Relation between the PMNS phase and proton decay in SUSY SO(10) models

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Talk at SUSY2016, Melbourne, Australia (2016.7.5)

Based on the works with B. Dutta and R. N. Mohapatra,

"Suppressing proton decay in the minimal SO(10) model"
Phys.Rev.Lett. 94 (2005) 091804

"Neutrino mixing predictions of a minimal SO(10) model with suppressed proton decay"
Phys.Rev. D72 (2005) 075009

"Proton decay and $\mu \to e+\gamma$ connection in a renormalizable SO(10) GUT for neutrinos"
Phys. Rev. D87 (2013) 7, 075008

The work with T. Fukuyama and K. Ichikawa,

"Revisiting fermion mass and mixing fits in the minimal SUSY SO(10) GUT"
arXiv:1508.07078

Work in progress...

Today's Topic

1. Introduction
   Neutrino Oscillations (Parameters : $\theta_{23} - \delta_{\mathrm{PMNS}}$)
   Flavor structure in the neutrino mass matrix

2. Connection to Grand Unified Theory (GUT)
   Dimension-five operators in SUSY GUTs
   Proton decay

3. Conclusion

History

A (nearly) maximal atmospheric neutrino mixing @Super-K (97-98)

A large solar neutrino mixing @SNO, KamLAND (~00-02)

A non-zero 13-mixing (reactor) @Daya Bay, RENO (2012)

CP phase (PMNS phase) (Combinations) (2013-present)

Reactor (Short baseline) + Accelerator (Long Baseline)

T2K, MINOS, NOvA

T2K MINOS NOvA

+

大亞灣 θ_{13} RENO ν θ_{13}

$$\delta_{\rm PMNS}$$

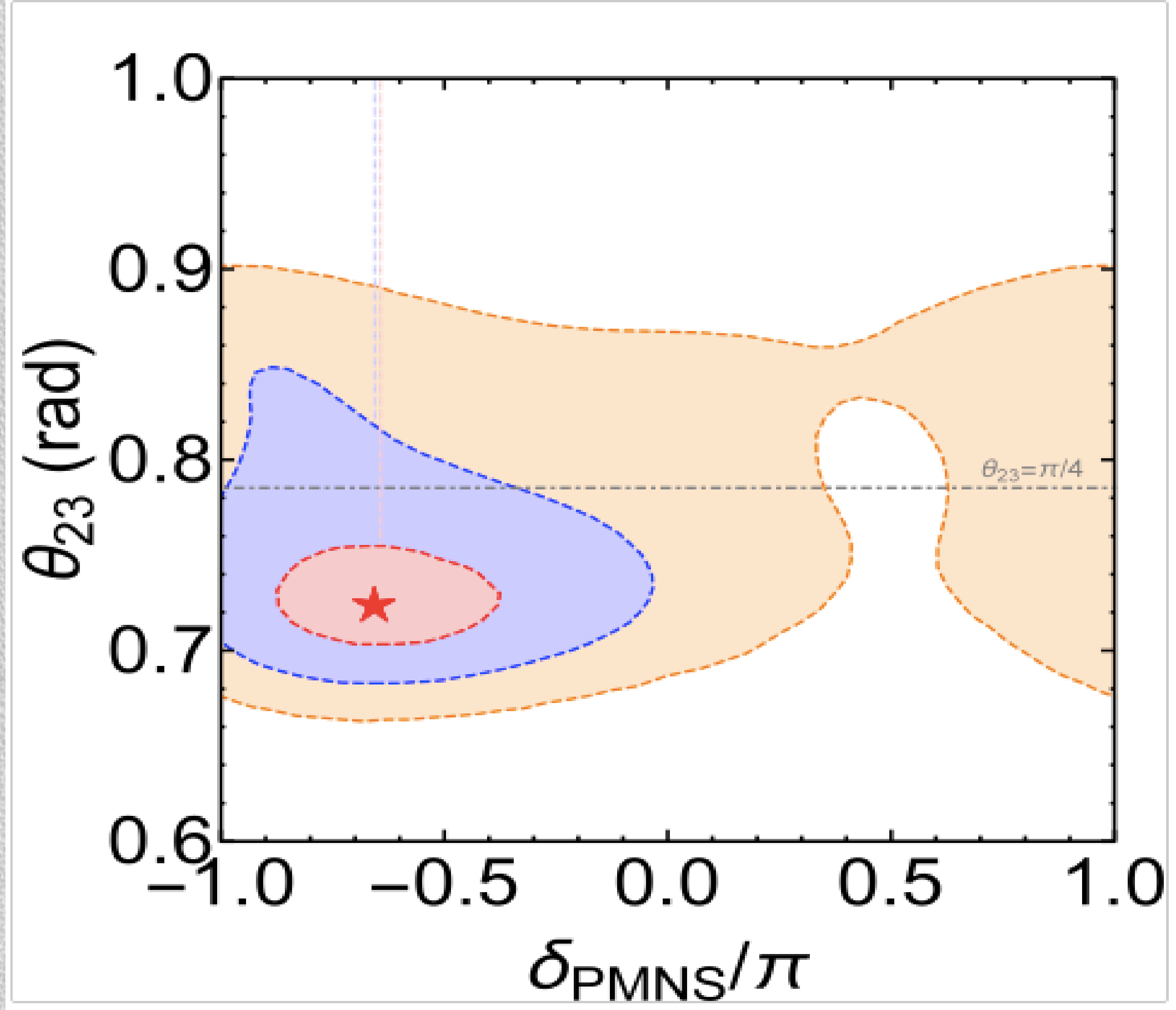
Table 1
Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1, 2 and 3σ ranges for the 3ν mass–mixing parameters. See also Fig. 1 for a graphical representation of the results. We recall that Δm^2 is defined as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH. The CP violating phase is taken in the (cyclic) interval $\delta/\pi \in [0, 2]$. The last row reports the (statistically insignificant) overall χ^2 difference between IH and NH.

Parameter	Hierarchy	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5}$ eV^2	NH or IH	7.37	7.21–7.54	7.07–7.73	6.93–7.97
$\sin^2\theta_{12}/10^{-1}$	NH or IH	2.97	2.81–3.14	2.65–3.34	2.50–3.54
$\Delta m^2/10^{-3}$ eV^2	NH	2.50	2.46–2.54	2.41–2.58	2.37–2.63
$\Delta m^2/10^{-3}$ eV^2	IH	2.46	2.42–2.51	2.38–2.55	2.33–2.60
$\sin^2\theta_{13}/10^{-2}$	NH	2.14	2.05–2.25	1.95–2.36	1.85–2.46
$\sin^2\theta_{13}/10^{-2}$	IH	2.18	2.06–2.27	1.96–2.38	1.86–2.48
$\sin^2\theta_{23}/10^{-1}$	NH	4.37	4.17–4.70	3.97–5.63	3.79–6.16
$\sin^2\theta_{23}/10^{-1}$	IH	5.69	4.28–4.91 ⊕ 5.18–5.97	4.04–6.18	3.83–6.37
δ/π	NH	1.35	1.13–1.64	0.92–1.99	0–2
δ/π	IH	1.32	1.07–1.67	0.83–1.99	0–2
$\Delta\chi^2_{\text{I-N}}$	IH–NH	+0.98			

Note : Atmospheric neutrino oscillations depend on $\sin^2 2\theta_{23}$.
Long baseline experiments (nu_e appearance) depend on $\sin\theta_{23}$.

$$\theta_{23} > \pi/4 \quad \text{or} \quad < \pi/4 \; ??$$

Global analysis (by Capozzi et al)



Bi-large mixing

(Normal hierarchy)

$$\mathcal{M}_\nu = \begin{pmatrix} \sim 0 & a\lambda & b\lambda \\ a\lambda & 1 & 1 \\ b\lambda & 1 & 1+\epsilon \end{pmatrix} m$$

$\boldsymbol{\epsilon \sim \lambda}$ ➡ A large solar mixing

$$\epsilon^2 \sim \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \qquad \sin\theta_{13} \sim \sqrt{\frac{\Delta m_{12}^2}{\Delta m_{23}^2}}$$

Bi-large mixing

$$\mathcal{M}_\nu = \begin{pmatrix} \sim 0 & a\lambda & b\lambda \\ a\lambda & 1 & 1 \\ b\lambda & 1 & 1+\epsilon \end{pmatrix} m$$

$$(a,b) \perp (1,1) \quad \Rightarrow \quad \sin\theta_{13} = 0$$

In general, $\sin\theta_{13} = 0 - 0.2$

Bi-large mixing

$$\mathcal{M}_\nu = \begin{pmatrix} \sim 0 & a\lambda & b\lambda \\ a\lambda & 1 & 1 \\ b\lambda & 1 & 1+\epsilon \end{pmatrix} m$$

$$(a,b) \perp (1,1) \quad \Rightarrow \quad \sin\theta_{13} = 0$$

Now we know $\sin^2 2\theta_{13} = 0.084 \pm 0.005$

$$(\sin\theta_{13} = 0.146 \pm 0.004)$$

Bi-large mixing

$$\mathcal{M}_\nu = \begin{pmatrix} \sim 0 & a\lambda & b\lambda \\ a\lambda & 1 & 1 \\ b\lambda & 1 & 1+\epsilon \end{pmatrix} m$$

If $a \to 0$, ➡ $\sin\theta_{13} \neq 0$

"Suppressing proton decay in the minimal SO(10) model"
Dutta-YM-Mohapatra, **PRL 94 (2005) 091804**

To suppress proton decay, (1,1) and (1,2) elements have to be small in type II seesaw.

$\sin\theta_{13} \neq 0$ $\qquad$ $\sin\theta_{13} \sim 0.1$

Flavor structure

$$\mathcal{M}_\nu = \begin{pmatrix} \sim 0 & a\lambda & b\lambda \\ a\lambda & 1 & 1 \\ b\lambda & 1 & 1+\epsilon \end{pmatrix} m$$

Q. Can the current experimental data allow the small (1,1) and (1,2) elements in the neutrino mass matrix ?

Flavor structure

$$\mathcal{M}_\nu = U \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U^T$$

$$U = U(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\rm PMNS})_{\rm PDG}$$

Note : (m1,m2,m3) have two independent Majorana phases.

Flavor structure

$$(\mathcal{M}_\nu)_{11} = (m_1 \cos^2\theta_{12} + m_2 \sin^2\theta_{12}) \cos^2\theta_{13} + e^{-2i\delta} m_3 \sin^2\theta_{13}$$

$$(\mathcal{M}_\nu)_{12} = \cos\theta_{13} \big[(m_2 - m_1) \cos\theta_{12} \sin\theta_{12} \cos\theta_{23} + e^{-i\delta} m_3 \sin\theta_{13} \sin\theta_{23} \\ - e^{i\delta} (m_1 \cos^2\theta_{12} + m_2 \sin^2\theta_{12}) \sin\theta_{13} \sin\theta_{23} \big]$$

$$(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12} = 0$$

Two-zero texture : Fritzsch-Xing-Zhou, Haba-Horita-Kaneta-YM, Geng-Huang-Tsai, ...

