From $E_8$-inspired SUSY trinification to a L-R symmetric theory

A. P. Morais$^{1,2}$
J. E. Camargo-Molina$^2$ A. Ordell$^2$ R. Pasechnik$^2$ M. O. P. Sampaio$^1$ J. Wessen$^2$

$^1$Center for Research and Development in Mathematics and Applications (CIDMA)
Aveiro University, Aveiro, Portugal

$^2$Theoretical High Energy Physics (THEP)
Lund University, Lund, Sweden

July 4, 2016

SUSY 2016 – The University of Melbourne
Outline

1. Motivations and issues

2. The Model

3. Symmetry breaking

4. Conclusions and outlook
Outline

1. Motivations and issues
2. The Model
3. Symmetry breaking
4. Conclusions and outlook
Main features (Glashow, Georgi and De Rujula 1984)

- LR gauge interactions and well motivated by $E_6$
- $SU(3)_L \times SU(3)_R \times SU(3)_C$ with $\mathbb{Z}_3 \rightarrow$ gauge unification
Main features (Glashow, Georgi and De Rujula 1984)

- LR gauge interactions and well motivated by $E_6$

- $SU(3)_L \times SU(3)_R \times SU(3)_C$ with $\mathbb{Z}_3 \rightarrow$ gauge unification

- All matter can be elegantly arranged in bi-fundamental representations for each generation

\[ 27^i = (3, \bar{3}, 1)^i \otimes (1, 3, \bar{3})^i \otimes (\bar{3}, 1, 3)^i \equiv L \otimes Q_R \otimes Q_L \]

- The model can accommodate any quark and lepton masses and mixing angles (Sayre et al. 2006)

- Naturally light neutrinos via radiative seesaw with split-SUSY (Cauet et al. 2011)
Main features (Glashow, Georgi and De Rujula 1984)

- LR gauge interactions and well motivated by $E_6$

- $SU(3)_L \times SU(3)_R \times SU(3)_C$ with $\mathbb{Z}_3 \rightarrow$ gauge unification

- All matter can be elegantly arranged in bi-fundamental representations for each generation

$$27^i = (3, \bar{3}, 1)^i \otimes (1, 3, \bar{3})^i \otimes (\bar{3}, 1, 3)^i \equiv L \otimes Q_R \otimes Q_L$$

- The model can accommodate any quark and lepton masses and mixing angles (Sayre et al. 2006)

- Naturally light neutrinos via radiative seesaw with split-SUSY (Cauet et al. 2011)

- Gauge symmetry preserves baryon number $\rightarrow$ Stable proton. (Achiman and Stech, 1978) (Glashow and Kang 1984)

- Well motivated as low energy versions of $E_8 \times E_8$ heterotic string theory (Gross et al. 1985), $E_6$ orbifold (Braam et al. 2010) or $N = 8$ supergravity (Cremmer et al. 1979).
Motivations and issues

Issues of standard trinification

- GUT scale lepton masses through the $L \cdot L \cdot L$ operator
  - Higher dimensional operators or large Higgs representations (Cauet et al. 2011)
  - Unnatural tuning among theory parameters
Issues of standard trinification

- GUT scale lepton masses through the $L \cdot L \cdot L$ operator
  - Higher dimensional operators or large Higgs representations (Cauet et al. 2011)
  - Unnatural tuning among theory parameters
- Considerable amount of particles and many couplings involved
  - Realistic calculations cumbersome
- Unmotivated Hierarchy between the trinification and the EW breaking scales (common to most GUTs)
Issues of standard trinification

- GUT scale lepton masses through the $L \cdot L \cdot L$ operator
  - Higher dimensional operators or large Higgs representations (Cauet et al. 2011)
  - Unnatural tuning among theory parameters
- Considerable amount of particles and many couplings involved
  - Realistic calculations cumbersome
- Unmotivated Hierarchy between the trinification and the EW breaking scales (common to most GUTs)

Trinification-based models were left as the least developed GUT scenarios
Our proposal

Novel solution including

1. A new SU(3)$_F$ flavour global symmetry inspired by $E_8$

2. Unification of the Higgs and lepton sectors via a common chiral supermultiplet

Our aim: Fix the lepton masses problem and show that the vacuum is stable
1 Motivations and issues

