The anomalous magnetic moment of the muon in a GUT model with A4 × Z5 family symmetry

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arXiv:1605.02072

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SUSY 2016

4th July 2016

The anomalous magnetic moment of the muon

The magnetic moment of the muon is related to its spin by $\boldsymbol{M} = g_\mu \frac{e}{2m_\mu} \boldsymbol{S}$.

The Dirac equation predicts a *gyromagnetic ratio* $g_\mu = 2$ but quantum corrections lead to a deviation quantified by

$$a_\mu = \frac{g_\mu - 2}{2}$$

New Fermilab and J-PARC experiments will improve uncertainty by a factor of 4.

Experimental value: $a_\mu^{\rm exp} = 116592091(63) \times 10^{-11}$ [Muon g-2 Col. 2006]

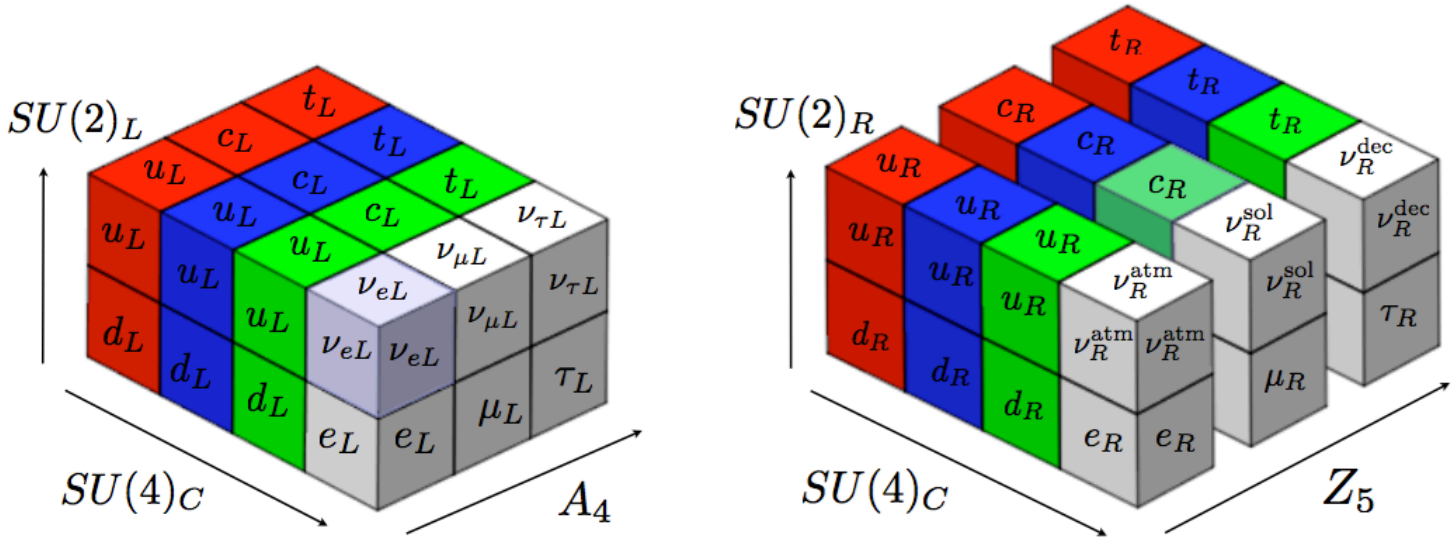
SM prediction: $a_\mu^{\rm SM} = 116591803(49) \times 10^{-11}$ [PDG, 2014]

Deviation between experiment and SM: $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 288(80) \times 10^{-11}$ $(3.6\,\sigma)$

The A to Z GUT Model

The Model has a Pati-Salam gauge group with extra family symmetries. [King, 2014]

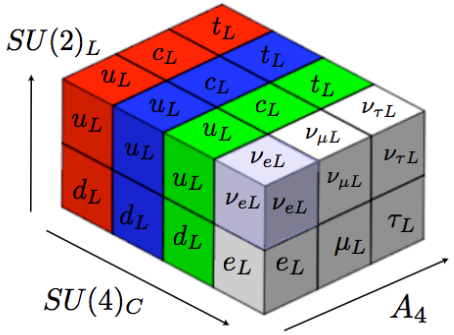
$$\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times A_4 \times Z_5$$

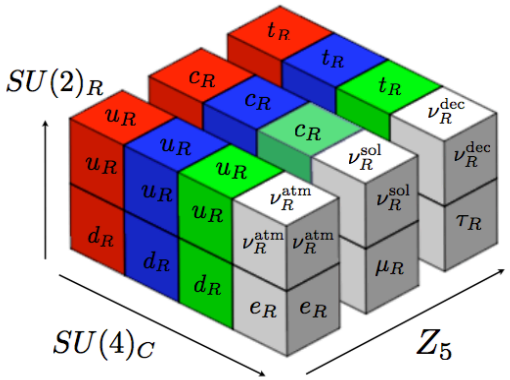


This model is motivated by an excellent description of quark and lepton masses, mixing and CP violation.

Left-handed quarks and leptons form an A_4 triplet:

$$F = (4,2,1)_i = \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_i \rightarrow (Q_i, L_i)$$





This has a GUT scale mass m.

But right-handed fields are A_4 singlets distinguished by Z_5 charges $(\alpha,\ \alpha^3,\ 1)$.

$$F_i^c = (\bar{4},1,2)_i = \begin{pmatrix} u^c & u^c & u^c & \nu^c \\ d^c & d^c & d^c & e^c \end{pmatrix}_i \rightarrow (u_i^c, d_i^c, \nu_i^c, e_i^c)$$

These have GUT scale masses $m_1,\ m_2,\ m_3$.

This is less constrained than the cMSSM.

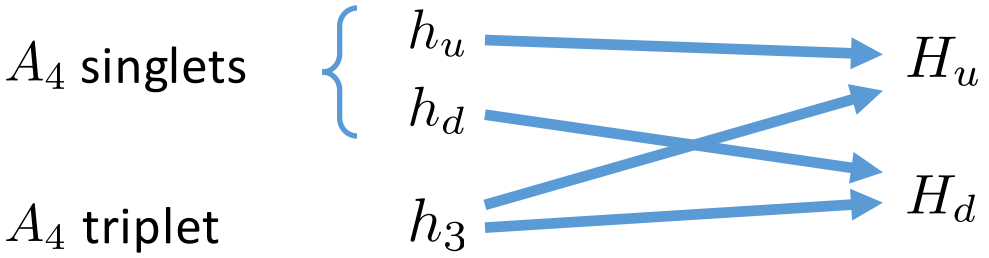
The Pati-Salam symmetry is broken to the SM at the GUT scale by PS Higgs fields

$$H^c = (u^c_H, d^c_H, \nu^c_H, e^c_H) \in (\bar{4}, 1, 2)$$
$$\overline{H}^c = (\bar{u}^c_H, \bar{d}^c_H, \bar{\nu}^c_H, \bar{e}^c_H) \in (4, 1, 2)$$

which gain a Vev in the right-handed neutrino direction $\langle H^c \rangle = \langle \nu^c_H \rangle = \langle \overline{H^c} \rangle = \langle \bar{\nu}^c_H \rangle \sim 2 \times 10^{16}$ GeV

(These vevs are kept equal to maintain gauge coupling unification.)