A relation among $\delta_{\rm PMNS}, \theta_{12}, \theta_{23}, \theta_{13}, \Delta m^2_{12}/\Delta m^2_{23}$

(as well as |m1| and two Majorana phases)

Flavor structure

$$\mathcal{M}_\nu \propto \begin{pmatrix} 0 & 0 & b\lambda \\ 0 & x & y \\ b\lambda & y & z \end{pmatrix}$$

$$(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12} = 0$$

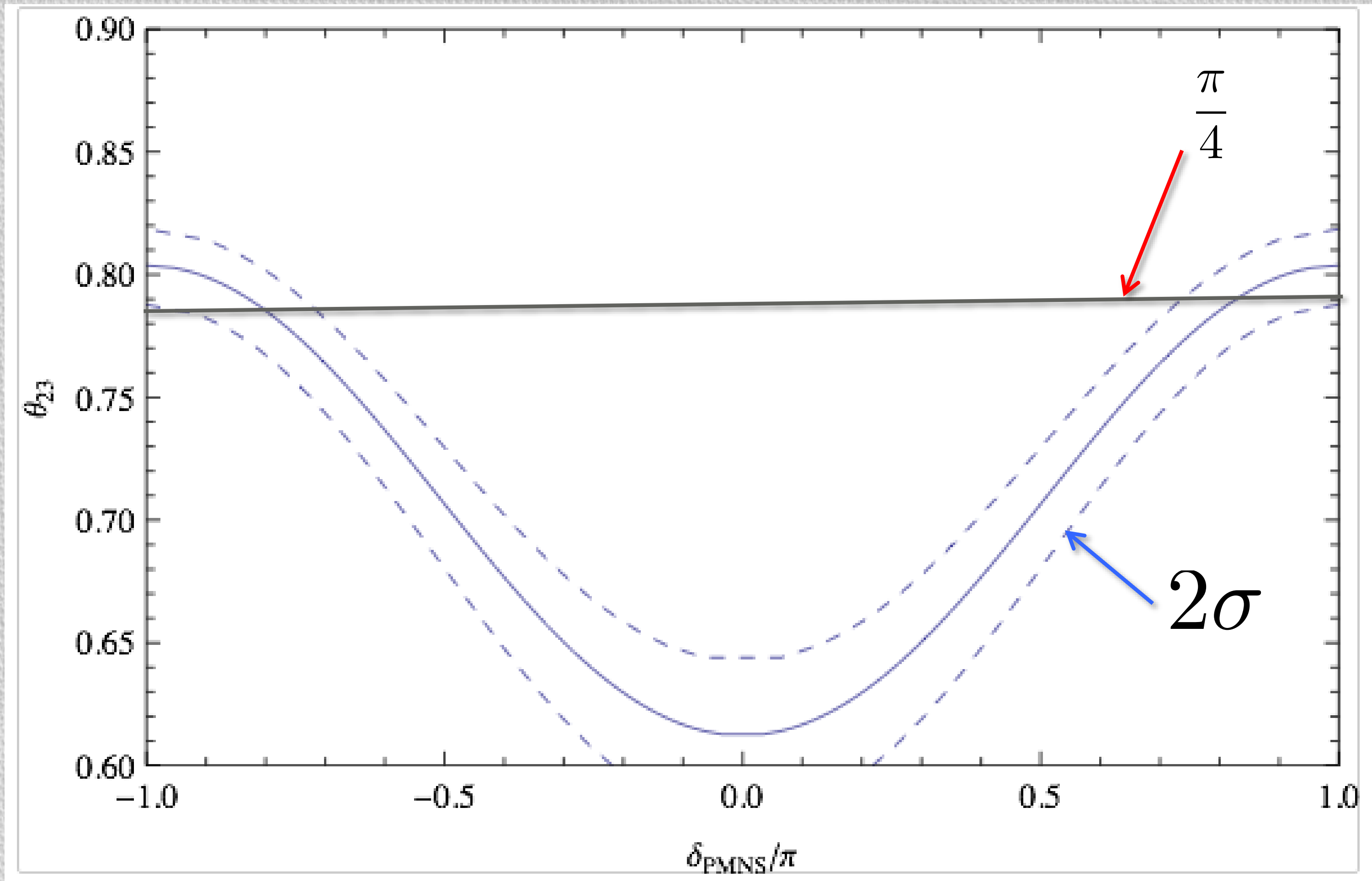
$$\cos\delta_{\rm PMNS} = \frac{\frac{\Delta m_{12}^2}{\Delta m_{23}^2}\cos 2\theta_{13}\sin^2 2\theta_{12} - 4\sin^2\theta_{13}\left(\frac{\Delta m_{12}^2}{\Delta m_{23}^2}\cos^4\theta_{12} + \cos 2\theta_{12}\right)\tan^2\theta_{23}}{4\sin^3\theta_{13}\left(1 + \frac{\Delta m_{12}^2}{\Delta m_{23}^2}\cos^2\theta_{12}\right)\sin 2\theta_{12}\tan\theta_{23}}.$$

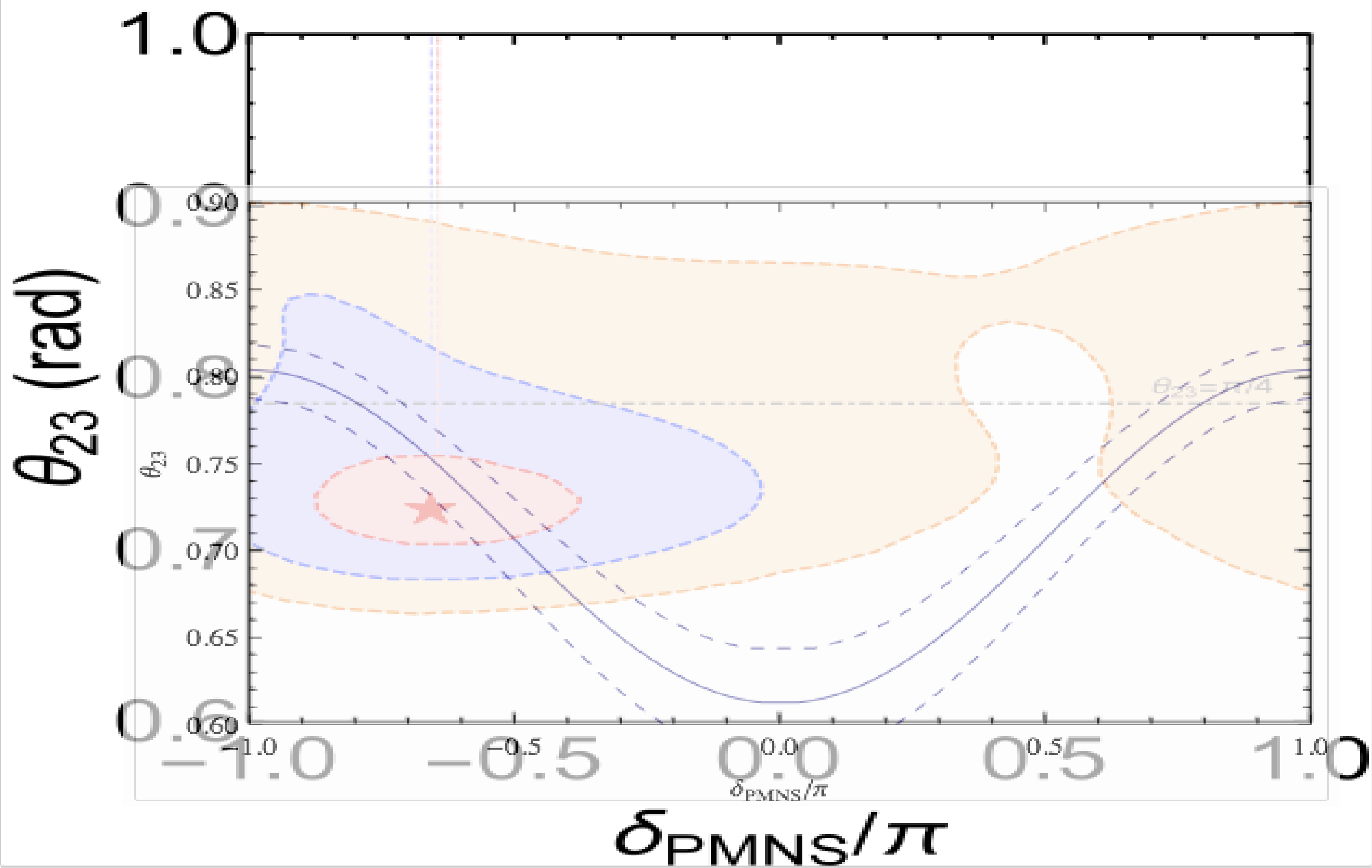
(Haba-Horita-Kaneta-YM)

The PMNS phase is sensitive to 23-mixing.

Flavor structure

$$(\mathcal{M}_\nu)_{11} = (\mathcal{M}_\nu)_{12} = 0$$



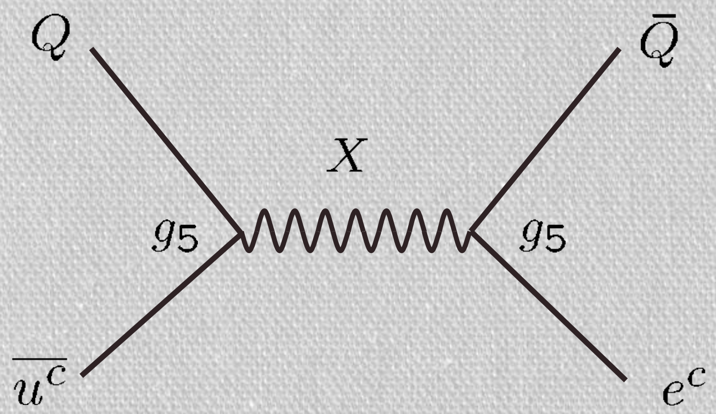


1.0
0.9
0.8
0.7
0.6
θ_{23} (rad)
0.90
0.85
0.80
0.75
0.70
0.65
0.60
θ_{23}
$\theta_{23}=\pi/4$
−1.0
−0.5
0.0
0.5
1.0
δ_{PMNS}/π
δ_{PMNS}/π

Connection to GUT

Proton decay (Important prediction of GUTs)

