Minimal Model of Majoronic Dark Radiation and Dark Matter and its Phenomenology

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Minimal Model of Majoronic Dark Radiation and Dark Matter and its Phenomenology

- Introduction
- The Model
- Numerical study
- Phenomenology
Need New Physics-1

- New phys. beyond the SM are called for: (1) \( m_\nu \neq 0 \) and (2) \( \Omega_{DM} h^2 \sim 0.12 \)
- (Too) Many models. The key is the Weinberg operator \((LH)^2\) at the low energy which breaks \( U(1)_L \).
- Accidental global \( U(1)_L \) is free in SM and it connects to neutrino masses.
- Majorona mass is controlled by the scale of \( U(1)_L \) SSB in the type-I (and inverse see-saw) (PLB730, 347.)

\[
y \bar{N}^c N S_L \rightarrow m_N = y \langle S_L \rangle
\]

- We need something electrically charge neutral and (meta)stable.
- DM is stabilized by the Krauss-Wilczek, \( U(1)_L \rightarrow Z_2 \).
- Goldstone is built in global SSB \( U(1)_l \) DM-\( m_\nu \) model, and it contributes to radiation energy density.
Neutrons decouple at $T \sim 1$MeV ($\Gamma_{\text{weak}} \lesssim H$). Later, the photons were heated up by $e^+e^-$ annihilation,

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \sim 2K$$

The present relativistic energy density of the universe

$$\rho_{rad} = g_\gamma \frac{\pi^2}{30} T_\gamma^4 + g_\nu \frac{\pi^2 7}{30 8} T_\nu^4 = \left[1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \rho_\gamma$$

Taking into account the incomplete decoupling, $N_{\text{eff}}^{SM} = 3.046$ (Mangano et al. 2005). Nonzero $\Delta N_{\text{eff}} = N_{\text{eff}} - 3.046$ call for new relativistic DOF beyond the SM.

This new DOF is coined as dark radiation.
\( N_{\text{eff}} \) After Planck 2015

- Planck 2015, 1502.01589, \( N_{\text{eff}} = 3.15(46) \) at 95\%CL. Although the SM seems OK, the statistical significance to rule out DR is still very poor.

- \( 2.5 < N_{\text{eff}} < 3.5 \) at 95\%CL, Valentino et al, PRD93,083523.
- \( \Delta N_{\text{eff}} = 0.29(15) \), G. Steigman, June 2015, INT talk.
- \( \Delta N_{\text{eff}} = 0.4 - 1.0 \), Riess et al(WFC3 on HST), 1604.01424
One more thing to be taken into account

- $\mu_{VS}^{SM} \simeq 10^{10-12} \text{ GeV}$

- Operational see-saw needs stable vacuum at the $\Lambda$ scale $\mu_{LV}$, or $\mu_{LV} < \mu_{VS}^{SM}$.

$$
\begin{pmatrix}
\nu^c & \nu_R
\end{pmatrix}
\begin{pmatrix}
0 & y_D v_{SM} \\
y_D v_{SM} & M_N (= y_s v_I)
\end{pmatrix}
\begin{pmatrix}
\nu \\
\nu^c_R
\end{pmatrix}
$$

- For $\phi_A, \phi_B$ in $V = \lambda_A \phi_A^4 + \lambda_B \phi_B^4 + \lambda_{AB} \phi_A^2 \phi_B^2 + \ldots$, $\lambda_A > 0$, $\lambda_B > 0$, $\lambda_{AB} > -2\sqrt{\lambda_A \lambda_B}$ at any given energy scale. RGE study is necessary.
Motivation and experimental facts

The Model

Numerical study

Phenomenology
Model

Particle content:

<table>
<thead>
<tr>
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<th>$L, Z_2$</th>
<th>$SU(2)$</th>
<th>$U(1)_Y$</th>
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</thead>
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<tr>
<td>$S$</td>
<td>$2_{SSB,+}$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>1 (DM candidate)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$H$</td>
<td>$0_{SSB,+}$</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$N_{iR}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$L_i$</td>
<td>1</td>
<td>2</td>
<td>$-\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Renormalizable Lagrangian: (8 new parameters)

$$
\mathcal{L}_{scalar} = (D_\mu H)^\dagger (D^\mu H) + (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) + (\partial_\mu S)^\dagger (\partial^\mu S) - V(H, S, \Phi)
$$