For EW symmetry breaking introduce bi-doublet (1,2,2) Higgs bosons, including



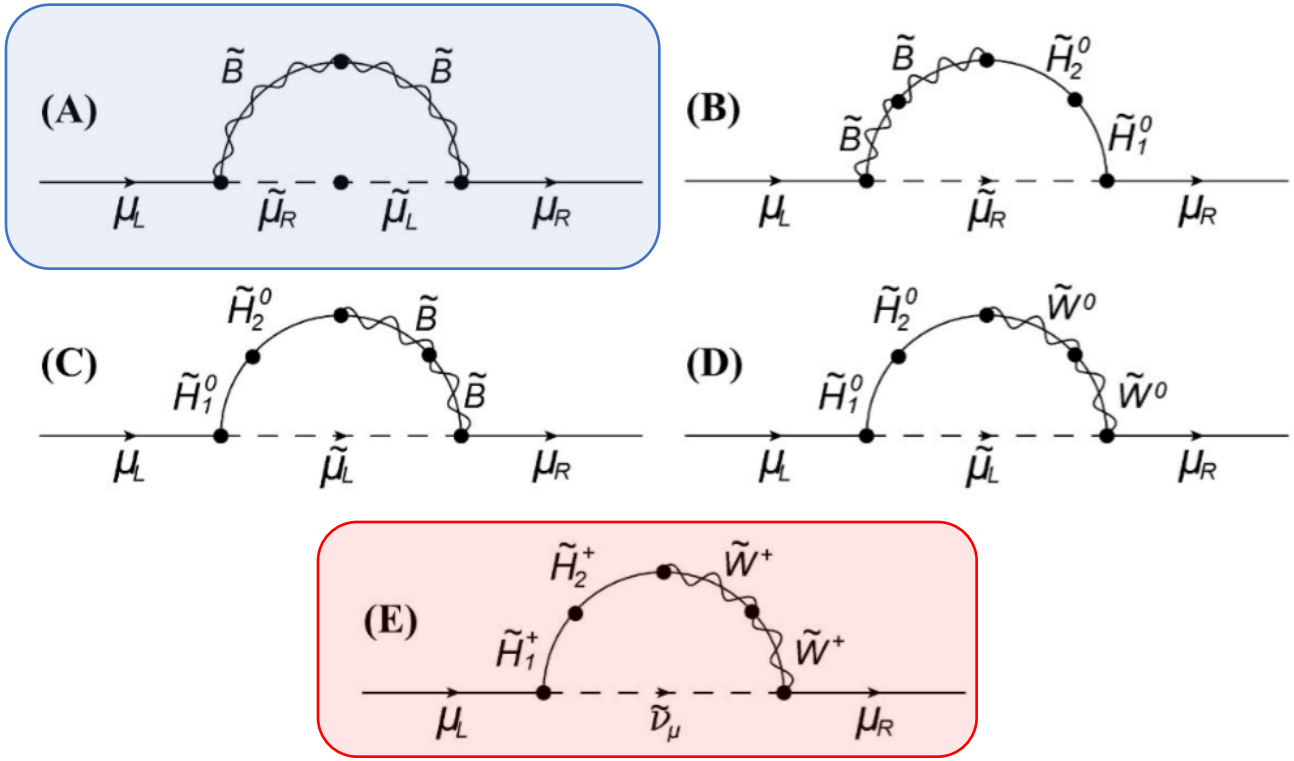
At PS breaking, most of these Higgs fields become heavy. Only two combinations, H_u and H_d become light.

Model Parameters

GUT Scale Parameters

m_0	Left-handed squarks and sleptons (all 3 generations)
$m_1,\ m_2,\ m_3$	Right-handed squarks and sleptons (each generation independent)
$M_1,\ M_2,\ M_3$	Gaugino masses
$m_{H_u},\ m_{H_d}$	Higgs boson masses
A_{tri}	Trilinear couplings (in principle, one for each generation)
$\tan\beta$	EW Scale parameters
$\mathrm{sgn}(\mu)$	

$(g-2)_\mu$ at one-loop



$$\Delta a_\mu^{(A)} = \boxed{\left(\frac{M_1\mu}{m^2_{\tilde{\mu}_L} m^2_{\tilde{\mu}_R}}\right)} \frac{\alpha_1}{4\pi} m_\mu^2 \tan\beta \cdot f_{\rm N}^{(A)}\left(\frac{m^2_{\tilde{\mu}_L}}{M_1^2}, \frac{m^2_{\tilde{\mu}_R}}{M_1^2}\right),$$

$$\Delta a_\mu^{(B)} = -\left(\frac{1}{M_1\mu}\right) \frac{\alpha_1}{4\pi} m_\mu^2 \tan\beta \cdot f_{\rm N}^{(B)}\left(\frac{M_1^2}{m^2_{\tilde{\mu}_R}}, \frac{\mu^2}{m^2_{\tilde{\mu}_R}}\right),$$

$$\Delta a_\mu^{(C)} = \left(\frac{1}{M_1\mu}\right) \frac{\alpha_1}{8\pi} m_\mu^2 \tan\beta \cdot f_{\rm N}^{(C)}\left(\frac{M_1^2}{m^2_{\tilde{\mu}_L}}, \frac{\mu^2}{m^2_{\tilde{\mu}_L}}\right),$$

$$\Delta a_\mu^{(D)} = -\left(\frac{1}{M_2\mu}\right) \frac{\alpha_2}{8\pi} m_\mu^2 \tan\beta \cdot f_{\rm N}^{(D)}\left(\frac{M_2^2}{m^2_{\tilde{\mu}_L}}, \frac{\mu^2}{m^2_{\tilde{\mu}_L}}\right),$$

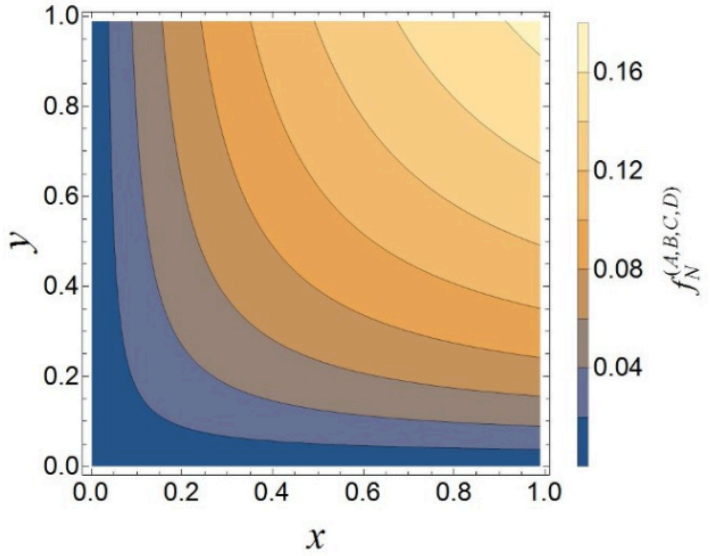
$$\Delta a_\mu^{(E)} = \left(\frac{1}{M_2\mu}\right) \frac{\alpha_2}{4\pi} m_\mu^2 \tan\beta \cdot \boxed{f_{\rm C}^{(E)}\left(\frac{M_2^2}{m^2_{\tilde{\nu}_\mu}}, \frac{\mu^2}{m^2_{\tilde{\nu}_\mu}}\right)},$$

If μ is large and smuons are light, then contribution **A** is enhanced.

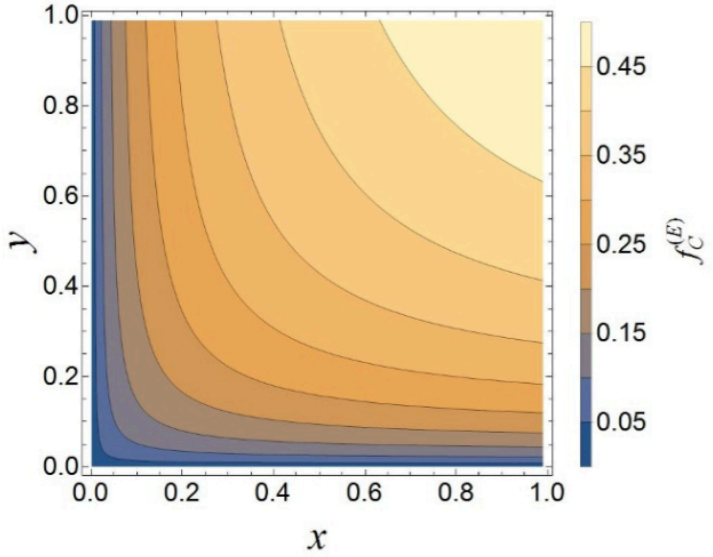
[Stöckinger, 2006; Endo et al, 2014]

However, the function $f_C^{(E)}$ may be large, enhancing contribution **E**.