- Dimension 6 operator

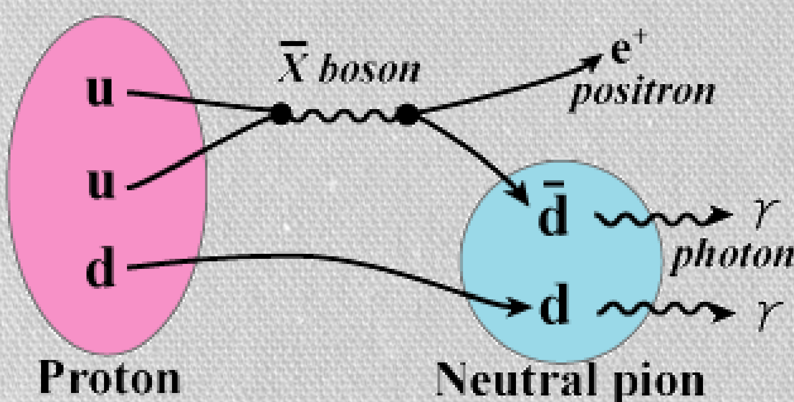


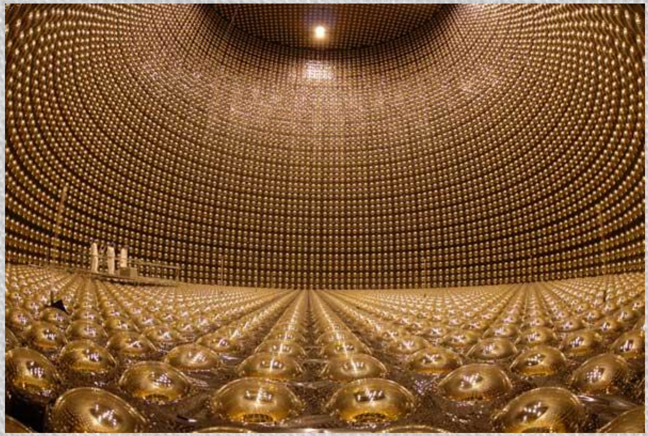
dominant mode : $p \to \pi^0 e^+$

$$A = \frac{g_5^2}{M_X^2}$$ (Up to Hadron matrix element)

$$M_X \sim 2 \times 10^{16} \text{ GeV}$$

$$\tau_p \sim 10^{34} - 10^{36} \text{ years}$$





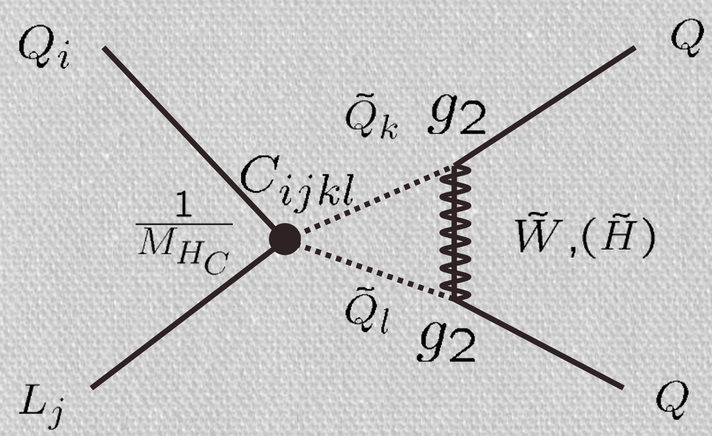
Super-Kamiokande

$$\tau(p \to \pi^0 e^+)^{\text{SK,EXP}} > 1.01 \times 10^{34} \text{ years}$$

Dangerous dimension-5 operator in SUSY GUTs

(Weinberg, Sakai-Yanagida)

- Dimension 5 operator with gaugino or Higgsino dressing



dominant mode : $p \to K^+ \bar{\nu}$

$$A = \frac{\alpha_2}{4\pi M_{H_C} m_{\mathrm{SUSY}}} C_{ijkl}$$

M_{H_C} : colored Higgs(ino) mass

Severe constraints to models

$$\tau(p \to K\bar{\nu}) > 59 \times 10^{32} \text{ years}$$
$$\tau(n \to \pi\bar{\nu}) > 4.4 \times 10^{32} \text{ years}$$
$$\tau(n \to K\bar{\nu}) > 1.8 \times 10^{32} \text{ years}$$

Dimension-5 Proton decay amplitude

$$A \simeq \frac{\alpha_2 \beta_p}{4\pi} \frac{1}{M_{H_C}} \frac{m_{\tilde{W}}}{m_{\tilde{q}}^2} C_{ijkl}$$

Wino mass

Colored Higgs mass

Squark mass

Function of Yukawa couplings

What is important to calculate the amplitude?

1. Masses of SUSY particles (squarks & gauginos).
2. Mass of Colored Higgs(ino)
3. Structure of Yukawa coupling to Colored Higgs

SO(10) Model with renormalizable Yukawa couplings

(Babu-Mohapatra,1992)

$\mathbf{10} + \overline{\mathbf{126}}$

Quarks & Leptons: $\psi(\mathbf{16})$

$$\mathbf{16} \times \mathbf{16} = \mathbf{10} + \mathbf{126} + \mathbf{120}$$

Higgs fields which couple to fermions :

$H(\mathbf{10})$, $\bar{\Delta}(\overline{\mathbf{126}})$ and $D(\mathbf{120})$

Renormalizable Yukawa terms:

$$\frac{1}{2}\bar{h}_{ij}\psi_i\psi_j H + \frac{1}{2}\bar{f}_{ij}\psi_i\psi_j\bar{\Delta} + \frac{1}{2}\bar{h}'_{ij}\psi_i\psi_j D$$

$\bar{h}, \bar{f}$: symmetric

$\bar{h}'$: anti-symmetric

Neutrino Mass

$$m_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T$$

Type II   Type I

$$M_L = 2\sqrt{2} \bar{f} \langle \bar{\Delta}_L \rangle \qquad M_R = 2\sqrt{2} \bar{f} \langle \bar{\Delta}_R \rangle$$

$$\bar{\Delta}_L : (\mathbf{1}, \mathbf{3}, 1) \qquad \bar{\Delta}_R : (\mathbf{1}, \mathbf{1}, 0)$$

$SU(2)_L$ triplet

$$W_Y = \frac{1}{2} \bar{h}_{ij} \psi_i \psi_j H + \frac{1}{2} \bar{f}_{ij} \psi_i \psi_j \bar{\Delta} + \frac{1}{2} \bar{h}'_{ij} \psi_i \psi_j D \qquad [H(\mathbf{10}),\ \bar{\Delta}(\overline{\mathbf{126}}),\ D(\mathbf{120})]$$

$$\psi \psi \bar{\Delta} \supset \ell \ell \bar{\Delta}_L + \bar{\nu} \bar{\nu} \bar{\Delta}_R$$

$$W_Y = \frac{1}{2}\bar{h}_{ij}\psi_i\psi_j H + \frac{1}{2}\bar{f}_{ij}\psi_i\psi_j\bar{\Delta} + \frac{1}{2}\bar{h}'_{ij}\psi_i\psi_j D \quad [H(\mathbf{10}),\ \bar{\Delta}(\overline{\mathbf{126}}),\ D(\mathbf{120})]$$

$$[\Delta(\mathbf{126}),\ \Phi(\mathbf{210})]$$

Higgs doublets

$$\varphi_d = (H_d^{10}, D_d^1, D_d^2, \bar{\Delta}_d, \Delta_d, \Phi_d) \qquad \varphi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \bar{\Delta}_u, \Phi_u)$$

$$\text{mass term}\ :\ (\varphi_d)_a(M_{\text{doub.}})_{ab}(\varphi_u)_b \qquad U M_{\text{doub.}} V^{\mathsf{T}} = M_{\text{doub.}}^{\text{diag}}$$

Light doublets

$$H_d = U_{1a}^*(\varphi_d)_a \qquad H_u = V_{1a}^*(\varphi_u)_a$$

U, V : unitary matrices

$$Y_u = h + r_2 f + r_3 h'$$
$$Y_d = r_1(h + f + h')$$
$$Y_e = r_1(h - 3f + c_e h')$$
$$Y_\nu = h - 3r_2 f + c_\nu h'$$

$$r_1 \sim \tan\beta\, m_b/m_t$$

$$h = V_{11}\bar{h} \qquad f = U_{14}/(\sqrt{3}r_1)\bar{f}$$
$$h' = (U_{12} + U_{13}/\sqrt{3})/r_1\bar{h}'$$
$$r_1 = \frac{U_{11}}{V_{11}}$$
$$r_2 = r_1\frac{V_{15}}{U_{14}} \qquad r_3 = r_1\frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}$$
$$c_e = \frac{U_{12} - \sqrt{3}U_{13}}{U_{12} + U_{13}/\sqrt{3}} \qquad c_\nu = r_1\frac{V_{12} + \sqrt{3}V_{13}}{U_{12} + U_{13}/\sqrt{3}}$$

Dimension 5 operators : $-W_5 = \frac{1}{2} C_L^{ijkl} \underbrace{q_k q_l} q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c$

symmetric

Higgs triplets $(\bar{\mathbf{3}}, \mathbf{1}, 1/3) + c.c.$

$$\varphi_{\bar{T}} = (H_{\bar{T}}, D_{\bar{T}}, D'_{\bar{T}}, \bar{\Delta}_{\bar{T}}, \Delta_{\bar{T}}, \Delta'_{\bar{T}}, \Phi_{\bar{T}})$$

$$\varphi_T = (H_T, D_T, D'_T, \Delta_T, \bar{\Delta}_T, \bar{\Delta}'_T, \Phi_T)$$

mass term : $(\varphi_{\bar{T}})_a (M_T)_{ab} (\varphi_T)_b$

$$X M_T Y^{\mathsf{T}} = M_T^{\text{diag}}$$

H_T and $\bar{\Delta}_T$ have opposite D-parity.

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}} (X_{a1} h + X_{a4} f + \sqrt{2} X_{a3} h')_{ij} (Y_{a1} h + Y_{a5} f)_{kl}$$

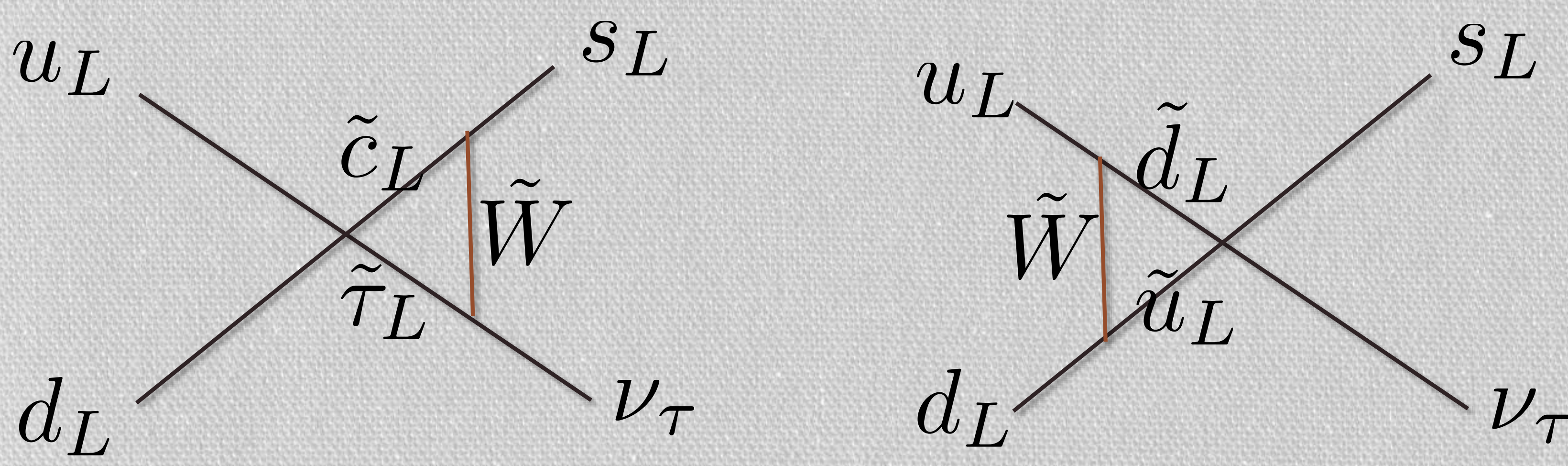
Opposite signature

no h'

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}} (X_{a1} h - X_{a4} f + \sqrt{2} X_{a2} h')_{ij} (Y_{a1} h - (Y_{a5} - \sqrt{2} Y_{a6}) f + \sqrt{2} (Y_{a3} - Y_{a2}))_{kl}$$

$$-W_5 = \frac{1}{2} C_L^{ijkl} q_k q_l q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c$$

Diagrams which contribute to $p \to K\bar{\nu}$



$$+(d \leftrightarrow s)$$

In principle, if the dimension five operator can be

$$C_L \sim \frac{1}{M_T}(Y_u + \epsilon_1(f, h'))_{ij}(Y_u + \epsilon_2 f)_{kl}$$

$$C_R \sim \frac{1}{M_T}(Y_u + \epsilon_3(f, h'))_{ij}(Y_u + \epsilon_4(f, h'))_{kl}$$

the nucleon decay amplitudes can be suppressed to satisfy the current experimental bound roughly.