$$
V(H, S, \Phi) = -\mu^2 H^\dagger H - \mu_s^2 S^\dagger S + m_\Phi^2 \Phi^\dagger \Phi + \lambda_H (H^\dagger H)^2 + \lambda_\Phi (\Phi^\dagger \Phi)^2 \\
+ \lambda_s (S^\dagger S)^2 + \lambda_{SH} (S^\dagger S)(H^\dagger H) + \lambda_{PH} (\Phi^\dagger \Phi)(H^\dagger H) \\
+ \lambda_{FS} (S^\dagger S)(\Phi^\dagger \Phi) + \frac{\kappa}{\sqrt{2}} \left[ (\Phi^\dagger)^2 S + S^\dagger \Phi^2 \right]
$$

and we take $\kappa$ to be real, $m_\Phi^2 > 0$, and define $\bar{\kappa} \equiv \lambda_{FS} v_s + \kappa$. 
After SSB, $\langle S \rangle \neq 0$ and $\langle H \rangle \neq 0$, the fields are expanded as
\[
\Phi = \frac{1}{\sqrt{2}}(\rho + i\chi), \quad S = \frac{1}{\sqrt{2}}(v_s + s + i\omega)
\]
and for the Higgs
\[
H = (0, \frac{v + h}{\sqrt{2}})^T.
\]
$\omega$ is the massless Goldstone or Majoron.

$\langle S \rangle$ is inv. under a $U(1)_l$ $\pi -$rotation, a $Z_2$ parity remains:
\[
s, \omega, h \rightarrow s, \omega, h \\
\rho, \chi \rightarrow -\rho, -\chi
\]

Due to mixing, the physical mass eigenstates are
\[
\begin{pmatrix}
h_1 \\
h_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
h \\
s
\end{pmatrix}
\]
with mixing angle $\tan 2\theta = \frac{\lambda_{HS}v_Hv_S}{\lambda_Sv_S^2 - \lambda_Hv_H^2}$.

Identify $h_1 \equiv h_{SM}$ with a mass of 125 GeV, $h_2$ is a new neutral scalar (just call them $H$ and $S$).
In terms of the physical masses and mixing, we have

\[ \lambda_H = \frac{\cos^2 \theta M_H^2 + \sin^2 \theta M_S^2}{2v_H^2}, \quad \nu_S = -\frac{\sin \theta \cos \theta (M_H^2 - m_S^2)}{v_H \lambda_{SH}}. \]

\[ \lambda_S = \frac{\sin^2 \theta M_H^2 + \cos^2 \theta M_S^2}{2v_S^2}, \quad \nu_S = \frac{\sqrt{2}M_N}{v_S}. \]

\[ \nu_H = 246 \text{GeV}, \quad m_H = 125 \text{ GeV} \]

for a given set of \( \{M_S, \theta, \lambda_{SH}\} \), \( \nu_S \) and \( \lambda_S \) are determined.

No solution found for \( M_N < 0.5 \text{TeV} \), not sensitive otherwise. We take \( M_N = 1 \text{TeV} \) as a benchmark value.

- leptons interact with the Majoron via

\[ \frac{1}{2\nu_S}(\partial_\mu \omega \bar{\psi}_1 \gamma^\mu \psi_1) \]

- couplings of the Majoron to charged fermions generated at the one-loop level. Very small, no constraints from stellar cooling
Majoron decoupling

- However a dim-5 int.

\[ \mathcal{L}_{f\omega} = -{\frac{\lambda_{HS} m_f}{M_h^2 M_s^2}} \bar{f} f \partial^\mu \omega \partial_\mu \omega \]

can be generated through scalar mixing:

- Order of magnitude estimation gives

\[ \Gamma(f \bar{f} \leftrightarrow \omega \omega) \sim {\frac{\lambda^2_{HS} m_f^2}{M_H^4 M_S^4}} \times T_{\text{dec}}^7 \times N_c^f \]

- Since \( H \sim T_{\text{dec}}^2 / M_{\text{Pl}} \),

\[ \frac{N_c \lambda^2_{HS} m_{\text{eff}}^2 T_{\text{dec}}^5 M_{\text{Pl}}}{M_H^4 M_S^4} \approx 1. \]
\( \Delta N_{\text{eff}} \) and \( T_{\text{dec}} \)

- Conservation of Entropy in the co-moving volume give:

\[
\Delta N_{\text{eff}} = \frac{4}{7} \left( \frac{g_{*}(T_{\nu}^{+})}{g_{*}(T_{\omega}^{-})} \right)^{\frac{4}{3}}
\]

where \( g_{*} \) is the effective number of relativistic DOF.

