

Nonperturbative Dynamics in Dark Matter Freezeout

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SUSY Conference, University of Melbourne

July 4, 2016



The Niels Bohr
International Academy

Based on 1412.5660, 1501.03153
and ongoing work
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The Scale of New Physics

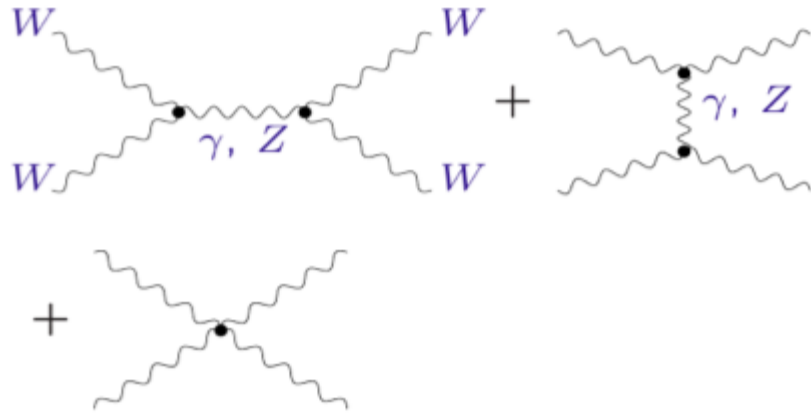
- Historically in HEP, we've often known where we were going
 - Fermi theory of weak decays needed new bosons
 - Precision measurements pointed to the top quark
 - Heavy bosons needed symmetry breaking
- After the Higgs discovery, we have no map
 - The Standard Model is stubbornly good
- Where are we going, and how far away is it?

The Scale of New Physics

- Naturalness
 - The Higgs mass is subject to corrections from new physics, these corrections are potentially huge
 - The SM as a UV theory requires cancellations in these corrections to 1 part in 10^{30}
 - New symmetry could enforce this if it happens low enough, but bounds on scale depend on amount of tuning deemed acceptable
- New Phenomena
 - Neutrino masses need at least the Weinberg operator, but that can be at scales far beyond what we'll see
 - Dark matter is the other new particle we need experimentally

The LHC No-Lose Theorem

- The Higgs (or something else doing its job) had to be there
 - Symmetry has to break to make gauge bosons massive
- Why did it have to be within the LHC's reach?

$$\mathcal{M}_V =$$


The diagram shows three Feynman diagrams for the scattering of two W bosons into two W bosons. The first diagram (top left) shows a t-channel exchange of a photon or Z boson, with external lines labeled W and internal lines labeled γ, Z. The second diagram (top right) shows a u-channel exchange of a photon or Z boson, also with external lines labeled W and internal lines labeled γ, Z. The third diagram (bottom) shows a four-point contact interaction between four W bosons. The diagrams are separated by plus signs, indicating they are summed together to form the total amplitude \mathcal{M}_V .

This amplitude (missing a Higgs contribution) grows with energy, and predicts scattering probabilities greater than 1 beyond energies of about 800 GeV

Basics of Unitarity

- We start with a scattering matrix

$$S = 1 + iT$$

- Unitarity gives the optical theorem

$$S^\dagger S = I \Rightarrow \frac{1}{2}(T - T^\dagger) = |T_{ii}^2|$$

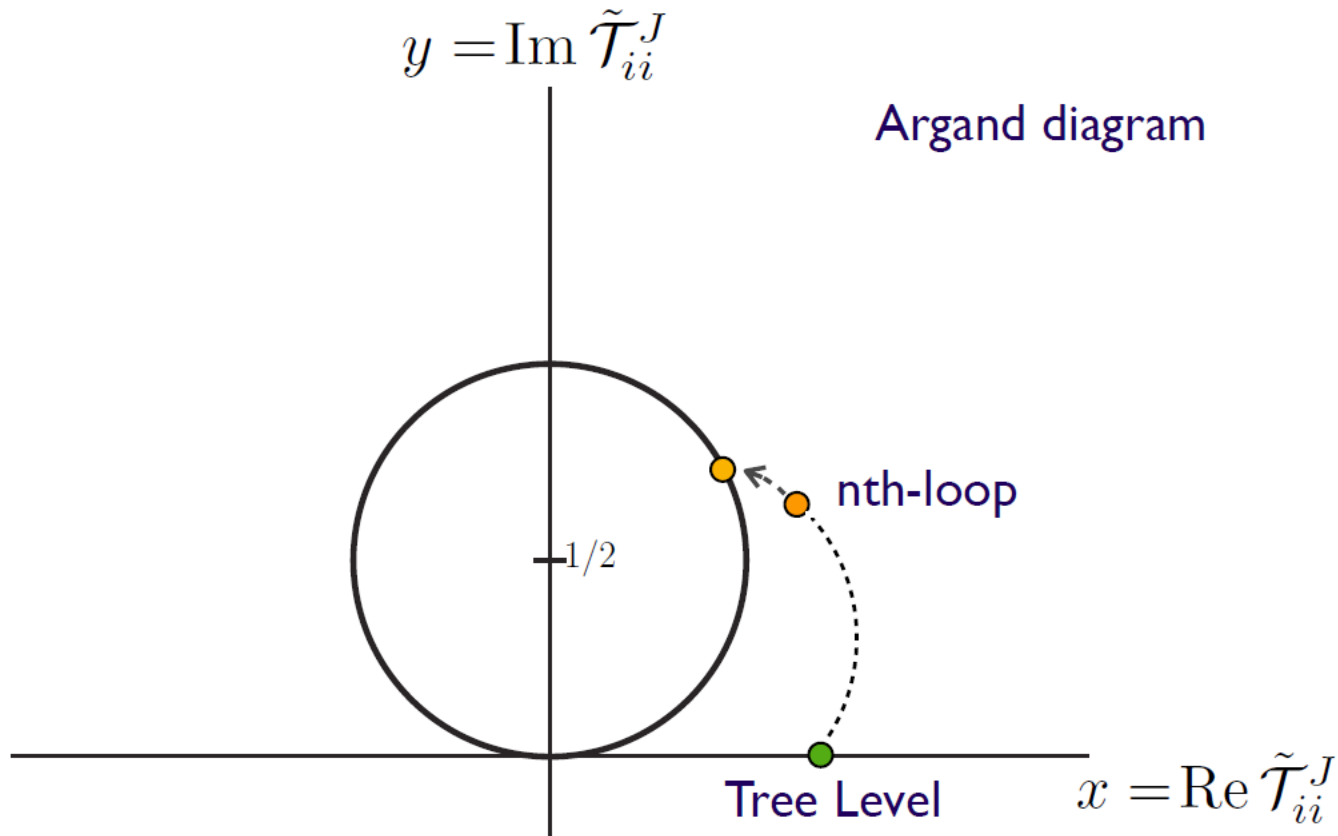
- Expanding in partial waves

$$\tilde{T}_{ij}^J = \frac{\lambda_i^{1/4} \lambda_f^{1/4}}{32\pi s} \int_{-1}^1 T_{ij} P_J(\cos \theta) d \cos \theta$$

- We find that

$$\text{Im } \tilde{T}_{ii} = |\tilde{T}_{ii}|^2 \Rightarrow |\text{Re } \tilde{T}_{ii}| < \frac{1}{2}$$