However, due to the opposite signatures in

$$C_L^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}h + X_{a4}f + \sqrt{2}X_{a3}h')_{ij}(Y_{a1}h + Y_{a5}f)_{kl}$$

no h'

Opposite signature

$$C_R^{ijkl} = \sum_a \frac{1}{M_{T_a}}(X_{a1}h - X_{a4}f + \sqrt{2}X_{a2}h')_{ij}(Y_{a1}h - (Y_{a5} - \sqrt{2}Y_{a6})f + \sqrt{2}(Y_{a3} - Y_{a2}))_{kl}$$

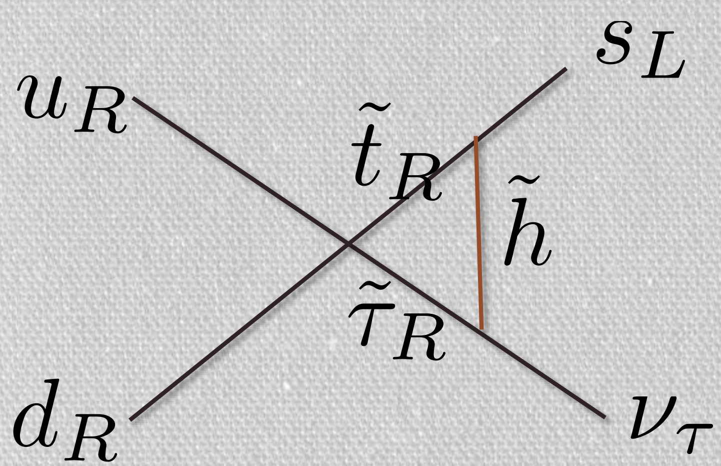
such a choice of the colored Higgs mixings is not realized in general.

$$-W_5 = \frac{1}{2} C_L^{ijkl} q_k q_l q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c$$

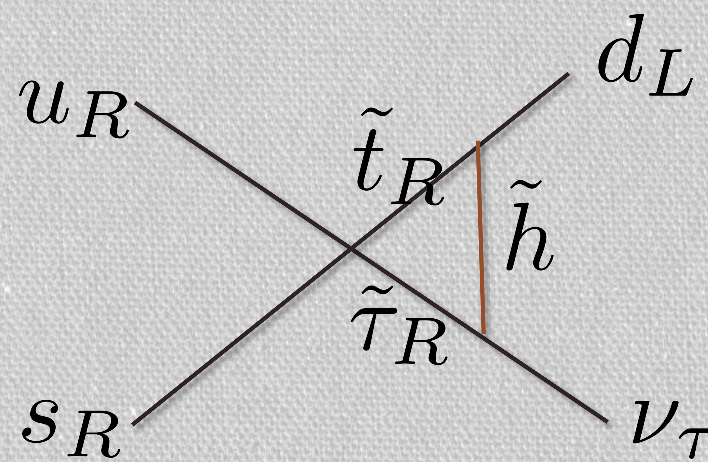
The criteria to suppress the amplitudes is that the contribution

$$\frac{1}{M_T} f_{ij} (Y_u + af)_{kl}$$

in the Right-handed operator is suppressed.



$$\sim f_{ud} \, y_t^2 y_\tau V_{ts}$$



$$\sim f_{us} \, y_t^2 y_\tau V_{td}$$

Triplet-dominant type II seesaw

$$\mathcal{M}_\nu \propto f$$

See
"Proton decay and μ→e+γ connection in a renormalizable SO(10) GUT for neutrinos"
Dutta-YM-Mohaptra, **Phys. Rev. D87 (2013) 7, 075008**

$$m_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T$$

Type II   Type I

$$M_L = 2\sqrt{2}\bar{f}\langle\bar{\Delta}_L\rangle \qquad M_R = 2\sqrt{2}\bar{f}\langle\bar{\Delta}_R\rangle$$

$$\bar{\Delta}_L : (\mathbf{1}, \mathbf{3}, 1) \qquad \bar{\Delta}_R : (\mathbf{1}, \mathbf{1}, 0)$$

$SU(2)_L$ triplet

"Revisiting fermion mass and mixing fits in the minimal SUSY SO(10) GUT"
Fukuyama-Ichikawa-YM, **arXiv:1508.07078**

If $\langle \Delta_R \rangle > 10^{16}$ GeV,

the coupling *f* can be written as a linear combination of

$$\mathcal{M}_\nu, \quad M_e, \quad \mathcal{M}_\nu M_e^{-1} \mathcal{M}_\nu .$$

$$m_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T$$

Type II — Type I

$$M_L = 2\sqrt{2} \bar{f} \langle \bar{\Delta}_L \rangle \qquad M_R = 2\sqrt{2} \bar{f} \langle \bar{\Delta}_R \rangle$$

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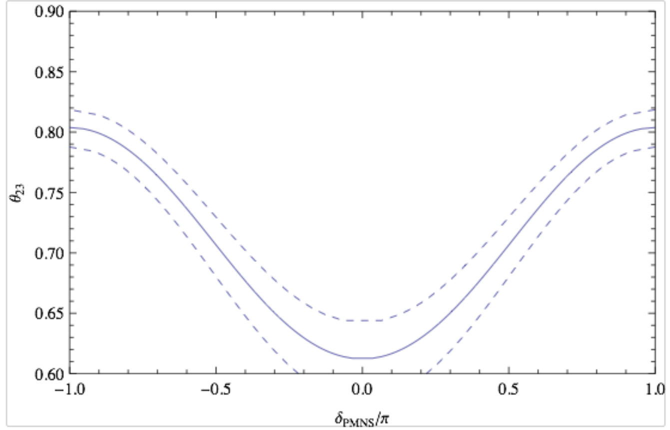
$$\mathcal{M}_\nu, \quad M_e, \quad \mathcal{M}_\nu M_e^{-1} \mathcal{M}_\nu .$$

Prediction from the small (1,1) and (1,2) elements still holds.



Note :

Type I seesaw contribution usually requires

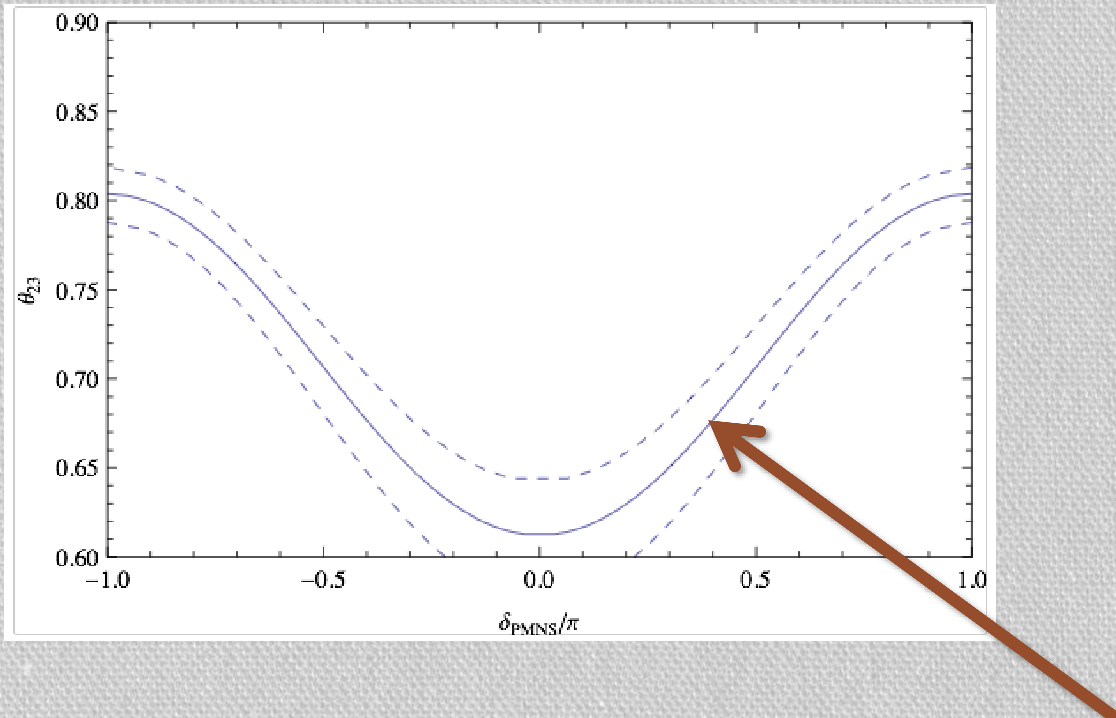
$$\langle \Delta_R \rangle \sim 10^{13-14}\ \mathrm{GeV}\ \text{for}\ (Y_\nu^D)_{33} \sim 1.$$

However, the type I part can generate sub-eV size of neutrino masses even for $\langle \Delta_R \rangle > 10^{16}\ \mathrm{GeV}$ if the right-handed mass matrix is nearly singular.

$$m_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T$$

If $\langle \Delta_R \rangle > 10^{16}$ GeV,

Prediction from the small (1,1) and (1,2) elements still holds.