\( \Delta N_{\text{eff}} = \{0.39, 0.055, 0.0451, 0.0423\} \) for \( T_{\text{dec}} = \{m_{\mu}, 1\,\text{GeV}, m_{c}, m_{\tau}\} \) respectively.

- Due to scalar mixing, \( H \) can always decays into a pair of invisible \( \omega \)'s,

\[
\Gamma_{\omega\omega} = \frac{1}{32\pi} \frac{\sin^{2}\theta M_{H}^{3}}{v_{S}^{2}}
\]

\[
\Gamma_{\omega\omega} \leq \Gamma_{H}^{\text{inv}} < 0.8\,\text{MeV}
\]

\( \) gives \( M_{S}^{\text{max}} \) via

\[
\frac{M_{S}^{4}}{(M_{H}^{2} - M_{S}^{2})^{2}} \leq \cos^{2}\theta \frac{32\pi m_{\text{eff}}^{2} T_{\text{dec}}^{5} M_{\text{Pl}}^{7}}{v_{H}^{2} M_{H}^{7}} \Gamma_{H}^{\text{inv}}
\]
$M_S$, $T_{dec}$, and $\sin \theta^2$

- LHC-I, $\mu = 1.1 \pm 0.11$ gives indirect bound $\sin \theta^2 < 0.13$ at 2 $\sigma$. Direct search from OPAL $e^+ e^- \rightarrow hZ$.

- From rare B decay, $|\theta| < 0.002$ for $M_S < 2\text{GeV}$.
- With this, the decoupling condition yields

$$\lambda_{SH} \sim \frac{M_H^2 M_S^2}{T_{dec}^3 \sqrt{T_{dec} M_{pl}}} \ll 1$$
Relic density/ SI scattering can be calculated.

Thermal $\Omega_{DM} h^2 \sim \frac{0.1 \text{pb}}{\langle \sigma v \rangle}$, $\langle \sigma v \rangle = 2.5(1) \times 10^{-9} (\text{GeV})^{-2}$.

Limit from LUX ($M_\rho \sim \mathcal{O}(\text{TeV})$)
1-loop beta function

\[
16\pi^2 \frac{d\lambda_H}{dt} = 12\lambda_H^2 + 6\lambda_H y_t^2 - 3y_t^4 - \frac{3}{2} \lambda_H (3g_2^2 + g_1^2) + \frac{3}{16} [(g_1^2 + g_2^2)^2 + 2g_2^4]
+ \frac{1}{2} (\lambda_{HS}^2 + \lambda_{\Phi H}^2)
\]

\[
16\pi^2 \frac{d\lambda_\Phi}{dt} = 10\lambda_\Phi^2 + \lambda_{\Phi H}^2 + \frac{1}{2} \lambda_{\Phi S}^2
\]

\[
16\pi^2 \frac{d\lambda_S}{dt} = 10\lambda_S^2 + \lambda_{HS}^2 + \frac{1}{2} \lambda_{\Phi S}^2 - 8Y_S^4 + 4Y_S^2 \lambda_S
\]

\[
16\pi^2 \frac{d\lambda_{HS}}{dt} = 2\lambda_{HS}^2 + \lambda_{HS} (6\lambda_H + 4\lambda_S) + 2\lambda_{HS} Y_S^2 - \frac{3}{4} \lambda_{HS} (3g_2^2 + g_1^2)
+ 3\lambda_{HS} y_t^2 + \lambda_{\Phi S} \lambda_{\Phi H}
\]

\[
16\pi^2 \frac{d\lambda_{\Phi H}}{dt} = 2\lambda_{\Phi H}^2 + \lambda_{\Phi H} (6\lambda_H + 4\lambda_\Phi) - \frac{3}{4} \lambda_{\Phi H} (3g_2^2 + g_1^2)
+ 3\lambda_{\Phi H} y_t^2 + \lambda_{\Phi S} \lambda_{HS}
\]

\[
16\pi^2 \frac{d\lambda_{\Phi S}}{dt} = 2\lambda_{\Phi S}^2 + 4\lambda_{\Phi S} (\lambda_\Phi + \lambda_S) + 2\lambda_{\Phi S} Y_S^2 + 2\lambda_{\Phi H} \lambda_{HS}
\]

\[
16\pi^2 \frac{dY_S}{dt} = 3Y_S^3, \quad 16\pi^2 \frac{dK}{dt} = \kappa (2\lambda_{\Phi S} + 2\lambda_\Phi + Y_S^2)
\]