A, B, C, D



E



$$f_{\mathrm{N}}^{(A,B,C,D)}(x,y) = xy\left[\frac{-3+x+y+xy}{(x-1)^2(y-1)^2} + \frac{2x\log x}{(x-y)(x-1)^3} - \frac{2y\log y}{(x-y)(y-1)^3}\right]$$

$$f_{\mathrm{C}}^{(E)}(x,y) = xy\left[\frac{5-3(x+y)+xy}{(x-1)^2(y-1)^2} - \frac{2\log x}{(x-y)(x-1)^3} + \frac{2\log y}{(x-y)(y-1)^3}\right]$$

For example for $x = y = 0.2$ we have $f_C^{(E)} \sim 10 \times f_N^{(A,B,C,D)}$.

What do we want?

Dark Matter:

Prefer a DM Relic Density $\Omega h^2 = 0.1198 \pm 0.0026$ [Planck, 2014]
but we will also allow $\Omega h^2 \in [0.06, 0.1224]$

Also require Direct Detection bounds $\sigma^{DD-SI} \leq 7.6 \times 10^{-10}\mathrm{pb}$ [Lux, 2014]
(This is the 90% bound for a WIMP with mass 33 GeV – away from this mass it is weaker.)

Collider Constraints:

Constrain $b \to s\gamma$ and $B_s \to \mu^+\mu^-$:

$$\mathrm{Br}(b \to s\gamma) = (3.29 \pm 0.19 \pm 0.48) \times 10^{-4}$$ [BaBar, 2012]

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = 3.0^{+1.0}_{-0.9} \times 10^{-9}$$ [CMS, 2013]

Allow a 1.5 GeV theoretical error on the Higgs boson mass, i.e. $m_H = (125.09 \pm 1.5)\,\mathrm{GeV}$

Tools

SoftSUSY for RGE running from the GUT scale to the ElectroWeak scale.

MadGraph + Pythia + Delphes + CheckMate to test (pre-summer) ATLAS and CMS direct searches.

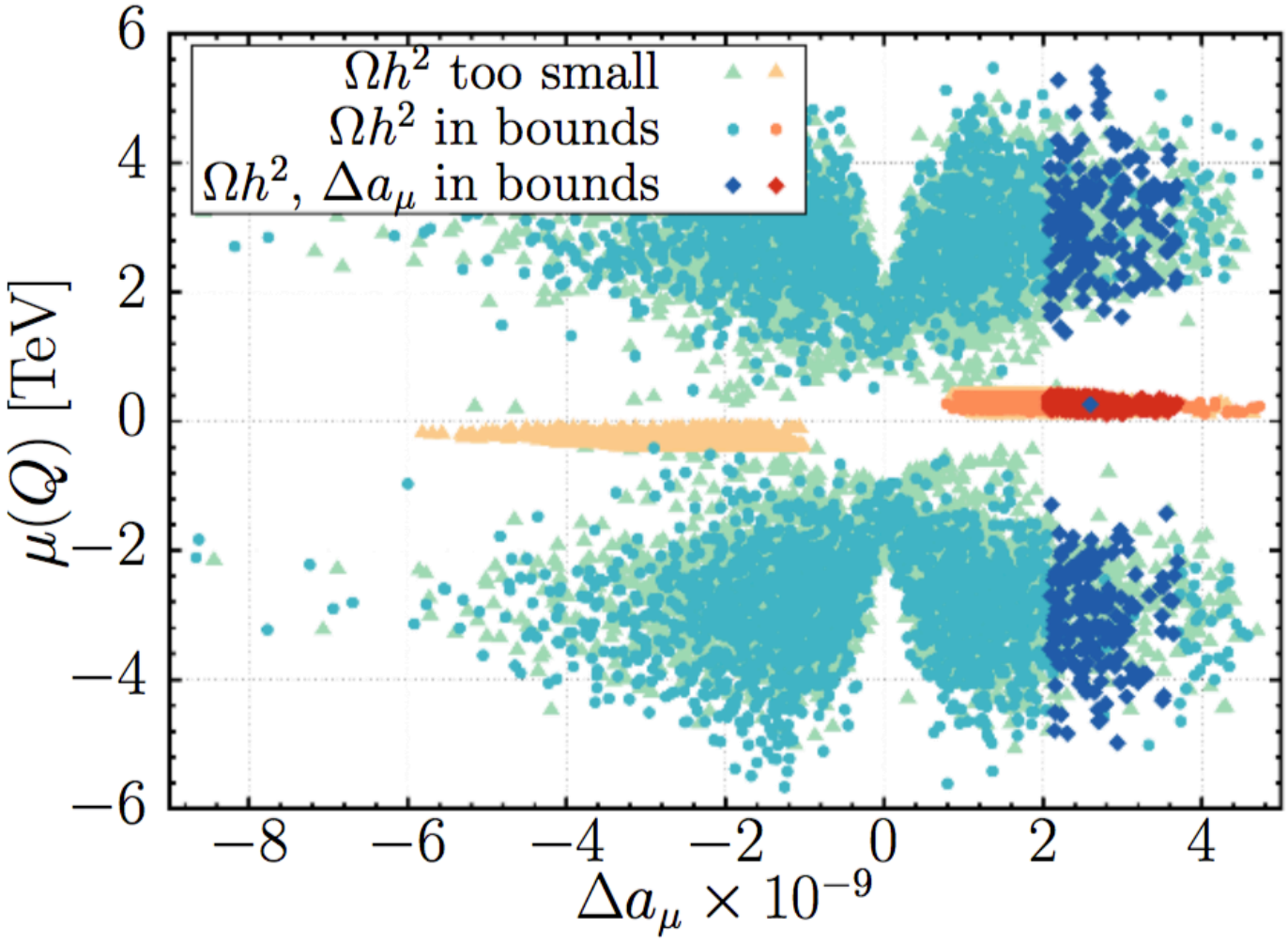
MicrOMEGAs to test $b \to s\gamma$, $B_s \to \mu^+\mu^-$ and Dark Matter constraints.

Vevacious to check vacuum stability (see later).

Inclusive scan

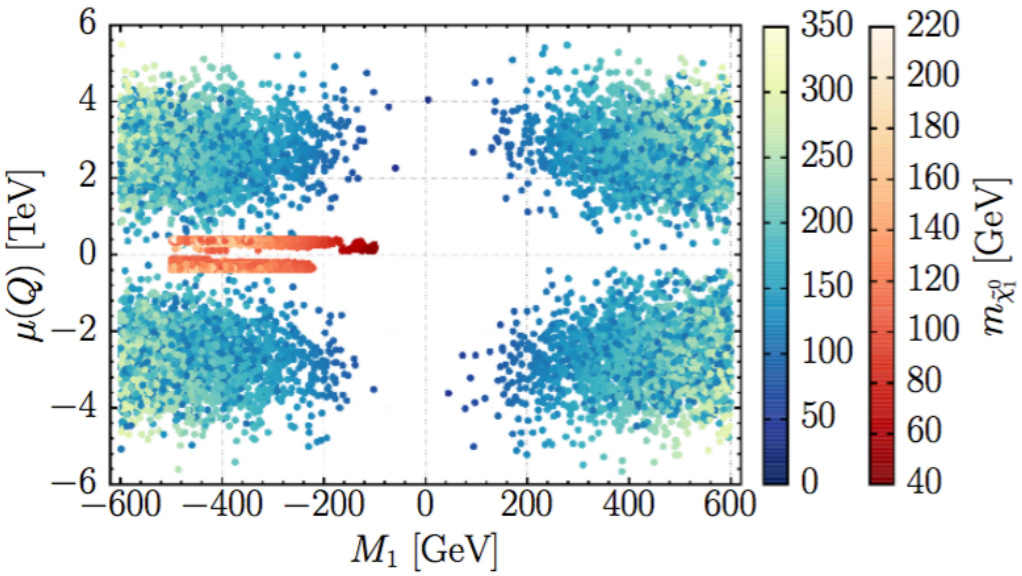
Parameter	range
$\lvert A_{\text{tri}}\rvert$	1 – 3000
$m_0,\ m_1,\ m_2$	1 – 500
m_3	1 – 3000
$m_{H_1},\ m_{H_2}$	1 – 3000