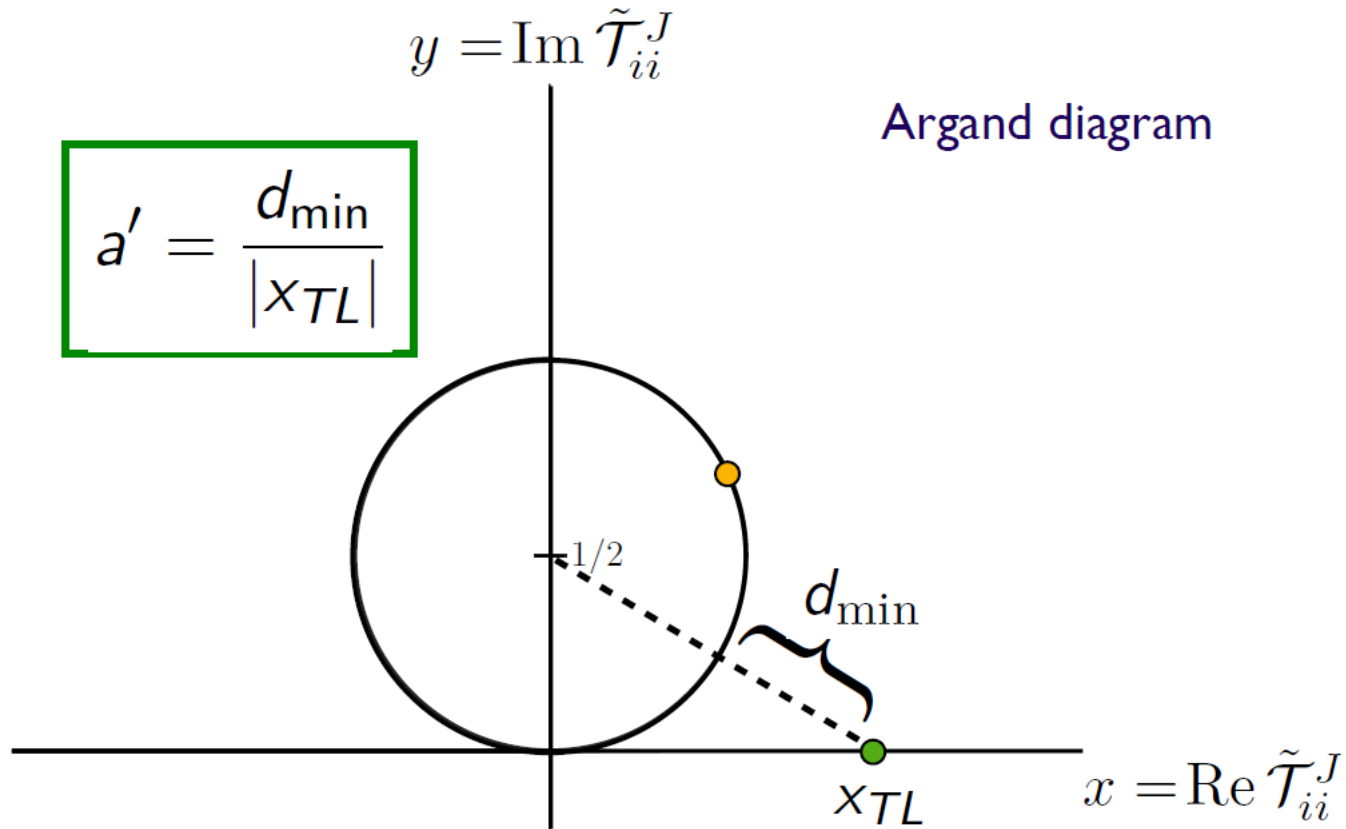
A Picture of Unitarity



Schuessler and Zeppenfeld 0710.5175

7/4/2015 Aydemir, Anber, Donoghue 1203.5153 Willmott-Shepherd, NBIA

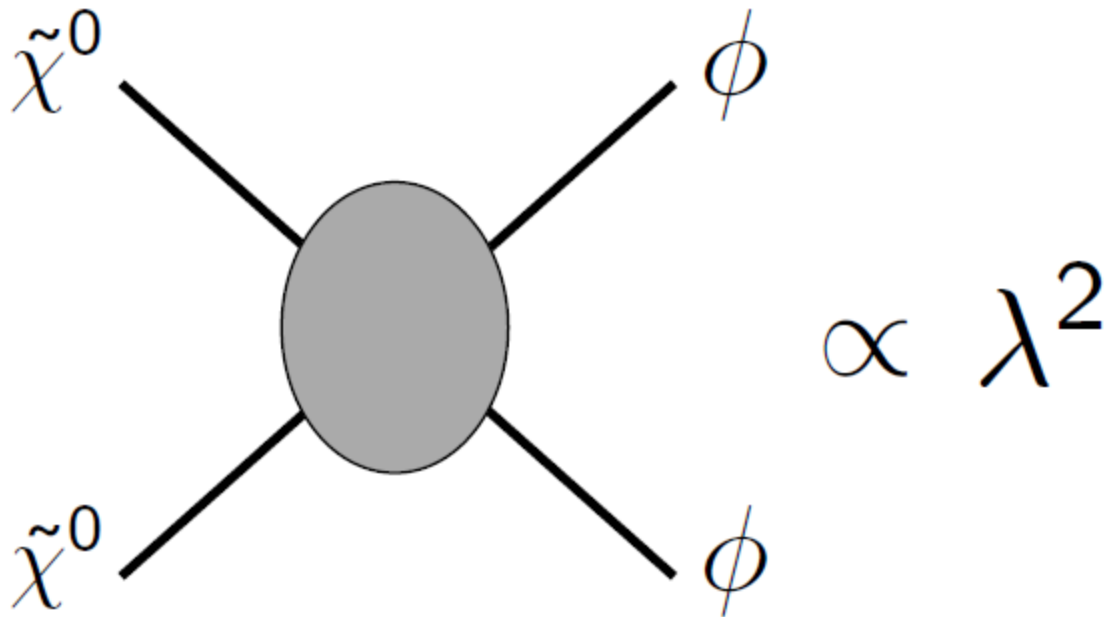
A Picture of Unitarity



Schuessler and Zeppenfeld 0710.5175

Unitarity and Dark Matter

- By insisting on unitarity in a general dark matter scenario, we can bound dark matter to be lighter than 120 TeV for coupling below 4π



Gauge Portal Dark Matter

- This model is characterized by the Lagrangian

$$\mathcal{L}_{DM} \supset g' \bar{\chi} \gamma^\mu \gamma_5 Z'_\mu \chi - \lambda_\chi \bar{\chi} \Phi \chi$$

$$\mathcal{L}_{gauge} \supset -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{\sin \delta}{2} Z'_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{Higgs} \supset |D_\mu \Phi|^2 + V(H, \Phi; \lambda_1, \lambda_2, \lambda_3)$$