2 The Model

3 Symmetry breaking

4 Conclusions and outlook
$\textbf{The Model}$

$\textbf{SU}(3)_F \times E_6$ is a maximal subgroup of $E_8$

- Branching rules for the fundamental representation of $E_8$ (Slansky)
  \[
  248 = (8, 1) \oplus (1, 78) \oplus (3, 27) \oplus (\overline{3}, \overline{27})
  \]

- Branching rules for the adjoint representation of $E_6$ down to trinification
  \[
  78 = (8, 1, 1) \oplus (1, 8, 1) \oplus (1, 1, 8) \oplus (3, 3, \overline{3}) \oplus (\overline{3}, \overline{3}, 3)
  \]
  (i) Tri-triplets do not couple to standard matter at tree-level
  (ii) For proof of concept we consider $(\overline{3}, 27)$-plet heavy (for now)
  (iii) For simplicity $SU(3)_F$ is global in our study (for now!)
**SU(3)_F × E_6 is a maximal subgroup of E_8**

- Branching rules for the fundamental representation of E_8 (Slansky)
  \[ 248 = (8, 1) \oplus (1, 78) \oplus (3, 27) \oplus (\bar{3}, \bar{27}) \]

- Branching rules for the adjoint representation of E_6 down to trinification
  \[ 78 = (8, 1, 1) \oplus (1, 8, 1) \oplus (1, 1, 8) \oplus (3, 3, \bar{3}) \oplus (\bar{3}, \bar{3}, 3) \]

(i) Tri-triplets do not couple to standard matter at tree-level
(ii) For proof of concept we consider (\bar{3}, \bar{27})-plet heavy (for now!)
(iii) For simplicity SU(3)_F is global in our study (for now!)
\[ \left[ \text{SU}(3)_L \times \text{SU}(3)_R \times \text{SU}(3)_C \right] \times \mathbb{Z}_3 \times \text{SU}(3)_F, \]

- \( \mathbb{Z}_3 \) is a permutation symmetry that guarantees gauge coupling unification

### Chiral Supermultiplet Fields

<table>
<thead>
<tr>
<th>Superfield</th>
<th>( \text{SU}(3)_C )</th>
<th>( \text{SU}(3)_L )</th>
<th>( \text{SU}(3)_R )</th>
<th>( \text{SU}(3)_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>( (L^i)_r )</td>
<td>( 1 )</td>
<td>( 3^i )</td>
<td>( \bar{3}_r )</td>
</tr>
<tr>
<td>Right-Quark</td>
<td>( (Q^i)_r )</td>
<td>( \bar{3}_x )</td>
<td>( 1 )</td>
<td>( 3^r )</td>
</tr>
<tr>
<td>Left-Quark</td>
<td>( (Q^i)_l )</td>
<td>( 3^x )</td>
<td>( \bar{3}_l )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Colour-adjoint</td>
<td>( \Delta^a_C )</td>
<td>( 8^a )</td>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Left-adjoint</td>
<td>( \Delta^a_L )</td>
<td>( 1 )</td>
<td>( 8^a )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Right-adjoint</td>
<td>( \Delta^a_R )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 8^a )</td>
</tr>
<tr>
<td>Flavour-adjoint</td>
<td>( \Delta^a_F )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
</tbody>
</table>

### Gauge Supermultiplet Fields

<table>
<thead>
<tr>
<th>Superfield</th>
<th>( \text{SU}(3)_C )</th>
<th>( \text{SU}(3)_L )</th>
<th>( \text{SU}(3)_R )</th>
<th>( \text{SU}(3)_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gluon</td>
<td>( G_C^{ua}, \lambda_C^a )</td>
<td>( 8^a )</td>
<td>( 1 )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Left-Gluon</td>
<td>( G_L^{ua}, \lambda_L^a )</td>
<td>( 1 )</td>
<td>( 8^a )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Right-Gluon</td>
<td>( G_R^{ua}, \lambda_R^a )</td>
<td>( 1 )</td>
<td>( 1 )</td>
<td>( 8^a )</td>
</tr>
</tbody>
</table>
Fundamental tri-triplets:

\[(L^i)^l_r = \begin{pmatrix} H_{11} & H_{12} & \nu_L \\ H_{21} & H_{22} & e_L \\ \nu_R & e_R & \phi \end{pmatrix}^i, (Q_R^i)^r_x = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ D_R^1 & D_R^2 & D_R^3 \end{pmatrix}^i, (Q_L^i)^x_l = \begin{pmatrix} u_L^1 & d_L^1 & D_L^1 \\ u_L^2 & d_L^2 & D_L^2 \\ u_L^3 & d_L^3 & D_L^3 \end{pmatrix}^i\]

