From E_8-inspired SUSY trinification to a L-R symmetric theory

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July 4, 2016

SUSY 2016 – The University of Melbourne

Outline

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Main features (Glashow, Georgi and De Rujula 1984)

- LR gauge interactions and well motivated by E_6
- $SU(3)_L \times SU(3)_R \times SU(3)_C$ with $\mathcal{Z}_3 \rightarrow$ gauge unification

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- All matter can be elegantly arranged in bi-fundamental representations for each generation

$$\mathbf{27}^i = \left(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}\right)^i \otimes \left(\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}\right)^i \otimes \left(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}\right)^i \equiv \boldsymbol{L} \otimes \boldsymbol{Q_R} \otimes \boldsymbol{Q_L}$$

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- Naturally light neutrinos via radiative seesaw with split-SUSY **(Cauet et al. 2011)**

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- Naturally light neutrinos via radiative seesaw with split-SUSY **(Cauet et al. 2011)**
- Gauge symmetry preserves baryon number $\to$ Stable proton. **(Achiman and Stech, 1978) (Glashow and Kang 1984)**
- Well motivated as low energy versions of $E_8 \times E_8$ heterotic string theory **(Gross et al. 1985)**, E_6 orbifold **(Braam et al. 2010)** or $N = 8$ supergravity **(Cremmer et al. 1979)**.

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Trinification-based models were left as the least developed GUT scenarios

Our proposal

Novel solution including

1. A new $\mathrm{SU}(3)_\mathrm{F}$ flavour global symmetry **inspired** by E_8
2. **Unification of the Higgs and lepton sectors** via a common chiral supermultiplet

Our aim: Fix the lepton masses problem and show that the vacuum is stable

Outline

The Model

$SU(3)_F \times E_6$ is a maximal subgroup of E_8

- Branching rules for the fundamental representation of E_8 (Slansky)

$$\mathbf{248} = (\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{78}) \oplus (\mathbf{3}, \mathbf{27}) \oplus \left(\overline{\mathbf{3}}, \overline{\mathbf{27}}\right)$$

- Branching rules for the adjoint representation of E_6 down to trinification

$$\mathbf{78} = (\mathbf{8}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8}) \oplus \left(\mathbf{3}, \mathbf{3}, \overline{\mathbf{3}}\right) \oplus \left(\overline{\mathbf{3}}, \overline{\mathbf{3}}, \mathbf{3}\right)$$

(i) Tri-triplets do not couple to standard matter at tree-level

(ii) For proof of concept we consider $\left(\overline{\mathbf{3}}, \overline{\mathbf{27}}\right)$-plet heavy (for now)

(iii) For simplicity $SU(3)_F$ is global in our study (for now!)

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$$[\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R} \times \mathrm{SU}(3)_\mathrm{C}] \times \mathbb{Z}_3 \times \mathrm{SU}(3)_\mathrm{F}\,,$$

- $\mathbb{Z}_3$ is a permutation symmetry that guarantees gauge coupling unification

Chiral Supermultiplet Fields

Superfield		$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\mathrm{SU}(3)_\mathrm{F}$
Lepton	$(L^i)^l{}_r$	$\mathbf{1}$	$\mathbf{3}^l$	$\bar{\mathbf{3}}_r$	$\mathbf{3}^i$
Right-Quark	$(Q_R^i)^r{}_x$	$\bar{\mathbf{3}}_x$	$\mathbf{1}$	$\mathbf{3}^r$	$\mathbf{3}^i$
Left-Quark	$(Q_L^i)^x{}_l$	$\mathbf{3}^x$	$\bar{\mathbf{3}}_l$	$\mathbf{1}$	$\mathbf{3}^i$
Colour-adjoint	Δ_C^a	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left-adjoint	Δ_L^a	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-adjoint	Δ_R^a	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$
Flavour-adjoint	Δ_F^a	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$