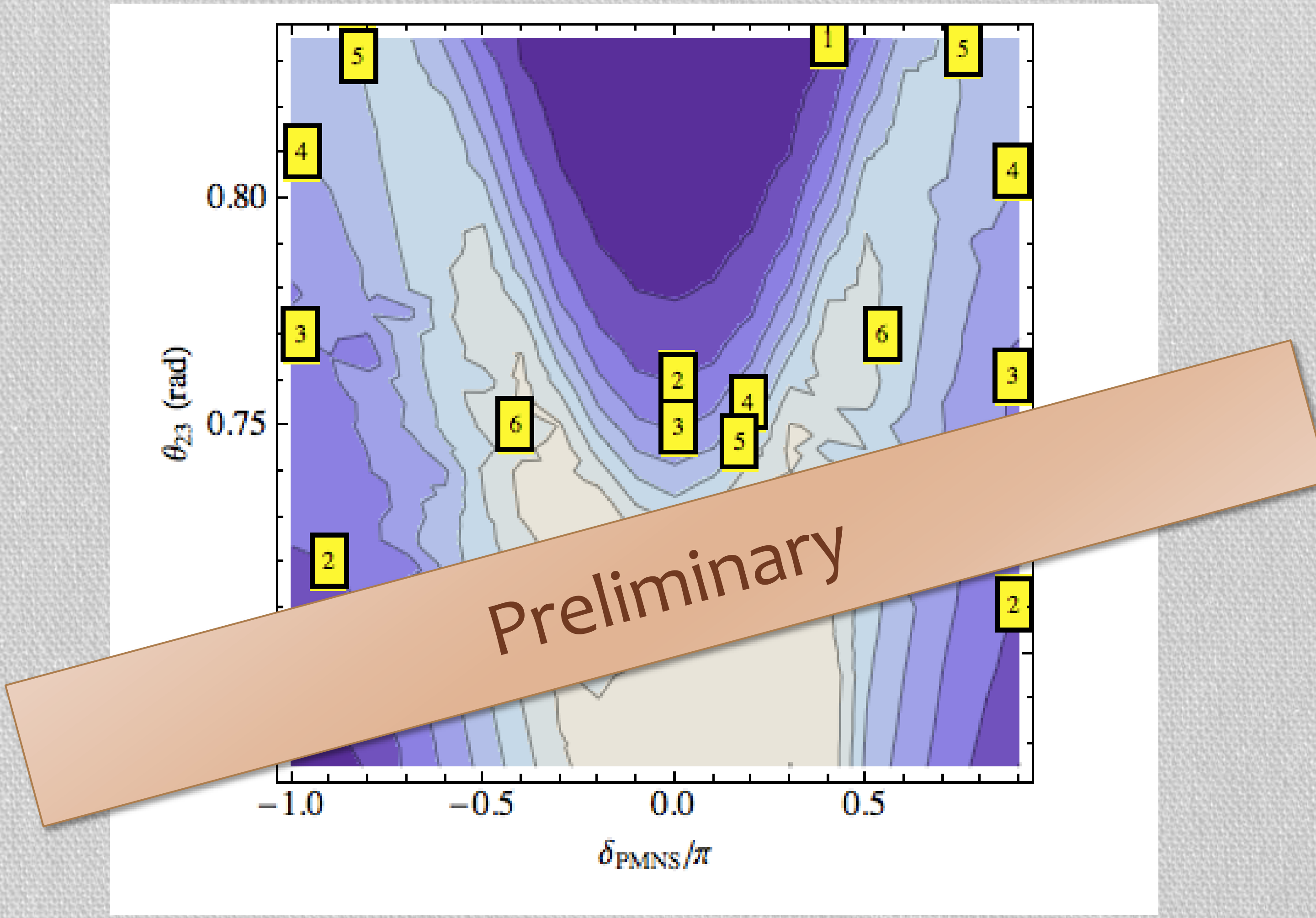


Proton decay amplitudes become smaller along this curve.

It can shift depending on the detail fits of quark masses & mixings.

Plot the proton lifetime using the fit data in the minimal SO(10) model (10+126) for $\langle \Delta_R \rangle > 10^{16}\ \mathrm{GeV}$

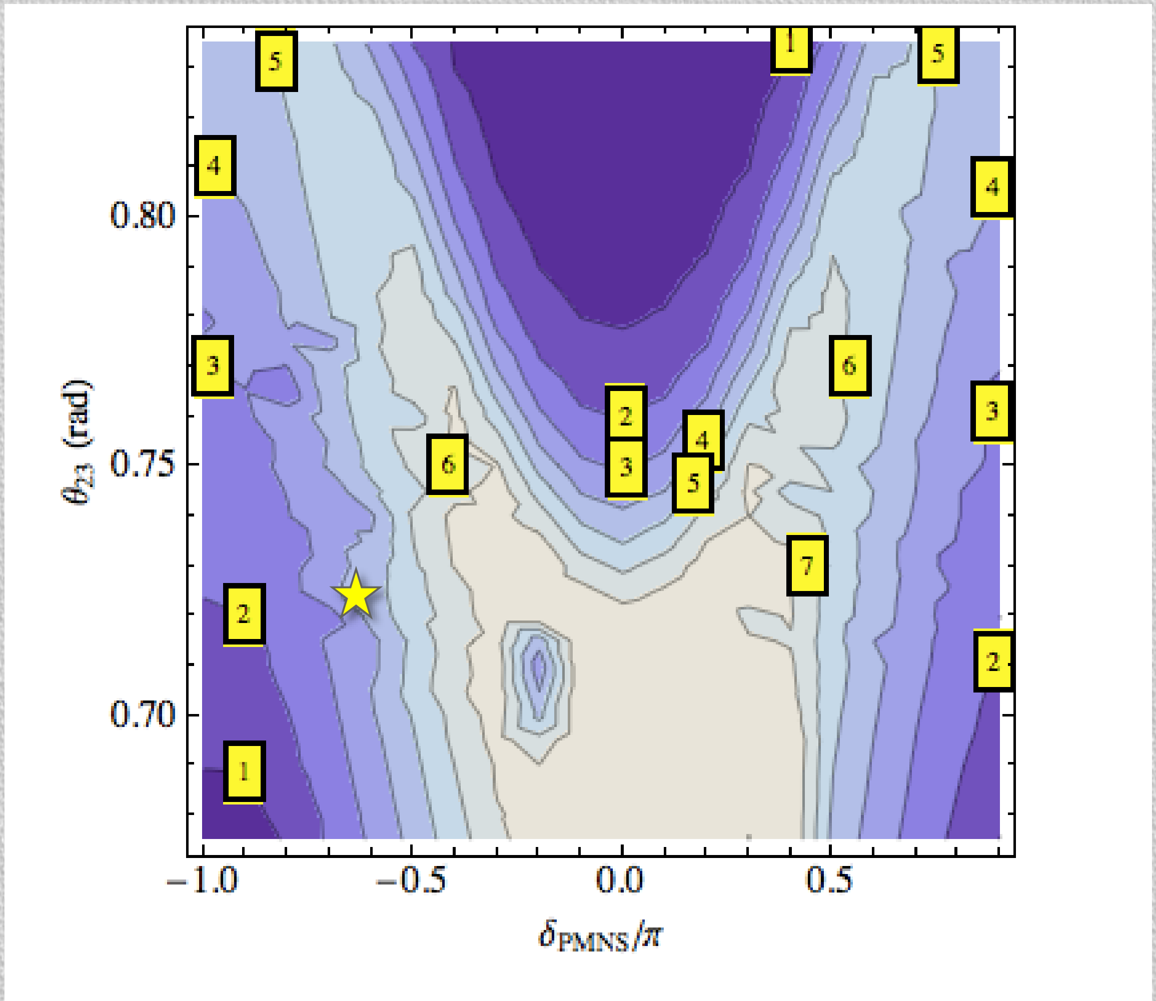
$$\tau(p \to K\bar{\nu})[10^{34} \text{ years}]$$



Left-handed contribution is canceled. $\tan\beta = 10$ $m_{\tilde{q}} = 2$ TeV

$$\tau(p \to K\bar{\nu})^{\text{EXP}} > 0.59 \times 10^{34} \text{ years}$$

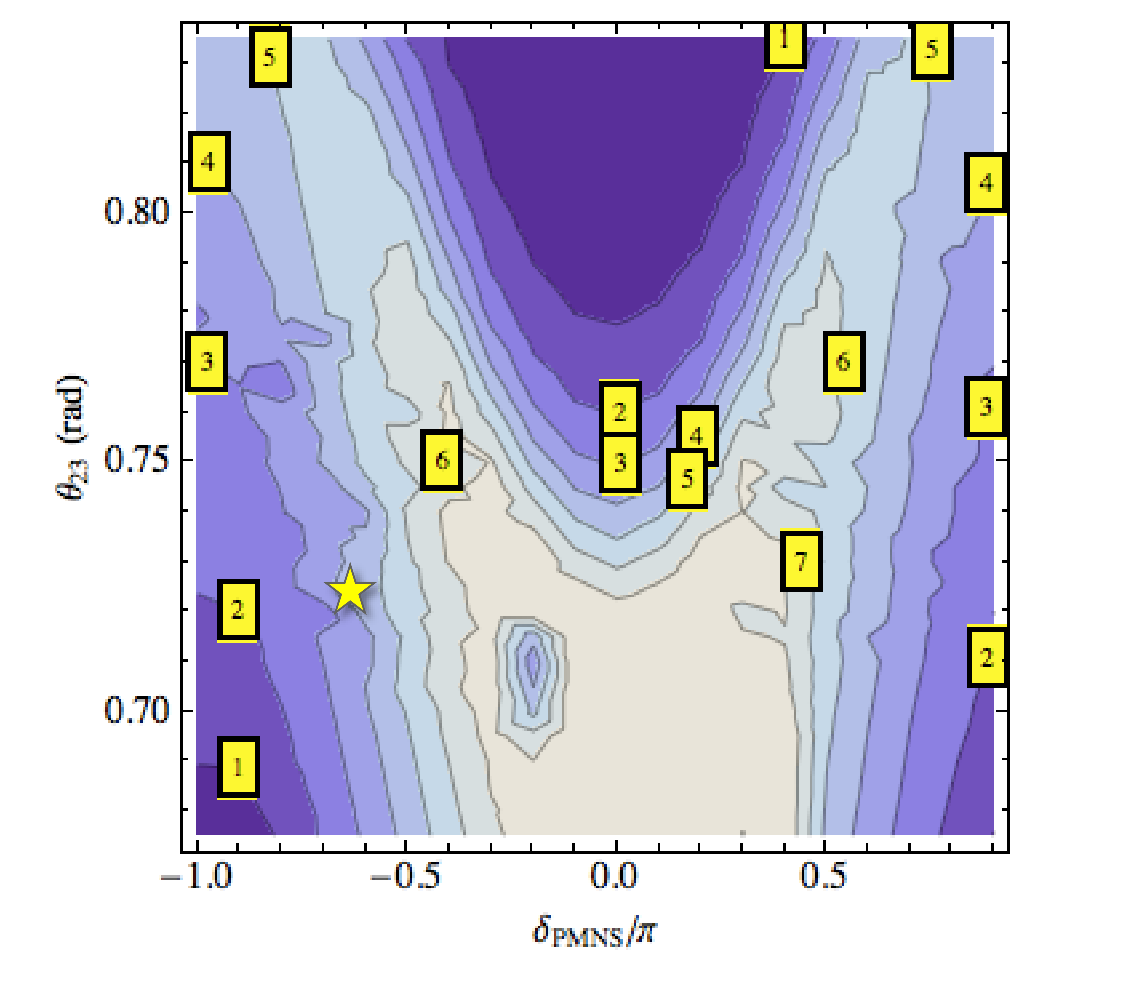
$$\tau(p \to K\bar{\nu})[10^{34}\ \text{years}]$$



