Theoretical uncertainty of the supersymmetric dark matter relic density from scale and scheme variations

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based on Phys. Rev. D 93, 114023 (2016), arXiv:1602.08103 [hep-ph]

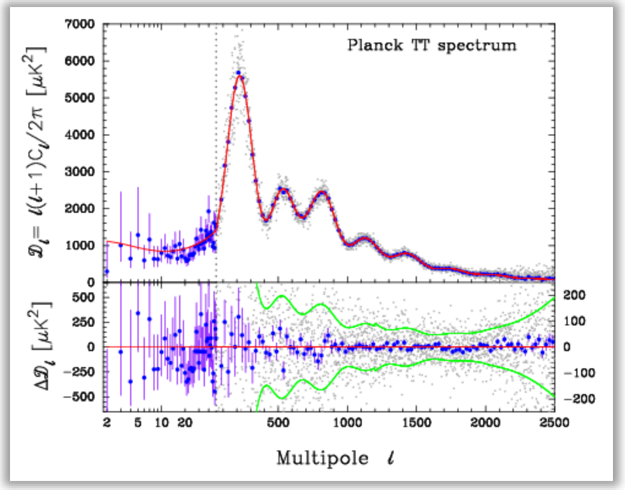
SUSY 2016, Melbourne

04/07/2016

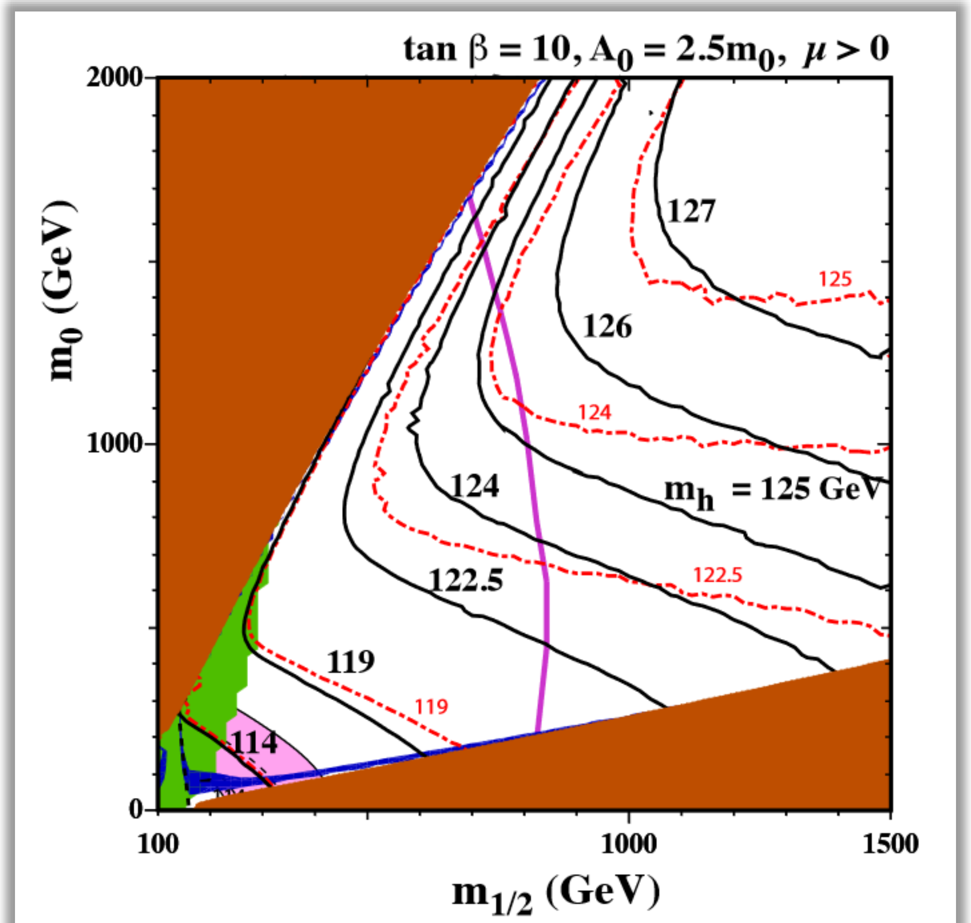
I. Motivation & Introduction

Constrain Parameter Space by DM Relic Density

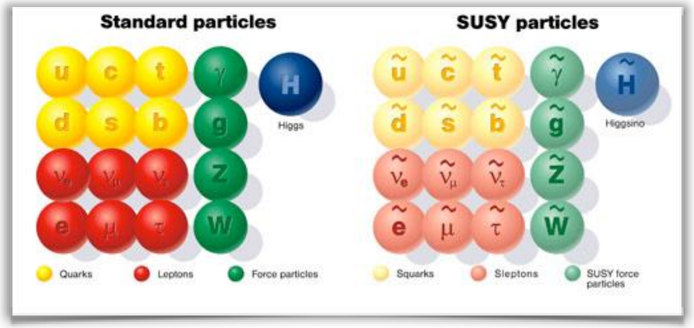
dark matter relic density allows for constraining the MSSM parameter space strongly



Planck Collaboration, arXiv:1303.5076



J. Ellis, K. Olive, J. Zheng, Eur. Phys. J. C74 (2014) 2947



Minimal Supersymmetric Standard Model (MSSM)

precise determination of the relic density by PLANCK to

$$\Omega_{\mathrm{CDM}} h^2 = 0.1198 \pm 0.0015$$

P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.01589 [astro-ph.CO]

assume lightest neutralino $\tilde{\chi}_1^0$ being the LSP and thus the DM candidate

$$\Omega_{\mathrm{CDM theoret.}}$$

- need of precise theoretical prediction to meet experimental precision
- measure of theoretical uncertainty needed for a realistic estimation

Theoretical Prediction of the Relic Density

- number density of DM in the early universe can be described by the Boltzmann equation

$$\dot{n} + 3Hn = -\langle\sigma_{\rm eff} v\rangle(n^2 - n_{\rm eq}^2)$$

  1) Thermal equilibrium regime (T>>m)

  annihilation and production of DM in thermal equilibrium

  2) Annihilation regime (T ~ m/10)

  SM particles not energetic enough to create DM particles

  3) Freeze-out (T~m/30)

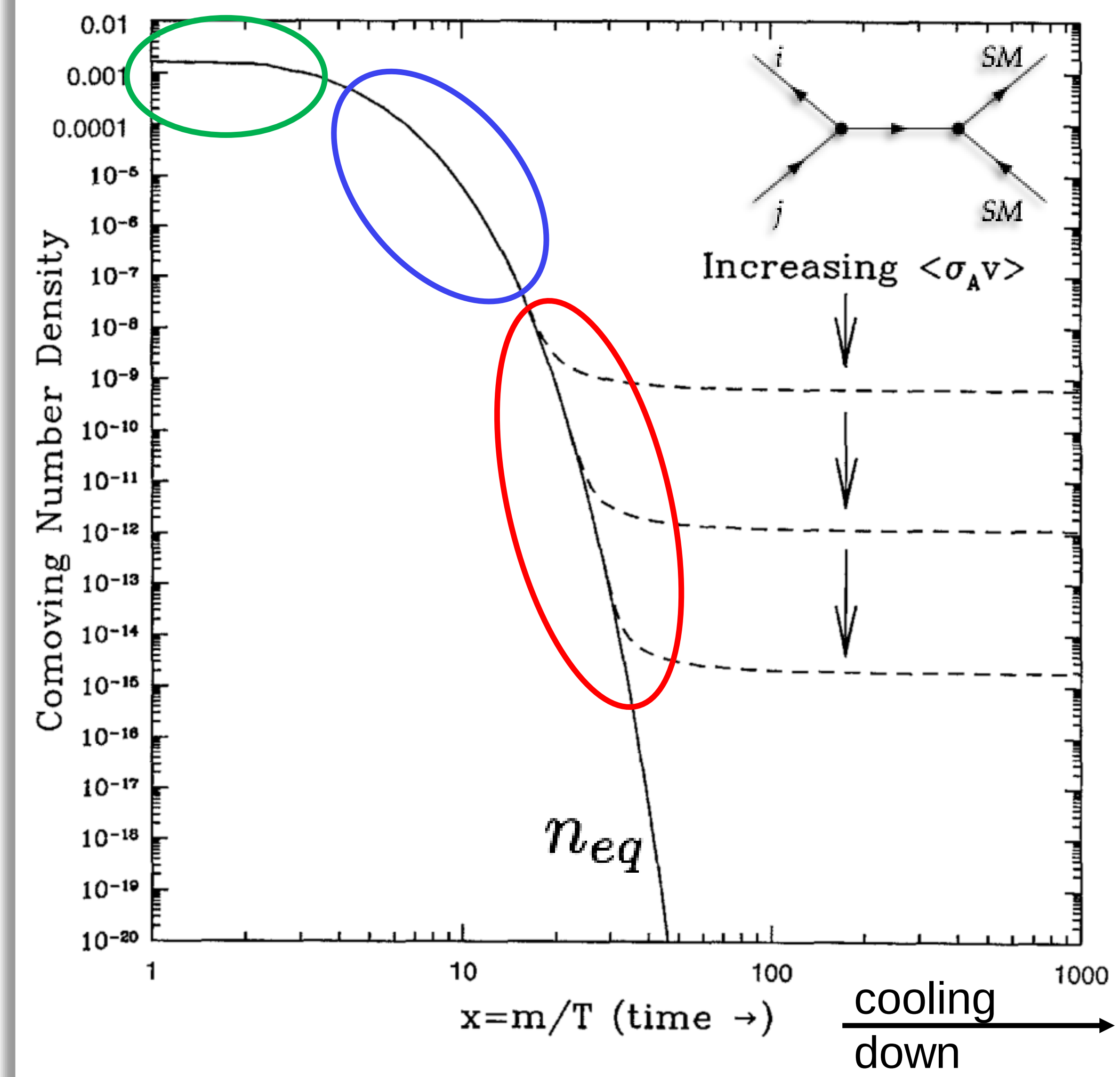
  Annihilation rate falls behind expansion rate → DM abundance

- relic density proportional to cross section

$$\Omega_\chi h^2 = \frac{n_\chi m_\chi}{\rho_{\rm crit}} \propto \frac{1}{<\sigma_{\rm eff} v>}$$

particle physics

Jungman, Kamionkowski, Griest, Phys. Reports 267 (1995)



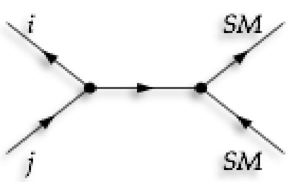
$\chi\,\chi \leftrightarrow$ SM SM $\quad \chi\,\chi \rightarrow$ SM SM $\quad \chi\,\chi \not\rightarrow$ SM SM

Theoretical Prediction of the Relic Density

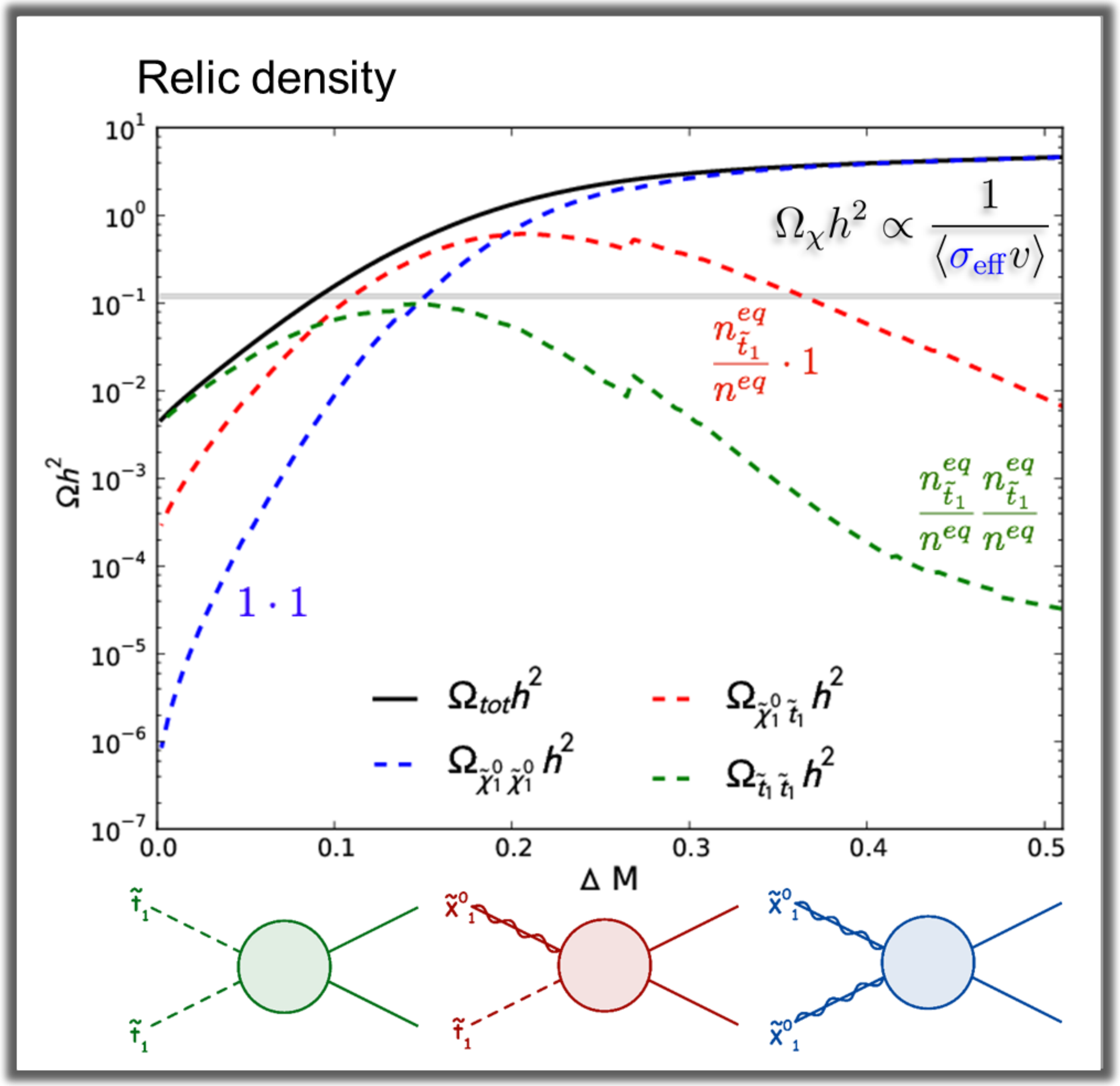
number density of DM in the early universe can be described by the Boltzmann equation

$$\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2)$$

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}}$$



with $\frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}} = \exp^{\frac{-(m_i - m_\chi)}{x \ m_\chi}}$



- sizeable contributions of AX → SM when particle A is almost degenerate in mass with particle X (DM candidate)
- various different processes can be important in the MSSM parameter space:
  → pair annihilation of neutralinos
  → coannihilation with other neutralinos, light stops, taus, etc…

we define $\Delta M = \frac{m_{\tilde{t}_1} - m_{\tilde{\chi}_1}}{m_{\tilde{\chi}_1}}$

Theoretical Uncertainties

Arising from Cosmology

- choice of cosmological model

  Hamann, Hannestad, et al. , Phys. Rev. D (2007)

Parameter set	Data set	$\Omega_c h^2$
B	WMAP+SDSS	$0.092 \rightarrow 0.136$
B	WMAP+SDSS+SNLS+BAO	$0.100 \rightarrow 0.123$
C	WMAP+SDSS+SNLS+BAO	$0.100 \rightarrow 0.123$
A	WMAP+SDSS+SNLS+BAO	$0.094 \rightarrow 0.136$