\(\mathbb{Z}_3\) cyclic permutations:

\[L \xrightarrow{\mathbb{Z}_3} Q_L,\]

\[Q_L \xrightarrow{\mathbb{Z}_3} Q_R,\]

\[Q_R \xrightarrow{\mathbb{Z}_3} L.\]
Superpotential

\[ W = \sum_{A=L,R,C} \left( \lambda_{78} d_{abc} \Delta^a_A \Delta^b_A \Delta^c_A + \mu_{78} \delta_{ab} \Delta^a_A \Delta^b_A \right) + \left( \lambda_1 d_{abc} \Delta^a_F \Delta^b_F \Delta^c_F + \mu_1 \delta_{ab} \Delta^a_F \Delta^b_F \right) \]

\[ + \lambda_{27} \varepsilon_{ijk} (Q^i_L)^x (Q^j_R)^l (L^K)_r, \text{ with } d_{abc} = 2 \text{Tr} \left[ [T_a, T_b] T_c \right] \]

- \( i, j \) and \( k \) \( \rightarrow \) flavour indices
- \( x, l \) and \( r \) \( \rightarrow \) colour, left-chirality and right-chirality respectively
- \( a, b \) and \( c \) \( \rightarrow \) adjoint indices.
- **One single Yukawa coupling for standard matter,** \( \lambda_{27} \)**
The Model

Superpotential

\[ W = \sum_{A=L,R,C} (\lambda_{78} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b) + (\lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b) \]
\[ + \lambda_{27} \varepsilon_{ijk} (Q_L^i)^x (Q_R^j)^r (L^k)^l , \text{ with } d_{abc} = 2\text{Tr} \left[ [T_a, T_b] T_c \right] \]

- \( i, j \) and \( k \) → flavour indices
- \( x, l \) and \( r \) → colour, left-chirality and right-chirality respectively
- \( a, b \) and \( c \) → adjoint indices.

- **One single Yukawa coupling for standard matter**, \( \lambda_{27} \)

- In a minimal \( E_6 \)-inspired model with flavour \( SU(3)_F \) the superpotential would be just the last term
The Model

Superpotential

\[ W = \sum_{A=L,R,C} \left( \lambda_{78} d_{abc} \Delta_A^a \Delta_B^b \Delta_C^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b \right) + \left( \lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b \right) + \lambda_{27} \epsilon_{ijk} \left( Q_i^L \right)^x_l \left( Q_j^R \right)^r_x \left( L^k \right)^l_r, \quad \text{with} \quad d_{abc} = 2 \text{Tr} \left[ \left[ T_a, T_b \right] T_c \right] \]

- \( i, j \) and \( k \) → flavour indices
- \( x, l \) and \( r \) → colour, left-chirality and right-chirality respectively
- \( a, b \) and \( c \) → adjoint indices.
- **One single Yukawa coupling for standard matter, \( \lambda_{27} \)**
- In a minimal \( E_6 \)-inspired model with flavour \( SU(3)_F \) the superpotential would be just the last term

- **Why then \( E_8 \)?**
- Minimal SUSY trinification with \( SU(3)_F \) and Higgs-lepton unification does not have a stable vacuum → \( SU(3)_C \) and \( SU(2)_L \) fully broken at GUT scale
The Model

Superpotential

\[ W = \sum_{A=L,R,C} \left( \lambda_{78} d_{abc} \Delta_A^a \Delta_A^b \Delta_A^c + \mu_{78} \delta_{ab} \Delta_A^a \Delta_A^b \right) + \left( \lambda_1 d_{abc} \Delta_F^a \Delta_F^b \Delta_F^c + \mu_1 \delta_{ab} \Delta_F^a \Delta_F^b \right) \]

\[ + \lambda_{27} \varepsilon_{ijk} (Q_L^i)^x (Q_R^j)^r (L^k)^l, \text{ with } d_{abc} = 2 \text{Tr} \left[ [T_a, T_b] T_c \right] \]

- \(i, j, k \rightarrow\) flavour indices
- \(x, l, r \rightarrow\) colour, left-chirality and right-chirality respectively
- \(a, b, c \rightarrow\) adjoint indices.