Left-handed contribution is canceled. $\tan\beta = 10$ $m_{\tilde{q}} = 2$ TeV

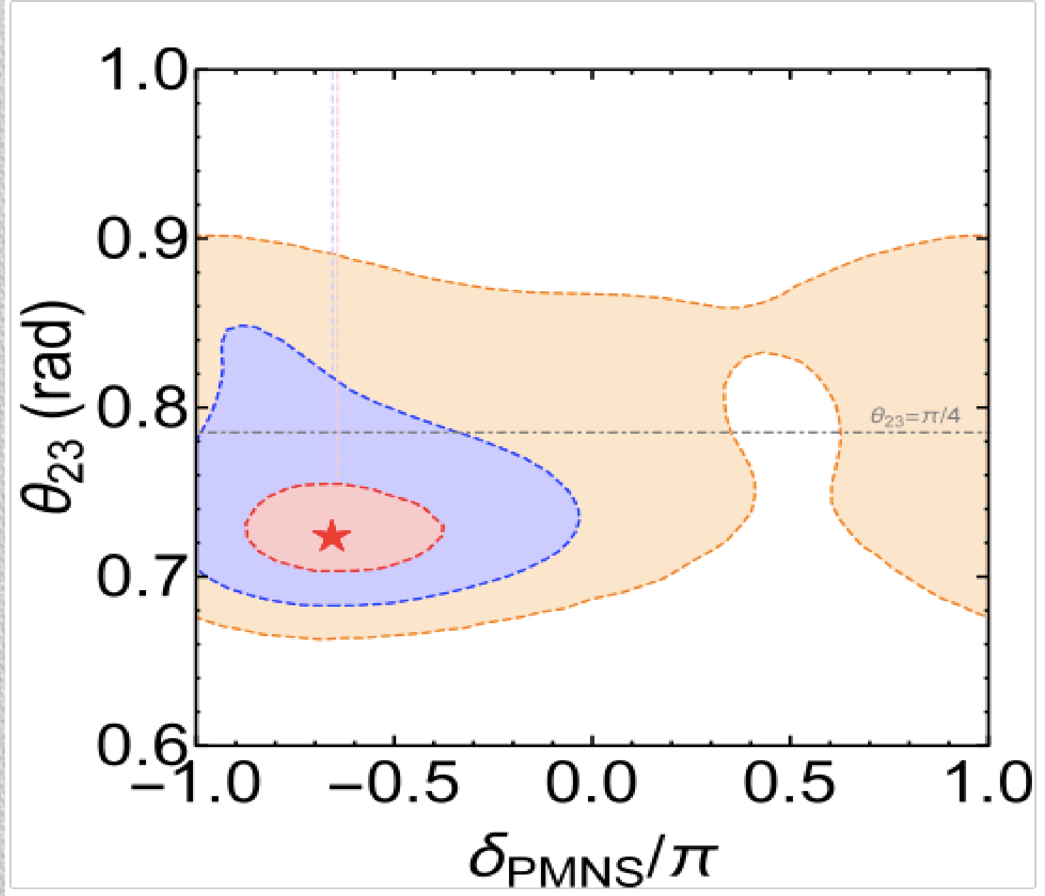
$$\tau(p \to K\bar{\nu})^{\rm EXP} > 0.59 \times 10^{34}\ \text{years}$$

$$\tau(p \to K\bar{\nu})[10^{34} \text{ years}]$$

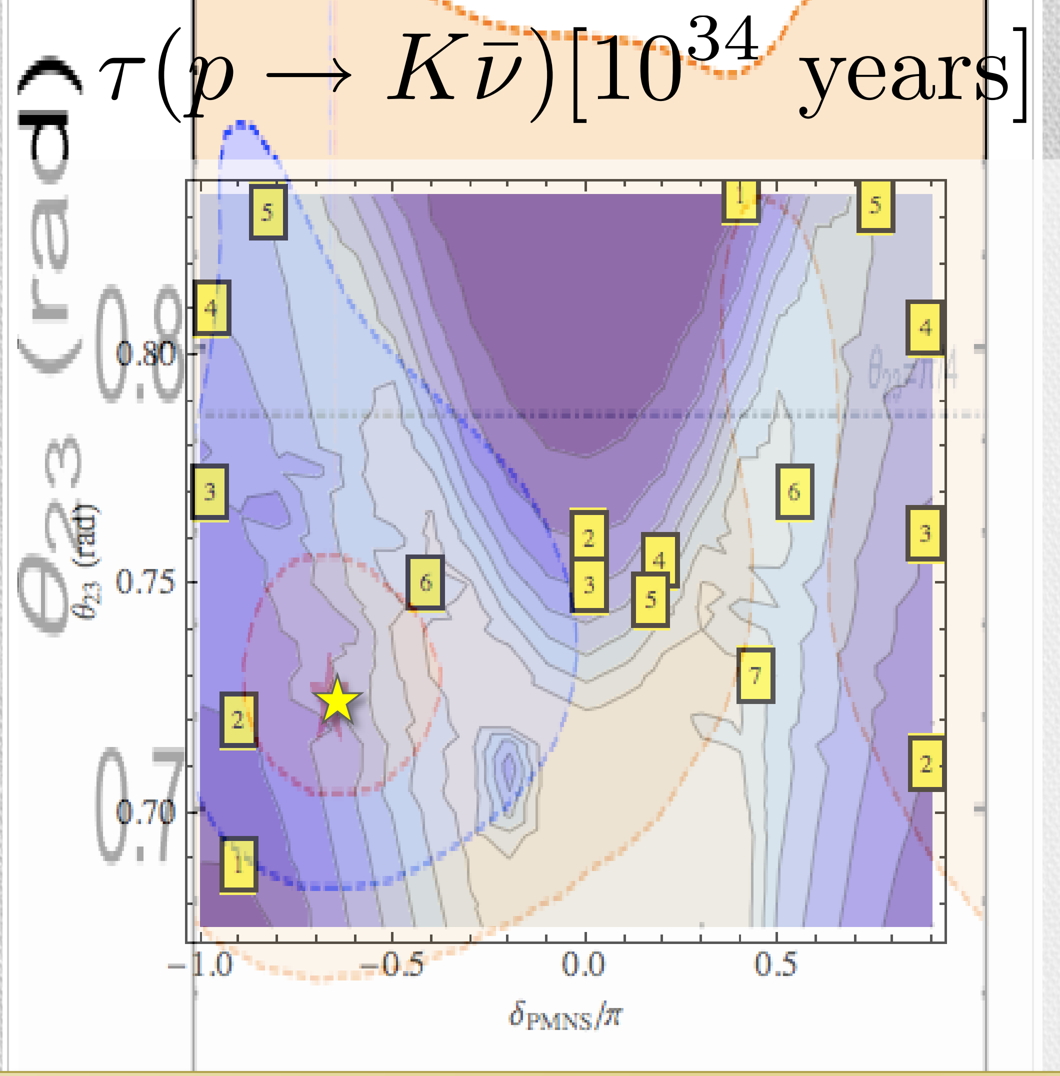


Qualitative behavior is model-independent.

Global analysis (by Capozzi et al)



The precise measurements of the PMNS phase (as well as 23 mixing) is important!!

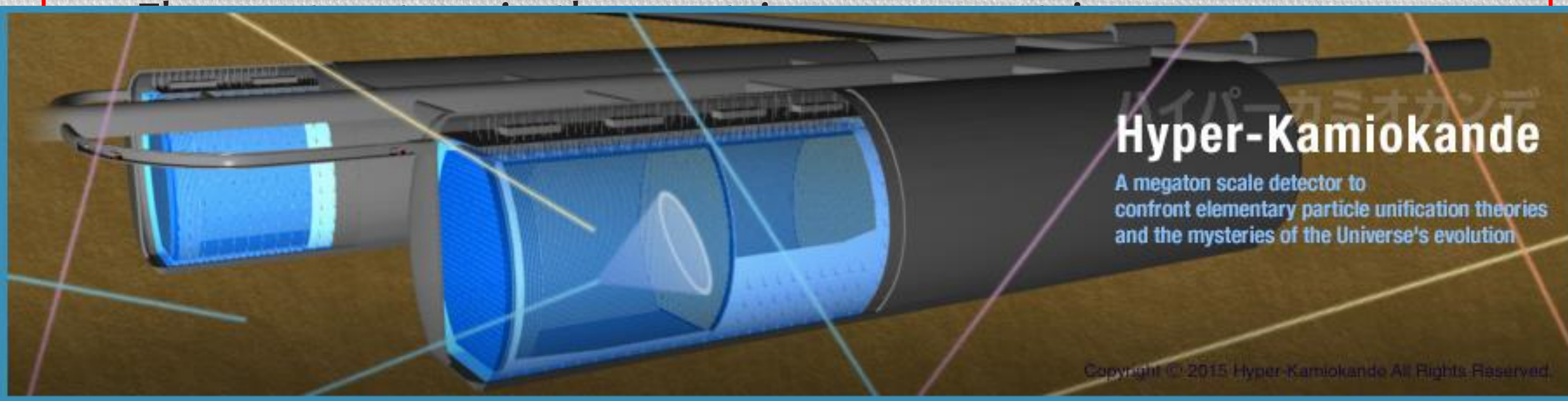


The precise measurements of the PMNS phase (as well as 23 mixing) is important!!

Summary

- Flavor structure in the neutrino mass matrix.
  Small (1,1) and (1,2) elements are preferred for the proton decay suppression.
- Predictions to the relations between the PMNS phase and 23-mixing.
  It is consistent with the current experimental data.
- The grand unified model : SO(10)
  Seesaw with $\langle \Delta_R \rangle > 10^{16}\ \mathrm{GeV}$
- We hope to have more data of the Long Baseline neutrino oscillations + proton decay.