- With breaking of the new symmetry by

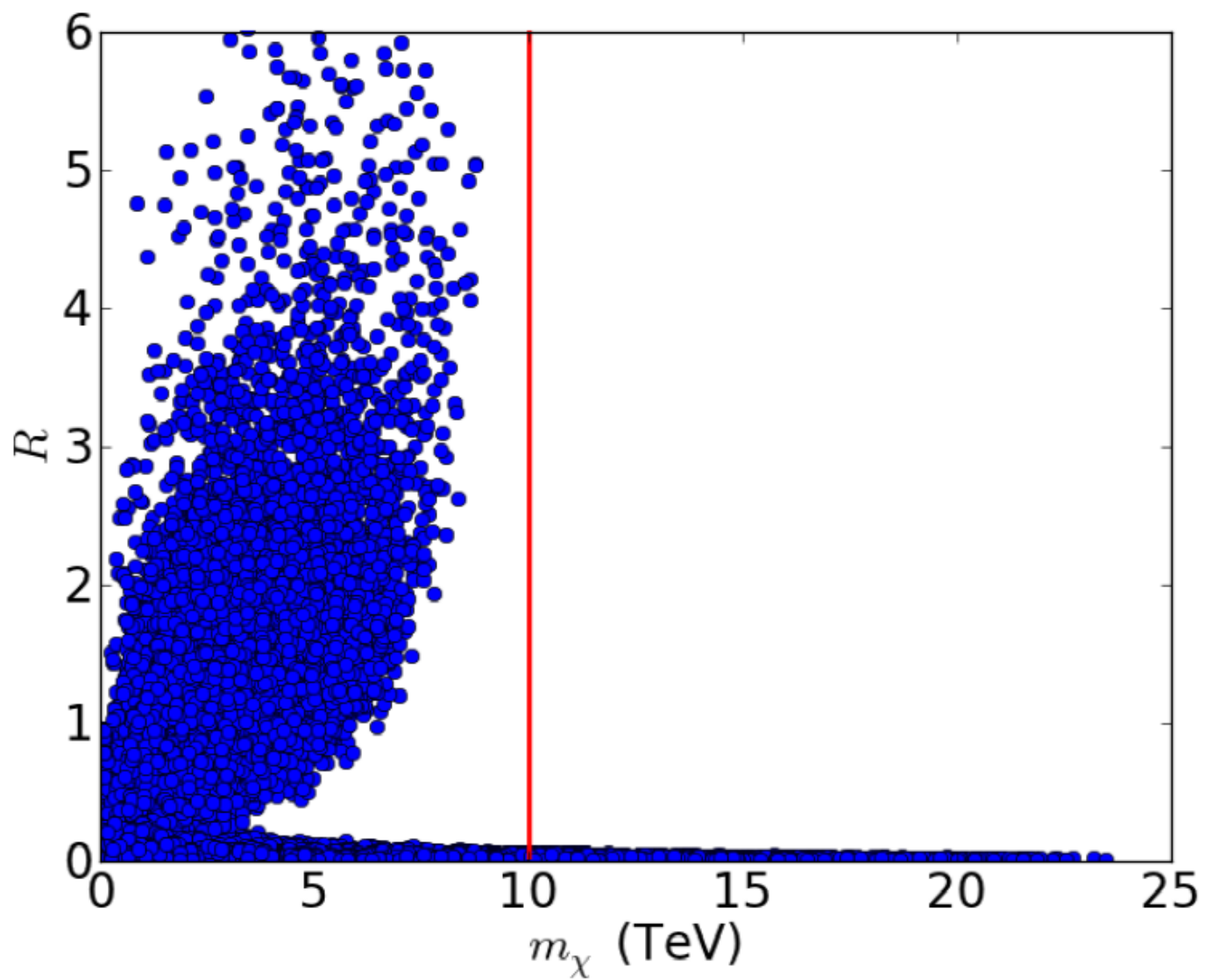
$$\Phi = \frac{1}{\sqrt{2}} (\mathbf{u} + \phi^0)$$

Gauge Portal Dark Matter

- This gives us 6 parameters and 1 new scale:

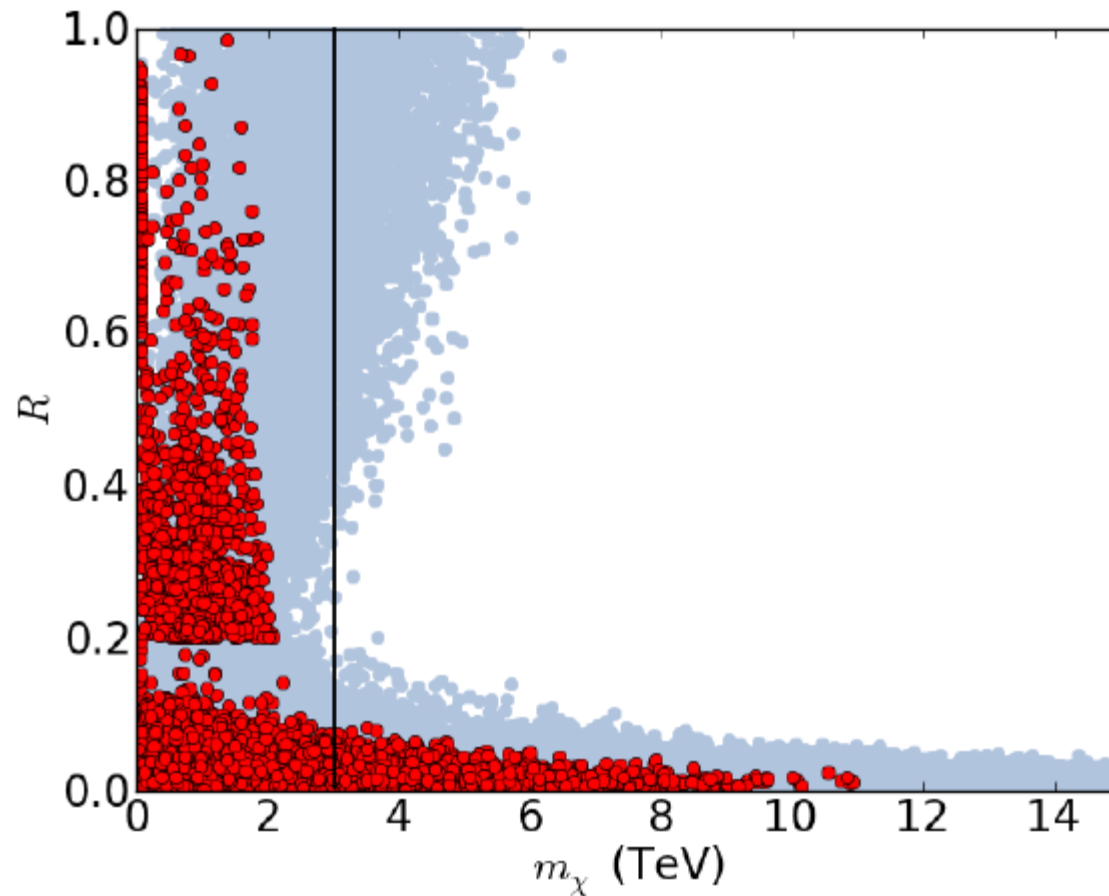
$$g', \lambda_\chi, \lambda_1, \lambda_2, \lambda_3, \sin \delta, u$$