Gauge Supermultiplet Fields

Superfield		$\mathrm{SU}(3)_\mathrm{C}$	$\mathrm{SU}(3)_\mathrm{L}$	$\mathrm{SU}(3)_\mathrm{R}$	$\mathrm{SU}(3)_\mathrm{F}$
Gluon	$G_C^{\mu a},\ \lambda_C^a$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Left-Gluon	$G_L^{\mu a},\ \lambda_L^a$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$	$\mathbf{1}$
Right-Gluon	$G_R^{\mu a},\ \lambda_R^a$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}^a$	$\mathbf{1}$

Fundamental tri-triplets:

$$(L^i)^l{}_r = \begin{pmatrix} H_{11} & H_{12} & \nu_L \\ H_{21} & H_{22} & e_L \\ \nu_R & e_R & \phi \end{pmatrix}^i, (Q_R^i)^r{}_x = \begin{pmatrix} u_R^{\bar{1}} & u_R^{\bar{2}} & u_R^{\bar{3}} \\ d_R^{\bar{1}} & d_R^{\bar{2}} & d_R^{\bar{3}} \\ D_R^{\bar{1}} & D_R^{\bar{2}} & D_R^{\bar{3}} \end{pmatrix}^i, (Q_L^i)^x{}_l = \begin{pmatrix} u_L^1 & d_L^1 & D_L^1 \\ u_L^2 & d_L^2 & D_L^2 \\ u_L^3 & d_L^3 & D_L^3 \end{pmatrix}^i$$

$\mathbb{Z}_3$ cyclic permutations:

$$L \xrightarrow{\mathbb{Z}_3} Q_{\mathrm{L}},$$
$$Q_{\mathrm{L}} \xrightarrow{\mathbb{Z}_3} Q_{\mathrm{R}},$$
$$Q_{\mathrm{R}} \xrightarrow{\mathbb{Z}_3} L.$$

Superpotential

$$W = \sum_{A=L,R,C} \left(\lambda_{78} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b\right) + \left(\lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b\right)$$
$$+ \lambda_{27} \varepsilon_{ijk} \left(Q_L^i\right)^x{}_l \left(Q_R^j\right)^r{}_x \left(L^k\right)^l{}_r , \quad \text{with} \quad d_{abc} = 2\mathrm{Tr}\left[\{T_a, T_b\} T_c\right]$$

- i, j and k $\rightarrow$ flavour indices
- x, l and r $\rightarrow$ colour, left-chirality and right-chirality respectively
- a, b and c $\rightarrow$ adjoint indices.
- **One single Yukawa coupling for standard matter, λ_{27}**

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    - **Why then E_8?**
    - Minimal SUSY trinification with $\mathrm{SU}(3)_{\mathrm{F}}$ and Higgs-lepton unification does not have a stable vacuum $\to$ $\mathbf{SU(3)_C}$ **and** $\mathbf{SU(2)_L}$ **fully broken at GUT scale**

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- **One single Yukawa coupling for standard matter, $\lambda_{\mathbf{27}}$**
- In a minimal E_6-inspired model with flavour $\mathrm{SU(3)_F}$ the superpotential would be just the last term
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- ***E_8*-inspired trinification is the minimal SUSY working model!**

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Symmetry breaking

Scalar potential

$$V = V_{\mathcal{F}} + V_{\mathcal{D}} + V_{\text{soft}} \qquad (\text{F} - \text{terms from the superpotential})$$