\textbf{One single Yukawa coupling for standard matter, \(\lambda_{27}\)}

- In a minimal \(E_6\)-inspired model with flavour \(SU(3)_F\) the superpotential would be just the last term

\textbf{Why then \(E_8\)?}

- Minimal SUSY trinification with \(SU(3)_F\) and Higgs-lepton unification does not have a stable vacuum \(\rightarrow SU(3)_C\) and \(SU(2)_L\) fully broken at GUT scale

\textbf{\(E_8\)-inspired trinification is the minimal SUSY working model!}
Outline

1. Motivations and issues
2. The Model
3. Symmetry breaking
4. Conclusions and outlook
Symmetry breaking

Scalar potential

\[ V = V_F + V_D + V_{\text{soft}} \quad (F - \text{terms from the superpotential}) \]
Symmetry breaking

Scalar potential

\[ V = V_T + V_D + V_{\text{soft}} \quad (F - \text{terms from the superpotential}) \]

(1) D-terms

\[
V_D = -\frac{1}{2} g_U^2 \left\{ \sum_c \left( \tilde{\Delta}_L^{a*} f_{abc} \tilde{\Delta}_L^b \right) \left( \tilde{\Delta}_L^{d*} f_{dec} \tilde{\Delta}_L^e \right) \\
- \frac{1}{2} g_U^2 \left[ T^a \right]_{l_1}^{l_2} \left[ T_a \right]_{l_3}^{l_4} \left[ (\tilde{L}_*)^{r_1} l_1 (T^c)^{l_1} l_2 (\tilde{L}_i)^{l_2} r_1 - (\tilde{Q}_L)^{x_1} l_3 (T^c)^{l_3} l_2 (\tilde{Q}_L^*)^{l_2} x_1 \right] \right\}
\]

- D-term interactions between adjoint and fundamental scalars

- Symmetry breaking

\[ g_L = g_R = g_C \equiv g_U \]
(2) Soft SUSY-breaking terms

\[ V_{\text{gauge soft}} = m_{27}^2 \left[ (\tilde{L}_i)^l_r (\tilde{L}_i^*)^r_l \right] + \delta_{ab} \left[ b_{78} \tilde{\Delta}_L^a \tilde{\Delta}_L^b + m_{78}^2 \tilde{\Delta}_L^*a \tilde{\Delta}_L^b + c.c. \right] \\
+ d_{abc} \left[ A_{78} \tilde{\Delta}_L^a \tilde{\Delta}_L^b \tilde{\Delta}_L^c + C_{78} \tilde{\Delta}_L^*a \tilde{\Delta}_L^b \tilde{\Delta}_L^c + c.c. \right] \\
+ A_G \left[ \tilde{\Delta}_L^a \left ( T_a \right )_{l_2}^{l_1} (\tilde{L}_i^*)^r_l (\tilde{L}_i^l)^l \right] + c.c. \\
+ A_{27} \left[ \varepsilon_{ijk} (\tilde{Q}_L^i)^x_l (\tilde{Q}_R^j)^x_r (\tilde{L}_k^l)^r + c.c. \right] + Z_3 \text{ permutations} \]

\[ V_{\text{global soft}} = \delta_{ab} \left[ b_1^2 \tilde{\Delta}_F^a \tilde{\Delta}_F^b + m_1^2 \tilde{\Delta}_F^*a \tilde{\Delta}_F^b + c.c. \right] + A_1 d_{abc} \left[ \tilde{\Delta}_F^a \tilde{\Delta}_F^b \tilde{\Delta}_F^c + c.c. \right] \\
+ A_F \left[ \tilde{\Delta}_F^a \left ( T_a \right )^i_j (\tilde{L}_i^*)^r_l (\tilde{L}_i^l)^l \right] + c.c. \right] + Z_3 \text{ permutations} \]
(2) Soft SUSY-breaking terms