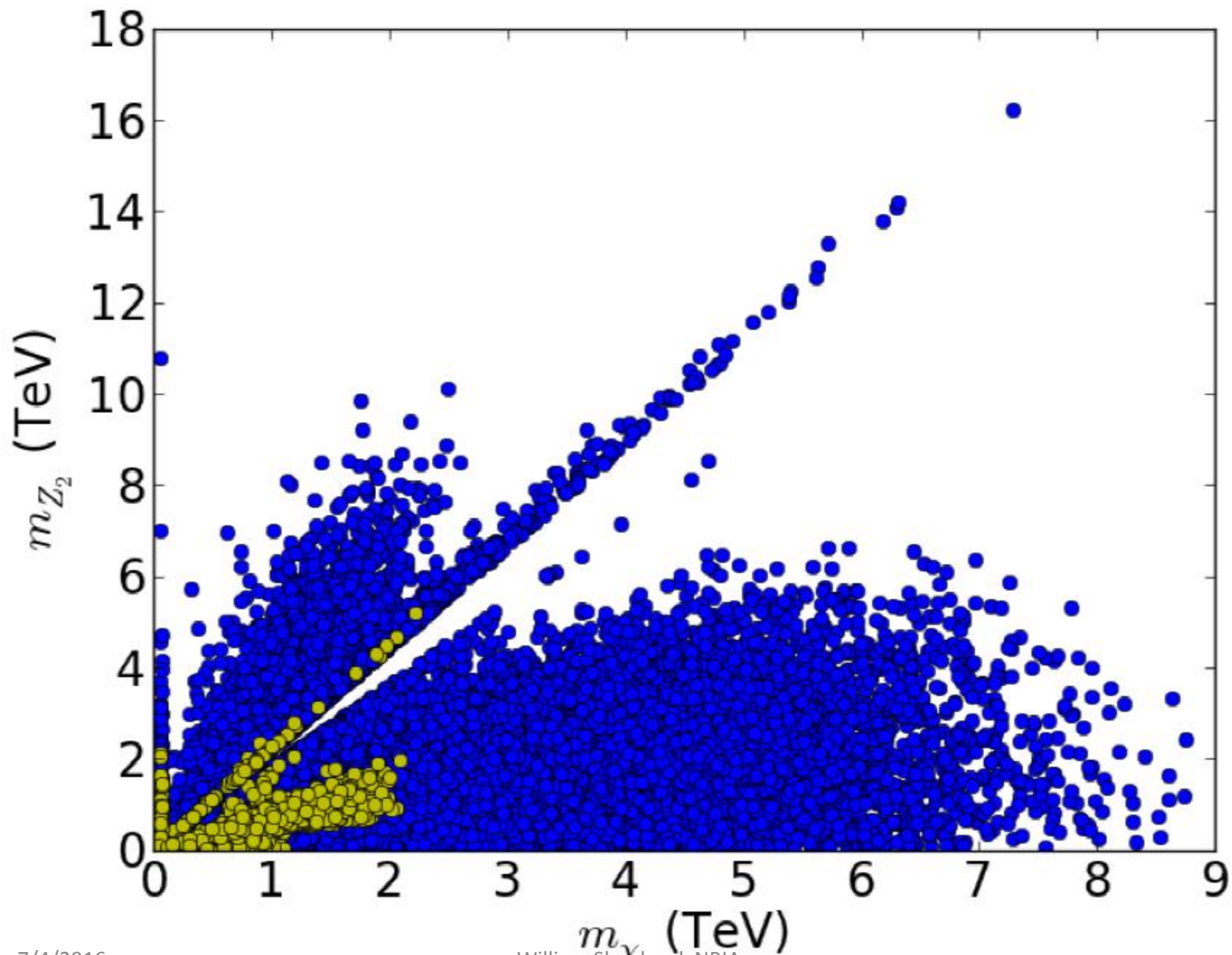
- The dimensionless couplings can be constrained directly from unitarity, but only ratios of scales can be constrained
- Here, the annihilation rate will set upper bounds on the scale of symmetry breaking



$$R = \min \left\{ \frac{|2m_\chi - m_{Z'}|}{m_\chi}, \frac{|2m_\chi - m_{\phi^0}|}{m_\chi} \right\}$$

‘Pure Vector’ Interactions





Colored Scalars and Dark Matter

- In a SUSY-inspired model, we add

$$\tilde{u}_R = (\tilde{u}_R, \tilde{c}_R, \tilde{t}_R)$$

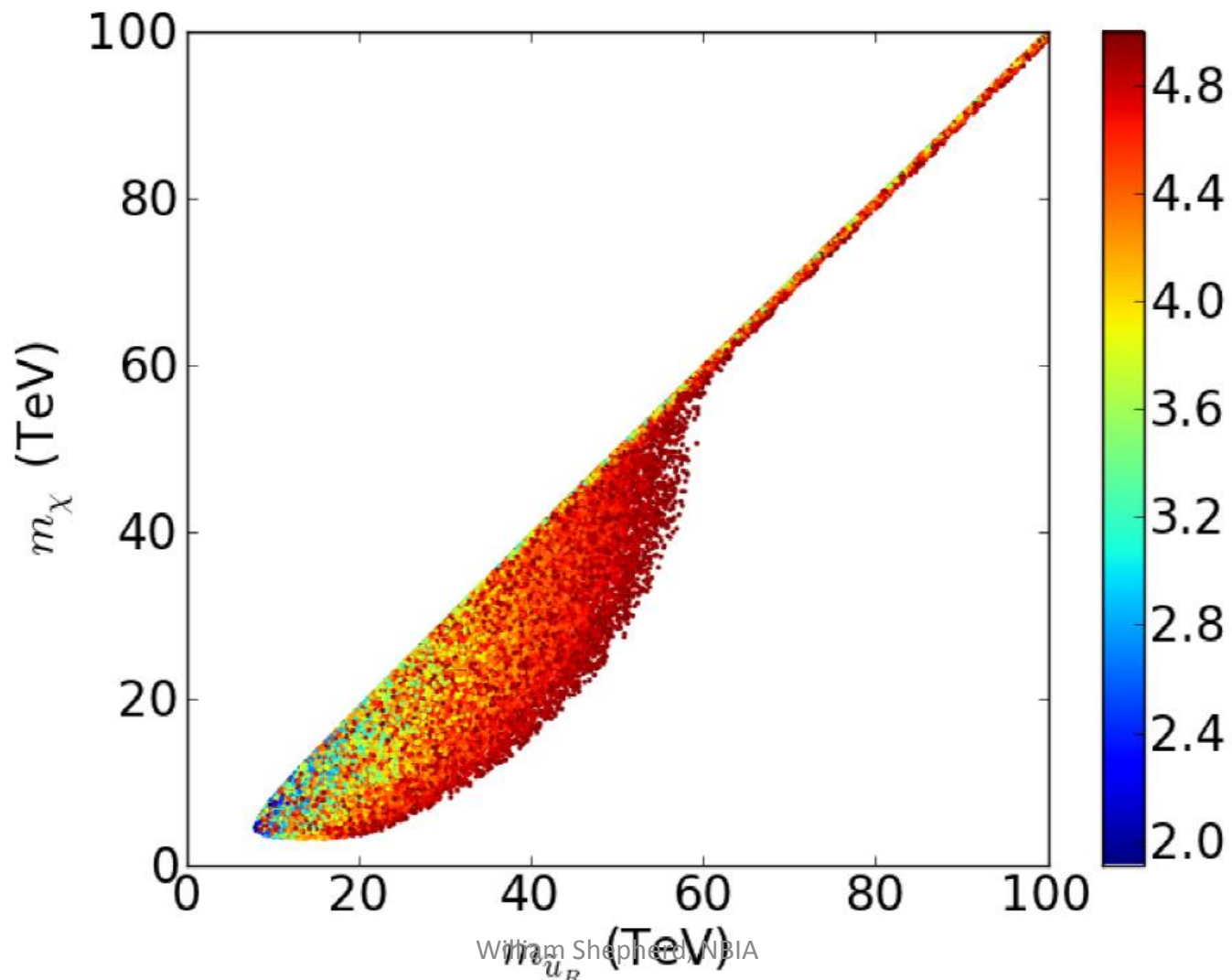
- And the Lagrangian terms

$$\mathcal{L} \supset \frac{1}{2} M_\chi \bar{\chi} \chi + \frac{1}{2} M_{\tilde{u}}^2 \tilde{u}^* u + \lambda_{\text{dark}} \tilde{u}^* \bar{\chi} P_R u$$