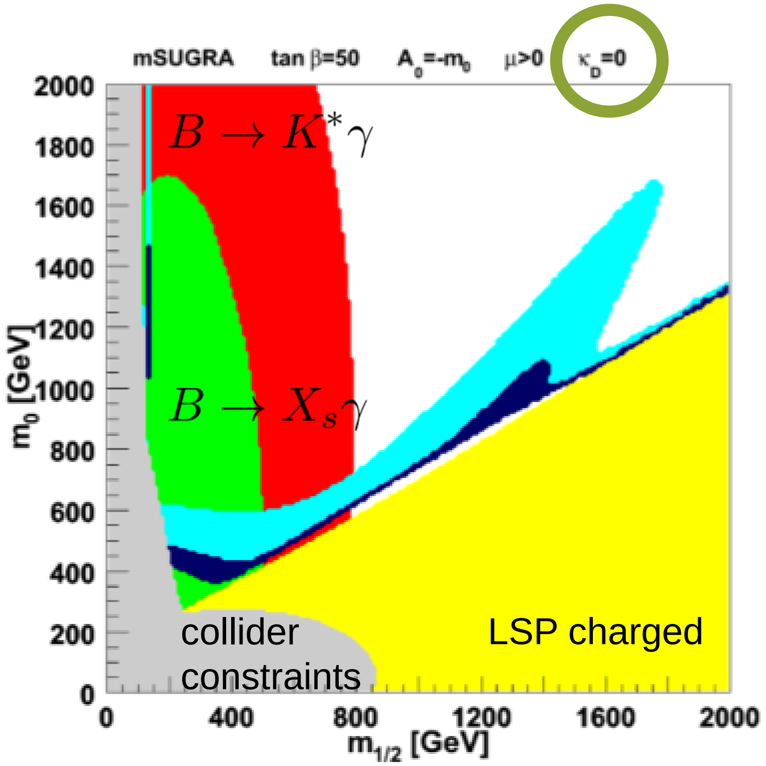
- variation in hubble expansion rate

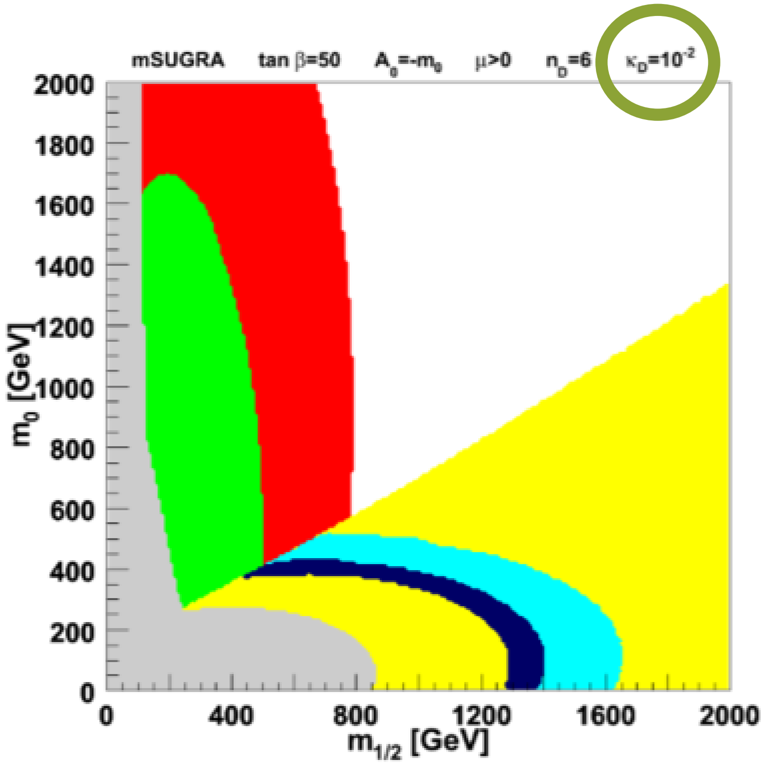
  Arbey, Mahmoudi, Phys. Lett. B (2008)

$$H^2 = \frac{8\pi G}{2}(\rho_{\rm rad} + \rho_D)$$

$$\kappa_D \equiv \frac{\rho_D(T_0)}{\rho_{\rm rad}(T_0)}$$

$$\Omega_{\rm LSP} h^2 < 0.135$$





- effective degrees of freedom of the Universe

  Hindmarsh, Philipsen, Phys. Rev. D (2005)

Precision data from CMB measurements
PLANCK: ~ 1.5% uncertainty

Theoretical Uncertainties

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\mathrm{eff}} v \rangle}$$

particle physics

Arising from Particle Physics

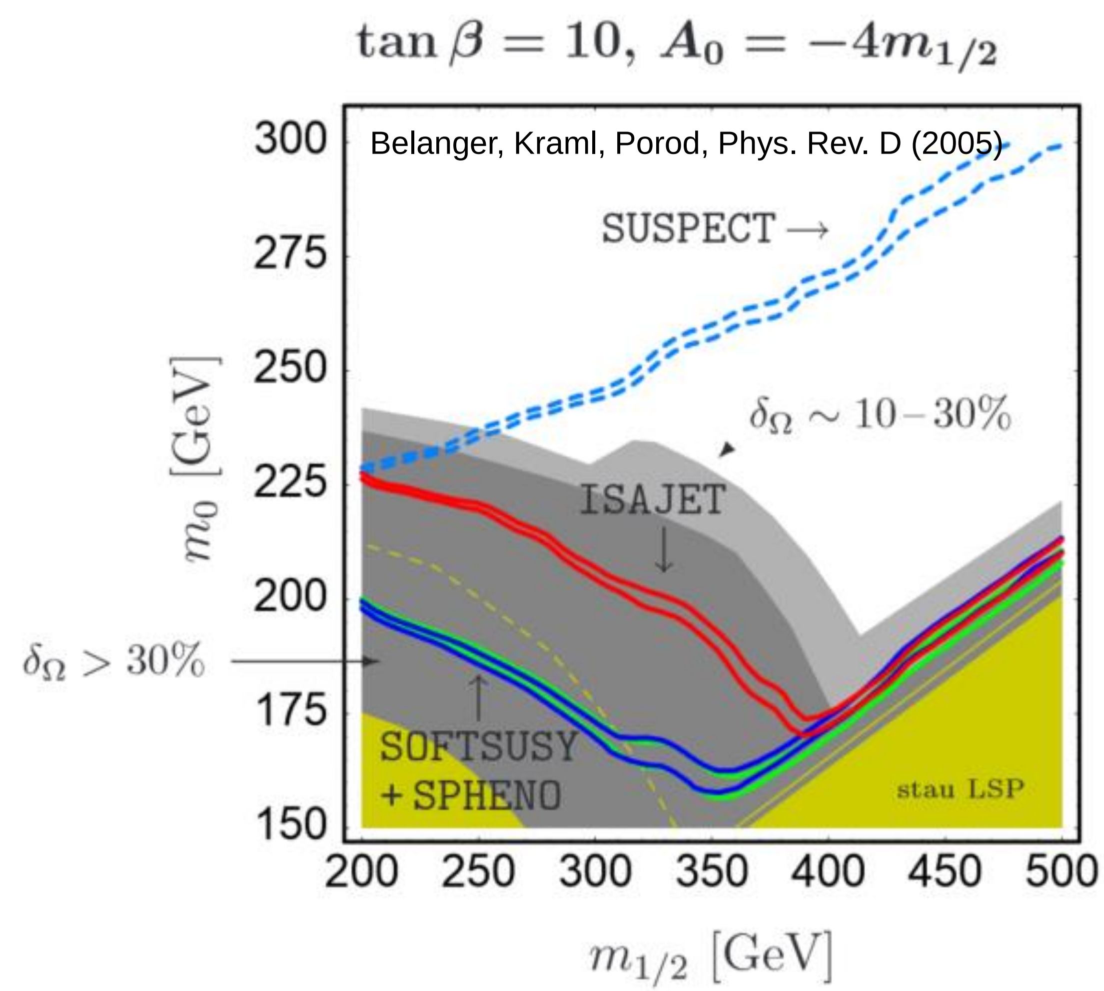
- determination of particle mass spectrum

  Allanach, Kraml, Porod, JHEP (2003)
  Allanch, Belanger, JHEP (2004)
  Belanger, Kraml, Porod, Phys. Rev. D (2005)

- accuracy of effective cross section calculation

MicrOMEGAs Belanger, Boudjema, et al. , CPC (2002)	DarkSUSY Gondolo, Edsjö, et al. , JCAP (2004)
SuperIso Relic Arbey, Mamoudi, et al. , CPC (2010)	MadDM Backovic, Kong, et al. , (2013)

tools use so far just (effective) tree level calculation

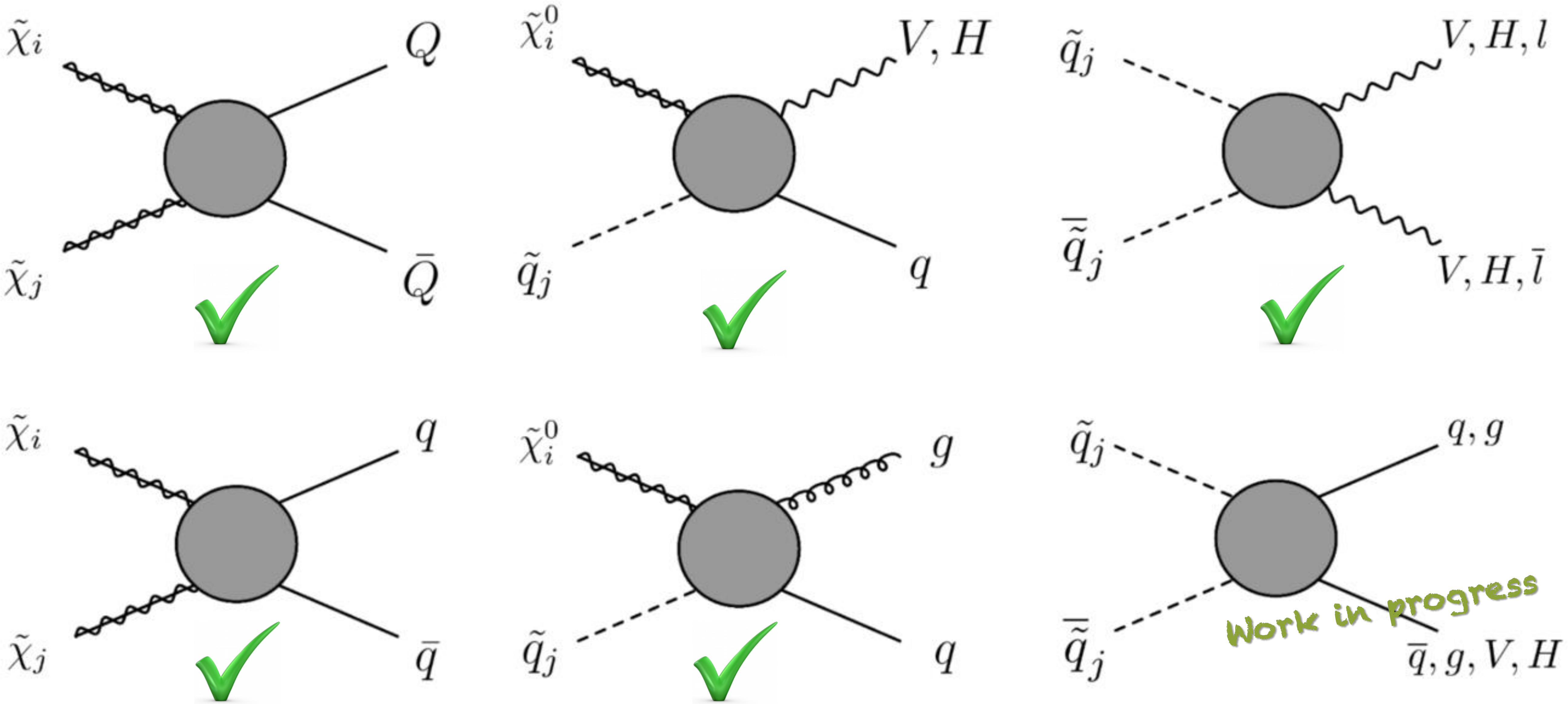


For enhancing the precision and getting a first quantitative estimate of the theoretical predicted relic density
→ calculate SUSY-QCD NLO corrections to σ_{eff}

DM@NLO

II. Processes & Technicalities

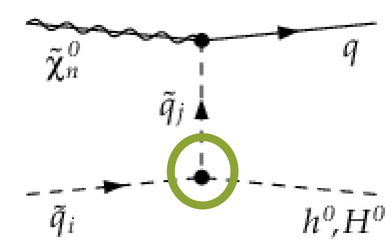
Classes of Diagrams



- Calculating SUSY-QCD corrections to all relevant processes for the relic density calculation
- providing a tool which extends public tools like micrOMEGAs (and DarkSUSY)

Renormalisation

aim: Renormalisation scheme which is valid over a wide parameter space for all (co)annihilation processes

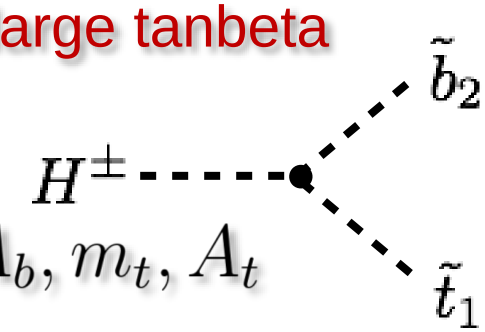


- relevant parameters: $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_t, m_b, A_t, A_b, \theta_{\tilde{t}}, \theta_{\tilde{b}}$

relevant as incoming (co)annihilating particles

known masses

crucial for coannihilating scenarios; as dependent parameter problem of large effects for large tanbeta



$\sim m_b, A_b, m_t, A_t$

- hybrid on-shell / $\overline{\text{DR}}$ renormalisation scheme

$$m_{\tilde{t}_1}^{\text{OS}}, m_{\tilde{b}_1}^{\text{OS}}, m_{\tilde{b}_2}^{\text{OS}} \quad A_t^{\overline{\text{DR}}}, A_b^{\overline{\text{DR}}} \quad m_t^{\text{OS}}, m_b^{\overline{\text{DR}}} \;\Longrightarrow\; m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$$

input parameters — dependent parameters

$$\begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix} = U^{\tilde{q}} \begin{pmatrix} M_{\tilde{Q}}^2 + (I_q^{3L} - e_q s_W^2) \cos 2\beta \, m_Z^2 + m_q^2 & m_q \left(A_q - \mu (\tan\beta)^{-2I_q^{3L}}\right) \\ m_q \left(A_q - \mu (\tan\beta)^{-2I_q^{3L}}\right) & M_{\{\tilde{U},\tilde{D}\}}^2 + e_q s_W^2 \cos 2\beta \, m_Z^2 + m_q^2 \end{pmatrix} (U^{\tilde{q}})^\dagger$$