\[ V_{\text{gauge soft}}^{soft} = m_{27}^2 \left[ \left( \tilde{L}_i \right)_r^l \left( \tilde{L}_i^* \right)_l^r \right] + \delta_{ab} \left[ b_{78} \tilde{\Delta}_L^a \tilde{\Delta}_L^b + m_{78}^2 \tilde{\Delta}_L^* \tilde{\Delta}_L^b + c.c. \right] \\
+ d_{abc} \left[ A_{78} \tilde{\Delta}_L^a \tilde{\Delta}_L^b \tilde{\Delta}_L^c + C_{78} \tilde{\Delta}_L^* \tilde{\Delta}_L^b \tilde{\Delta}_L^c + c.c. \right] \\
+ A_F \left[ \tilde{\Delta}_L^a \left( T_a \right)_{l_1}^{l_2} \left( \tilde{L}_i^* \right)_l^r \left( \tilde{L}_i \right)_l^r + c.c. \right] \\
+ A_{27} \left[ \varepsilon_{ijk} \left( \tilde{Q}_L^i \right)_x^l \left( \tilde{Q}_R^j \right)_r^x \left( \tilde{L}_k \right)_l^r + c.c. \right] + \mathbb{Z}_3 \text{ permutations} \]

\[ V_{\text{global soft}}^{soft} = \delta_{ab} \left[ b_{1}^2 \tilde{\Delta}_F^a \tilde{\Delta}_F^b + m_{1}^2 \tilde{\Delta}_F^* \tilde{\Delta}_F^b + c.c. \right] + A_1 d_{abc} \left[ \tilde{\Delta}_F^a \tilde{\Delta}_F^b \tilde{\Delta}_F^c + c.c. \right] \\
+ A_F \left[ \tilde{\Delta}_F^a \left( T_a \right)_{j}^{i} \left( \tilde{L}_i^* \right)_l^r \left( \tilde{L}_i \right)_l^r + c.c. \right] + \mathbb{Z}_3 \text{ permutations} \]

- Only fundamental VEVs \( \implies \) **Unstable vacuum**
- Need adjoint VEVs to have a stable minimum
Symmetry breaking

Vacuum choice:

- Assign vevs to $\tilde{\Delta}_L^a$, $\tilde{\Delta}_R^a$ (gauge breaking) and $\tilde{\Delta}_F^a$ (flavour breaking):
Vacuum choice:

- Assign vevs to $\tilde{\Delta}_L^a$, $\tilde{\Delta}_R^a$ (gauge breaking) and $\tilde{\Delta}_F^a$ (flavour breaking):
- Eight component of $SU(3)$ generators $T^8$

$$T^8_{L,R,F} = \frac{1}{2\sqrt{2}}\begin{pmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2\end{pmatrix},$$

- Choosing $\langle \tilde{\Delta}_L^8 \rangle = \nu$ and $\langle \tilde{\Delta}_F^8 \rangle = \nu_F$ we break trinification to a rank-6 LR effective model:

$$[SU(3)_C \times SU(3)_L \times SU(3)_R] \times \mathbb{Z}_3 \times SU(3)_F \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \times SU(2)_F \times U(1)_F$$
Symmetry breaking

Vacuum choice:

- Assign vevs to $\tilde{\Delta}_L^a$, $\tilde{\Delta}_R^a$ (gauge breaking) and $\tilde{\Delta}_F^a$ (flavour breaking):

- Eight component of SU(3) generators $T^8$

$$T_{L,R,F}^8 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

- Choosing $\langle \tilde{\Delta}_L^8 \rangle = v$ and $\langle \tilde{\Delta}_F^8 \rangle = v_F$ we break trinification to a rank-6 LR effective model:

$$[SU(3)_C \times SU(3)_L \times SU(3)_R] \times \mathbb{Z}_3 \times SU(3)_F \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R \times SU(2)_F \times U(1)_F$$

Minimization:

1. Positive mass spectrum for the full scalar sector $\rightarrow$ STABLE MINIMUM

2. 8 gauge goldstones in the adjoint sector

$$\left| D^\mu \langle \tilde{\Delta}_L^b \rangle \right|^2 = \frac{3}{4} g_v^2 v^2 \sum_{a=4} \eta_{\mu\nu} G_{L,R}^{\mu a} G_{L,R}^{\nu a},$$

3. 4 flavour goldstones (absorbed by flavour gauge bosons or decouple according to Burgess [hep-ph/9812468])
Fermion masses

Scalar-fermion terms

\[ \mathcal{L}_{\text{fermion}} = \mathcal{L}_{\mathcal{F}}^{\text{fermion}} + \mathcal{L}_{\mathcal{D}}^{\text{fermion}} + \mathcal{L}_{\text{soft}}^{\text{fermion}} \quad (F \text{ terms from the superpotential}) \]