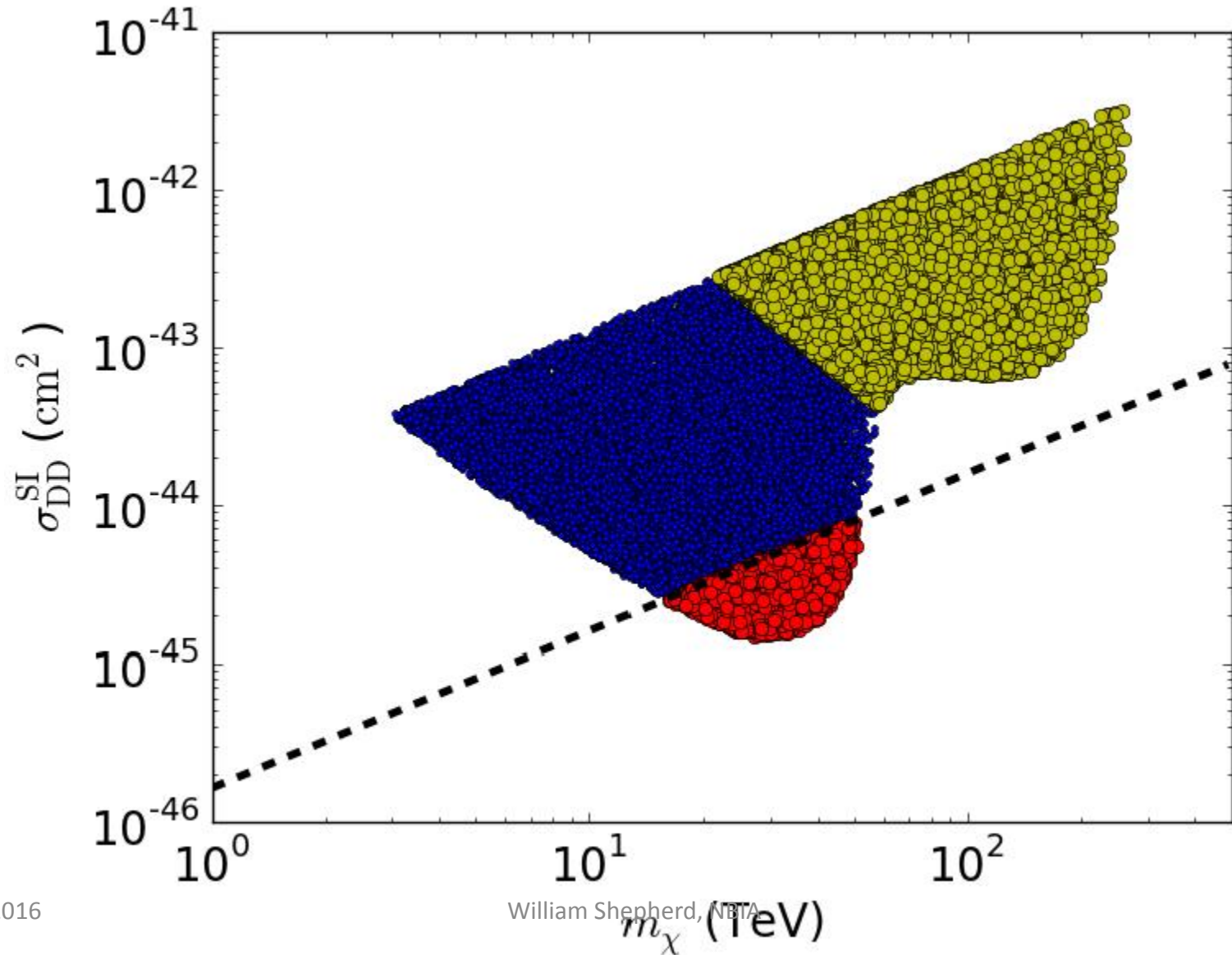
- This introduces the new parameter and scales