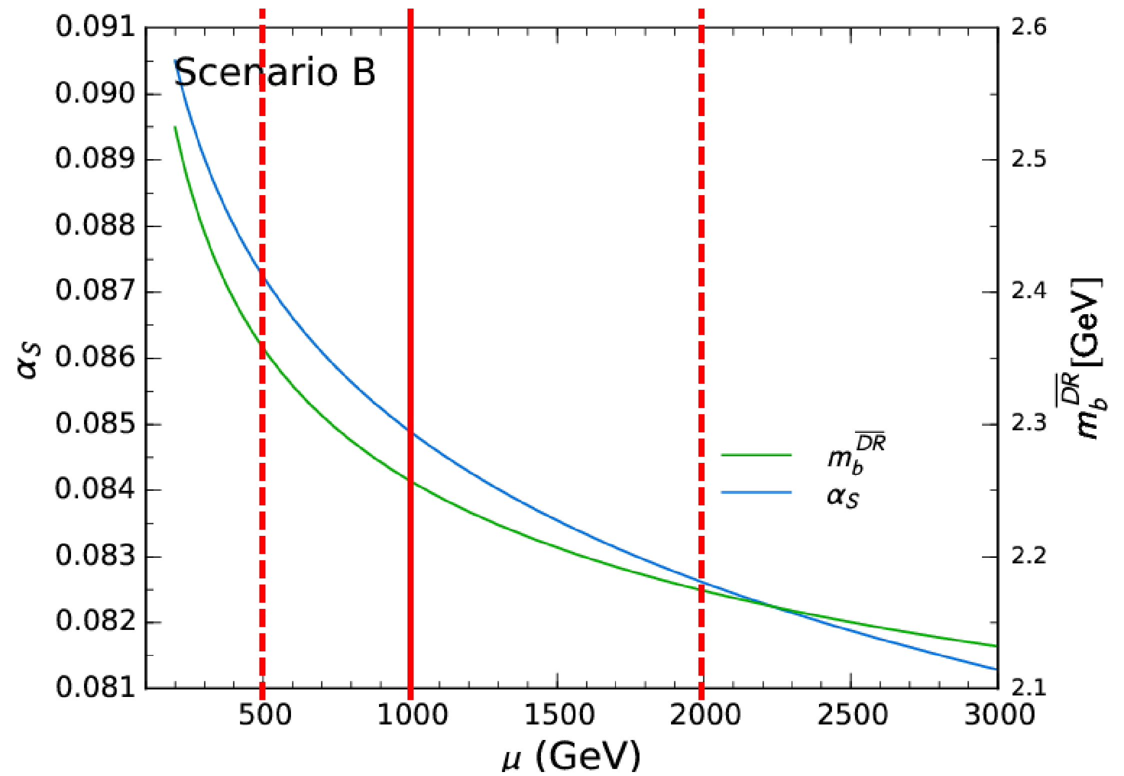
- stable choice important

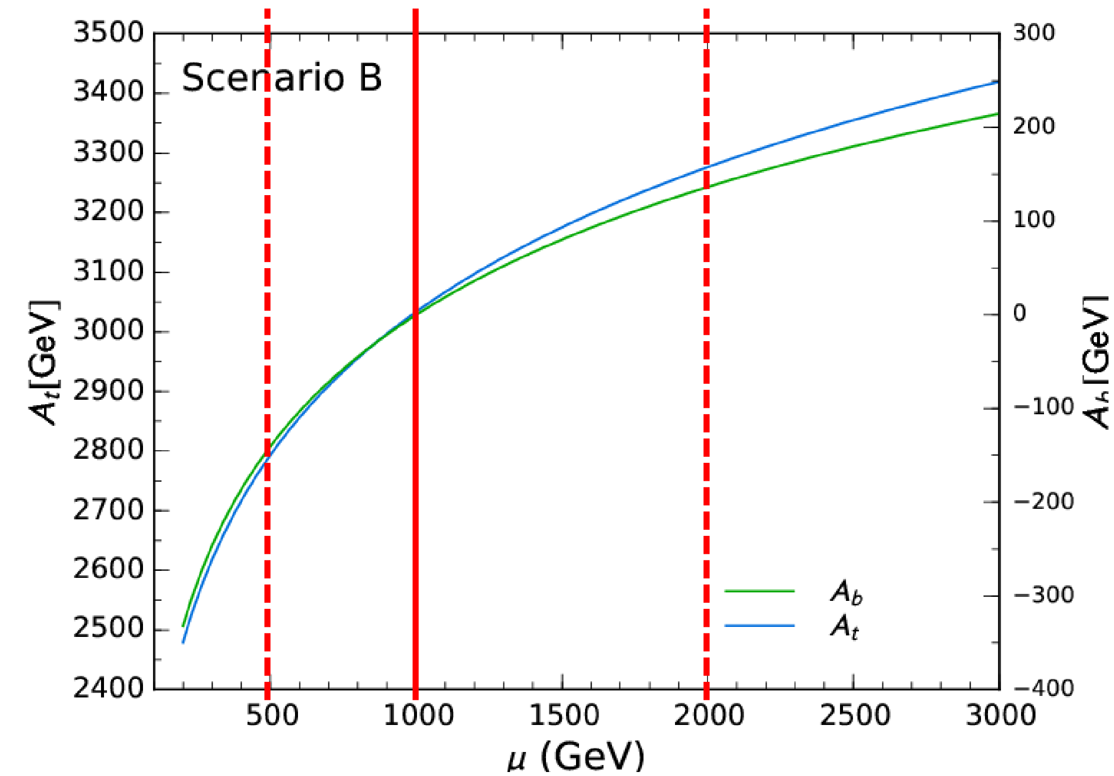
$$\delta\theta_{\tilde{q}} \propto \frac{1}{(U_{21}^{\tilde{q}} U_{12}^{\tilde{q}} + U_{11}^{\tilde{q}} U_{22}^{\tilde{q}})}$$

Estimate Theoretical Uncertainty

→ What is the theoretical uncertainty of the calculation?

- loop calculation contains explicit uncanceled logs of the renormalisation scale as well as scale implicitly scale dependent parameter $\alpha_s, \theta_{\tilde{t}}, \theta_{\tilde{b}}, A_t, A_b, m_b, m_{\tilde{t}_2}$
- scale variation gives an estimate of the uncertainty of the calculation $\frac{1}{2}\mu_R < \mu < 2\mu_R$





α_s less scale dependent as in TeV range

trilinear couplings more scale dependent

first quantitative study of theoretical uncertainty of Neutralino relic density calculation possible

III. Results & Phenomenology

Example Scenarios

For studying the uncertainty from scale & scheme variations we define 3 example scenarios:

gaugino annihilation

default case with interesting features depending on final state

		A	B	C
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to$	$t\bar{t}$	2%		16%
	$b\bar{b}$	9%		
$\tilde{\chi}_1^0\tilde{\chi}_2^0 \to$	$t\bar{t}$	3%		
	$b\bar{b}$	23%		
$\tilde{\chi}_1^0\tilde{\chi}_1^\pm \to$	$t\bar{b}, \bar{t}b$	43%		
$\tilde{\chi}_1^0\tilde{t}_1 \to$	th^0		1%	23%
	tg		6%	23%
	tZ^0			5%
	bW^+			11%
$\tilde{t}_1\tilde{t}_1^* \to$	h^0h^0		12%	5%
	h^0H^0		11%	
	Z^0A^0		7%	
	$W^\pm H^\mp$		13%	
	Z^0Z^0		8%	2%
	W^+W^-		14%	3%
Total		80%	72%	88%

Neutralino-stop Coannihilation

study of scheme dependence on cross section and combination of all channels

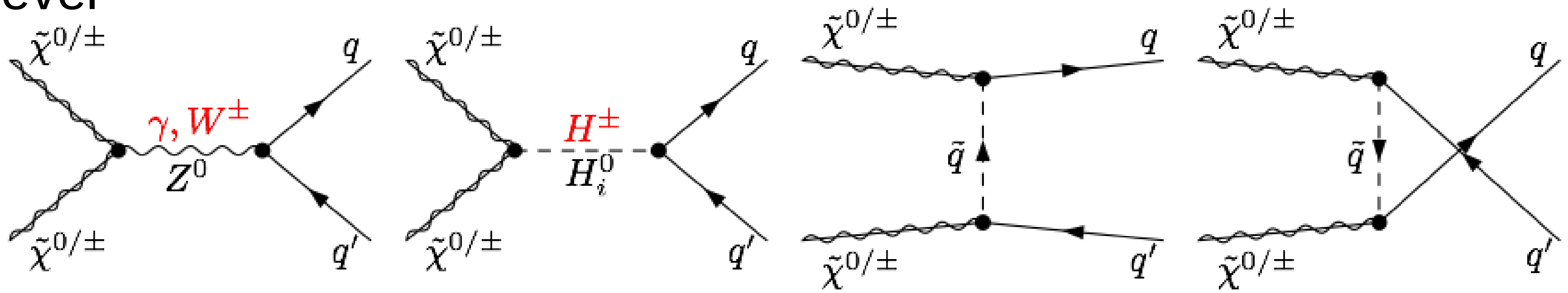
stop-antistop annihilation

receives huge corrections due to Coulomb enhancement effects

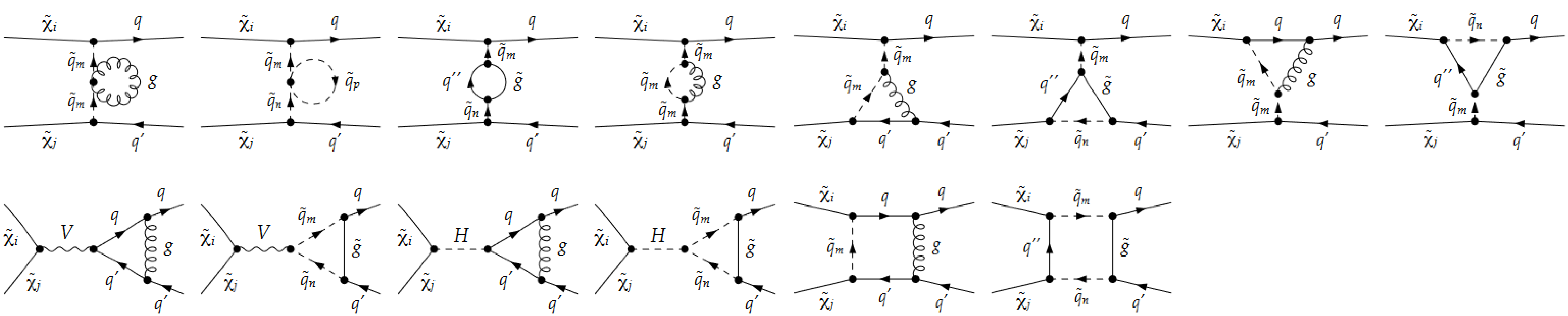
	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	m_{h^0}	$\Omega_{\tilde{\chi}_1^0}h^2$	$\mathrm{BR}(b \to s\gamma)$
A	738.1	802.5	802.4	1295.3	1032.1	1682.0	126.5	0.1248	$3.0 \cdot 10^{-4}$
B	1306.3	1827.0	1827.2	2640.0	1361.7	2157.3	123.7	0.1134	$3.1 \cdot 10^{-4}$
C	338.3	1996.6	1996.7	2909.0	376.3	1554.0	121.7	0.1193	$3.49 \cdot 10^{-4}$