- Fermionic $y^4$ contributions responsible for vacuum instability.
- In general, new scalar DOF’s help VS.
- Possible new Landau pole in $\lambda_\Phi$
Outline

- Motivation and experimental facts
- The Model
- **Numerical study**
- Phenomenology
Numerical Scan

- Comprehensive scan of the whole parameter space.
  Randomly scan $T_{\text{dec}}, M_S, \theta, M_\rho, \lambda_\phi S (\in [-4\sqrt{\pi \lambda_S}, 4\pi]), \bar{\kappa}, \lambda_{\Phi H}, \lambda_\phi$.

- Requirements and experimental constraints in our search:
  - Improve the SM vacuum stability, $\mu_{VS} > \mu_{VS}^{SM}$
    ($\mu_{VS1-\text{loop}}^{SM} = 2 \times 10^5 \text{GeV}$)
  - No Landau pole below $\mu_{VS}^{SM}$
  - $\Gamma_H^{inv} < 0.8 \text{MeV}$.
  - $T_{\text{dec}} \in [m_\mu, 2 \text{GeV}]$.
  - $\theta$ complies with all experimental bounds.
  - relic density $\langle \sigma v \rangle = 2.5(1) \times 10^{-9} (\text{GeV})^{-2}$.
  - Spin-independent direct DM search bound (LUX)

- The largest $R_{VS} \equiv \log_{10} \mu_{VS}/\mu_{VS}^{SM}$ we got $\sim 11$. New scalar DOF help to go up to GUT scale, but not $M_{pl}$.

- $T_{\text{dec}} > 1.3 \text{GeV}, 1.5 \text{TeV} < M_\rho < 4 \text{TeV}, M_S \in [20, 100] \text{GeV},$ $\nu_S, -\kappa \in [2 - 20] \text{TeV}$
Some qualitative understanding

- $T_{dec} < 2\text{GeV}$, small $\lambda_{SH}$.
- $\lambda_S$ V.S. needs large $v_S$ to suppress $y_S = \sqrt{2}M_N/v_s$.
- large $v_S \Rightarrow$ large mixing $\Rightarrow$ large $M_S$ and higher $T_{dec}$.
- To counter act $y_S$ in $\lambda_S$ V.S. $\Rightarrow$ sizable $\lambda_\Phi_S$.
- to improve $\lambda_H$ V.S. $\Rightarrow$ $\lambda_\Phi_H \sim O(1)$.
- $\lambda_\Phi_H \sim O(1) \Rightarrow M_\rho > 1.5\text{TeV}$.
- to avoid $\lambda_\Phi$ Landau pole $\Rightarrow$ small $\lambda_\Phi$, $\lambda_\Phi_S$ are preferred.
- small $\lambda_\Phi_S \Rightarrow M_\rho < 4\text{TeV}$, or too much DM.

\[ R_{VS} \ G: 2 - 4, \ B: 4 - 6, R: > 6. \]
### Numerical Scan: 2 examples

<table>
<thead>
<tr>
<th>Config.</th>
<th>$T_{\text{dec}}$</th>
<th>$M_S$</th>
<th>$\theta$</th>
<th>$M_\rho$</th>
<th>$\nu_S$</th>
<th>$R_{VS}$</th>
<th>$Br(\omega\omega)$</th>
<th>$Br(\bar{b}b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.94</td>
<td>27.3</td>
<td>-0.03</td>
<td>2.2</td>
<td>6.7</td>
<td>2.1</td>
<td>0.87</td>
<td>0.11</td>
</tr>
<tr>
<td>B</td>
<td>1.87</td>
<td>67.6</td>
<td>-0.32</td>
<td>1.8</td>
<td>12.1</td>
<td>10.0</td>
<td>0.07</td>
<td>0.78</td>
</tr>
</tbody>
</table>

$T_{\text{dec}}$ and $M_S$ ($M_\rho$ and $\nu_S$) are in GeV (TeV).