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(1) D-terms

$$\begin{aligned} V_{\mathcal{D}} = -\frac{1}{2} g_U^2 \Bigg\{ \sum_c & \left(\tilde{\Delta}_L^{a*} f_{abc} \tilde{\Delta}_L^b\right) \left(\tilde{\Delta}_L^{d*} f_{dec} \tilde{\Delta}_L^e\right) \\ & -\mathrm{i} \left(\tilde{\Delta}_L^{a*} f_{abc} \tilde{\Delta}_L^b\right) \left[\left(\tilde{L}_i^*\right)^{r_1}{}_{l_1} \left(T^c\right)^{l_1}{}_{l_2} \left(\tilde{L}^i\right)^{l_2}{}_{r_1} - \left(\tilde{Q}_L^i\right)^{x_1}{}_{l_3} \left(T^c\right)^{l_3}{}_{l_2} \left(\tilde{Q}_{Li}^*\right)^{l_2}{}_{x_1} \right] \Bigg\} \\ & + \frac{1}{2} g_U^2 \left[T^a\right]^{l_1}{}_{l_2} \left[T_a\right]^{l_3}{}_{l_4} \Big[\left(\tilde{L}_i^*\right)^{r_1}{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_{r_1} \left(\tilde{L}_j^*\right)^{r_2}{}_{l_3} \left(\tilde{L}^j\right)^{l_4}{}_{r_2} \\ & \qquad + \left(\tilde{Q}_L^i\right)^{x_1}{}_{l_1} \left(\tilde{Q}_{Li}^*\right)^{l_2}{}_{x_1} \left(\tilde{Q}_L^j\right)^{x_2}{}_{l_3} \left(\tilde{Q}_{Lj}^*\right)^{l_4}{}_{x_2} \\ & \qquad - 2 \left(\tilde{L}_i^*\right)^{r_1}{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_{r_1} \left(\tilde{Q}_L^j\right)^{x_2}{}_{l_3} \left(\tilde{Q}_{Lj}^*\right)^{l_4}{}_{x_2} \Big] \\ & \qquad + (\mathbb{Z}_3 \text{ permutations}) \end{aligned}$$

- $g_L = g_R = g_C \equiv g_U$
- D-term interactions between adjoint and fundamental scalars

(2) Soft SUSY-breaking terms

$$
\begin{aligned}
V_{\text{soft}}^{\text{gauge}} = & \, m_{\mathbf{27}}^2 \left[\left(\tilde{L}^i\right)^l{}_r \left(\tilde{L}^*_i\right)^r{}_l \right] + \delta_{ab} \left[b_{\mathbf{78}} \tilde{\Delta}^a_L \tilde{\Delta}^b_L + m_{\mathbf{78}}^2 \tilde{\Delta}^{*a}_L \tilde{\Delta}^b_L + c.c \right] \\
& + d_{abc} \left[A_{\mathbf{78}} \tilde{\Delta}^a_L \tilde{\Delta}^b_L \tilde{\Delta}^c_L + C_{\mathbf{78}} \tilde{\Delta}^{*a}_L \tilde{\Delta}^b_L \tilde{\Delta}^c_L + c.c. \right] \\
& + A_G \left[\tilde{\Delta}^a_L \left(T_a\right)_{l_1}^{l_2} \left(\tilde{L}^*_i\right)^r{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_r + c.c. \right] \\
& + A_{\mathbf{27}} \left[\varepsilon_{ijk} \left(\tilde{Q}^i_L\right)^x{}_l \left(\tilde{Q}^j_R\right)^r{}_x \left(\tilde{L}^k\right)^l{}_r + c.c. \right] + \mathcal{Z}_3 \text{ permutations}
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$$

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V_{\text{soft}}^{\text{global}} = & \, \delta_{ab} \left[b_{\mathbf{1}}^2 \tilde{\Delta}^a_F \tilde{\Delta}^b_F + m_{\mathbf{1}}^2 \tilde{\Delta}^{*a}_F \tilde{\Delta}^b_F + c.c \right] + A_{\mathbf{1}} d_{abc} \left[\tilde{\Delta}^a_F \tilde{\Delta}^b_F \tilde{\Delta}^c_F + c.c. \right] \\
& + A_F \left[\tilde{\Delta}^a_F \left(T_a\right)_j^i \left(\tilde{L}^*_i\right)^r{}_l \left(\tilde{L}^j\right)^l{}_r + c.c. \right] + \mathcal{Z}_3 \text{ permutations}
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$$+ d_{abc} \left[A_{\mathbf{78}} \tilde{\Delta}_L^a \tilde{\Delta}_L^b \tilde{\Delta}_L^c + C_{\mathbf{78}} \tilde{\Delta}_L^{*a} \tilde{\Delta}_L^b \tilde{\Delta}_L^c + c.c. \right]$$