Gaugino Annihilation

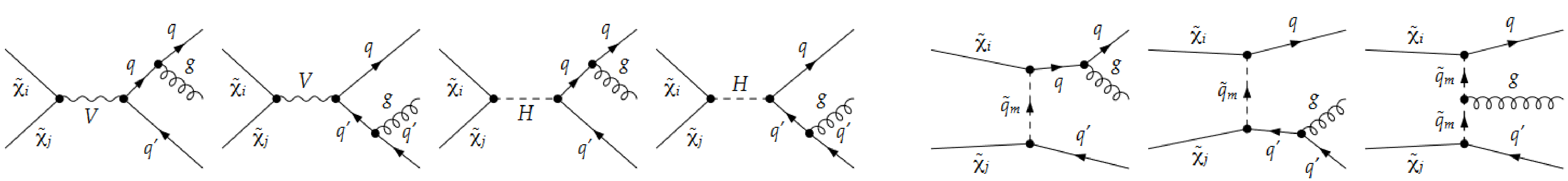
Tree level



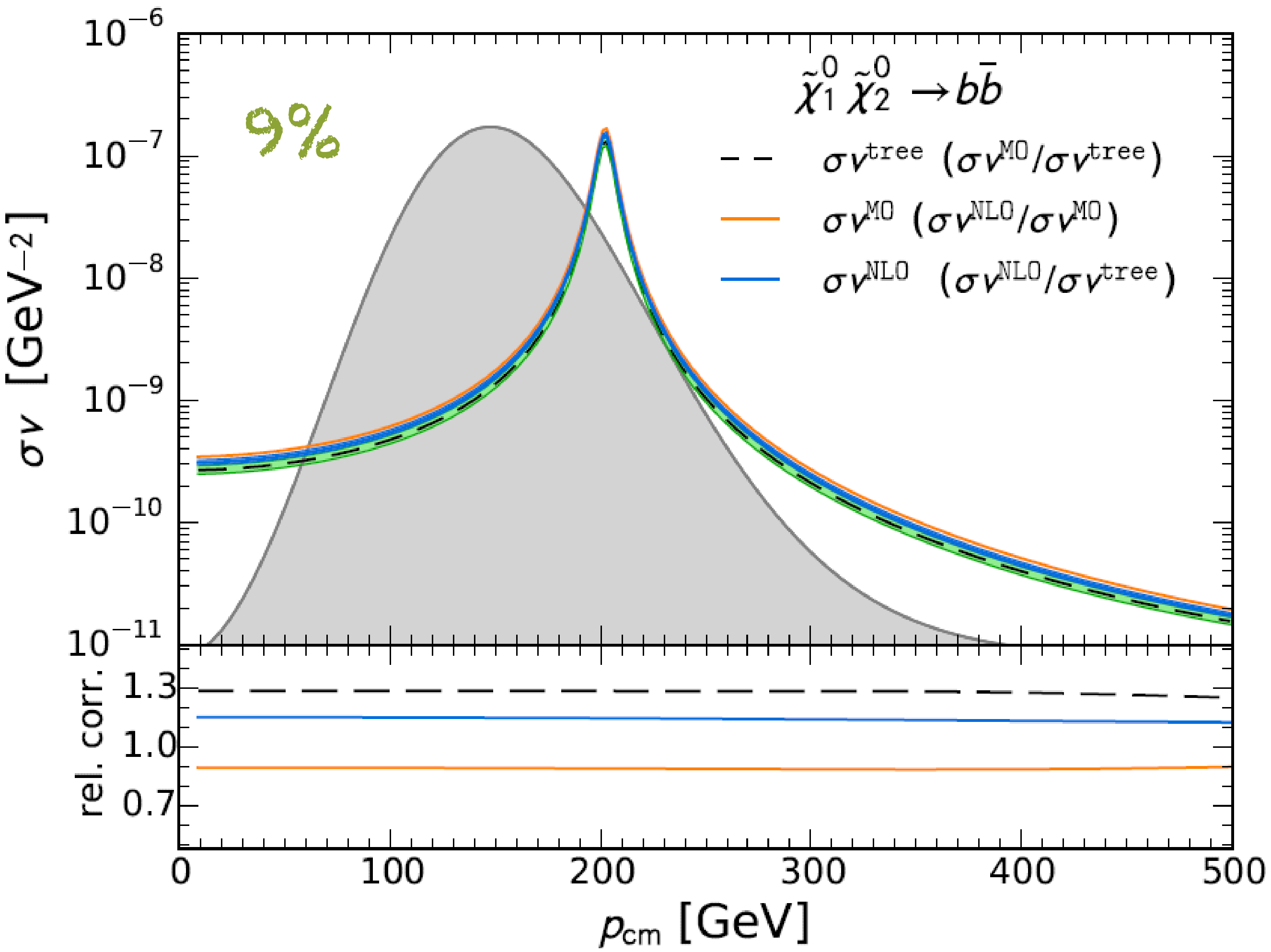
Virtual one-loop corrections

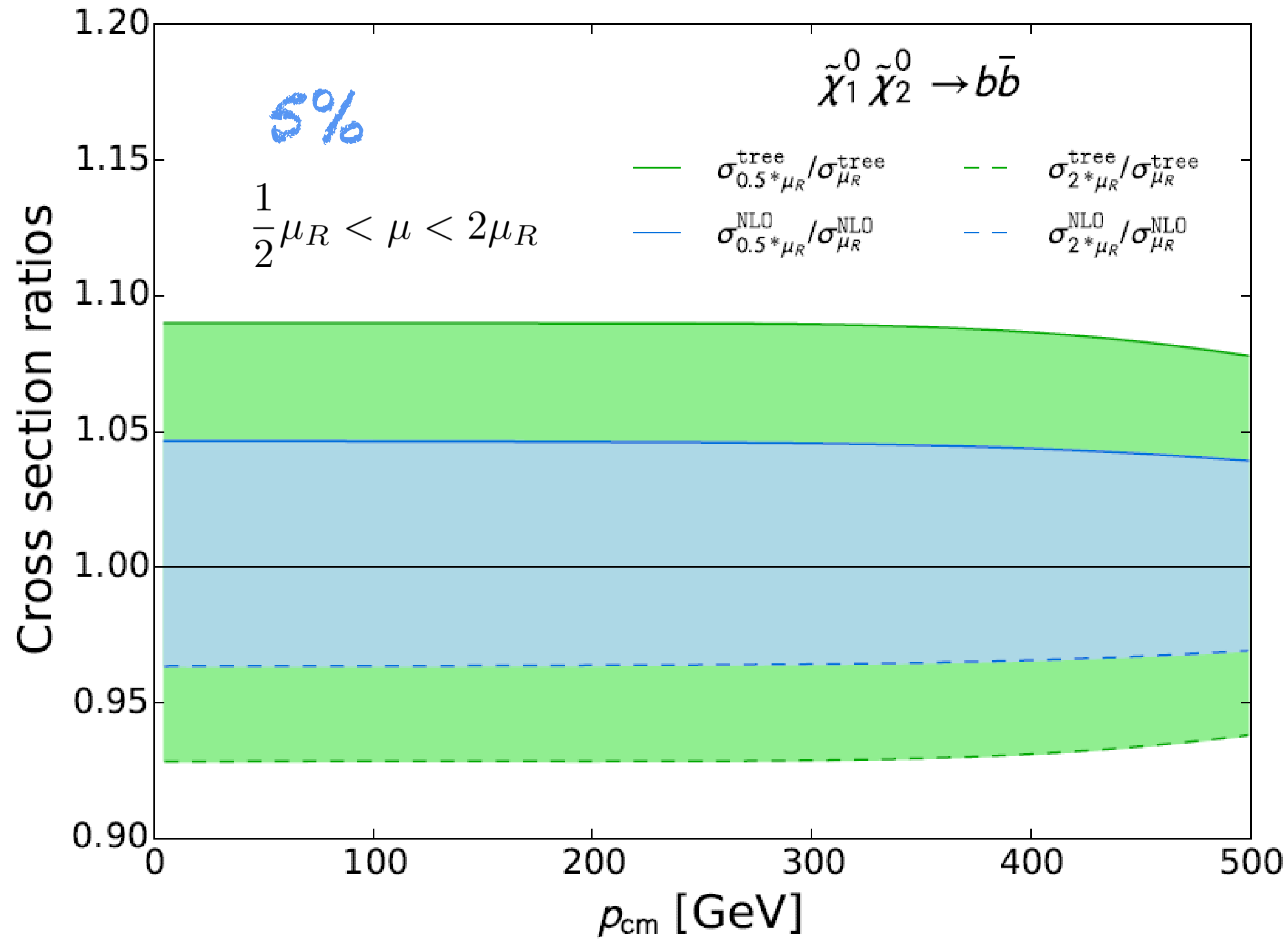


Real gluon emission processes

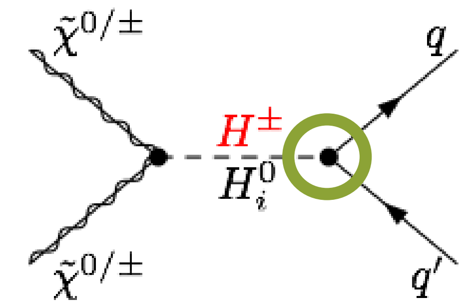


Gaugino Annihilation





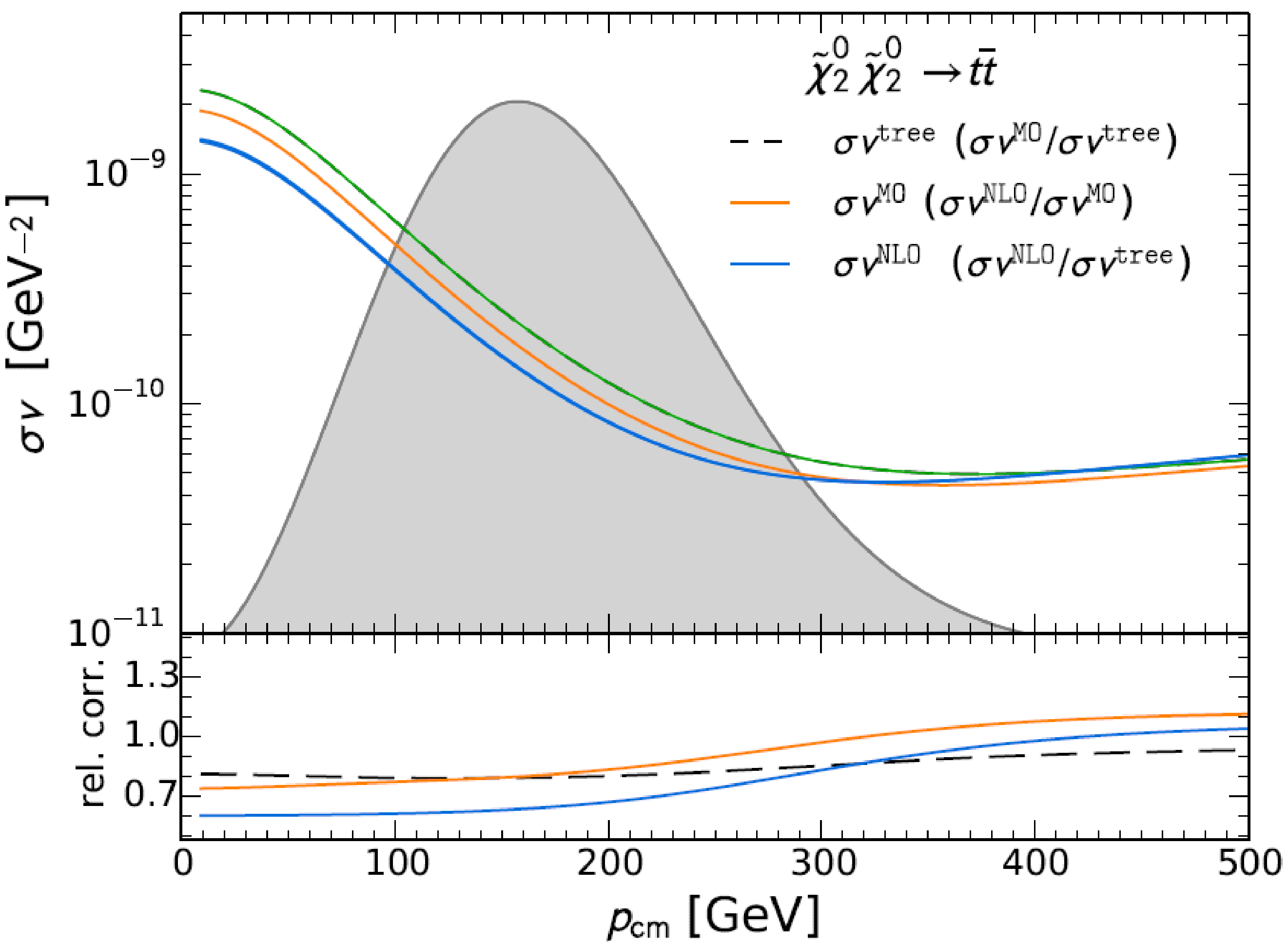
- NLO value does not lie within LO uncertainty band ← pure electroweak process
- **Larger** LO uncertainty band ← scale dependent $m_b^{\overline{DR}}$
- **Smaller** NLO uncertainty band ← scale dependence decreased by virtual corrections to $A^0 b\bar{b}$ coupling

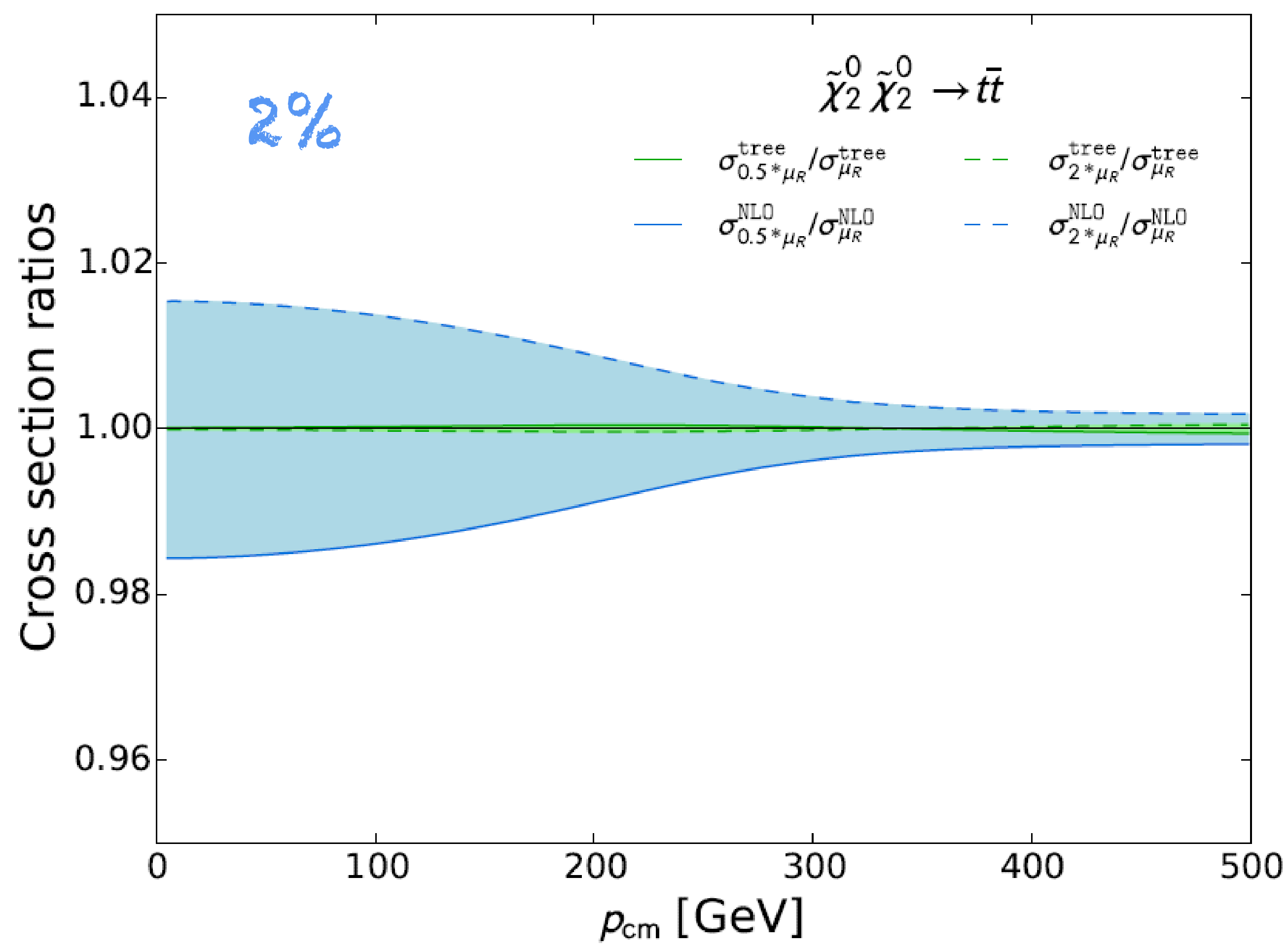


NLO calculation shows 10-15% correction with respect to default MicrOMEGAs value

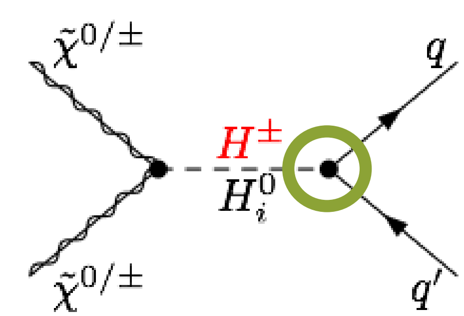
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler,, arXiv:1602.08103 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Gaugino Annihilation





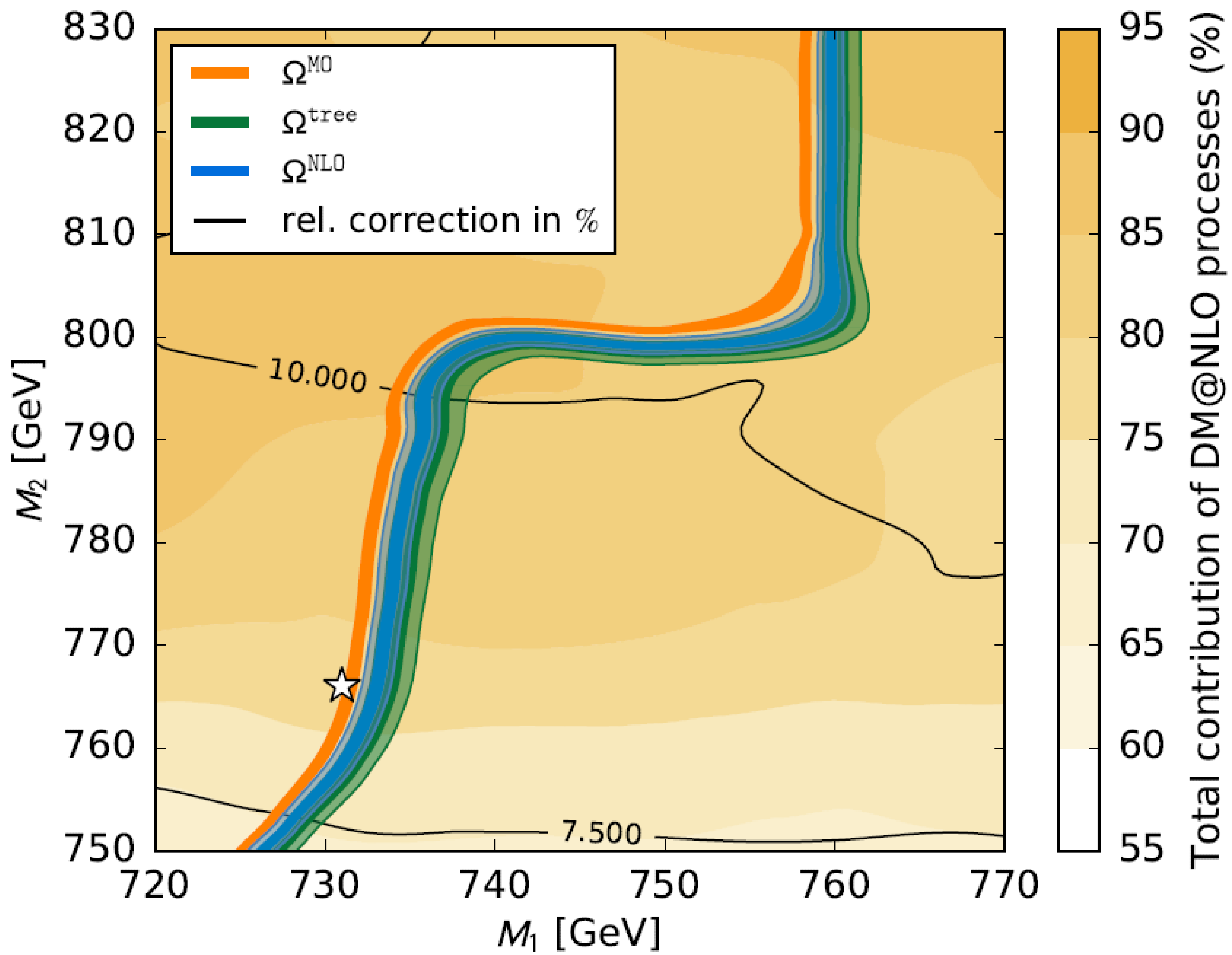
- Tiny scale dependence at LO and NLO
- **Smaller** LO uncertainty band ← scale independent $m_t^{\rm OS}$
- **Larger** NLO uncertainty band ← first appearance of scale dependent α_s