- No F-term interactions mixing adjoint and fundamental sectors
Symmetry breaking

Fermion masses

Scalar-fermion terms

\[ \mathcal{L}^{\text{fermion}} = \mathcal{L}^{\text{fermion}}_F + \mathcal{L}^{\text{fermion}}_D + \mathcal{L}^{\text{fermion}}_{\text{soft}} \quad (F - \text{terms from the superpotential}) \]

- No F-term interactions mixing adjoint and fundamental sectors

D-terms

\[ \mathcal{L}^{\text{fermion}}_D = - \sqrt{2} g_U \left[ (\tilde{L}_i^*)^r_{l_1} (T^a)^{l_1}_{l_2} (L^i)^{l_2}_{r_1} \tilde{\lambda}_L^a + (\tilde{L}_i^*)^r_{l_1} (T^a)^{r_1}_{r_2} (L^i)^{l}_{r_2} \tilde{\lambda}_R^a \right. \\
\left. - i f_{abc} \tilde{\Delta}_L^a \Delta_L^c \tilde{\lambda}_L^a \right] + (Z_3 \text{ permutations}) . \]

Soft SUSY-breaking terms

\[ \mathcal{L}^{\text{fermion}}_{\text{soft}} = - \frac{1}{2} M_0 \delta_{ab} \tilde{\lambda}_L^a \tilde{\lambda}_L^b - M_0' \delta_{ab} \tilde{\lambda}_L^a \Delta_L^b + h.c. + (Z_3 \text{ permutations}) , \]

- The model naturally contains Dirac gaugino mass terms
### # of Weyl spinors \((mass)^2\)

<table>
<thead>
<tr>
<th># of Weyl spinors</th>
<th>((mass)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(81 (= 27 \times 3))</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>(\frac{1}{6} \left( v_F^2 \lambda_1^2 - 2\sqrt{6} v_F \lambda_1 \mu_1 + 6\mu_1^2 \right))</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{1}{6} \left( v_F^2 \lambda_1^2 + 2\sqrt{6} v_F \lambda_1 \mu_1 + 6\mu_1^2 \right))</td>
</tr>
<tr>
<td>4</td>
<td>(\frac{1}{24} \left( v_F^2 \lambda_1^2 - 4\sqrt{6} v_F \lambda_1 \mu_1 + 24\mu_1^2 \right))</td>
</tr>
<tr>
<td>8</td>
<td>(\frac{1}{2} \left( X_C^8 - \sqrt{Y_C^8 + Z_C^8} \right))</td>
</tr>
<tr>
<td>8</td>
<td>(\frac{1}{2} \left( X_C^8 + \sqrt{Y_C^8 + Z_C^8} \right))</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{24} \left( X_{L,R}^1 - \sqrt{Y_{L,R}^1 + Z_{L,R}^1} \right))</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{1}{24} \left( X_{L,R}^1 + \sqrt{Y_{L,R}^1 + Z_{L,R}^1} \right))</td>
</tr>
<tr>
<td>6</td>
<td>(\frac{1}{24} \left( X_{L,R}^3 - \sqrt{Y_{L,R}^3 + Z_{L,R}^3} \right))</td>
</tr>
<tr>
<td>6</td>
<td>(\frac{1}{24} \left( X_{L,R}^3 + \sqrt{Y_{L,R}^3 + Z_{L,R}^3} \right))</td>
</tr>
<tr>
<td>8</td>
<td>(\frac{1}{24} \left( X^2_{L,R} - \sqrt{Y^2_{L,R} + Z^2_{L,R}} \right))</td>
</tr>
<tr>
<td>8</td>
<td>(\frac{1}{24} \left( X^2_{L,R} + \sqrt{Y^2_{L,R} + Z^2_{L,R}} \right))</td>
</tr>
</tbody>
</table>

### Massless SM fermions → due to \(SU(3)_F\)

- **X, Y and Z** are functions of the theory parameters.