$$+ A_G \left[\tilde{\Delta}_L^a \left(T_a\right)_{l_1}^{l_2} \left(\tilde{L}_i^*\right)^r{}_{l_1} \left(\tilde{L}^i\right)^{l_2}{}_r + c.c. \right]$$

$$+ A_{\mathbf{27}} \left[\varepsilon_{ijk} \left(\tilde{Q}_L^i\right)^x{}_l \left(\tilde{Q}_R^j\right)^r{}_x \left(\tilde{L}^k\right)^l{}_r + c.c. \right] + \mathcal{Z}_3 \text{ permutations}$$

$$V_{\rm soft}^{\rm global} = \delta_{ab} \left[b_{\mathbf{1}}^2 \tilde{\Delta}_F^a \tilde{\Delta}_F^b + m_{\mathbf{1}}^2 \tilde{\Delta}_F^{*a} \tilde{\Delta}_F^b + c.c \right] + A_{\mathbf{1}} d_{abc} \left[\tilde{\Delta}_F^a \tilde{\Delta}_F^b \tilde{\Delta}_F^c + c.c. \right]$$

$$+ A_F \left[\tilde{\Delta}_F^a \left(T_a\right)_j^i \left(\tilde{L}_i^*\right)^r{}_l \left(\tilde{L}^j\right)^l{}_r + c.c. \right] + \mathcal{Z}_3 \text{ permutations}$$

- Only fundamental VEVs $\Longrightarrow$ **Unstable vacuum**
- **Need adjoint VEVs to have a stable minimum**

Vacuum choice:

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- Eight component of $\mathrm{SU}(3)$ generators T^8

$$T^8_{\mathrm{L,R,F}} = \frac{1}{2\sqrt{2}} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 0 & 0 & -2 \end{array} \right),$$

- Choosing $\langle \tilde{\Delta}^8_{L,R} \rangle = v$ and $\langle \tilde{\Delta}^8_F \rangle = v_F$ we break trinification to a rank-6 LR effective model:

$$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \times \mathbb{Z}_3 \times \mathrm{SU}(3)_\mathrm{F} \longrightarrow$$
$$\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{F} \times \mathrm{U}(1)_\mathrm{F}$$

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$$\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}} \times \mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{U}(1)_{\mathrm{F}}$$

Minimization:

1. **Positive mass spectrum for the full scalar sector→STABLE MINIMUM**
2. 8 gauge goldstones in the **adjoint sector**

$$\left| D^\mu \left\langle \tilde{\Delta}^b_{L,R} \right\rangle \right|^2 = \frac{3}{4} g_U^2 v^2 \sum_{a=4}^{7} \eta_{\mu\nu} G^{\mu a}_{L,R} G^{\nu a}_{L,R}\,,$$

3. 4 flavour goldstones (absorbed by flavour gauge bosons or decouple according to Burgess [hep-ph/9812468])

Fermion masses

Scalar-fermion terms

$$\mathcal{L}^{fermion} = \mathcal{L}_{\mathcal{F}}^{\text{fermion}} + \mathcal{L}_{\mathcal{D}}^{\text{fermion}} + \mathcal{L}_{\text{soft}}^{\text{fermion}} \qquad (\text{F} - \text{terms from the superpotential})$$