→ only at NLO it becomes possible to quantify the theoretical error for the first time

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Gaugino Annihilation

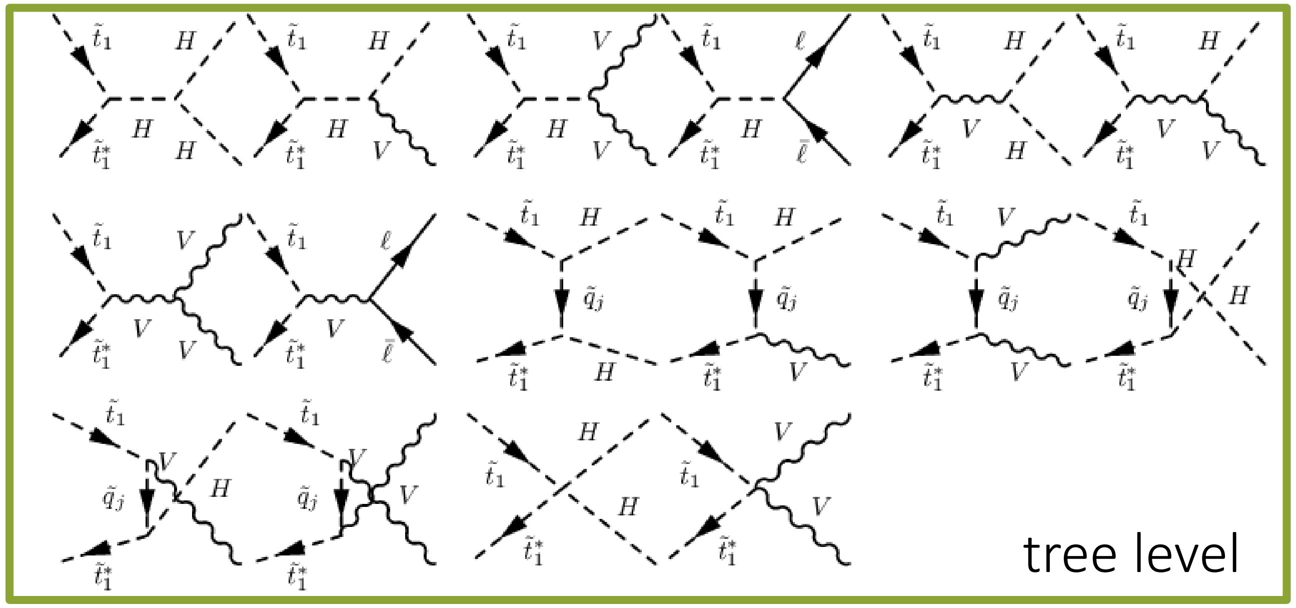


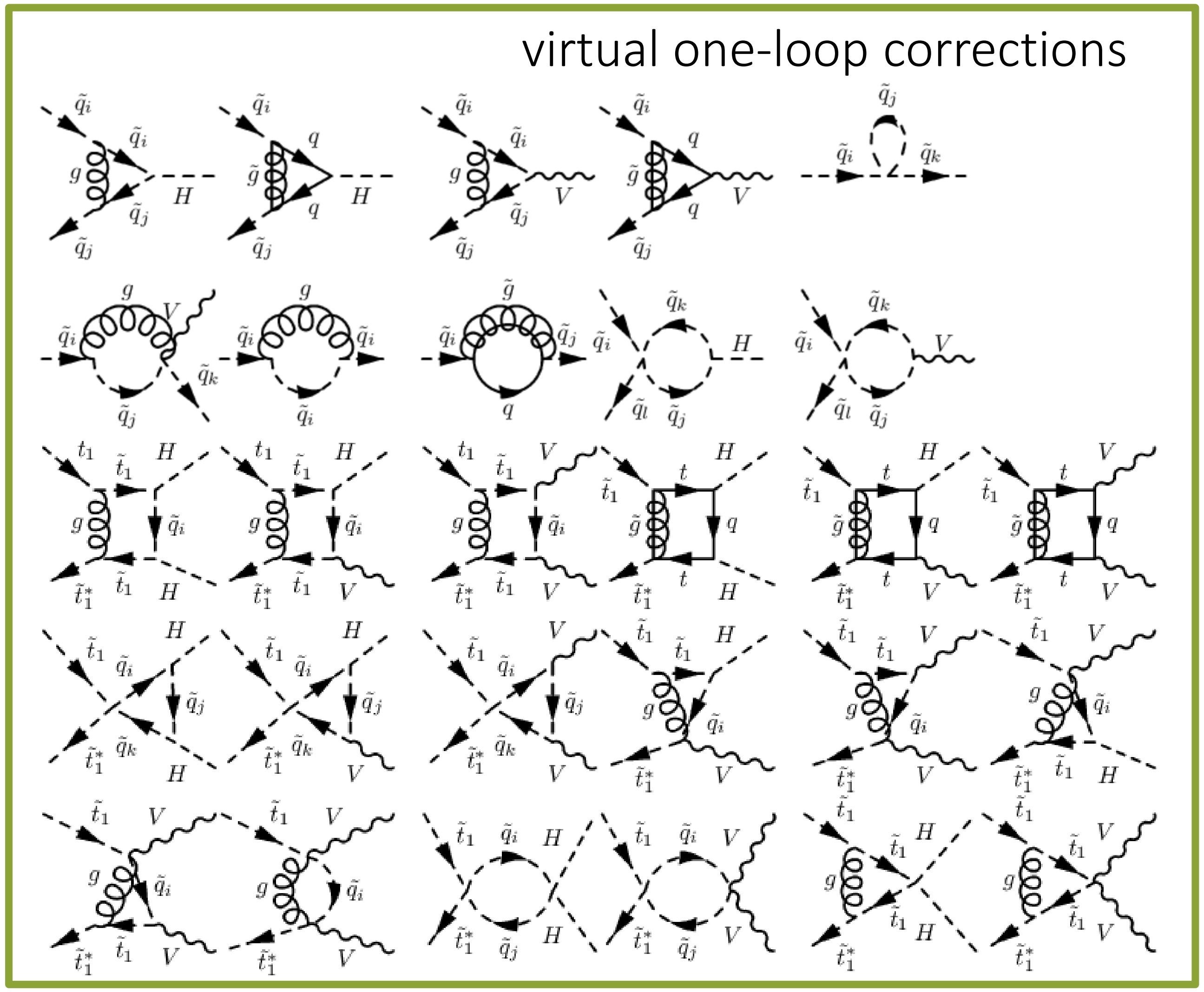
- scale dependence at LO mainly triggered by $m_b^{\overline{DR}}$
- decreased scale dependence at NLO

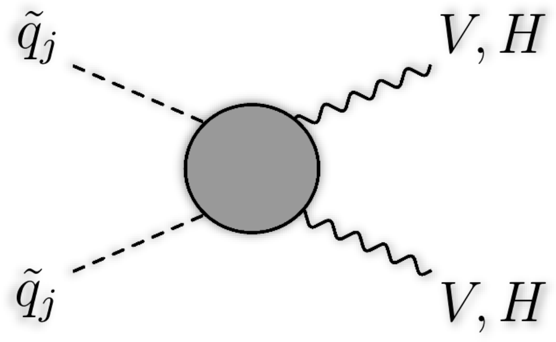
- NLO calculation shows 5-10% relative correction with respect to default MicrOMEGAs value
- including theoretical error, still shift in the parameter space with respect to the MO value
- **first quantitative estimation of theoretical uncertainty**

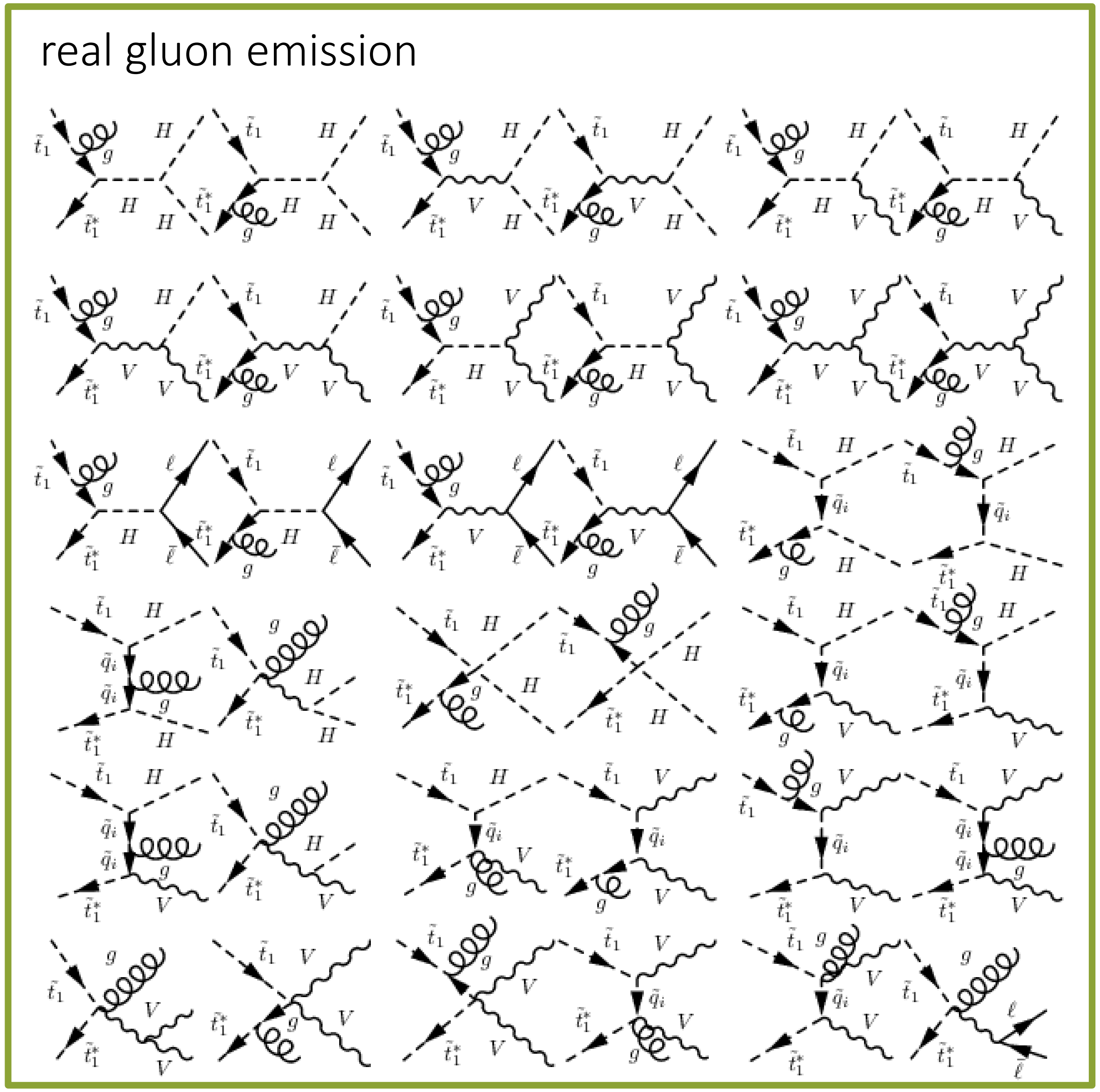
B. Herrmann, M. Klasen, K. Kovařík, M. Meinecke and P. Steppeler, Phys. Rev. D 89, 114012 (2014), arXiv:1404.2931 [hep-ph]

Stop-Antistop Annihilation





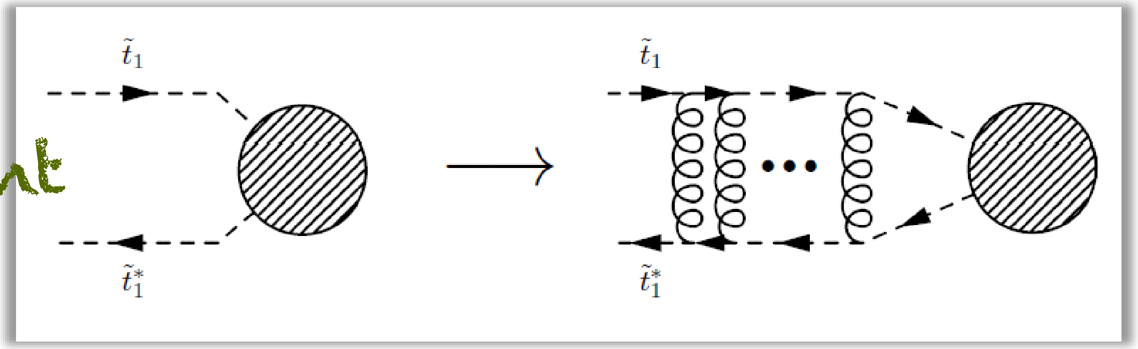




J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

Stop-Antistop Annihilation

+ Coulomb enhancement



- during freeze-out stop-antistop pair is moving slowly
- exchange of *n* gluons lead to a correction factor proportional to $(\frac{\alpha_s}{v})^n$
- with $\alpha_s/v > \mathcal{O}(1)$ Coulomb corrections can be become sizeable

⇨ resummation to all orders within framework of nonrelativistic QCD

M.J. Strassler and M.E.Peskin, Phys. Rev. D 43 1500 (1991)
Y. Kiyo et al. Eur. Phys. J. C. 60 375 (2009)

Would-be stoponium Schroedinger equation

$$\sigma^{\text{Coul.}}(\tilde{t}_1\tilde{t}_1^* \to \text{EW}) = \frac{4\pi}{v m_{\tilde{t}_1}^2} \Im\Big\{G^{[1]}(\mathbf{r}=0; \sqrt{s}+i\Gamma_{\tilde{t}_1}, \mu_C)\Big\} \times \sigma^{\text{LO}}(\tilde{t}_1\tilde{t}_1^* \to \text{EW})$$

$$\Big[H^{[1]} - (\sqrt{s}+i\Gamma_{\tilde{t}_1})\Big] G^{[1]}(\mathbf{r}; \sqrt{s}+i\Gamma_{\tilde{t}_1}, \mu_C) = \delta^{(3)}(\mathbf{r})$$

$$H^{[1]} = -\frac{1}{m_{\tilde{t}_1}}\Delta + 2m_{\tilde{t}_1} + V^{[1]}(\mathbf{r})$$