$$\lambda_{\text{dark}}, M_\chi, M_{\tilde{u}}$$

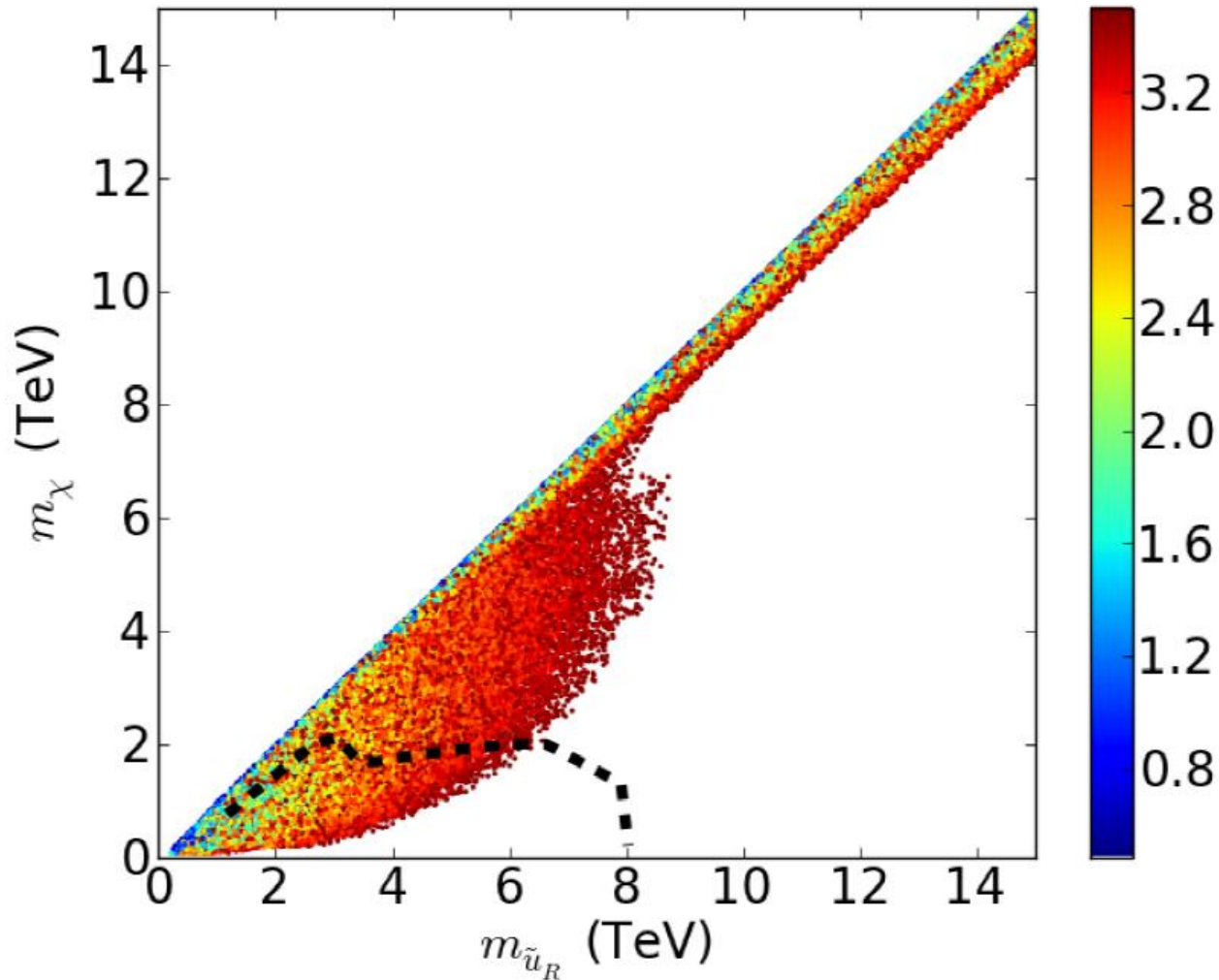
Dirac Dark Matter



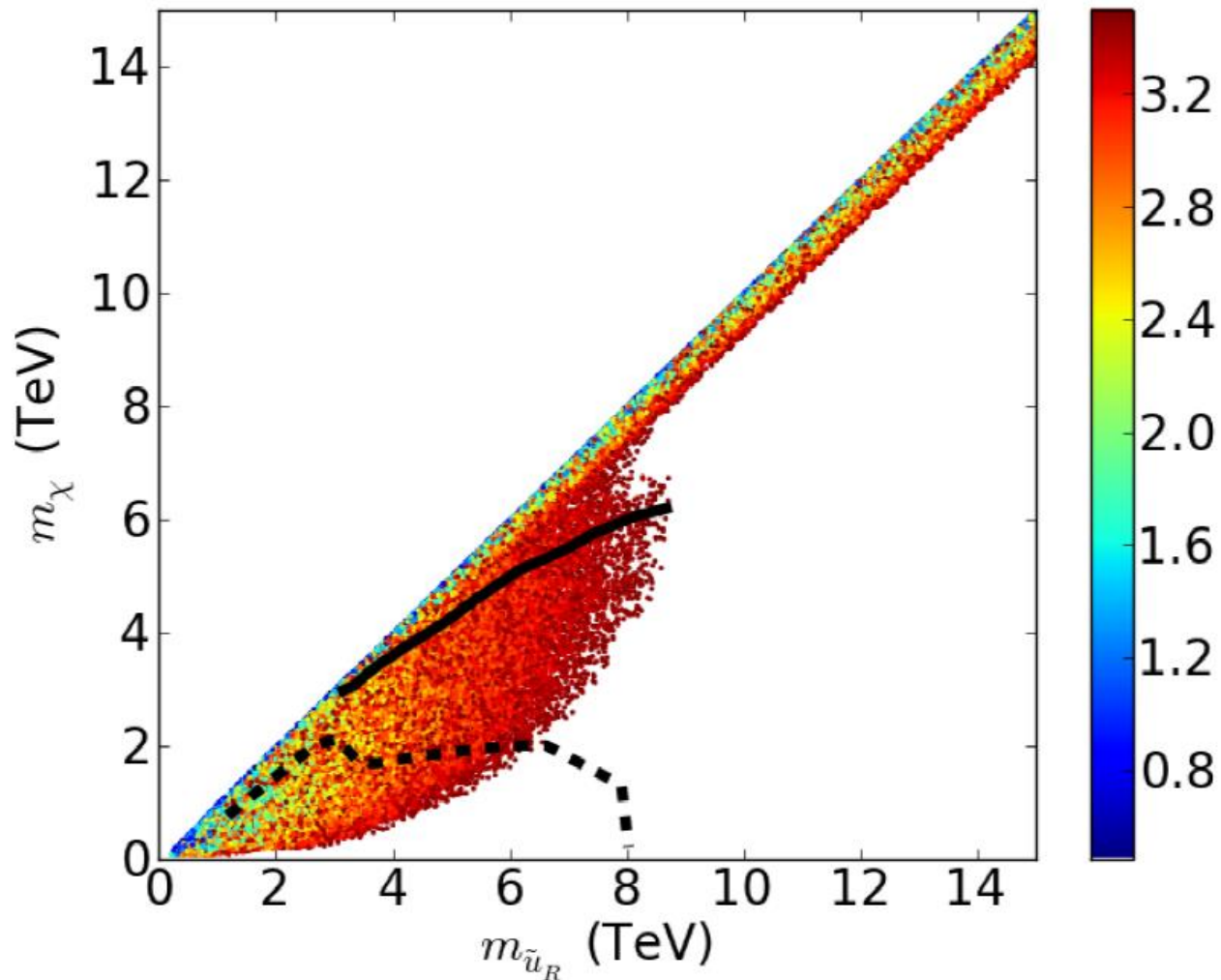
Direct Detection



Majorana Dark Matter

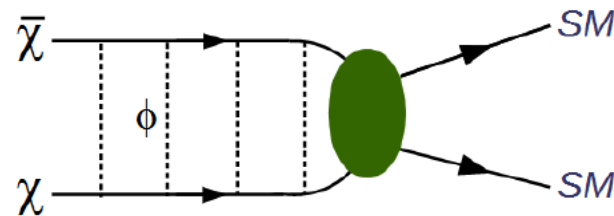
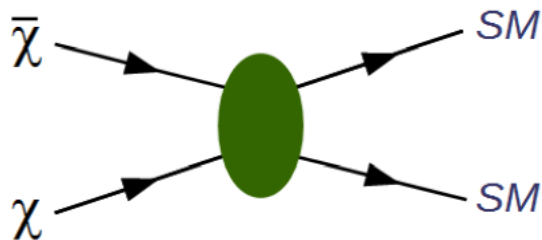