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D-terms

$$\mathcal{L}_{\mathcal{D}}^{\text{fermion}} = -\sqrt{2} g_U \left[\left(\tilde{L}_i^*\right)^r{}_{l_1} \left(T^a\right)^{l_1}{}_{l_2} \left(L^i\right)^{l_2}{}_r \tilde{\lambda}_L^a + \left(\tilde{L}_i^*\right)^{r_1}{}_l \left(T^a\right)^{r_2}{}_{r_1} \left(L^i\right)^l{}_{r_2} \tilde{\lambda}_R^a \right.$$
$$\left. - \mathrm{i} f_{abc} \tilde{\Delta}_L^{*b} \Delta_L^c \tilde{\lambda}_L^a \right] + \left(\mathcal{Z}_3 \text{ permutations}\right).$$

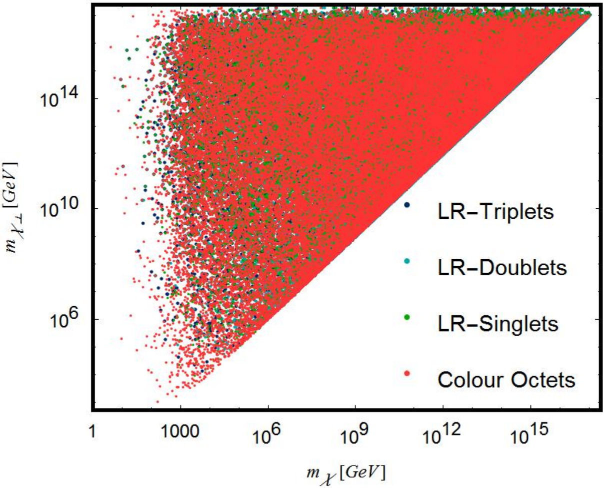
Soft SUSY-breaking terms

$$\mathcal{L}_{\text{soft}}^{\text{fermion}} = -\frac{1}{2} M_0 \delta_{ab} \tilde{\lambda}_L^a \tilde{\lambda}_L^b - M_0' \delta_{ab} \tilde{\lambda}_L^a \Delta_L^b + h.c. + \left(\mathcal{Z}_3 \text{ permutations}\right),$$

