mu}\right)^n$
• with $\alpha_s/\mu > \mathcal{O}(1)$ Coulomb corrections can be become sizeable

resummation to all orders within framework of nonrelativistic QCD


Would-be stoponium Schroedinger equation

$$\sigma^{\text{Coul.}}(\tilde{t}\tilde{\bar{t}} \to EW) = \frac{4\pi}{vm_{\tilde{t}}^2} \Im \left\{ G^{[1]}(r=0; \sqrt{s} + i\Gamma_{\tilde{t}} + \mu_C) \right\} \times \sigma^{\text{LO}} (\tilde{t}\tilde{\bar{t}} \to EW)$$

$$H^{[1]} - (\sqrt{s} + i\Gamma_{\tilde{t}}) \right] G^{[1]}(r; \sqrt{s} + i\Gamma_{\tilde{t}} + \mu_C) = \delta^{(3)}(r)$$

$$H^{[1]} = -\frac{1}{m_{\tilde{t}}} \Delta + 2m_{\tilde{t}} + V^{[1]}(r)$$

Coulomb potential @ NLO $\mathcal{O}\left((\alpha_s^2/\mu)^n\right)$

Stop-Antistop Annihilation

Scenario with a small mass gap between LSP and NLSP

- Scale dependence at LO triggered by trilinear coupling $A_t$
- Scale dependence at NLO due to explicit logarithms and implicit ($A_t \alpha_s$)
- Coulomb scale leads to 20% uncertainty

→ large correction (K-factor of 1-9) in relevant region, 20 % theoretical error

Stop-Antistop Annihilation

Scenario with a small mass gap between LSP and NLSP

- NLO calculation shows 50-60% relative difference with respect to default MicrOMEGAs value
- including theoretical error, still shift in the parameter space with respect to the MO value
- Mass shift of 30 GeV (2%) with an uncertainty of 0.5%

Neutralino-Stop Coannihilation

Tree level

Virtual one-loop corrections

Real gluon emission processes
• Scale dependence on LO triggered by trilinear coupling $A_t$
• Scale dependence at NLO due to explicit logarithmical and implicit ($A_t \alpha_s$)
• Scale uncertainty of 20%

→ K-factor of 1.05, correction of 40% w.r.t. MicrOMEGAs, 20% theoretical error

Effect of Scheme Variation ($m_{\text{top}}$)

Study effect of taking $m_{\text{top}}^{\text{DR}}$ instead of $m_{\text{top}}^{\text{OS}}$

- MO larger than LO cross section as MO uses effective top mass
- Larger K-factor of 1.4 in contrast to our scheme with 1.05

\[ \sigma_{\text{tree}} (\sigma_{\text{tree}}/\sigma_{\text{MO}}) \]
\[ \sigma_{\text{MO}} (\sigma_{\text{NLO}}/\sigma_{\text{MO}}) \]
\[ \sigma_{\text{NLO}} (\sigma_{\text{NLO}}/\sigma_{\text{tree}}) \]

\[ \rightarrow \text{Confirms our choice of taking top mass on-shell} \]

Relic density in the combined Scenario

- \tilde{\chi}\tilde{\chi}: K-factor of 1.15, 5-10% correction w.r.t. MO, 5% scale uncertainty (A)
- \tilde{\chi}_1^0\tilde{t}_1: K-factors of 1.05-1.5, 15% correction w.r.t. MO, 20% scale uncertainty (C)
- \tilde{t}_1\tilde{t}_1^*: K-factors of 1-9, up to 16% correction w.r.t. MO, 20% scale uncertainty (B)

Correction of almost 30% w.r.t. MO relic density calculation

Conclusions

• We presented the first quantitative estimate of the theoretical uncertainty coming from scale and scheme variations.

• Renormalisation scale dependence can differ significantly, however, usually reduced at NLO.

• We could demonstrate an enhanced stability in our mixed renormalisation scheme in comparison to pure \( \overline{\text{DR}} \).

• SUSY-QCD corrections induce important shifts up to 50% (\( \rightarrow \) Coulomb enhancement!).

• Theoretical uncertainty should be more reliably estimated to be six times larger than the experimental one.

• When extracting the pMSSM parameters from relic density measurement, shifts and uncertainties of a few percent have to be taken into account.
Thank you!