- **Massless SM fermions** → due to \(SU(3)_F\)
For each parameter point there is at least one stable vacuum solution

Natural fermion mass hierarchy up to 16 orders of magnitude
Outline

1. Motivations and issues
2. The Model
3. Symmetry breaking
4. Conclusions and outlook
Develop a $E_8$-inspired GUT trinification model containing a flavour $SU(3)_F$

- Solved the multi-TeV lepton mass problem
- Elegantly unified Higgs with leptons, gauge and Yukawa couplings
- Minimal (well motivated) SUSY version with a stable vacuum
  - Unless Non-SUSY version that works with just fundamental 27-plets!
    - see Jonas talk Thursday 15h00
● Develop a $E_8$-inspired GUT trinification model containing a flavour $SU(3)_F$

> Solved the multi-TeV lepton mass problem

> Elegantly unified Higgs with leptons, gauge and Yukawa couplings

> Minimal (well motivated) SUSY version with a stable vacuum

  ● Unless Non-SUSY version that works with just fundamental 27-plets!

see Jonas talk Thursday 15h00

● Diverse scenarios for effective models can be studied (future work)

> Integrate out heavy DOF

> Fundamental fermions massless and adjoint ones allow large hierarchies

> Certain scalars need to be lighter than the matching scale in order to consistently break the remaining symmetry
Develop a $E_8$-inspired GUT trinification model containing a flavour $SU(3)_F$

- Solved the multi-TeV lepton mass problem
- Elegantly unified Higgs with leptons, gauge and Yukawa couplings
- Minimal (well motivated) SUSY version with a stable vacuum
  - Unless Non-SUSY version that works with just fundamental 27-plets!
  
  *see Jonas talk Thursday 15h00*

Diverse scenarios for effective models can be studied (future work)

- Integrate out heavy DOF
- Fundamental fermions massless and adjoint ones allow large hierarchies
- Certain scalars need to be lighter than the matching scale in order to consistently break the remaining symmetry
- A vast landscape for a rich phenomenology
### Higgs-slepton and squark masses

<table>
<thead>
<tr>
<th># of real d.o.f.'s</th>
<th>$(mass)^2$</th>
<th>Scalar components</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>$m_{27}^2 - \frac{1}{\sqrt{6}} (A_G v + 2 A_F v_F)$</td>
<td>$\tilde{\nu}_R^{(3)}$, $\tilde{\nu}_L^{(3)}$, $\tilde{e}_R^{(3)}$, $\tilde{e}_L^{(3)}$</td>
</tr>
<tr>
<td>2</td>
<td>$m_{27}^2 - \frac{1}{\sqrt{6}} (4 A_G v + 2 A_F v_F)$</td>
<td>$\tilde{\phi}^{(3)}$</td>
</tr>
<tr>
<td>8</td>
<td>$m_{27}^2 + \frac{1}{\sqrt{6}} (2 A_G v - 2 A_F v_F)$</td>
<td>$H_{11}^{(3)}$, $H_{21}^{(3)}$, $H_{12}^{(3)}$, $H_{22}^{(3)}$</td>
</tr>
<tr>
<td>4</td>
<td>$m_{27}^2 - \frac{1}{\sqrt{6}} (4 A_G v - A_F v_F)$</td>
<td>$\tilde{\phi}^{(1,2)}$</td>
</tr>
<tr>
<td>16</td>
<td>$m_{27}^2 - \frac{1}{\sqrt{6}} (A_G v - A_F v_F)$</td>
<td>$\tilde{\nu}_R^{(1,2)}$, $\tilde{e}_R^{(1,2)}$, $\tilde{\nu}_L^{(1,2)}$, $\tilde{e}_L^{(1,2)}$</td>
</tr>
<tr>
<td>16</td>
<td>$m_{27}^2 + \frac{1}{\sqrt{6}} (2 A_G v + A_F v_F)$</td>
<td>$H_{11}^{(1,2)}$, $H_{21}^{(1,2)}$, $H_{12}^{(1,2)}$, $H_{22}^{(1,2)}$</td>
</tr>
<tr>
<td>24</td>
<td>$m_{27}^2 + \frac{1}{\sqrt{6}} (A_G v - 2 A_F v_F)$</td>
<td>$\tilde{u}_L^{(3)}$, $\tilde{d}_L^{(3)}$, $\tilde{u}_R^{(3)}$, $\tilde{d}_R^{(3)}$</td>
</tr>
<tr>
<td>12</td>
<td>$m_{27}^2 - \frac{1}{\sqrt{6}} (2 A_G v + 2 A_F v_F)$</td>
<td>$\tilde{\nu}_R^{(3)}$, $\tilde{\nu}_L^{(3)}$, $\tilde{e}_R^{(3)}$, $\tilde{e}_L^{(3)}$</td>
</tr>
<tr>
<td>48</td>
<td>$m_{27}^2 + \frac{1}{\sqrt{6}} (A_G v + A_F v_F)$</td>
<td>$\tilde{u}_L^{(1,2)}$, $\tilde{d}_L^{(1,2)}$, $\tilde{u}_R^{(1,2)}$, $\tilde{d}_R^{(1,2)}$</td>
</tr>
<tr>
<td>24</td>
<td>$m_{27}^2 - \frac{1}{\sqrt{6}} (2 A_G v + A_F v_F)$</td>
<td>$\tilde{D}_L^{(1,2)}$, $\tilde{D}_R^{(1,2)}$</td>
</tr>
</tbody>
</table>