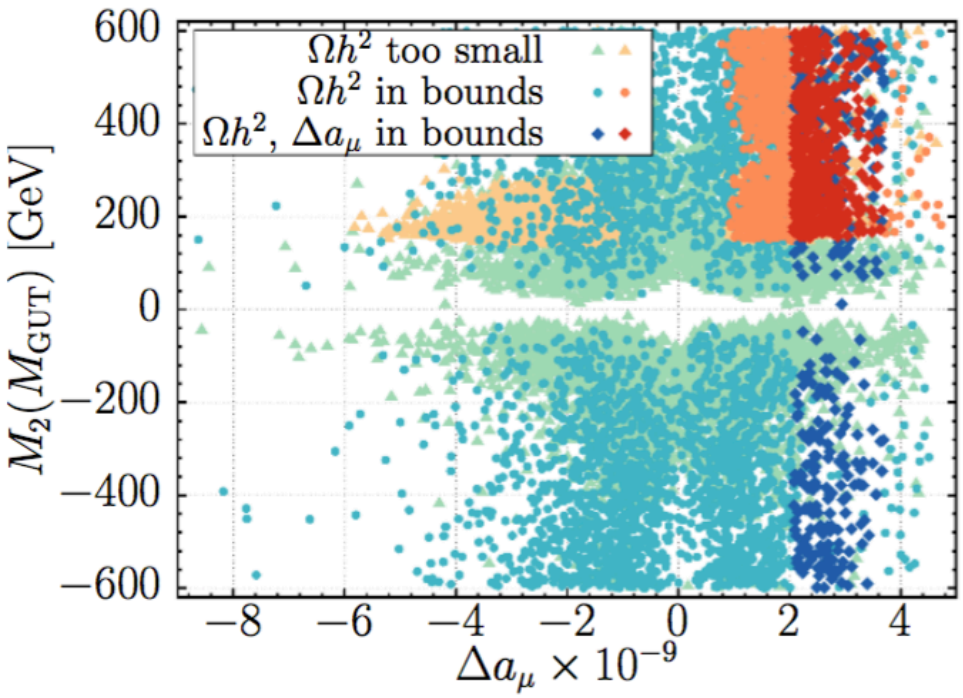
Parameter	range
$\lvert M_1\rvert,\ \lvert M_2\rvert$	1 – 600
$\lvert M_3\rvert$	1 – 6000
$\tan\beta$	5 – 50
$\text{sgn}\,(\mu)$	± 1



$$\Delta a_\mu^{(A)} = \left(\frac{M_1 \mu}{m^2_{\tilde{\mu}_L} m^2_{\tilde{\mu}_R}}\right) \frac{\alpha_1}{4\pi} m_\mu^2 \tan\beta \cdot f_{\mathrm{N}}^{(A)} \left(\frac{m^2_{\tilde{\mu}_L}}{M_1^2}, \frac{m^2_{\tilde{\mu}_R}}{M_1^2}\right)$$

$$\Delta a_\mu^{(E)} = \left(\frac{1}{M_2 \mu}\right) \frac{\alpha_2}{4\pi} m_\mu^2 \tan\beta \cdot f_{\mathrm{C}}^{(E)} \left(\frac{M_2^2}{m^2_{\tilde{\nu}_\mu}}, \frac{\mu^2}{m^2_{\tilde{\nu}_\mu}}\right)$$



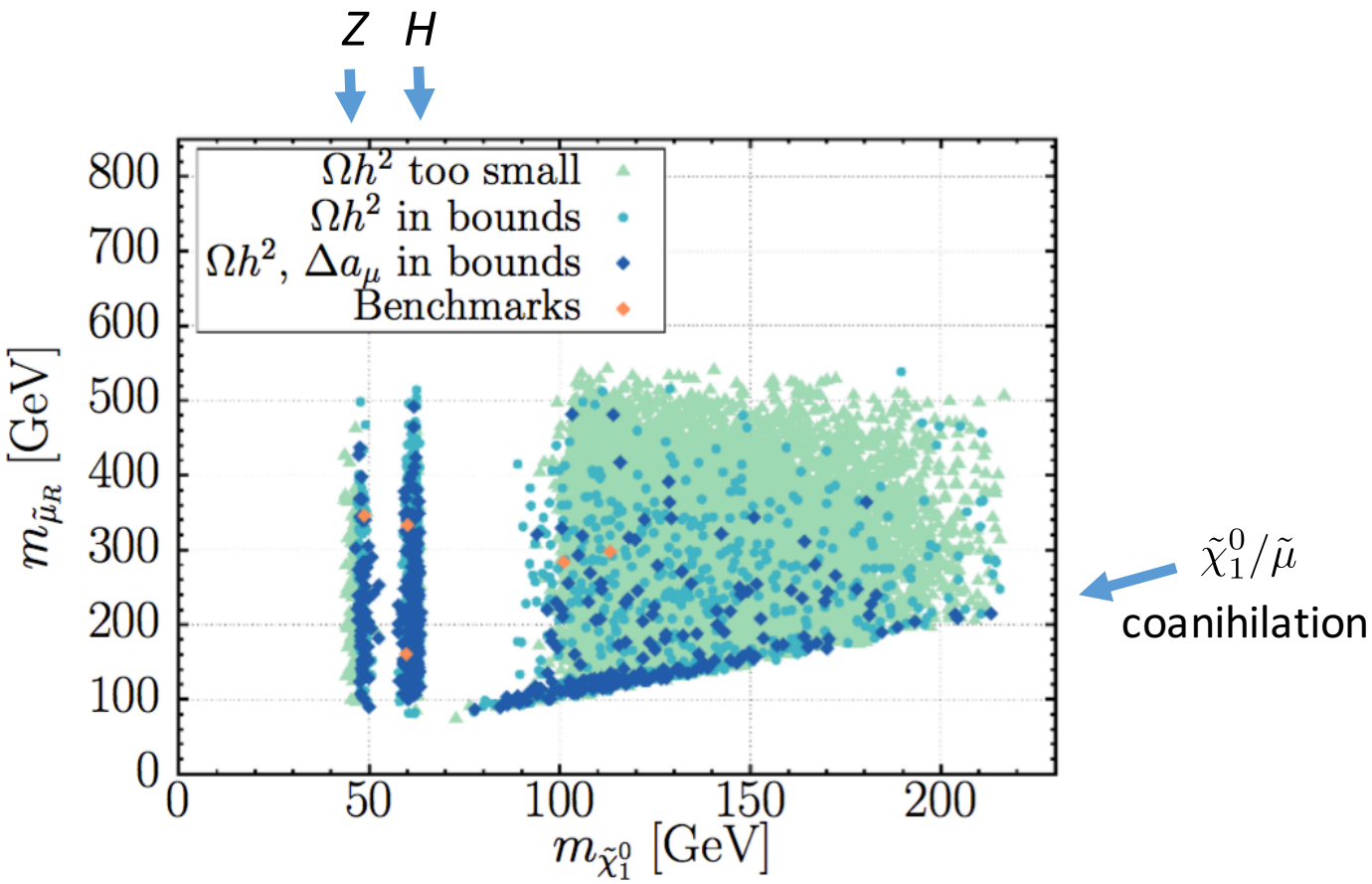


For low μ LSP is Bino-like.