True FCC Reach

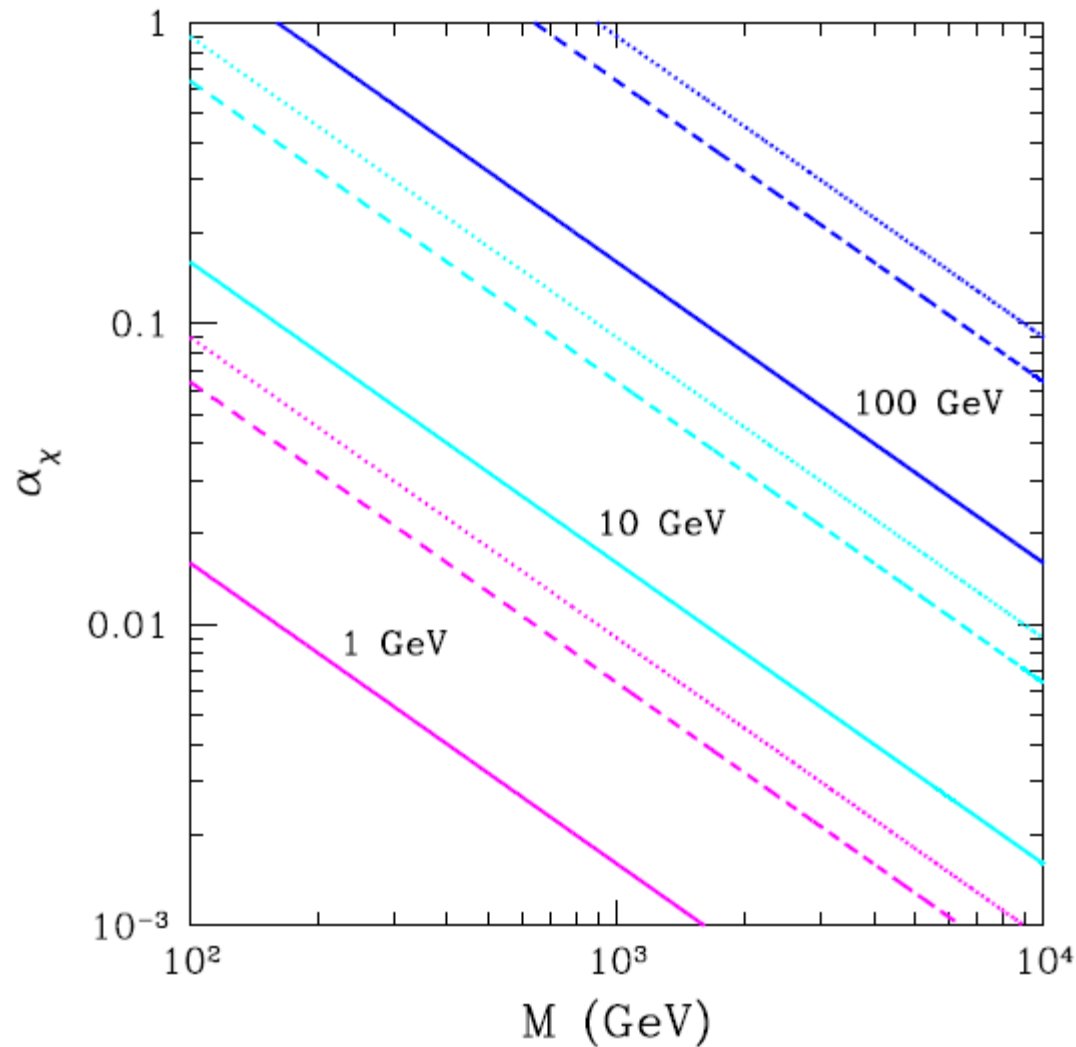


Strong Couplings and Bound States

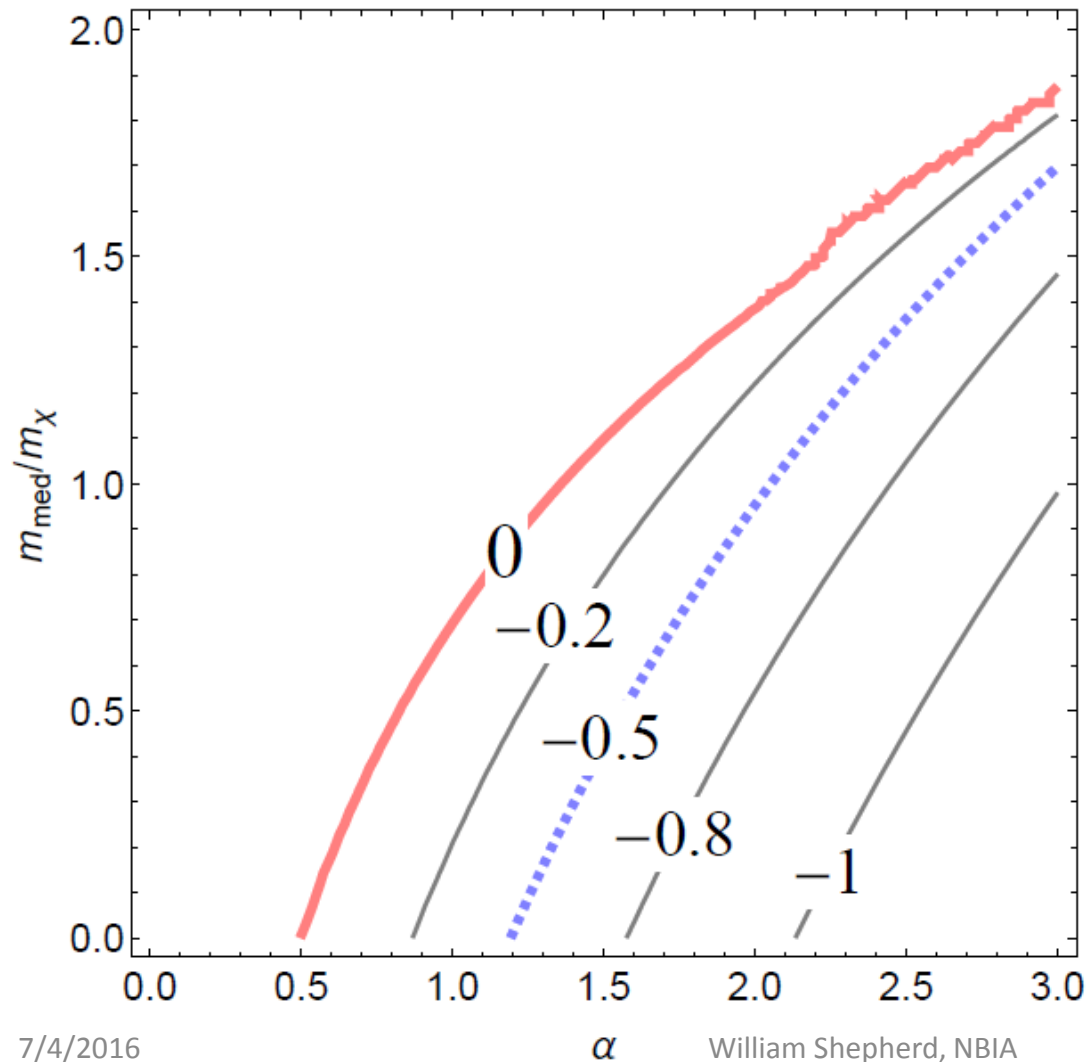
- All of this analysis has focused on the case of very strong couplings to get high allowed mass
- These large couplings can also lead to other effects that may be important
 - Sommerfeld enhancements
 - Dark matter bound states