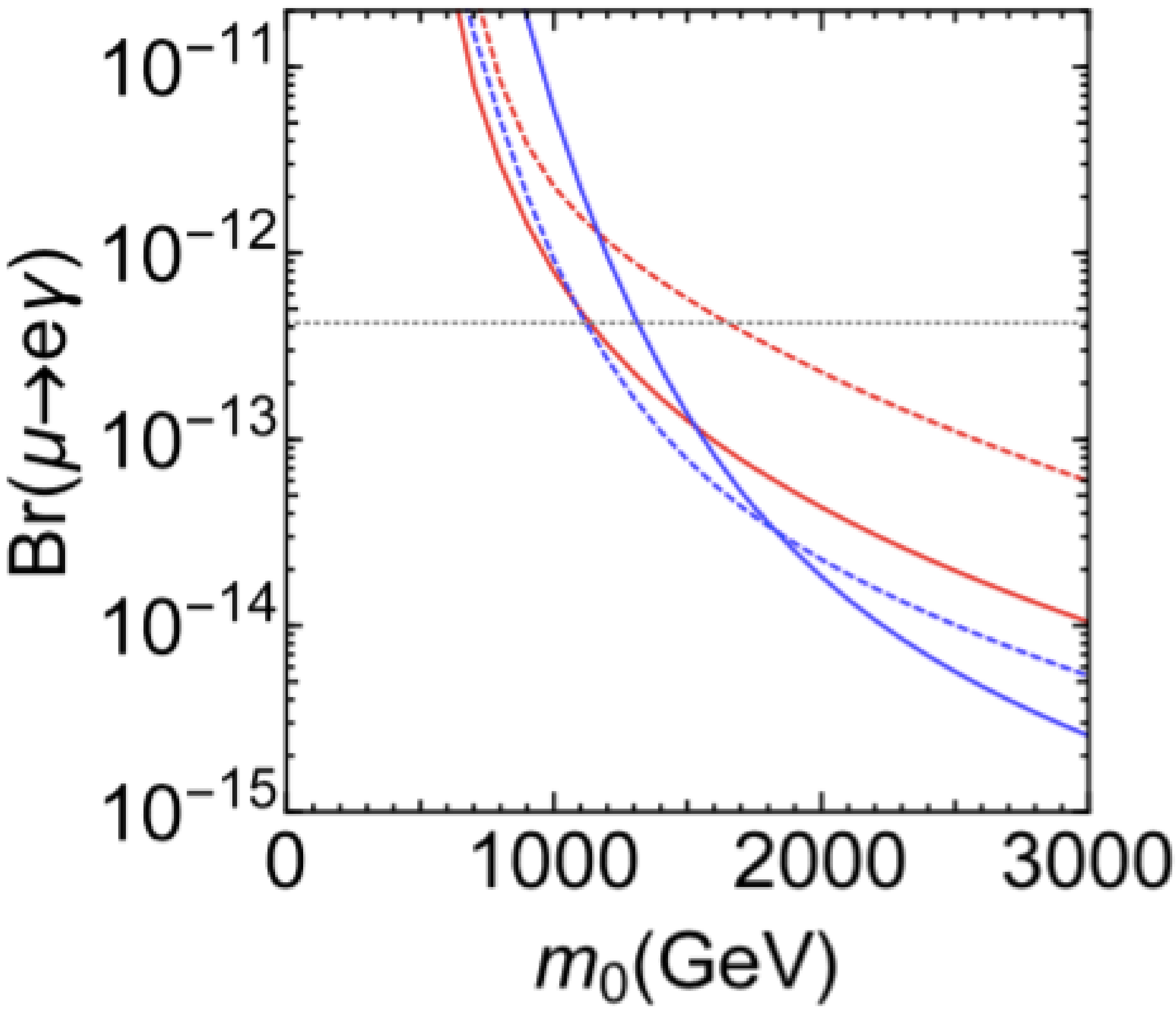
Summary



- The grand unified model : SO(10)
  Triplet ... ty ... II + Se ... aw ... h $\langle\Delta_R$ ... V
- More d ... a of the L ... g Baseline neu ... o os ... proton decay ... e neede ...

Thank you very much!

- CKM mixings are small.
- Approximate bottom-tau Yukawa unification.
- $3\frac{m_s}{m_b} \simeq \frac{m_\mu}{m_\tau}$
- $V_{cb} \sim \frac{m_s}{m_b}, \qquad V_{ub} \sim V_{cb}\frac{m_{\nu 2}}{m_{\nu 3}}$
- Atmospheric and solar neutrino mixings are generically O(1).
But, 13 mixing is related to mass ratio $\sin\theta_{13} \sim \frac{m_{\nu 2}}{m_{\nu 3}}$

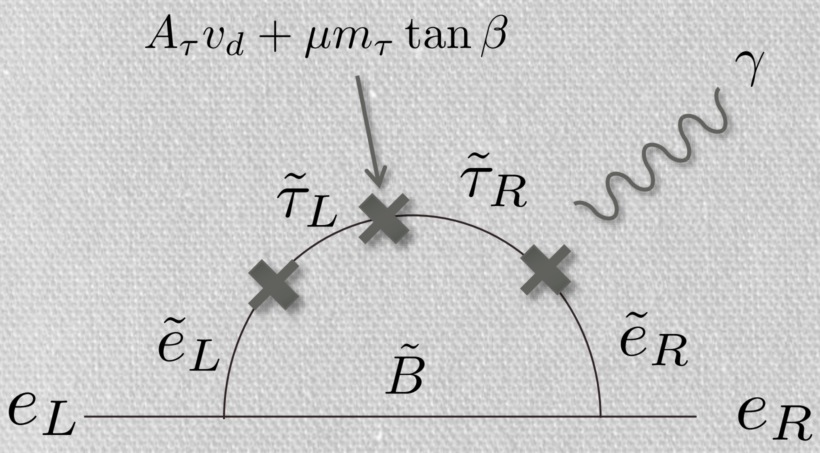


10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
Br($\mu \to e\gamma$)
0
1000
2000
3000
m_0(GeV)

Implication

If $\langle \Delta_R \rangle \sim 10^{13}$ GeV, the $e^c e^c \Delta_R^{--}, e^c \nu^c \Delta_R^{-}$ couplings induce FCNC in the right-handed sleptons (in addition to the left-handed ones from Dirac neutrino couplings)

The electron electric dipole moment is generated even if Higgsino mass and scalar trilinear coupling are real.

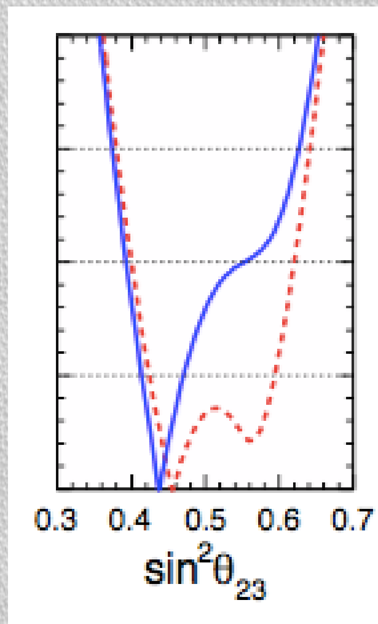


Experimental bound:

$$|d_e| < 8.7 \times 10^{-29}\ e \cdot \mathrm{cm}$$

$\Rightarrow$ $m_{\tilde{\ell}} > 2$ TeV

Exp. Status



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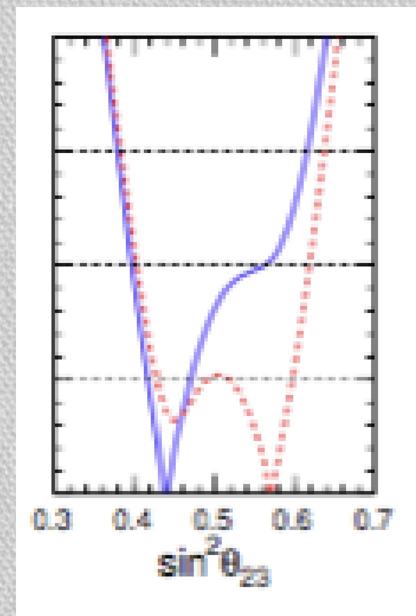


2016

NH

IH

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010$$

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$

(Daya Bay, 621 days data, 2015)

F. Capozzi et al. / Nuclear Physics B 00 (2016) 1–14

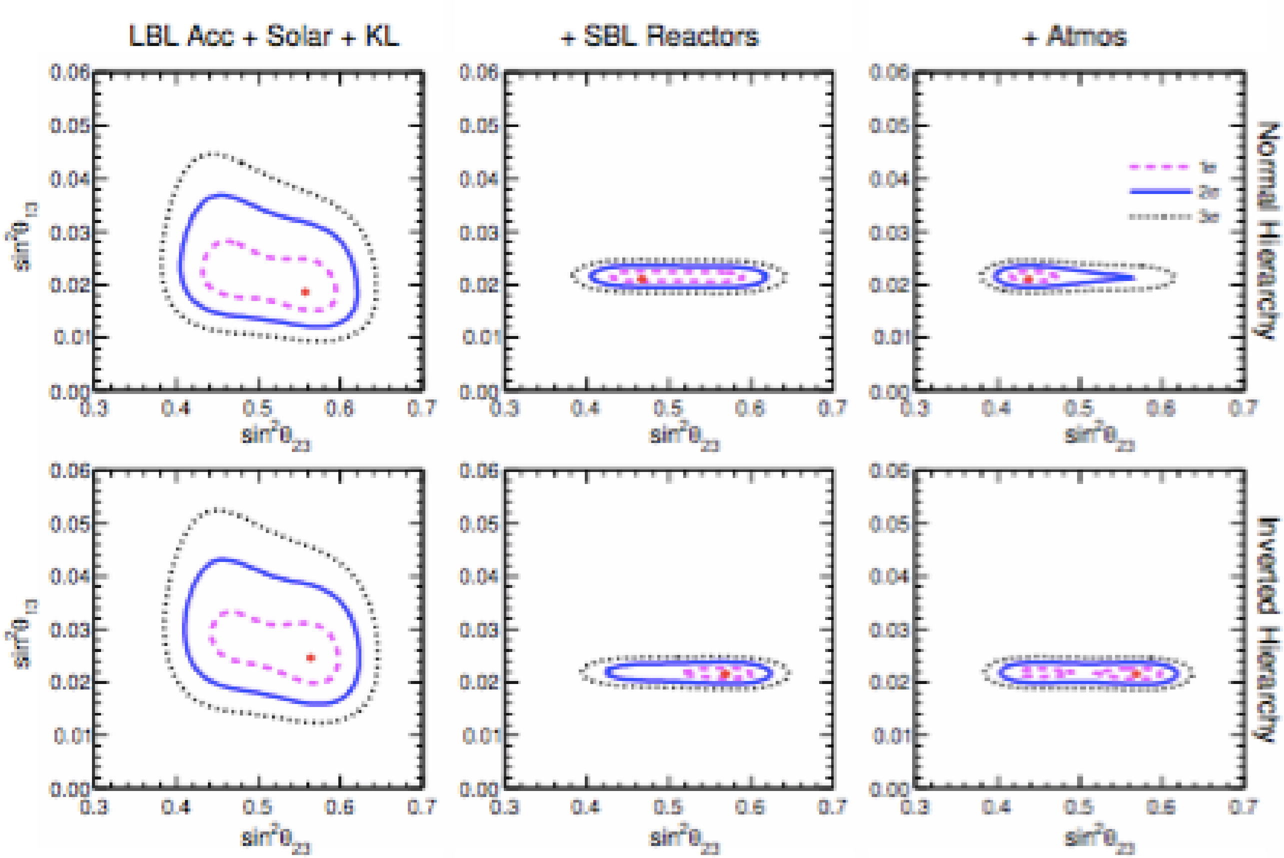


Fig. 5: As in Fig. 4, but for the ($\sin^2\theta_{23}$, $\sin^2\theta_{13}$) parameters.

Without the reactor data, the best fit lies at $\theta_{23} > \pi/4$.
Stay tuned to the long baseline experiments.