![Graph 1](image1)

![Graph 2](image2)
Numerical Scan: DM annihilation

For $\lambda_S, \lambda_{SH} \ll 1$ and $M_\rho \gg M_W, M_Z, M_H$, the total annihilation cross section of $\rho\rho \to W^+W^-, ZZ, HH$ can be estimated to be

$$\langle \sigma v \rangle_{W/Z/H} \equiv \langle \sigma v \rangle_{W^+W^-} + \langle \sigma v \rangle_{ZZ} + \langle \sigma v \rangle_{HH} \sim \frac{1}{64\pi} \frac{\lambda_{PH}^2}{M_\rho^2} \times [2 + 1 + 1]$$

$$\sim 5 \times 10^{-9} \text{GeV}^{-2} \left( \frac{\lambda_{PH}}{0.5} \right)^2 \left( \frac{1 \text{TeV}}{M_\rho} \right)^2.$$

The channels $\rho\rho \to SS, \omega\omega$ open only when $M_\rho > 2\text{TeV}$. 
Outline

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Indirect search at LHC

- A universal value $\cos^2 \theta$ for all signal strengths due to $H - S$ mixing. $M_S > 40\text{GeV}$ detectable at LHC14 with $3ab^{-1}$.
- The SM Higgs triple coupling $\lambda_{HHH}^{SM} = 3M_H^2/v_H$ is modified to $\lambda_{HHH} = 6\lambda_H v_H c_\theta^3 - 6\lambda_S v_S s_\theta^3 + 3\lambda_SH s_\theta c_\theta (v_H s_\theta - v_S c_\theta)$. And $\delta_{HHH} \equiv (\lambda_{HHH} - \lambda_{HHH}^{SM})/\lambda_{HHH}^{SM}$.
- Similarly, $\lambda_{4H}^{SM} = 6\lambda_H = 3(M_H/v_H)^2$ is replaced by $\lambda_{4H} = 6(\lambda_H c_\theta^4 + \lambda_S s_\theta^4)$ in this model. $\delta_{4H} \equiv (\lambda_{4H} - \lambda_{4H}^{SM})/\lambda_{4H}^{SM}$, XS for triple Higgs production is too small.
S decay

The relevant modes are $s$ into quarks, leptons, and $\omega$'s.

$$\Gamma(s \rightarrow \omega\omega) = \frac{1}{32\pi} \frac{c_\theta^2 M_s^3}{\nu_s^2}$$

$$\Gamma(s \rightarrow f\bar{f}) = \frac{M_s}{8\pi} N_f \beta_f^3 \left( \frac{m_f s_\theta}{\nu} \right)^2$$

where $\beta_f = \sqrt{1 - \frac{4m_f^2}{M_s^2}}$ for the fermion.
Future Circular Collider expects to have $10^{12-13}$ $Z$ bosons at $\sqrt{s} = M_Z$ with multi-$ab^{-1}$ luminosity. (JHEP1401,164)

Defining $y_f = \frac{M_{f\bar{f}}}{M_Z^2}$ we obtain

$$\frac{d\text{Br}(Z \rightarrow Sf\bar{f})}{dy} = \frac{g^2 \sin^2 \theta}{192\pi^2 \cos^2 \theta_W} \sqrt{y_f^2 - 2y_f(1 + r_Z^2) + (1 - r_Z^2)^2}$$

$$\times \left[ y_f^2 + 2y_f(5 - r_Z^2) + (1 - r_Z^2)^2 \right] \times \frac{1}{(1 - y_f)^2} \times \text{Br}(Z \rightarrow f\bar{f})$$

where $r_Z = \frac{M_S}{M_Z}$ and $0 \leq y_f \leq (1 - r_Z)^2$. The kinematic lower bound can be safely taken to be zero even for $y_b$. 

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Note the lower bound for each decay mode.

\[ Br(Z \rightarrow b\bar{b} E)_{SM} = 5.25 \times 10^{-8} \]

\[ Z \rightarrow S\bar{f}f \] signal stands out from the SM background.
DM bound state?

- DM $\rho - \rho$ interact by exchanging $t-$channel $S$ and $H$ and this force is attractive. The relevant terms are:

$$\mathcal{L} \supset \frac{1}{2} [\lambda_{\phi} v_H h + \bar{\kappa} s] \rho^2$$

- $\bar{\kappa} \gg \lambda_{\phi} v_H$, $s$ mediation dominates.
- $\bar{\kappa}/M_\rho \in [-1.0, 1.0] \Rightarrow$ DM may form bound state, $B_{\rho}$.
- Write the effective coupling between $B_{\rho}$ and $2\rho$ as

$$\mathcal{L} \sim \alpha_{B} B_{\rho} \rho^2.$$ 

By dimensional analysis, $\alpha_{B} \sim (\bar{\kappa}^2/M_\rho)$. 