- **The model naturally contains Dirac gaugino mass terms**

# of Weyl spinors		$(mass)^2$	Fermionic components
$81(=27\times 3)$		0	$L^{(1,2,3)}\,,Q_L^{(1,2,3)}\,,Q_R^{(1,2,3)}$
1		$\frac{1}{6}\left(v_F^2\lambda_{\mathbf{1}}^2-2\sqrt{6}v_F\lambda_{\mathbf{1}}\mu_{\mathbf{1}}+6\mu_{\mathbf{1}}^2\right)$	Δ_F^8
3		$\frac{1}{6}\left(v_F^2\lambda_{\mathbf{1}}^2+2\sqrt{6}v_F\lambda_{\mathbf{1}}\mu_{\mathbf{1}}+6\mu_{\mathbf{1}}^2\right)$	$\Delta_F^{1,2,3}$
4		$\frac{1}{24}\left(v_F^2\lambda_{\mathbf{1}}^2-4\sqrt{6}v_F\lambda_{\mathbf{1}}\mu_{\mathbf{1}}+24\mu_{\mathbf{1}}^2\right)$	$\Delta_F^{4,5,6,7}$
8	**gluinos**?	$\frac{1}{2}\left(X_C^{\mathbf{8}}-\sqrt{Y_C^{\mathbf{8}}+Z_C^{\mathbf{8}}}\right)$	$\left(\tilde\lambda_C^a\,,\Delta_C^a\right)\equiv\tilde g^a$
8		$\frac{1}{2}\left(X_C^{\mathbf{8}}+\sqrt{Y_C^{\mathbf{8}}+Z_C^{\mathbf{8}}}\right)$	$\left(\tilde\lambda_C^a\,,\Delta_C^a\right)\equiv\tilde g_\perp^a$
2		$\frac{1}{24}\left(X_{L,R}^{\mathbf{1}}-\sqrt{Y_{L,R}^{\mathbf{1}}+Z_{L,R}^{\mathbf{1}}}\right)$	$\left(\tilde\lambda_{L,R}^8\,,\Delta_{L,R}^8\right)\equiv S_{L,R}$
2		$\frac{1}{24}\left(X_{L,R}^{\mathbf{1}}+\sqrt{Y_{L,R}^{\mathbf{1}}+Z_{L,R}^{\mathbf{1}}}\right)$	$\left(\tilde\lambda_{L,R}^8\,,\Delta_{L,R}^8\right)\equiv S_{L,R}^\perp$
6		$\frac{1}{24}\left(X_{L,R}^{\mathbf{3}}-\sqrt{Y_{L,R}^{\mathbf{3}}+Z_{L,R}^{\mathbf{3}}}\right)$	$\left(\tilde\lambda_{L,R}^{1,2,3}\,,\Delta_{L,R}^{1,2,3}\right)\equiv T_{L,R}$
6		$\frac{1}{24}\left(X_{L,R}^{\mathbf{3}}+\sqrt{Y_{L,R}^{\mathbf{3}}+Z_{L,R}^{\mathbf{3}}}\right)$	$\left(\tilde\lambda_{L,R}^{1,2,3}\,,\Delta_{L,R}^{1,2,3}\right)\equiv T_{L,R}^\perp$
8		$\left(X_{L,R}^{\mathbf{2}}-\sqrt{Y_{L,R}^{\mathbf{2}}+Z_{L,R}^{\mathbf{2}}}\right)$	$\left(\Delta_{L,R}^{4,6}\,,\Delta_{L,R}^{5,7}\,,\tilde\lambda_{L,R}^{4,6}\,,\tilde\lambda_{L,R}^{5,7}\right)\equiv\tilde{\mathcal{H}}_{L,R}$
8		$\left(X_{L,R}^{\mathbf{2}}+\sqrt{Y_{L,R}^{\mathbf{2}}+Z_{L,R}^{\mathbf{2}}}\right)$	$\left(\Delta_{L,R}^{4,6}\,,\Delta_{L,R}^{5,7}\,,\tilde\lambda_{L,R}^{4,6}\,,\tilde\lambda_{L,R}^{5,7}\right)\equiv\tilde{\mathcal{H}}_{L,R}^\perp$

- X, Y and Z are functions of the theory parameters
- **Massless SM fermions→ due to $\mathbf{SU(3)_F}$**



Parameter	range		
v	10^5	–	10^{17}
M_0'	0	–	10^{17}
M_0	0	–	10^{17}
μ_{78}	0	–	10^{17}
g_U	0	–	1.2
λ_{78}	0	–	6

- v , M_0' , M_0 , μ_{78} in GeV
- $\chi \in \left\{ \tilde{g}^a, T, \tilde{\mathcal{H}}, S \right\}$

- **For each parameter point there is at least one stable vacuum solution**
- **Natural fermion mass hierarchy up to 16 orders of magnitude**

Outline

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  - > Solved the multi-TeV lepton mass problem
  - > Elegantly unified Higgs with leptons, gauge and Yukawa couplings
  - > Minimal (well motivated) SUSY version with a stable vacuum
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  - > **A vast landscape for a rich phenomenology**