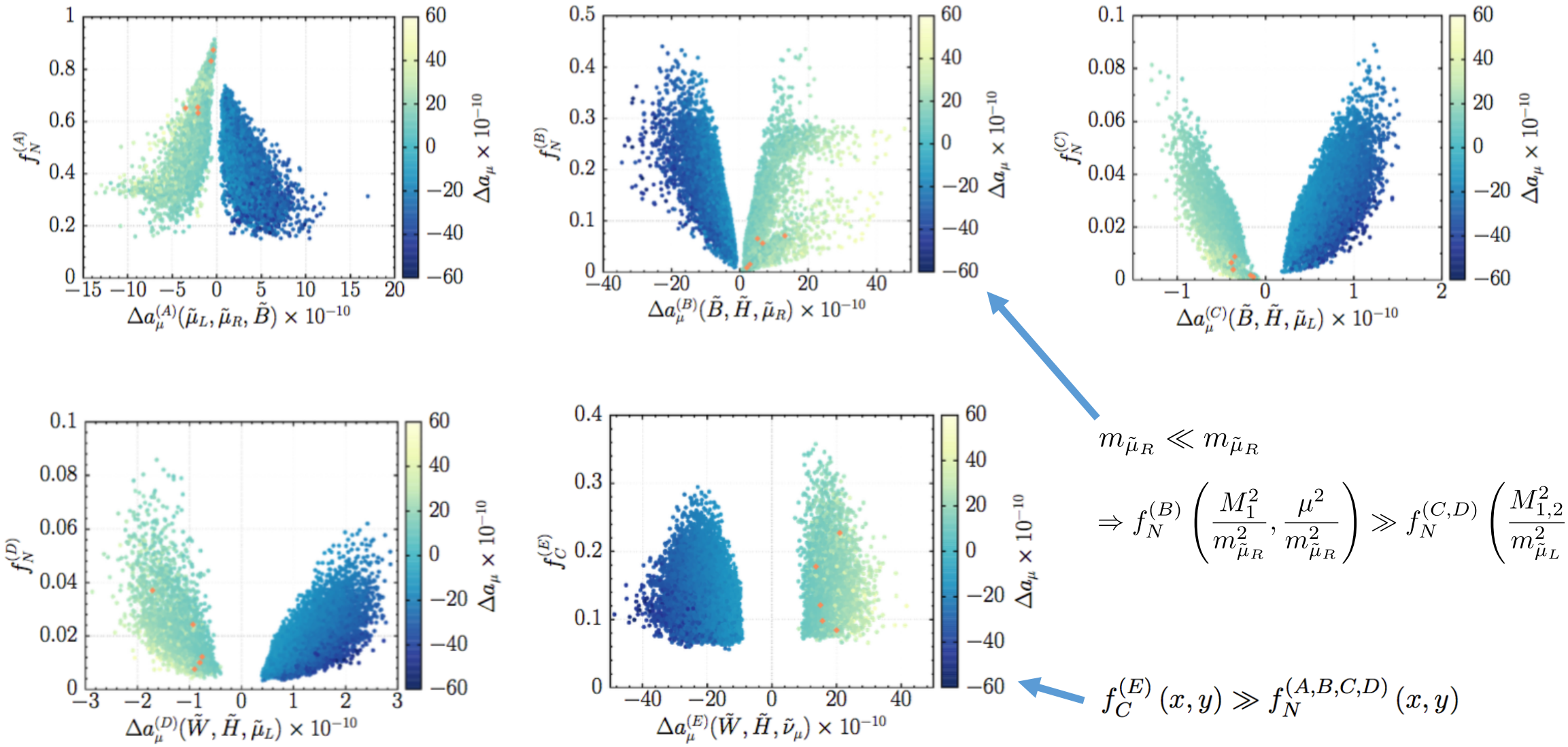
Small μ $(\mu \lesssim 400\,\mathrm{GeV})$

Parameter	Range
A_{tri}	-4000 – -2300
m_0	400 – 700
m_1	300 – 500
m_2	200 – 400
m_3	200 – 2000
m_{H_1}, m_{H_2}	1500 – 2500

Parameter	Range
M_1	-500 – -100
M_2	100 – 600
M_3	750 – 1200
$\tan\beta$	15 – 35
$\mathrm{sgn}\,(\mu)$	$+1$



Often large $\tilde{\chi}_1^0/\tilde{\mu}$ mass gap, giving a hard muon signal in smuon decays at the LHC.



$$m_{\tilde{\mu}_R} \ll m_{\tilde{\mu}_R}$$

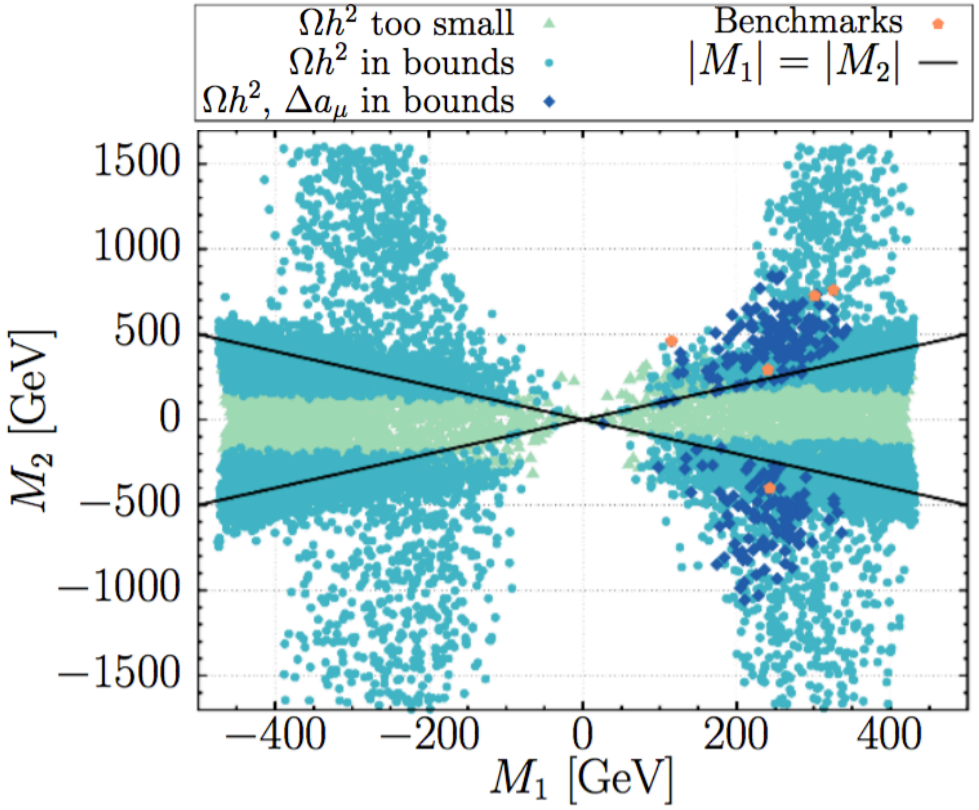
$$\Rightarrow f_N^{(B)}\left(\frac{M_1^2}{m_{\tilde{\mu}_R}^2}, \frac{\mu^2}{m_{\tilde{\mu}_R}^2}\right) \gg f_N^{(C,D)}\left(\frac{M_{1,2}^2}{m_{\tilde{\mu}_L}^2}, \frac{\mu^2}{m_{\tilde{\mu}_L}^2}\right)$$