Simple realization of successful fermion mass and mixings

If fermion mass matrices are rank 1 (h) + corrections (f, h')

$$Y_u = h + r_2 f + r_3 h' \qquad Y_d = r_1(h + f + h')$$

$$Y_e = r_1(h - 3f + c_e h') \qquad m_\nu^{\text{light}} \propto f \quad \text{(type II)}$$

2 large, 1 small neutrino mixings are obtained.

Quark mixings are small.

Dutta-YM-Mohapatra, PRD**80**, 095021 (2009)

Simple realization of successful fermion mass and mixings

Fermion mass matrices are rank 1 (h) + corrections (f, h')

f-diagonal basis. f_{33} small $\longrightarrow$ CKM small.

$$h = h_{33}\begin{pmatrix} c^2 & bc & ac \\ bc & b^2 & ab \\ ac & ab & a^2 \end{pmatrix}, \quad f = f_{33}\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In the limit $\lambda_1, \lambda_2 \to 0$

$$\tan^2\theta_{\rm atm} = \frac{b^2+c^2}{a^2} \qquad \tan^2\theta_{\rm sol} = \frac{c}{b} \qquad \sin\theta_{13} = 0.$$

2 large, 1 small neutrino mixings are natural.

$$Y_u = h + r_2 f + r_3 h' \qquad Y_d = r_1(h + f + h')$$

$$Y_e = r_1(h - 3f + c_e h') \qquad m_\nu^{\rm light} \propto f \text{ (type II)}$$

Suppressing proton decay by choosing Yukawa matrices

(Dutta-YM-Mohapatra)

Yukawa terms :

$$W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \bar{\Delta} \; + \; \frac{1}{2} h'_{ij} \psi_i \psi_j D \quad [H(\mathbf{10}),\ \bar{\Delta}(\overline{\mathbf{126}}),\ D(\mathbf{120})]$$

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda_t \end{pmatrix}, \quad f \simeq \begin{pmatrix} 0 & 0 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad h' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}$$

$$\lambda \sim 0.2$$

* This texture needs less cancellation and allows a large bottom Yukawa in SO(10).

⇒ Prediction to neutrino mixing parameters

Consistent with the current experiments!

In the minimal SO(10) model, $M_\nu^D = M_e + af$

$$\mathcal{M}_\nu^I \propto (M_e + af) f^{-1} (M_e + af)$$
$$= M_e f^{-1} M_e + 2a M_e + a^2 f$$

Quadratic equation in terms of $M_e^{-1/2} f M_e^{-1/2}$

In the limit of $\langle \Delta_R \rangle \to \infty$,

$$f \propto M_e^{1/2} (K + \sqrt{K^2}) M_e^{1/2}$$

where $K = M_e^{-1/2} \mathcal{M}_\nu M_e^{-1/2}$

Cf. $X(X - A) = 0$ → $X = \frac{1}{2}(A \pm \sqrt{A^2})$

$$f \propto M_e^{1/2}(K + \sqrt{K^2})M_e^{1/2}$$

where $K = M_e^{-1/2}\mathcal{M}_\nu M_e^{-1/2}$

Mathematics

$$K = V \mathrm{diag.}\,(\lambda_1, \lambda_2, \lambda_3) V^{-1}$$

$$\sqrt{K^2} = V \mathrm{diag.}\,(\pm\lambda_1, \pm\lambda_2, \pm\lambda_3) V^{-1}$$

Nearly singular solution in the limit of $\langle \Delta_R \rangle \to \infty$.

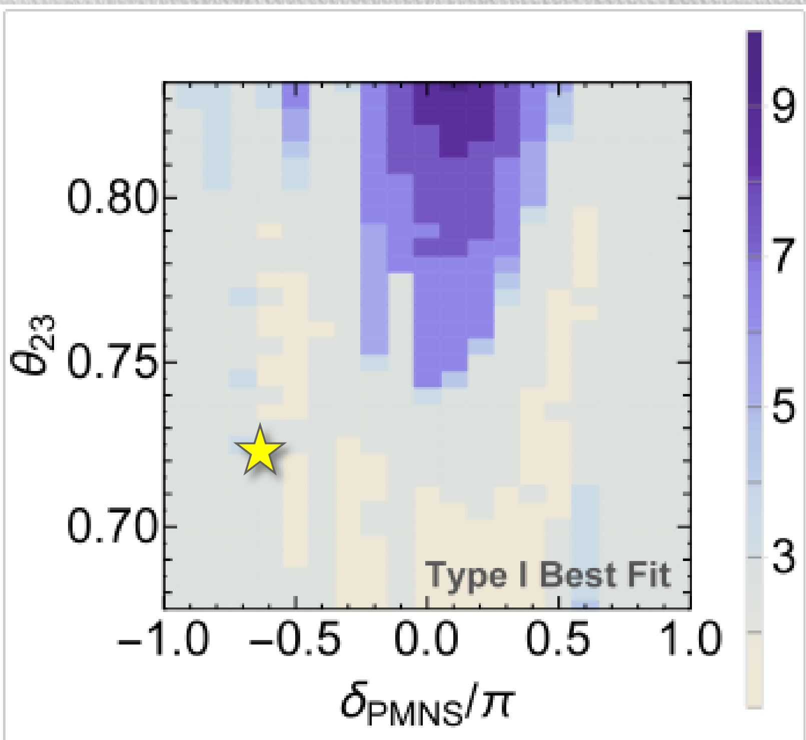
$$V \mathrm{diag.}\,(1,0,0) V^{-1} = \frac{K^2 - (\lambda_2 + \lambda_3)K + \lambda_2\lambda_3 \mathbf{1}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)}$$ etc.

The coupling *f* can be written as a linear combination of

$\mathcal{M}_\nu, \quad M_e, \quad \mathcal{M}_\nu M_e^{-1} \mathcal{M}_\nu.$

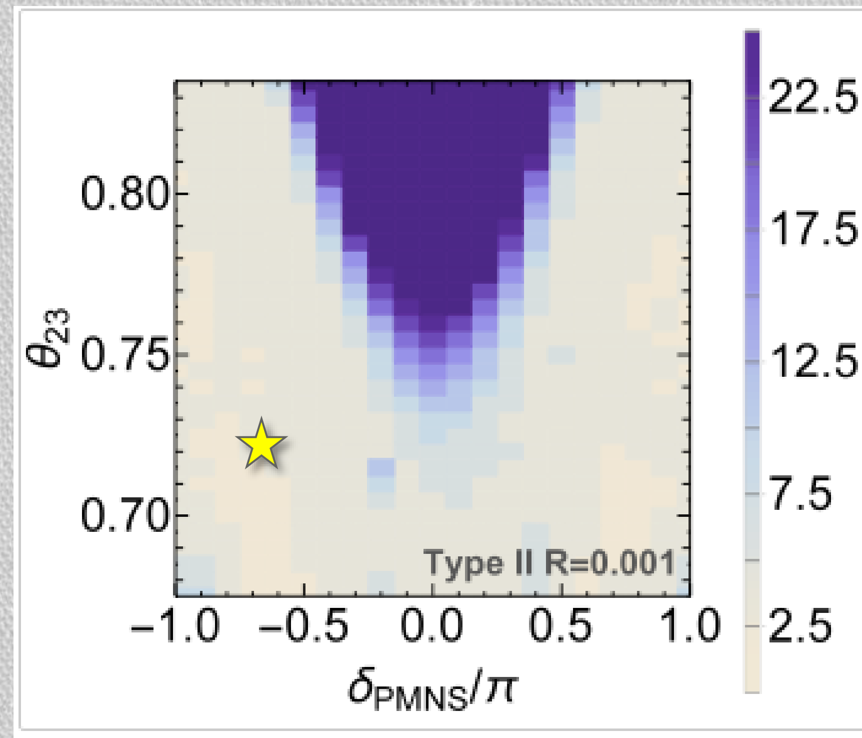
χ^2 fit results of fermion masses and mixings (in 10+126 Higgs model)

(arXiv:1508.07078)



Type I seesaw

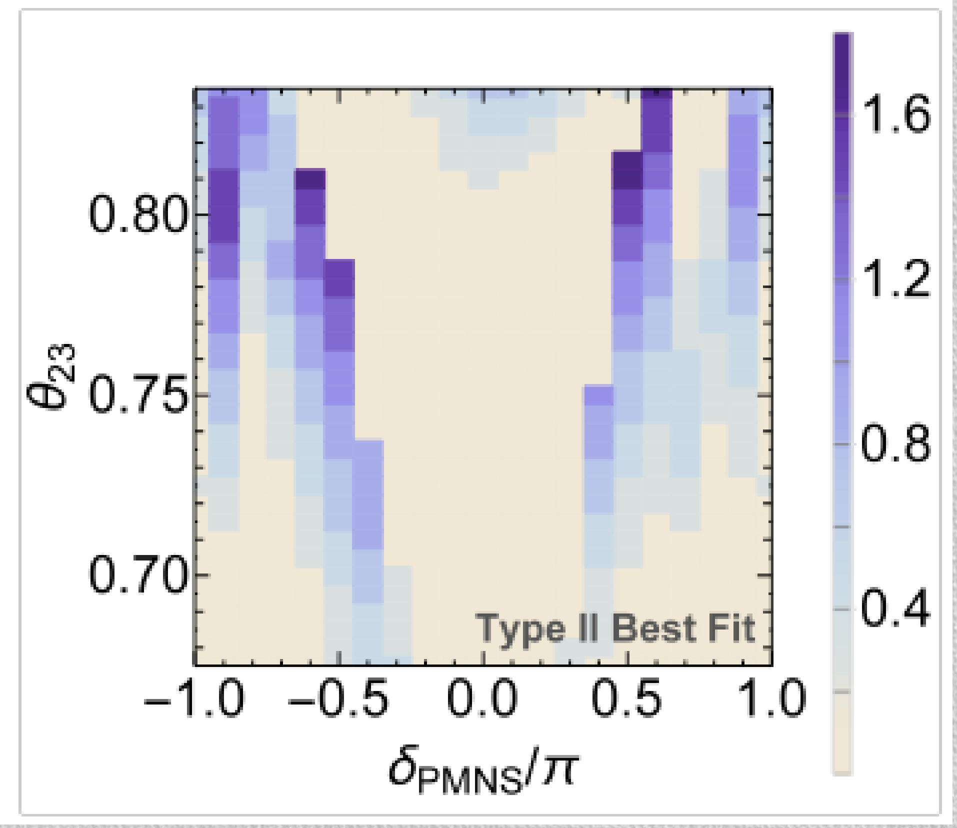
$(\langle \Delta_R \rangle = 10^{13}\ \mathrm{GeV}\ )$



Type I + II seesaw

$(\langle \Delta_R \rangle = 10^{16}\ \mathrm{GeV}\ )$

Exp best fit : $\begin{cases} \theta_{23} = 0.722 (= 41.4^\circ) \\ \delta_{\mathrm{PMNS}}/\pi = -0.65 \end{cases}$



Type II seesaw (best fit) $(\langle \Delta_R \rangle \sim 10^{13}\ \text{GeV}\ )$