Higgs-slepton and squark masses

# of real d.o.f.'s	$(mass)^2$	Scalar components
8	$m_{\mathbf{27}}^2 - \frac{1}{\sqrt{6}}\left(A_G v + 2A_F v_F\right)$	$\tilde{\nu}_R^{(3)}, \tilde{e}_R^{(3)}, \tilde{\nu}_L^{(3)}, \tilde{e}_L^{(3)}$
2	$m_{\mathbf{27}}^2 - \frac{1}{\sqrt{6}}\left(4A_G v + 2A_F v_F\right)$	$\tilde{\phi}^{(3)}$
8	$m_{\mathbf{27}}^2 + \frac{1}{\sqrt{6}}\left(2A_G v - 2A_F v_F\right)$	$H_{11}^{(3)}, H_{21}^{(3)}, H_{12}^{(3)}, H_{22}^{(3)}$
4	$m_{\mathbf{27}}^2 - \frac{1}{\sqrt{6}}\left(4A_G v - A_F v_F\right)$	$\tilde{\phi}^{(1,2)}$
16	$m_{\mathbf{27}}^2 - \frac{1}{\sqrt{6}}\left(A_G v - A_F v_F\right)$	$\tilde{\nu}_R^{(1,2)}, \tilde{e}_R^{(1,2)}, \tilde{\nu}_L^{(1,2)}, \tilde{e}_L^{(1,2)}$
16	$m_{\mathbf{27}}^2 + \frac{1}{\sqrt{6}}\left(2A_G v + A_F v_F\right)$	$H_{11}^{(1,2)}, H_{21}^{(1,2)}, H_{12}^{(1,2)}, H_{22}^{(1,2)}$
24	$m_{\mathbf{27}}^2 + \frac{1}{\sqrt{6}}\left(A_G v - 2A_F v_F\right)$	$\tilde{u}_L^{(3)}, \tilde{d}_L^{(3)}, \tilde{u}_R^{(3)}, \tilde{d}_R^{(3)}$
12	$m_{\mathbf{27}}^2 - \frac{1}{\sqrt{6}}\left(2A_G v + 2A_F v_F\right)$	$\tilde{D}_L^{(3)}, \tilde{D}_R^{(3)}$
48	$m_{\mathbf{27}}^2 + \frac{1}{\sqrt{6}}\left(A_G v + A_F v_F\right)$	$\tilde{u}_L^{(1,2)}, \tilde{d}_L^{(1,2)}, \tilde{u}_R^{(1,2)}, \tilde{d}_R^{(1,2)}$
24	$m_{\mathbf{27}}^2 - \frac{1}{\sqrt{6}}\left(2A_G v + A_F v_F\right)$	$\tilde{D}_L^{(1,2)}, \tilde{D}_R^{(1,2)}$

$$\left(\begin{array}{cc|c} H_{11} & H_{12} & \nu_L \\ H_{21} & H_{22} & e_L \\ \hline \nu_R & e_R & \phi \end{array}\right)^{(1,2|3)}, \quad \left(\begin{array}{ccc} u_R^{\bar{1}} & u_R^{\bar{2}} & u_R^{\bar{3}} \\ d_R^{\bar{1}} & d_R^{\bar{2}} & d_R^{\bar{3}} \\ \hline D_R^{\bar{1}} & D_R^{\bar{2}} & D_R^{\bar{3}} \end{array}\right)^{(1,2|3)}, \quad \left(\begin{array}{cc|c} u_L^1 & d_L^1 & D_L^1 \\ u_L^2 & d_L^2 & D_L^2 \\ u_L^3 & d_L^3 & D_L^3 \end{array}\right)^{(1,2|3)},$$

Adjoint scalar masses

$$\mathbf{8} \to \mathbf{3}_0 \oplus \mathbf{2}_1 \oplus \mathbf{2}_{-1} \oplus \mathbf{1}_0 \,,$$