Coulomb potential @ NLO $\mathcal{O}((\alpha_s^2/v)^n)$

$$\Im\Big\{G^{[1]}(0; \sqrt{s}+i\Gamma_{\tilde{t}_1}, \mu_C)\Big\} = m_{\tilde{t}_1}^2 \Im\Big\{\frac{v}{4\pi}\Big[i + \frac{\alpha_s(\mu_C) C_F}{v}\Big(\frac{i\pi}{2} + \ln\frac{\mu_C}{2m_{\tilde{t}_1} v}\Big) + \mathcal{O}(\alpha_s^2)\Big]\Big\}$$

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

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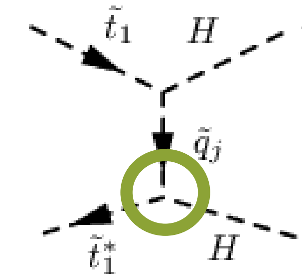
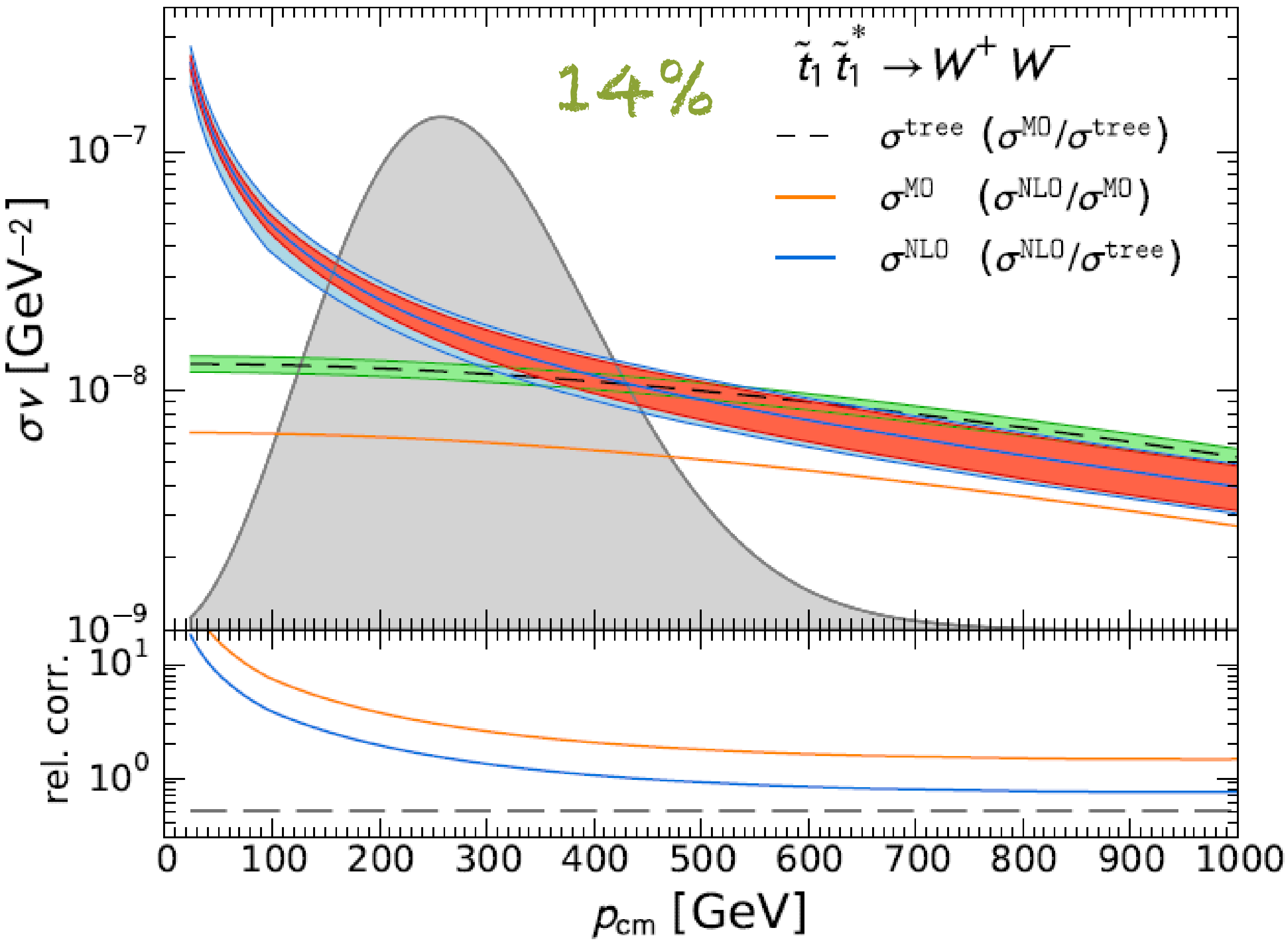
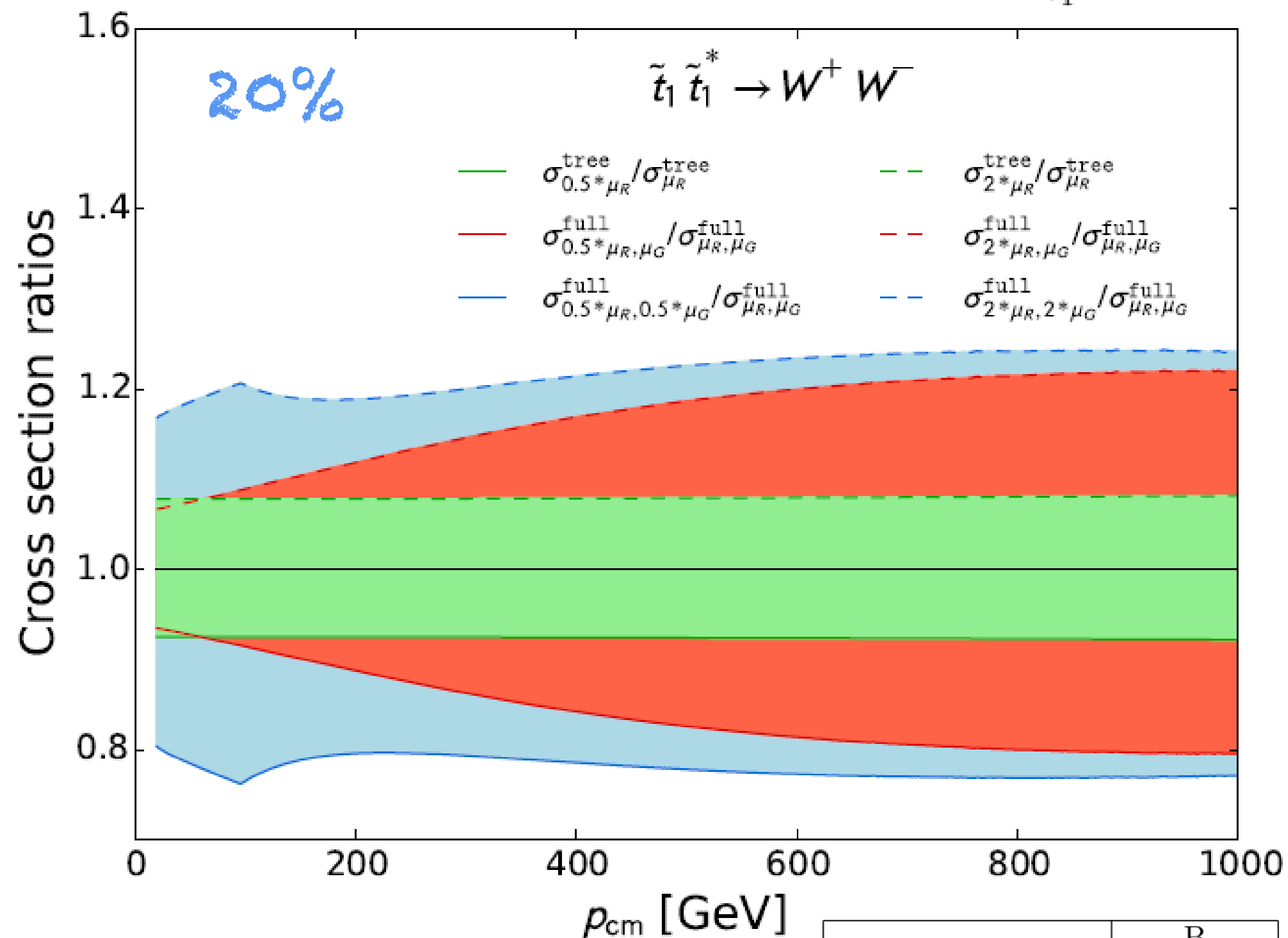


Scenario with a small mass gap between LSP and NLSP

	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$Z_{1\tilde{B}}$	$Z_{1\tilde{W}}$	$Z_{1\tilde{H}_1}$	$Z_{1\tilde{H}_2}$	m_{h^0}	$\Omega_{\tilde{\chi}_1^0}h^2$	BR($b \to s\gamma$)
B	1306.3	1827.0	1827.2	2640.0	1361.7	2157.3	-1.000	0.002	-0.024	0.013	123.7	0.1134	$3.1 \cdot 10^{-4}$





- Scale dependence at LO triggered by trilinear coupling A_t
- Scale dependence at NLO due to explicit logarithms and implicit ($A_t \, \alpha_s$)
- Coulomb scale leads to 20% uncertainty

→ large correction (K-factor of 1-9) in relevant region, 20 % theoretical error

		B
$\tilde{\chi}_1^0 \tilde{t}_1 \to$	th^0	1%
	tg	6%
$\tilde{t}_1 \tilde{t}_1^* \to$	$h^0 h^0$	12%
	$h^0 H^0$	11%
	$Z^0 A^0$	7%
	$W^\pm H^\mp$	13%
	$Z^0 Z^0$	8%
	$W^+ W^-$	14%
Total		72%

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

Stop-Antistop Annihilation

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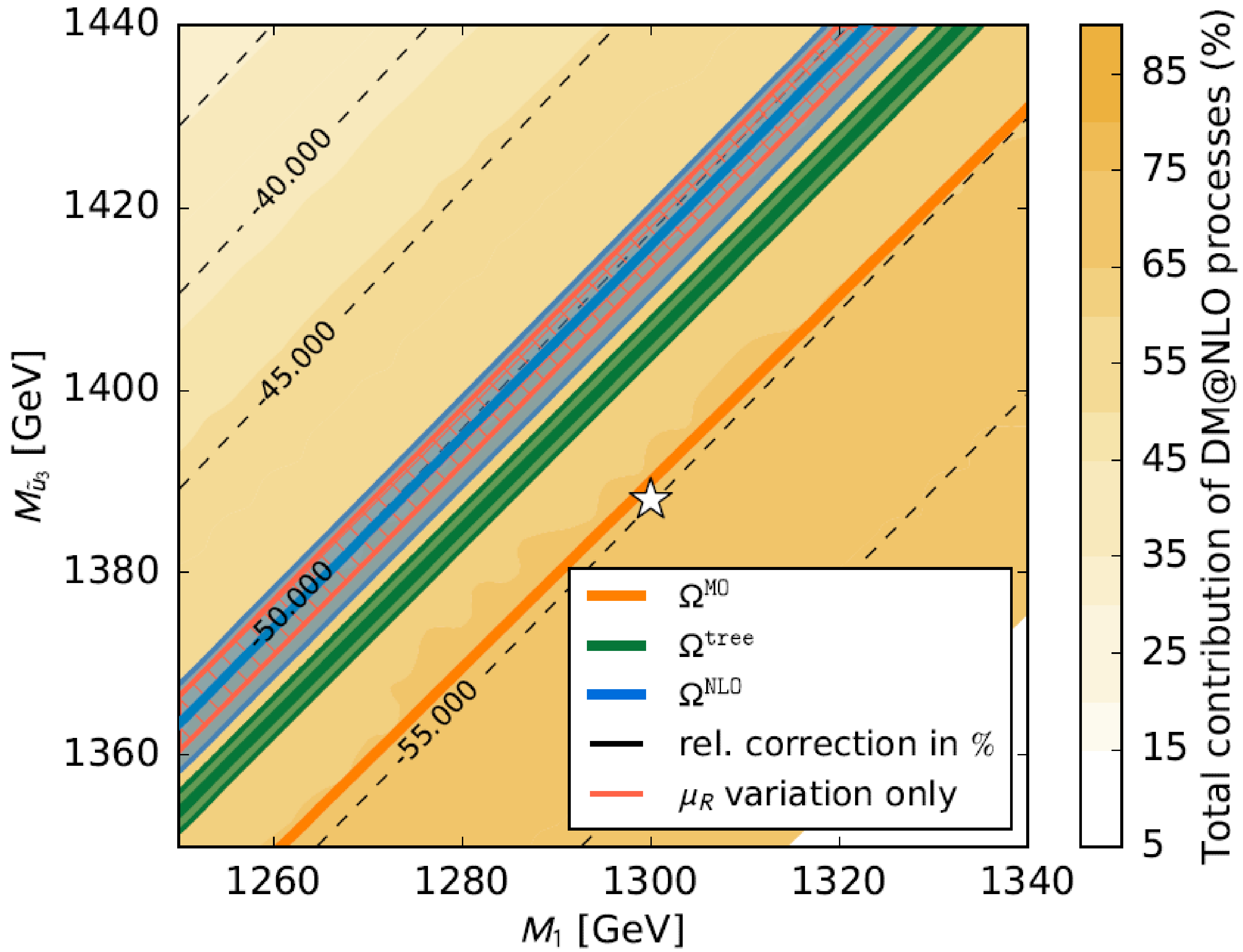


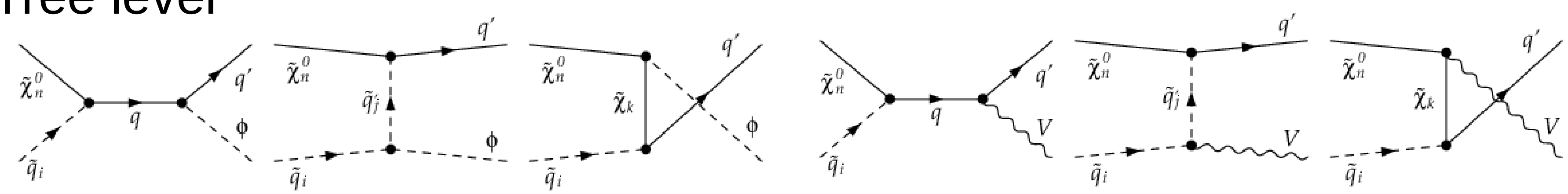
		B
$\tilde{\chi}_1^0\tilde{t}_1 \rightarrow$	th^0	1%
	tg	6%
$\tilde{t}_1\tilde{t}_1^* \rightarrow$	h^0h^0	12%
	h^0H^0	11%
	Z^0A^0	7%
	$W^{\pm}H^{\mp}$	13%
	Z^0Z^0	8%
	W^+W^-	14%
Total		72%

- NLO calculation shows 50-60% relative difference with respect to default MicrOMEGAs value
- including theoretical error, still shift in the parameter space with respect to the MO value
- Mass shift of 30 GeV (2 %) with an uncertainty of 0.5%