Yukawa Potential Bound States



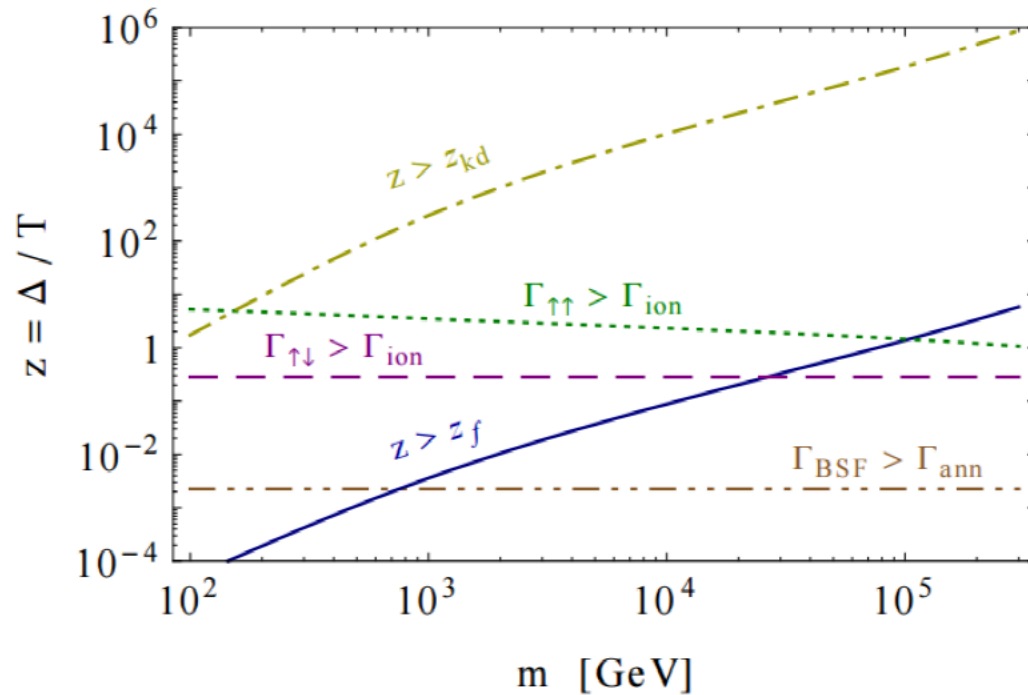
Yukawa Potential Bound States



$$V(r) = \alpha \frac{e^{-m_{\text{med}} r}}{r}$$

$$T_{\text{freeze}} \sim \frac{m_\chi}{20}$$

Cosmological Rates

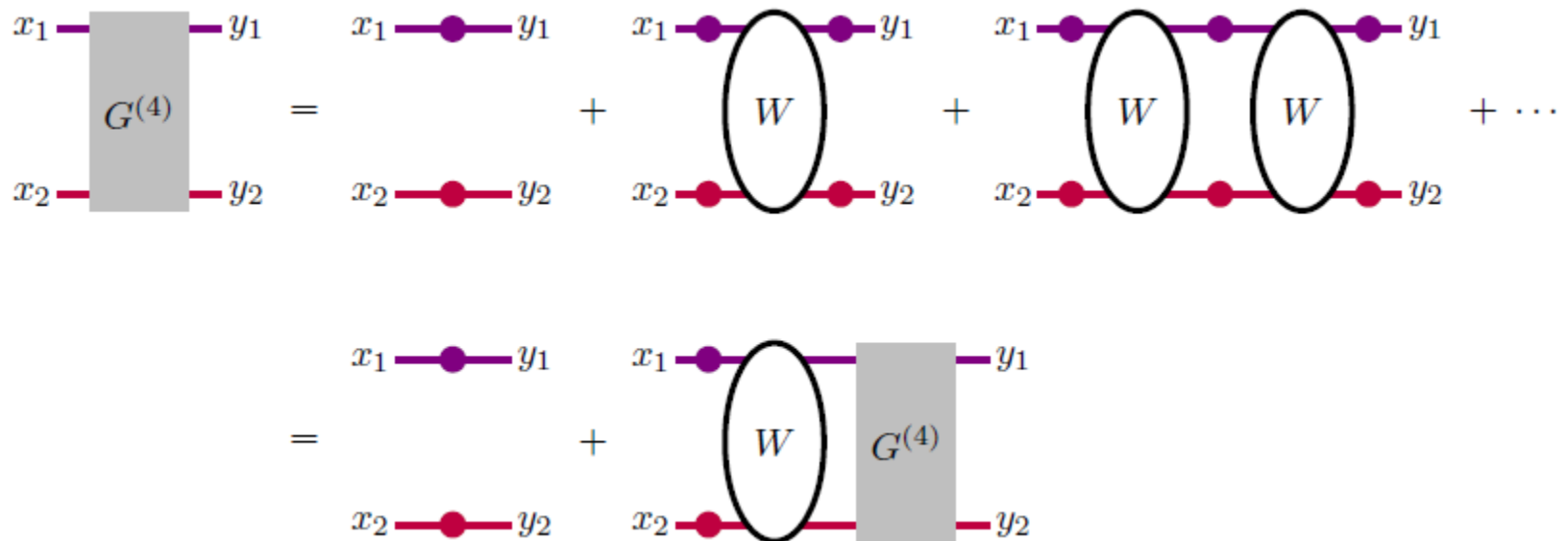


$$\Delta = \frac{m\alpha^2}{4}$$

$\uparrow\downarrow$ and $\uparrow\uparrow$ bound states

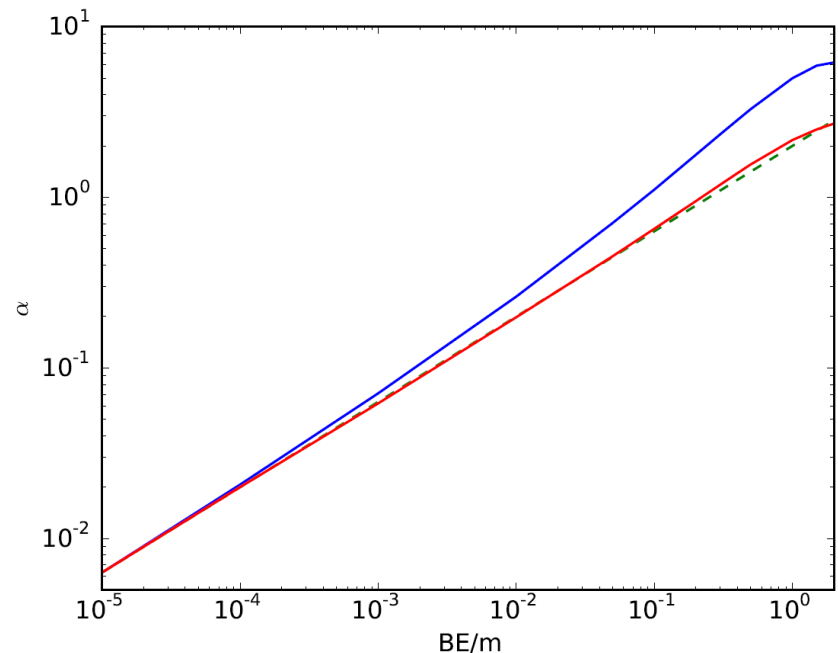
Bethe-Salpeter Equation

- States that are strongly bound enough to matter will have momenta high enough to require relativistic treatment
- If ladder diagrams are the dominant contribution to the binding the Bethe-Salpeter equation describes the physics



Relativistic Corrections

- States with binding energies of $M/10$ or larger require relativistic corrections to the coupling of a factor of 2 or more
- This will be an important shift in the cosmological implications of strong coupling



Outlook

- Perturbativity arguments can be made fully rigorous through unitarity considerations
- These unitarity bounds provide strong constraints on dark matter dynamics
- Combined with collider searches we will be able to place strong limits on WIMPs
- Models with strong coupling like these may already be affected by new phenomena due to bound state formation
 - Investigations of cosmological impact of bound state dynamics are in progress