$$f_C^{(E)}(x,y) \gg f_N^{(A,B,C,D)}(x,y)$$

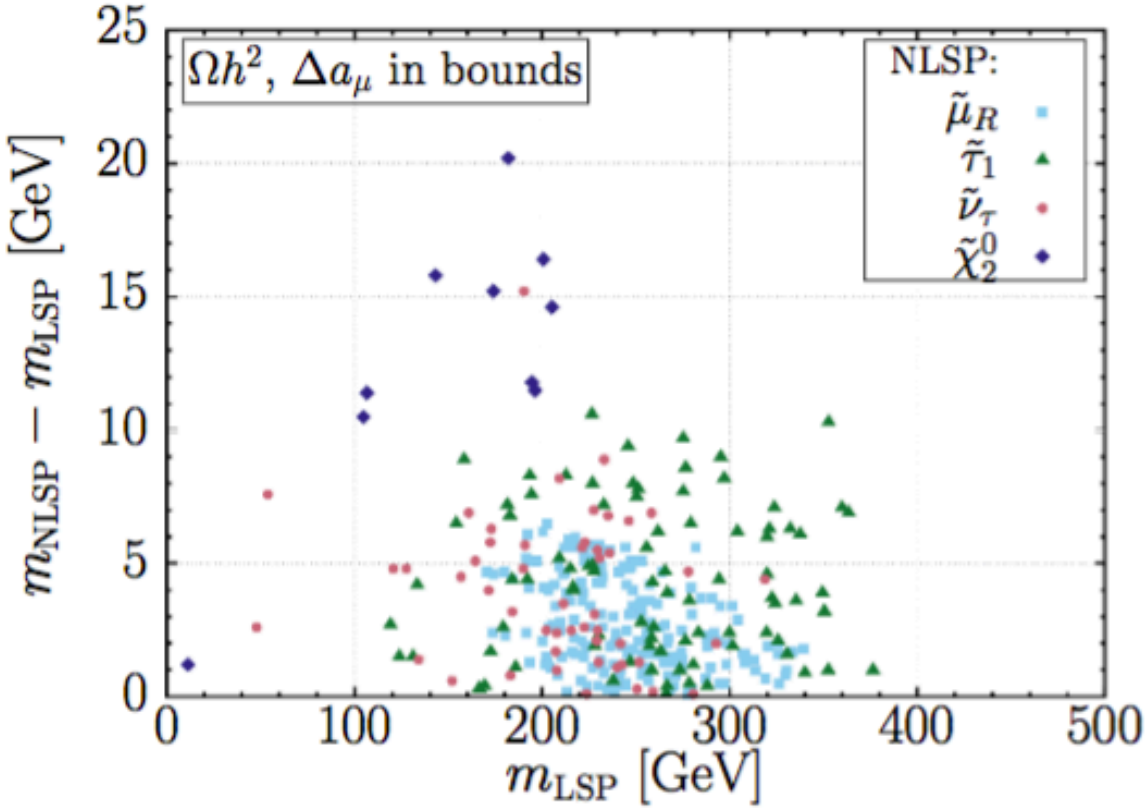
Large μ $(\mu \gtrsim 2\,\mathrm{TeV})$

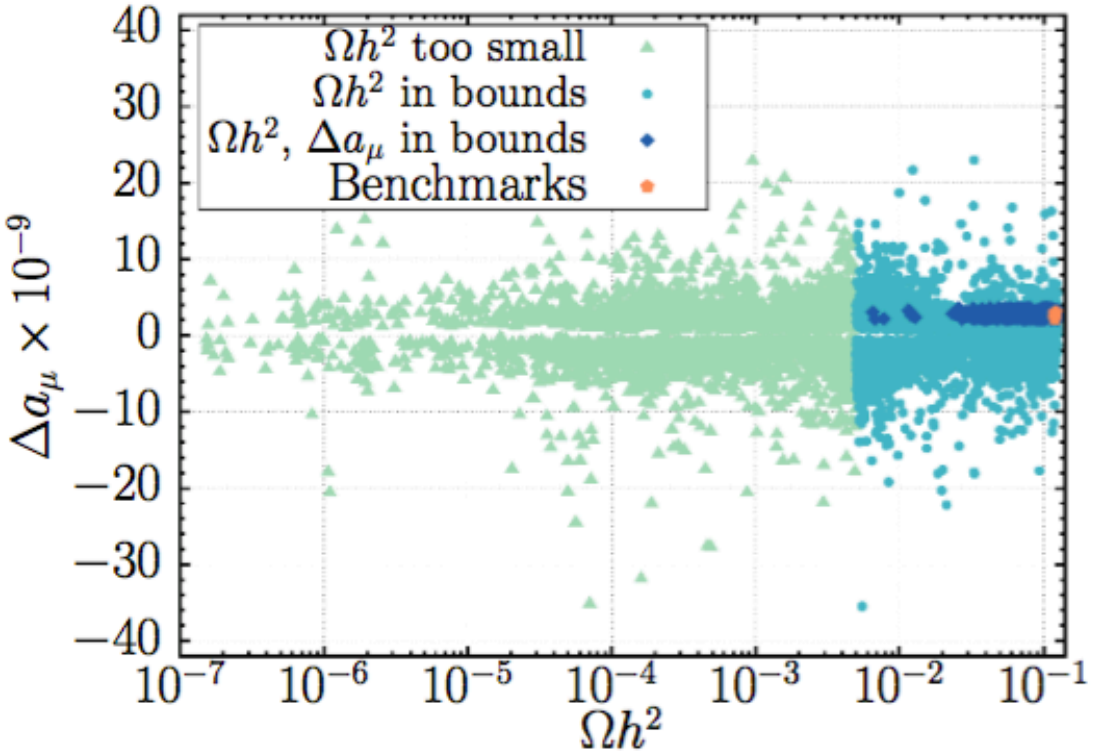
Parameter	Range
A_{tri}	-3000 – 0
m_0	100 – 300
m_1	500 – 1500
m_2	100 – 400
m_3	1000 – 2000
m_{H_1}, m_{H_2}	100 – 3000

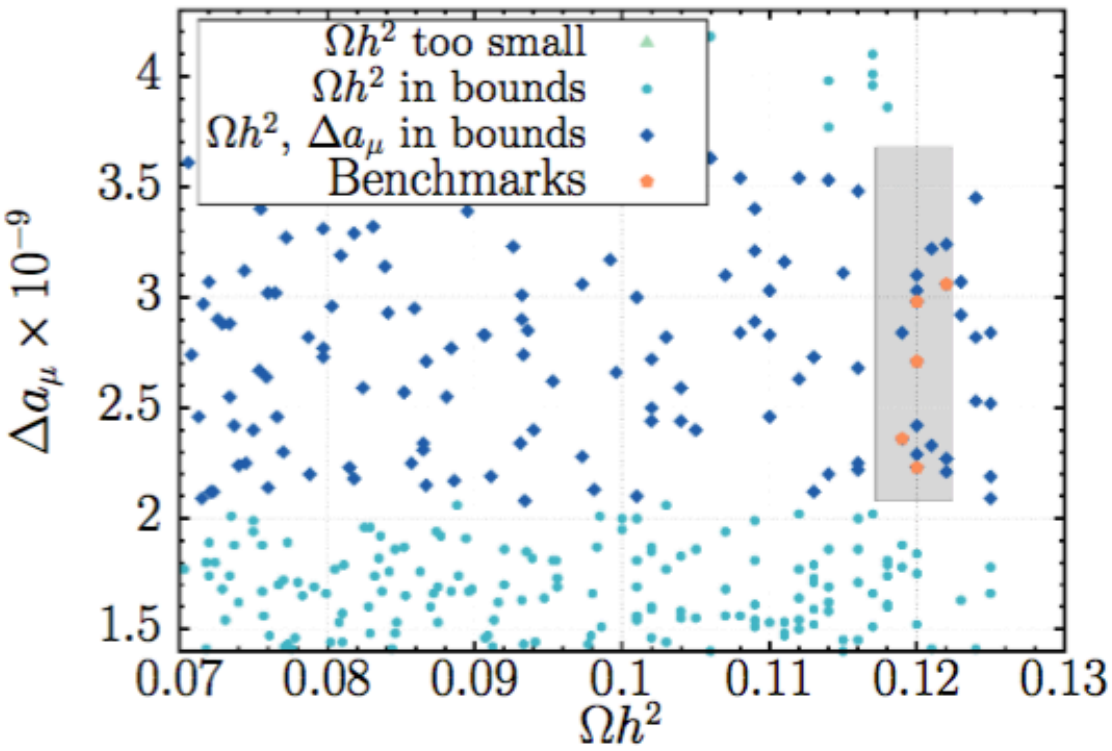
Parameter	Range
M_1	-1000 – 1000
M_2	-2000 – 2000
M_3	2000 – 3000
$\tan\beta$	5 – 50
$\mathrm{sgn}\,(\mu)$	± 1