# of real d.o.f.'s	$(mass)^2$	Scalar components
12	0	$Re[\tilde{\Delta}^{4,5,6,7}_{L,R,F}]$
3	$\sqrt{\frac{2}{3}}\frac{v_F}{2}\,(3\lambda_{\mathbf{1}}\mu_{\mathbf{1}}+A_{\mathbf{1}})$	$Re[\tilde{\Delta}^{1,2,3}_{F}]$
1	$\frac{v_F}{12}\left(2v_F\lambda_{\mathbf{1}}^2-3\sqrt{6}\lambda_{\mathbf{1}}\mu_{\mathbf{1}}-\sqrt{6}A_{\mathbf{1}}\right)$	$Re[\tilde{\Delta}^{8}_{F}]$
1	$-2b_{\mathbf{1}}+\frac{v_F}{12}\left(\sqrt{6}\lambda_{\mathbf{1}}\mu_{\mathbf{1}}+3\sqrt{6}A_{\mathbf{1}}\right)$	$Im[\tilde{\Delta}^{8}_{F}]$
4	$-2b_{\mathbf{1}}+\frac{v_F}{12}\left(2\sqrt{6}\lambda_{\mathbf{1}}\mu_{\mathbf{1}}-v_F\lambda_{\mathbf{1}}^2+2\sqrt{6}A_{\mathbf{1}}\right)$	$Im[\tilde{\Delta}^{4,5,6,7}_{F}]$
3	$-2b_{\mathbf{1}}+\frac{v_F}{12}\left(5\sqrt{6}\lambda_{\mathbf{1}}\mu_{\mathbf{1}}+2v_F\lambda_{\mathbf{1}}^2-\sqrt{6}A_{\mathbf{1}}\right)$	$Im[\tilde{\Delta}^{1,2,3}_{F}]$
6	$\sqrt{\frac{2}{3}}\frac{v}{2}\,(3\lambda_{\mathbf{78}}\mu_{\mathbf{78}}+A_{\mathbf{78}}+3C_{\mathbf{78}})$	$Re[\tilde{\Delta}^{1,2,3}_{L,R}]$
8	$\frac{v}{12}\left(-v\lambda_{\mathbf{78}}^2+3\sqrt{6}\lambda_{\mathbf{78}}\mu_{\mathbf{78}}+\sqrt{6}A_{\mathbf{78}}+3\sqrt{6}C_{\mathbf{78}}\right)$	$Re[\tilde{\Delta}^{1,\cdots,8}_{C}]$
2	$\frac{v}{12}\left(2v\lambda_{\mathbf{78}}^2-3\sqrt{6}\lambda_{\mathbf{78}}\mu_{\mathbf{78}}-\sqrt{6}A_{\mathbf{78}}-3\sqrt{6}C_{\mathbf{78}}\right)$	$Re[\tilde{\Delta}^{8}_{L,R}]$
2	$-2b_{\mathbf{78}}+\frac{\sqrt{6}}{12}v\,(\lambda_{\mathbf{78}}\mu_{\mathbf{78}}+3A_{\mathbf{78}}+C_{\mathbf{78}})$	$Im[\tilde{\Delta}^{8}_{L,R}]$
8 **L-R Higgs?**	$-2b_{\mathbf{78}}+\frac{3}{4}g_U^2v^2+\frac{v^2}{12}\lambda_{\mathbf{78}}^2+\frac{\sqrt{6}}{6}v\,(\lambda_{\mathbf{78}}\mu_{\mathbf{78}}+A_{\mathbf{78}}+C_{\mathbf{78}})$	$Im[\tilde{\Delta}^{4,5,6,7}_{L,R}]$
8	$-2b_{\mathbf{78}}+\frac{v^2}{12}\lambda_{\mathbf{78}}^2+\frac{\sqrt{6}}{12}v\,(3\lambda_{\mathbf{78}}\mu_{\mathbf{78}}+A_{\mathbf{78}}+3C_{\mathbf{78}})$	$Im[\tilde{\Delta}^{1,\cdots,8}_{C}]$
6	$-2b_{\mathbf{78}}+\frac{v^2}{6}\lambda_{\mathbf{78}}^2+\frac{\sqrt{6}}{12}v\,(5\lambda_{\mathbf{78}}\mu_{\mathbf{78}}-A_{\mathbf{78}}+5C_{\mathbf{78}})$	$Im[\tilde{\Delta}^{1,2,3}_{L,R}]$