![Diagram showing the interaction between DM particles and the exchange of bosons S and H]
DM bound state decay Width

- Decay width of $B_\rho$, $\Gamma_B \propto |\psi(0)|^2 \times |M_{B_\rho}|^2$.
- $|\psi(0)|^2 \sim \tilde{\kappa}^6 / M_\rho^3$ by dimension analysis.
- Rescale the decay amplitude square and make it dimensionless, broken into

$$|M_{B_\rho}|^2 = \gamma_{ss} + \gamma_{HH} + \gamma_{sH} + \gamma_{\omega\omega} + \gamma_{W, Z} + \gamma_{f\bar{f}}$$

subscripts label the decay final state.
- drop terms of $\mathcal{O}(v_H / M_\rho)$.

$$\gamma_{ss} \approx \left[ \lambda \Phi_S - \frac{\tilde{\kappa}^2}{M_\rho^2} \right]^2, \quad \gamma_{HH} \approx \lambda_{\Phi H}^2,$$

$$\gamma_{\omega\omega} \approx \left[ \lambda \Phi_S - \frac{\tilde{\kappa}^2}{M_\rho^2 - \kappa v_S} \right]^2, \quad \gamma_{W, Z} \approx 3\lambda_{\Phi H}^2$$

- Finally, the decay width:

$$\Gamma_B \sim M_\rho \left( \frac{\tilde{\kappa}}{M_\rho} \right)^6 \left[ \gamma_{ss} + \gamma_{\omega\omega} + 4\lambda_{\Phi H}^2 \right]$$
\[ \langle \sigma \nu \rangle \text{ and DM bound state} \]

- Put \( \Gamma_B \) into the propagator squared, annihilation XS due to \( B_\rho \) resonant

\[
\sigma \nu \sim \frac{\alpha_B^2 (\Gamma_B/M_B)}{(s - M_B^2)^2 + \Gamma_B^2 M_B^2}
\]

\( \Gamma_B/M_B \) takes care the nearly on-shell \( B_\rho \) decay.

- When \( \nu \ll 1, s \sim M_B^2 \), no temperature dependence,

\[
\langle \sigma \nu \rangle \sim \frac{\alpha_B^2}{M_B^3 \Gamma_B} \sim \frac{R_B}{M_B^2 \rho^2} [\gamma_{ss} + \gamma_{\omega\omega} + 4\lambda_{\Phi H}^2]
\]

and

\[
R_B \equiv \left( \frac{M_\rho}{\tilde{\kappa}} \right)^2 [\gamma_{ss} + \gamma_{\omega\omega} + 4\lambda_{\Phi H}^2]^{-2}
\]

is the boost factor for indirect DM detection.

- For \( |M_{B_\rho}|^2 \sim \mathcal{O}(10^0) \) and \( \tilde{\kappa} \sim 0.1 M_\rho \), the boost factor around 100.
DM annihilate into Majoron pair is a few to 40%. With boost factor 100, \(\langle \sigma v \rangle (DM + DM \rightarrow \omega \omega) \sim 10^{-26} - 10^{-24} (cm^3/s)\).

\(\omega\) could be a component of the ‘apparent’ neutrino flux at \(E_\nu = M_\rho\) in IceCube and other neutrino observatories.

Shower events, mostly from the Galactic center.
Minimal Majoron model with SM singlet scalars carrying lepton numbers takes care of DR+DM+$m_\nu$+V.S.

$\Delta N_{eff} \sim 0.05$, or $T_{dec} > m_c$ is preferred.

Scalar DM, $\rho$, of mass $1.5 - 4$ TeV is required by V.S. and an operational type-I see-saw.

$M_S \in [10, 100]$ GeV, mixing as large as 0.1.

$S$ mainly decays into $b \bar{b}$ and/or $\omega \omega$.

Sensitive search will be $Z \rightarrow S + f \bar{f}$, followed by $S$ into a pair of Majoron and/or b-quarks.

Possible DM bound state due to sizable $S\rho\rho$-coupling.

shower-like events with apparent neutrino energy at $M_\rho$ could contribute to the neutrino flux in underground neutrino detectors.