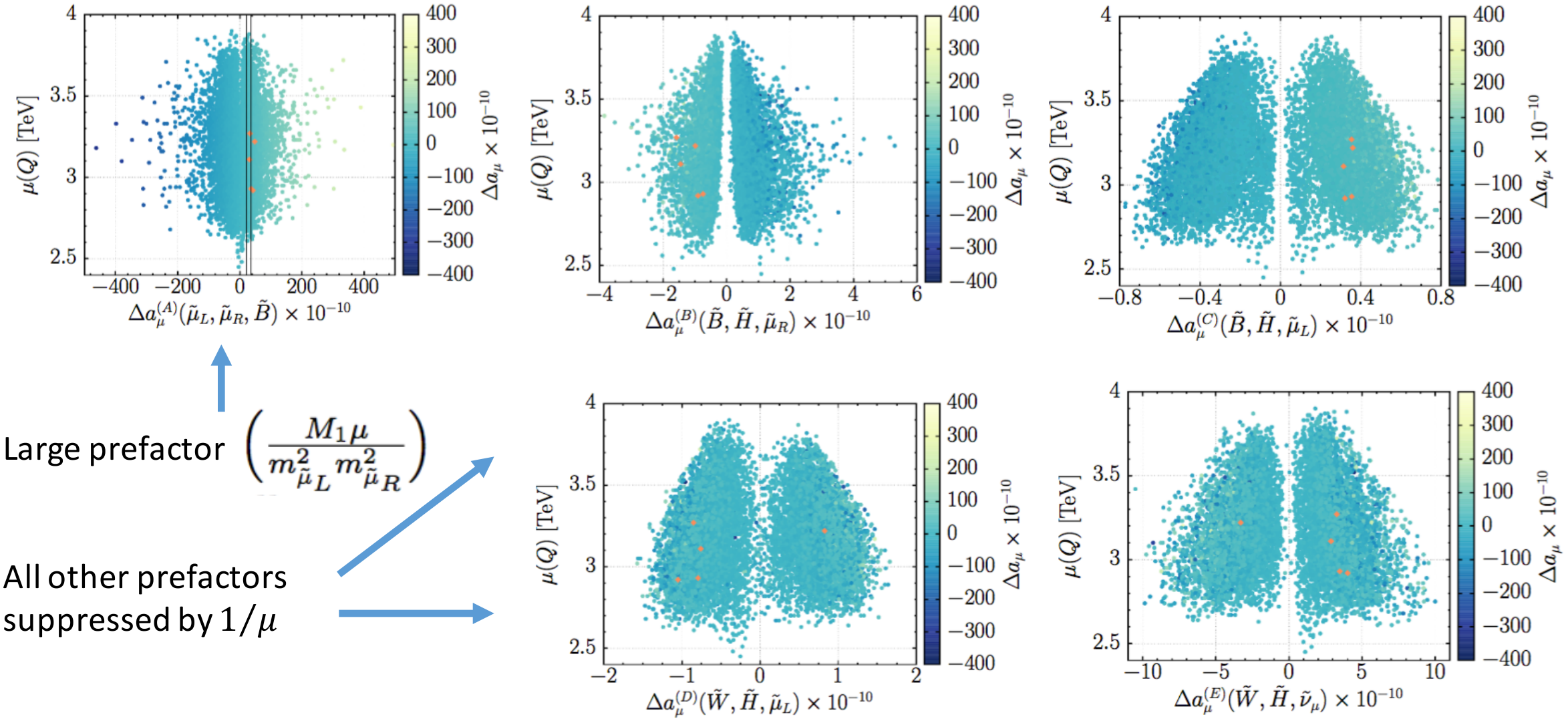
LSP is lightest neutralino







Grey rectangle contains points that get Δa_μ and Ωh^2 exactly right!



Large prefactor $\left(\frac{M_1 \mu}{m^2_{\tilde{\mu}_L} m^2_{\tilde{\mu}_R}}\right)$

All other prefactors suppressed by $1/\mu$

Vacuum Stability

SoftSUSY calculated EWSB using two-loop tadpoles, but ignores Charge and Colour Breaking (CCB) minima that may potentially provide a deeper vacuum.

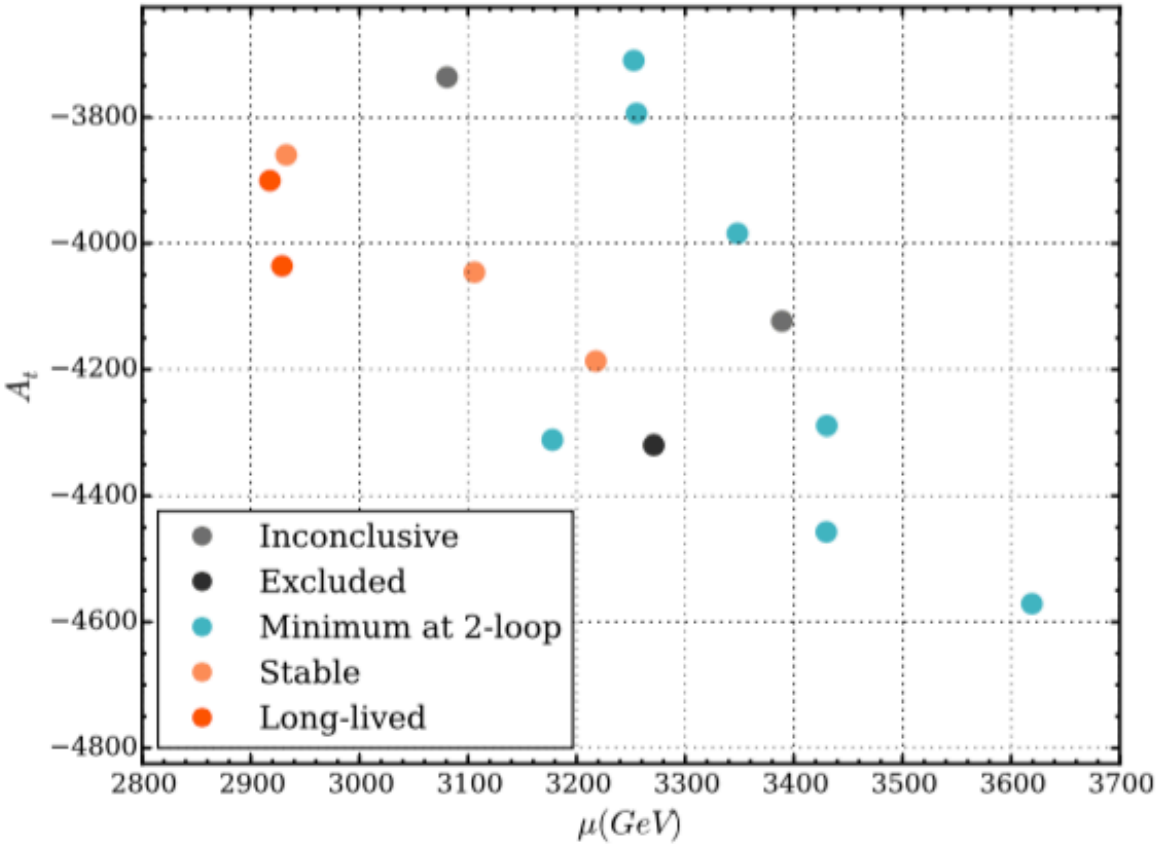
We use Vevacious to check interesting scenarios for these CCB minima, which used a on-loop effective potential.