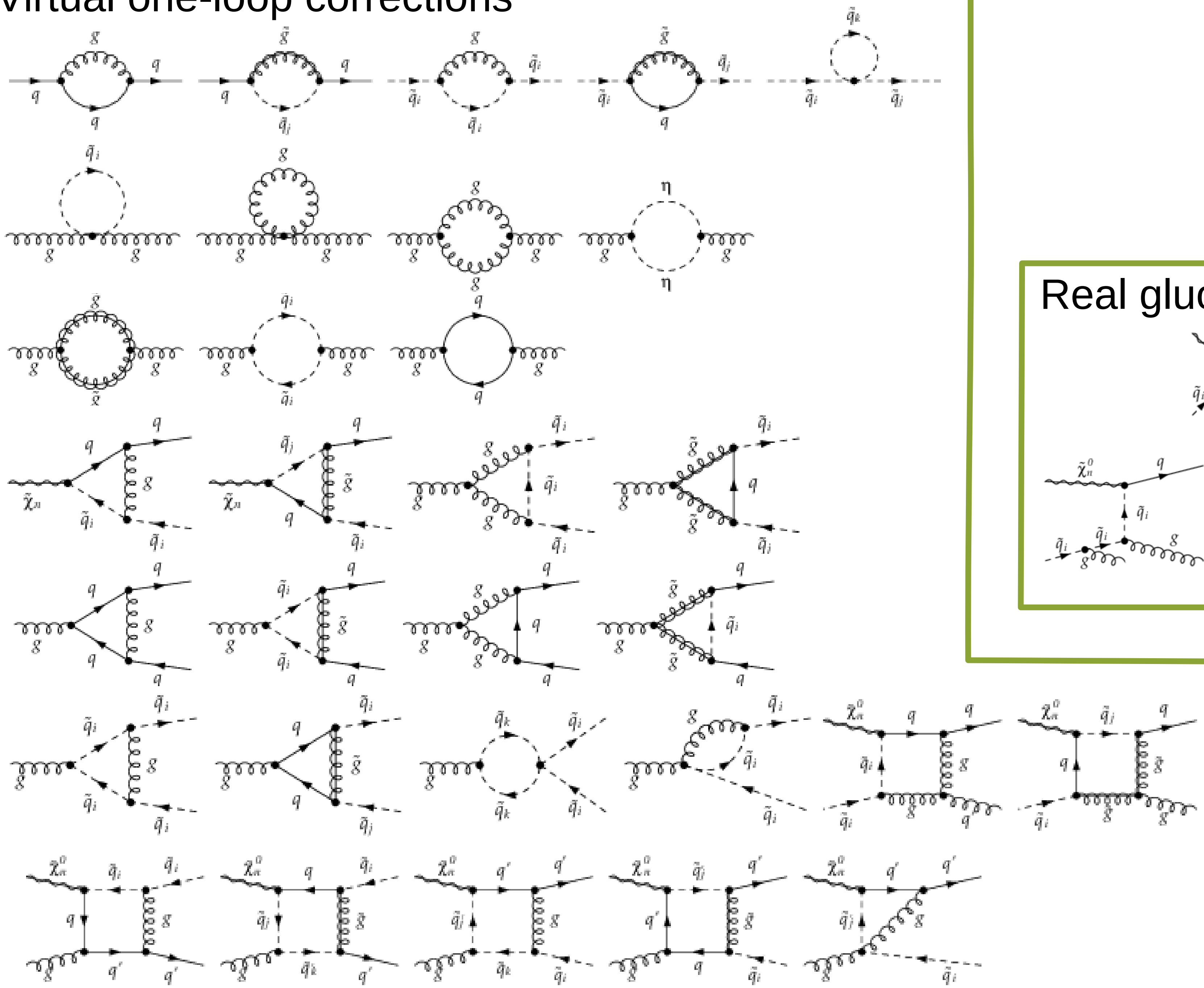
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and M. Meinecke, Phys. Rev D 91 034012 (2015), arXiv:1410.8063 [hep-ph]

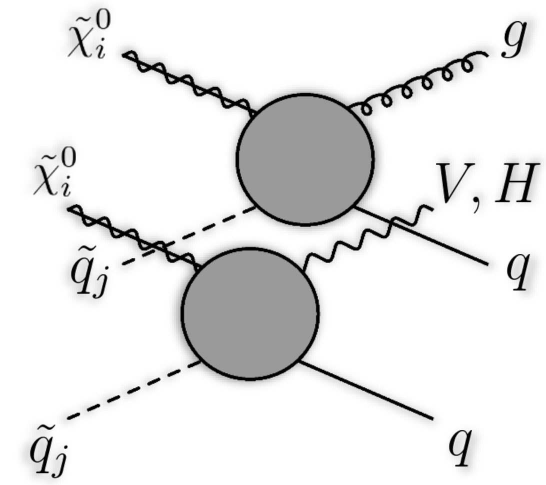
Neutralino-Stop Coannihilation

Tree level

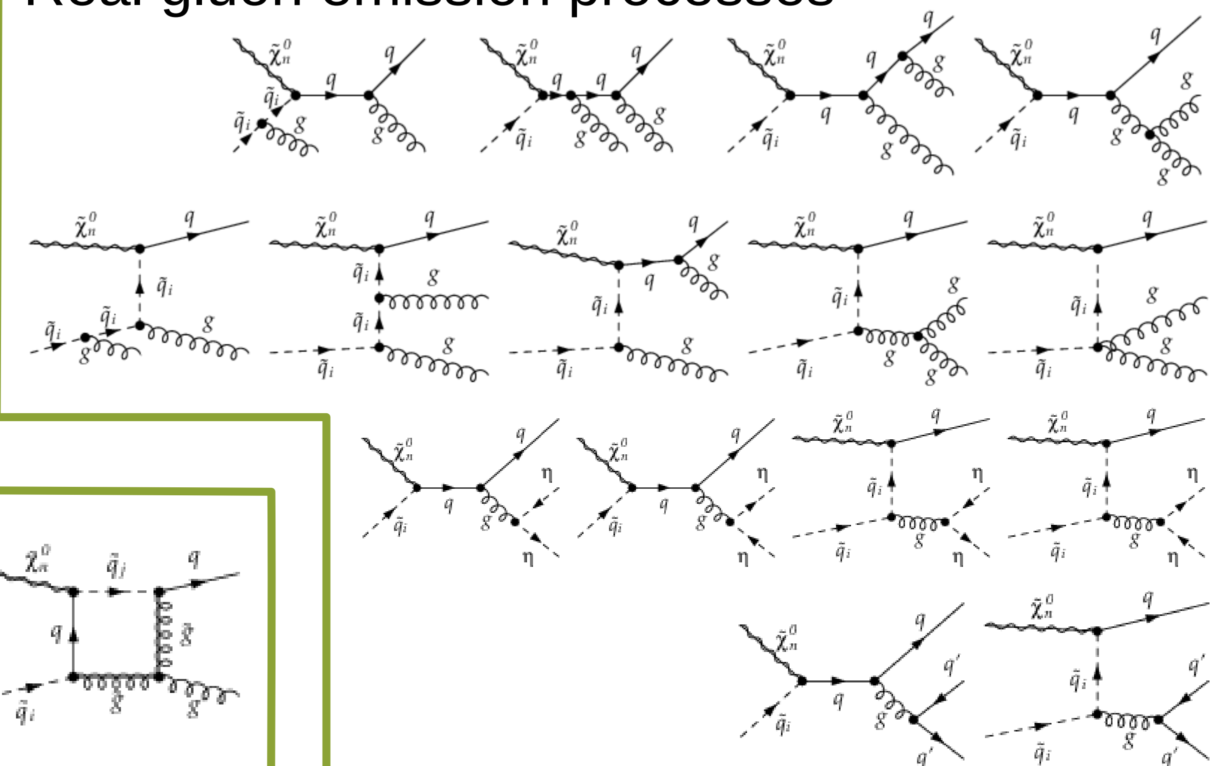


Virtual one-loop corrections

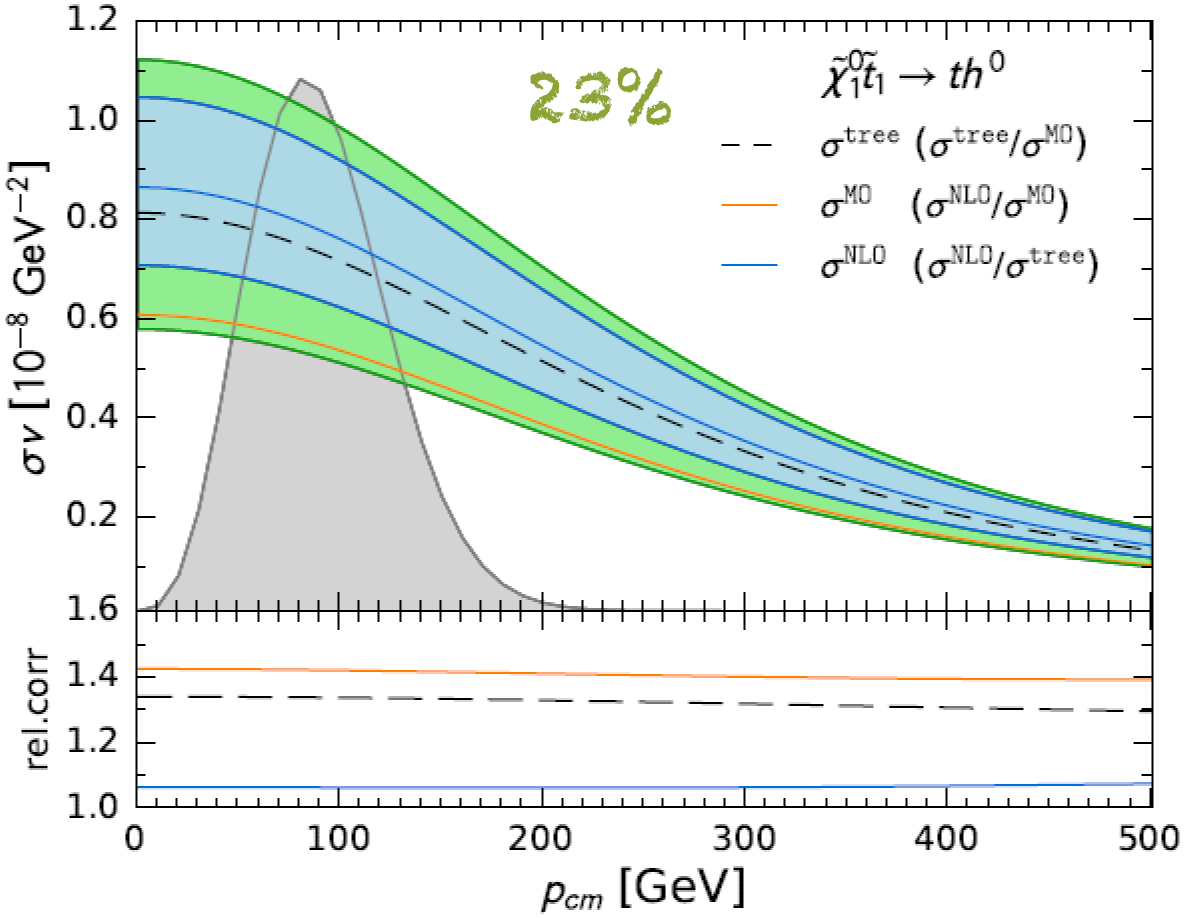


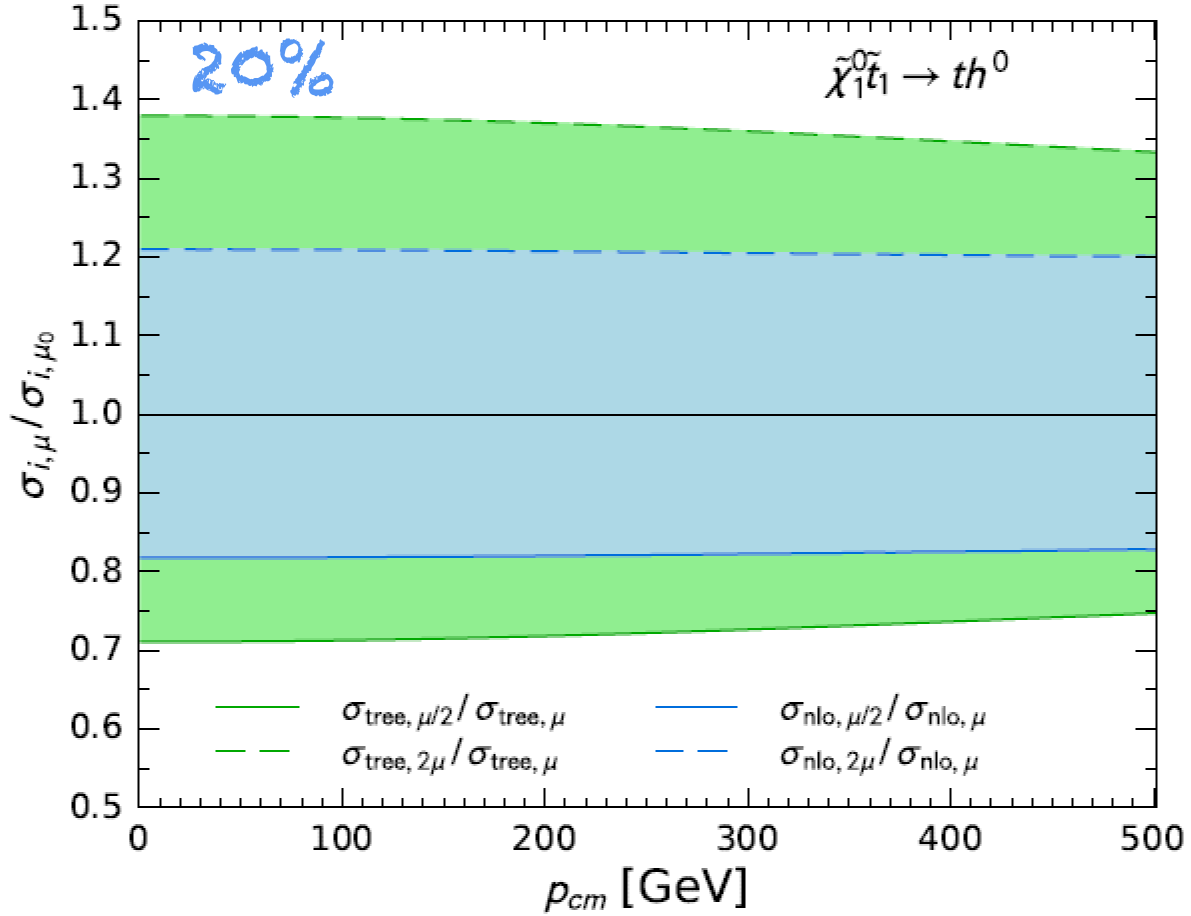


Real gluon emission processes

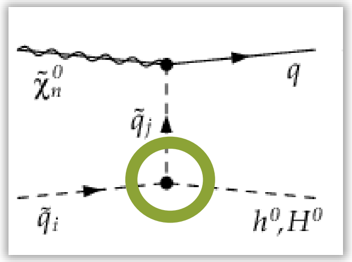


Neutralino-Stop Coannihilation





- Scale dependence on LO triggered by trilinear coupling A_t
- Scale dependence at NLO due to explicit logarithmical and implicit ($A_t \, \alpha_s$)
- Scale uncertainty of 20%

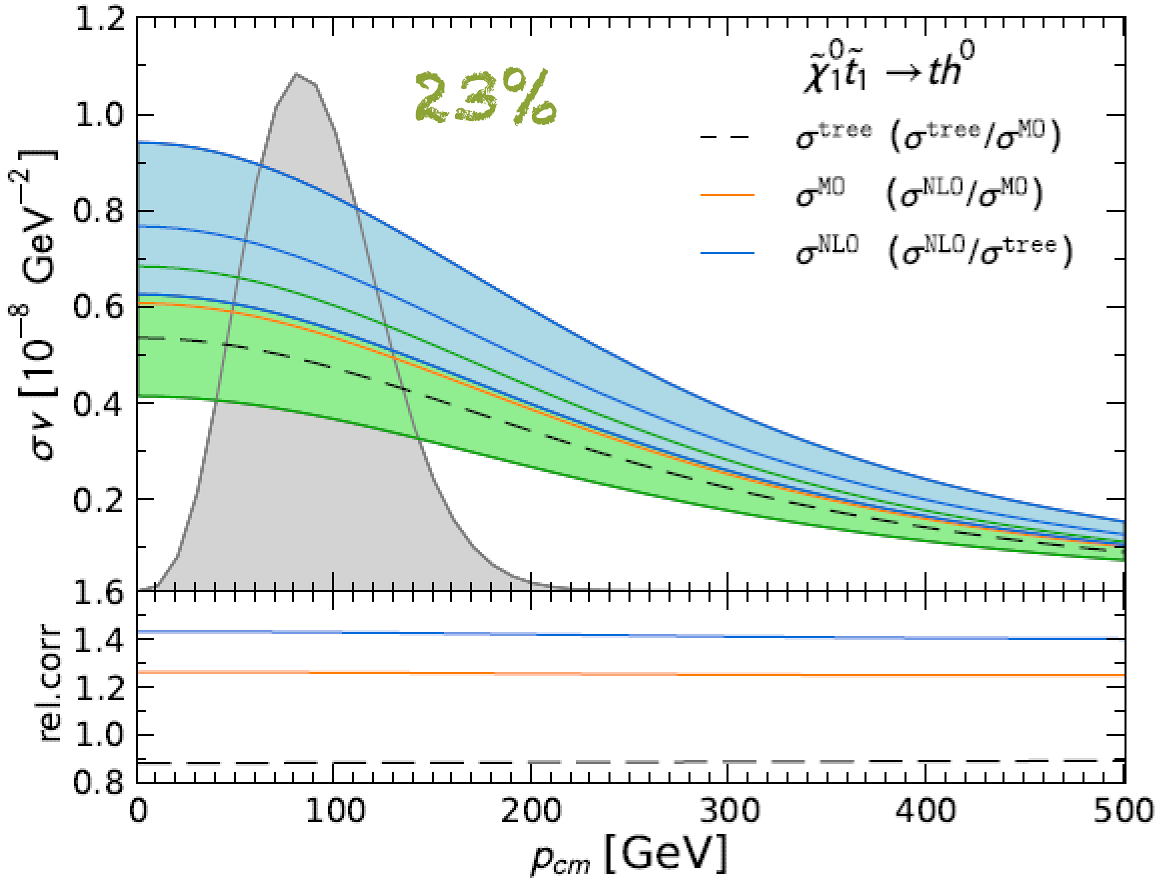


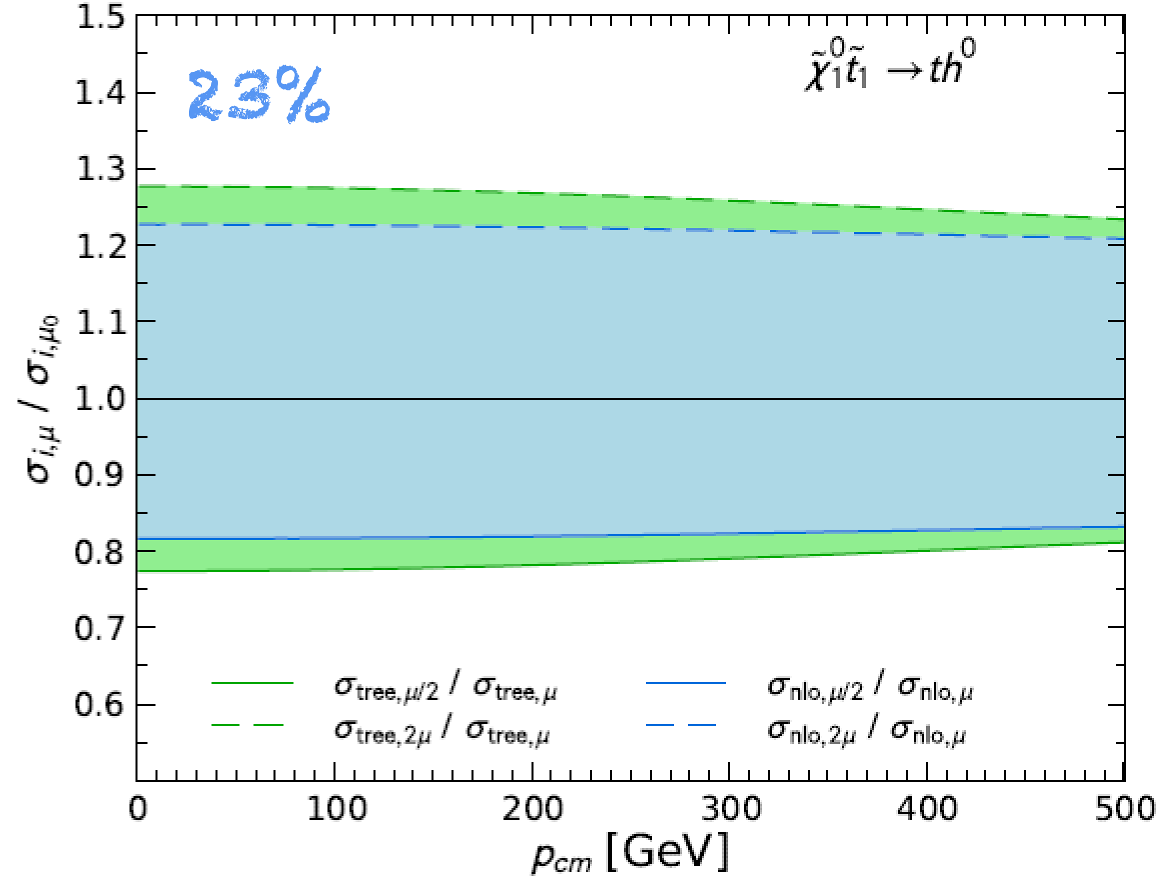
→ K-factor of 1.05, correction of 40% w.r.t. MicrOMEGAs, 20 % theoretical error

J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015), arXiv:1409.2898 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]

Effect of Scheme Variation (m_{top})

Study effect of taking $m_t^{\overline{\mathrm{DR}}}$ **instead of** m_t^{OS}



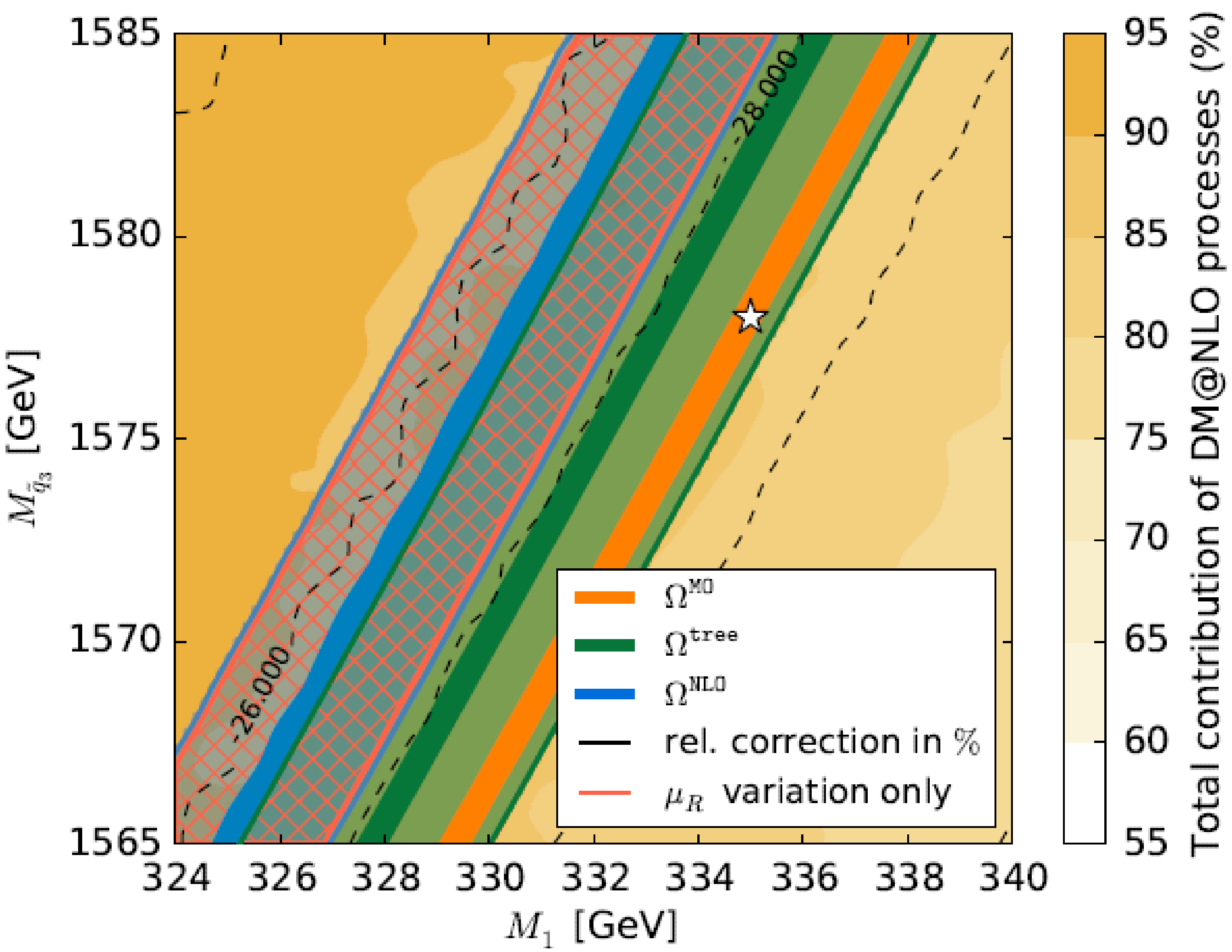


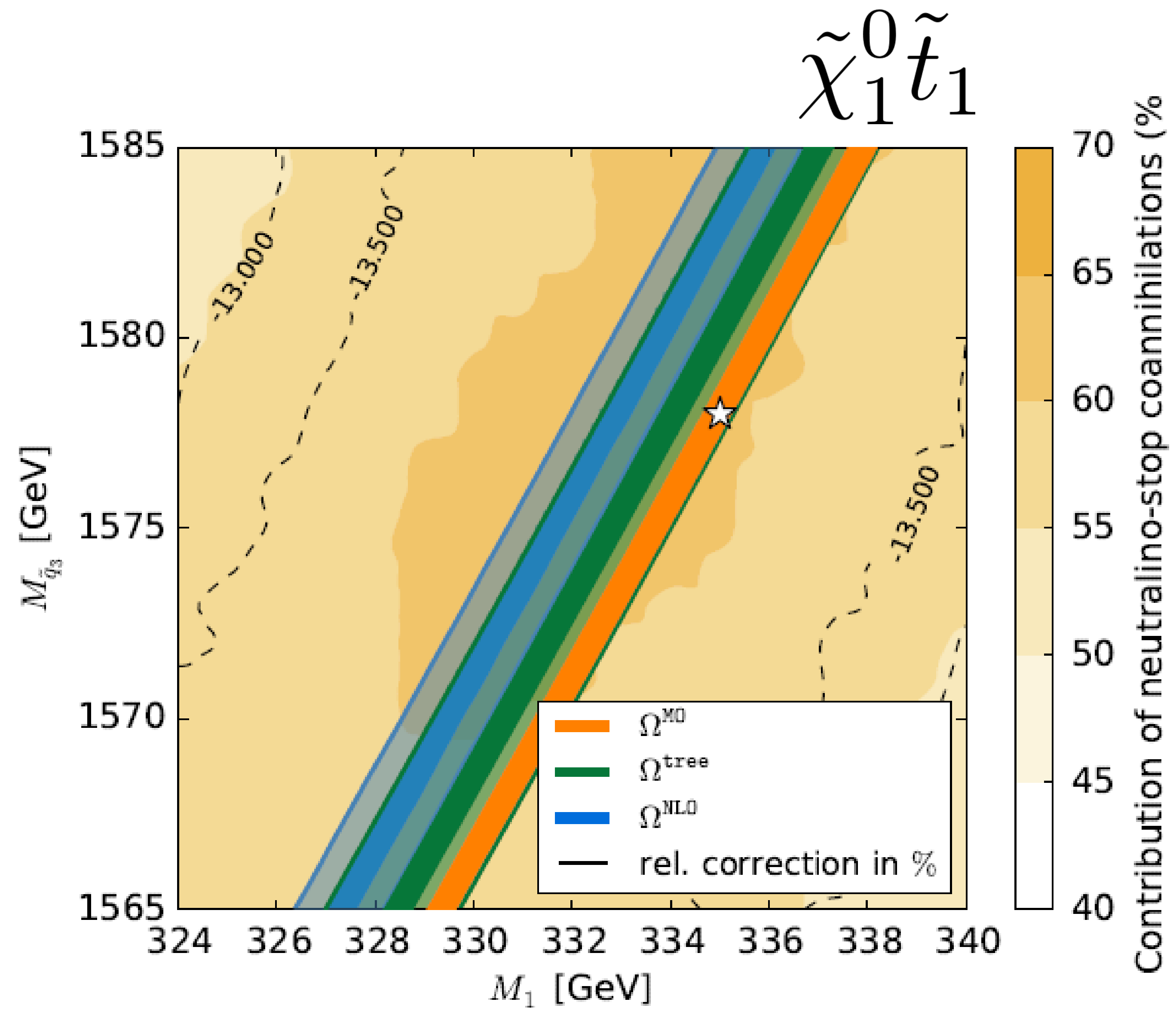
- MO larger than LO cross section as MO uses effective top mass
- Larger K-factor of 1.4 in contrast to our scheme with 1.05

→ Confirms our choice of taking top mass on-shell

J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013), arXiv:1212.5241 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015), arXiv:1409.2898 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]

Relic density in the combined Scenario





- $\tilde{\chi}\tilde{\chi}$ — K-factor of 1.15, 5-10% correction w.r.t. MO, 5% scale uncertainty (A)
- $\tilde{\chi}_1^0 \tilde{t}_1$ — K-factors of 1.05-1.5, 15% correction w.r.t. MO, 20% scale uncertainty (C)
- $\tilde{t}_1 \tilde{t}_1^*$ — K-factors of 1-9, up to 16% correction w.r.t. MO, 20% scale uncertainty (B)

→ Correction of almost 30 % w.r.t. MO relic density calculation

J. Harz, B. Herrmann, M. Klasen, K. Kovařík, and P. Steppeler, arXiv:1602.08103 [hep-ph]

Conclusions

- We presented the first quantitative estimate of the theoretical uncertainty coming from scale and scheme variations

- Renormalisation scale dependence can differ significantly, however, usually reduced at NLO

- We could demonstrate an enhanced stability in our mixed renormalisation scheme in comparison to pure $\overline{\text{DR}}$

- SUSY-QCD corrections induce important shifts up to 50% (→ Coulomb enhancement!)

- Theoretical uncertainty should be more reliably estimated to be six times larger than the experimental one

- When extracting the pMSSM parameters from relic density measurement, shifts and uncertainties of a few percent have to be taken into account

DM@NLO

Thank you!