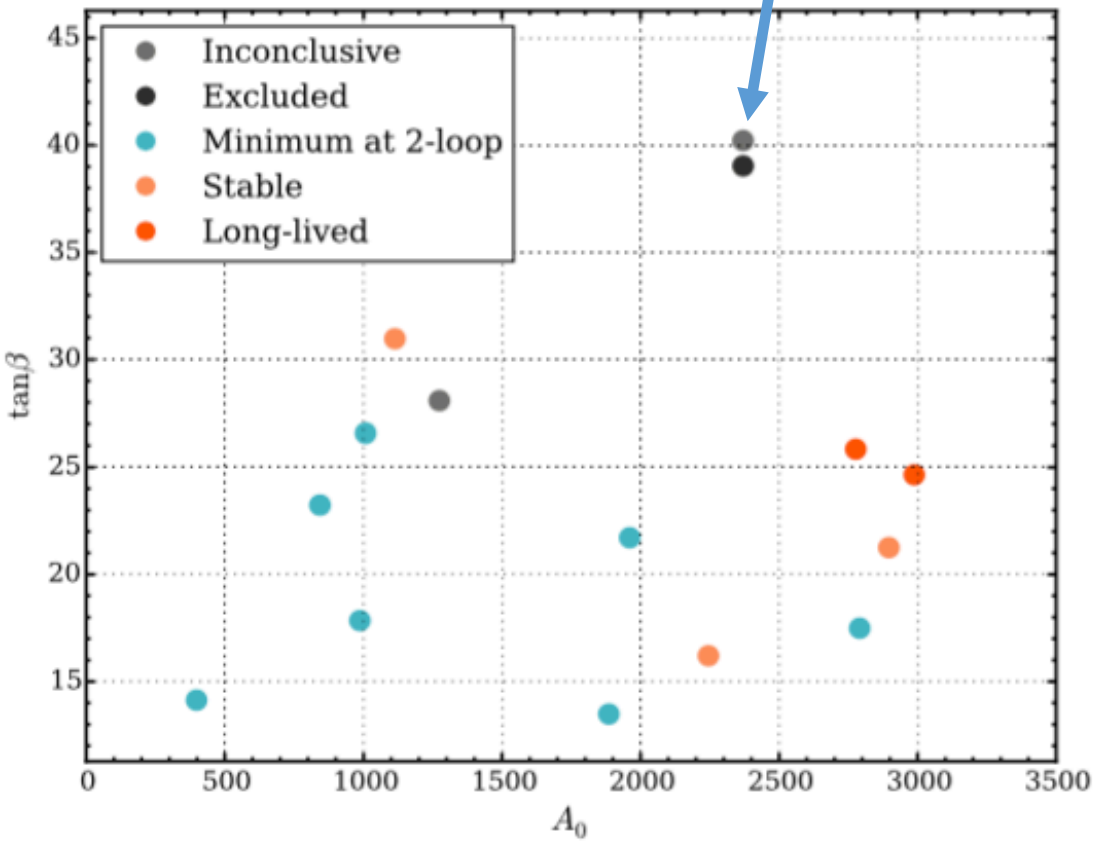
For small μ we found all our interesting scenarios had the correct global vacuum.

For large μ we found many of the scenarios only develop the correct ESWB vacuum at two-loop. We can't test these with Vevacious since this only uses the one-loop effective potential.

However, we were able to test on scenarios where the vacuum is present already at one-loop.

CCB at 1–loop but 2-loop EWSB vacuum





Conclusions

We have examined the "A to Z" Pati-Salam GUT model, which was introduced to explain quark and lepton masses, mixing and CP violation.

We have shown that this model can also explain the anomalous magnetic moment of the muon, as well as the Dark Matter relic density and all experimental constraints.

Two regions of μ are viable – large and small – where Δa_μ is dominated by different loop processes.

This is possible due to the non-universal sfermion masses at the GUT scale.

For small μ, smuons and LSP may have a large mass gap providing a clear LHC signal of smuon decay.

Both regions predict light smuons (100-300 GeV) which may be probed via leptonic signatures.

Benchmarks

	Benchmark:	BP1	BP2	BP3	BP4	BP5	
INPUT AT GUT SCALE	$\tan\beta$	26.48	21.20	22.89	29.52	25.88	
	$\mathrm{sgn}(\mu)$	+	+	+	+	+	
	m_0	681.1	490.4	689.0	691.4	688.4	[GeV]
	m_1	402.0	327.5	447.0	364.4	417.9	
	m_2	397.4	273.0	394.2	342.2	390.7	
	m_3	1204.7	871.8	1085.4	987.4	1192.3	
	M_1	-100.1	-124.1	-123.8	-224.9	-255.1	
	M_2	294.9	367.5	449.9	168.6	177.9	
	M_3	1004.6	1085.7	1109.8	1066.5	947.6	
	M_{h_1}	2204.8	2108.4	2246.6	2127.3	2007.2	
	M_{h_2}	2385.7	2350.9	2455.7	2330.2	2344.7	
	A_{tri}	-2839.1	-2762.5	-2838.5	-2764.0	-3090.0	
CONSTRAINTS	$\mathrm{Br}(b \to s\gamma)$	2.89×10^{-4}	2.91×10^{-4}	2.91×10^{-4}	3.25×10^{-4}	3.25×10^{-4}	
	$\mathrm{Br}(B_s \to \mu^+\mu^-)$	2.69×10^{-9}	2.97×10^{-9}	2.97×10^{-9}	3.06×10^{-9}	3.11×10^{-9}	
	$\sigma^{\mathrm{DD\ SI}}$	1.31×10^{-11}	1.28×10^{-11}	1.18×10^{-11}	2.42×10^{-11}	1.06×10^{-11}	[pb]
	Ωh^2	1.05×10^{-1}	1.25×10^{-1}	1.23×10^{-1}	8.32×10^{-2}	8.47×10^{-2}	
	Δa_μ	1.37×10^{-9}	2.28×10^{-9}	1.30×10^{-9}	1.99×10^{-9}	1.52×10^{-9}	

Benchmarks

	Benchmark:	BP6	BP7	BP8	BP9	BP10	
INPUT AT GUT SCALE	$\tan\beta$	16.96	26.88	32.15	22.21	40.22	
	$\mathrm{sgn}(\mu)$	+	+	+	+	+	
	m_0	238.8	149.6	106.5	271.5	137.5	[GeV]
	m_1	1426.7	1131.1	626.5	508.9	1470.7	
	m_2	239.2	302.7	125.3	193.5	178.4	
	m_3	1458.7	1631.9	1076.3	1434.2	1847.8	
	M_1	577.9	292.3	711.6	579.8	760.7	
	M_2	412.8	612.4	948.8	-436.4	982.8	
	M_3	2195.7	2055.2	2680.5	2456.0	2524.6	
	M_{h_1}	670.6	2924.4	577.0	1512.8	1577.3	
	M_{h_2}	814.9	925.9	918.8	1306.2	1362.7	
	A_{tri}	-2244.8	-2776.6	-1113.2	-2896.2	-2370.1	
CONSTRAINTS	$\mathrm{Br}(b \to s\gamma)$	3.32×10^{-4}	3.29×10^{-4}	3.30×10^{-4}	3.32×10^{-4}	3.28×10^{-4}	
	$\mathrm{Br}(B_s \to \mu^+\mu^-)$	3.07×10^{-9}	3.13×10^{-9}	3.14×10^{-9}	3.08×10^{-9}	3.32×10^{-9}	
	$\sigma^{\mathrm{DD\ SI}}$	9.69×10^{-13}	4.44×10^{-13}	6.65×10^{-13}	5.50×10^{-13}	6.31×10^{-13}	[pb]
	Ωh^2	1.20×10^{-1}	1.22×10^{-1}	1.20×10^{-1}	1.20×10^{-1}	1.19×10^{-1}	
	Δa_μ	2.71×10^{-9}	3.06×10^{-9}	2.23×10^{-9}	2.98×10^{-9}	2.36×10^{-9}	