\[
\begin{pmatrix}
H_{11} & H_{12} & \nu_L \\
H_{21} & H_{22} & e_L \\
\nu_R & e_R & \phi
\end{pmatrix}^{(1,2|3)},
\begin{pmatrix}
\bar{u}_R^1 & \bar{u}_R^2 & \bar{u}_R^3 \\
\bar{d}_R^1 & \bar{d}_R^2 & \bar{d}_R^3 \\
\bar{D}_R^1 & \bar{D}_R^2 & \bar{D}_R^3
\end{pmatrix}^{(1,2|3)},
\begin{pmatrix}
u_L^1 & d_L^1 \\
\nu_L^2 & d_L^2 \\
\nu_L^3 & d_L^3
\end{pmatrix}^{(1,2|3)}
Conclusions and outlook

Adjoint scalar masses

<table>
<thead>
<tr>
<th># of real d.o.f.'s</th>
<th>(mass)$^2$</th>
<th>Scalar components</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0</td>
<td>Re[$\tilde{\Delta}_{L,R,F}^4,5,6,7]$</td>
</tr>
<tr>
<td>3</td>
<td>$\sqrt{\frac{2}{3}} \frac{v_F}{2} (3\lambda_1 \mu_1 + A_1)$</td>
<td>Re[$\tilde{\Delta}_F^{1,2,3}$]</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{v_F}{12} \left(2v_F \lambda_1^2 - 3\sqrt{6} \lambda_1 \mu_1 - \sqrt{6}A_1\right)$</td>
<td>Re[$\tilde{\Delta}_F^8$]</td>
</tr>
<tr>
<td>1</td>
<td>$-2b_1 + \frac{v_F}{12} \left(\sqrt{6} \lambda_1 \mu_1 + 3\sqrt{6}A_1\right)$</td>
<td>Im[$\tilde{\Delta}_F^8$]</td>
</tr>
<tr>
<td>4</td>
<td>$-2b_1 + \frac{v_F}{12} \left(2\sqrt{6} \lambda_1 \mu_1 - v_F \lambda_1^2 + 2\sqrt{6}A_1\right)$</td>
<td>Im[$\tilde{\Delta}_F^{4,5,6,7}$]</td>
</tr>
<tr>
<td>3</td>
<td>$-2b_1 + \frac{v_F}{12} \left(5\sqrt{6} \lambda_1 \mu_1 + 2v_F \lambda_1^2 - \sqrt{6}A_1\right)$</td>
<td>Im[$\tilde{\Delta}_F^{1,2,3}$]</td>
</tr>
<tr>
<td>6</td>
<td>$\sqrt{\frac{2}{3}} \frac{v}{2} \left(3\lambda_{78} \mu_{78} + A_{78} + 3C_{78}\right)$</td>
<td>Re[$\tilde{\Delta}_{L,R}^{1,2,3}$]</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{v}{12} \left(-v \lambda_{78}^2 + 3\sqrt{6} \lambda_{78} \mu_{78} + \sqrt{6}A_{78} + 3\sqrt{6}C_{78}\right)$</td>
<td>Re[$\tilde{\Delta}_C^{1,\cdots,8}$]</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{v}{12} \left(2v \lambda_{78}^2 - 3\sqrt{6} \lambda_{78} \mu_{78} - \sqrt{6}A_{78} - 3\sqrt{6}C_{78}\right)$</td>
<td>Re[$\tilde{\Delta}_C^8$]</td>
</tr>
<tr>
<td>2</td>
<td>$-2b_{78} + \frac{\sqrt{6}}{12}v \left(\lambda_{78} \mu_{78} + 3A_{78} + C_{78}\right)$</td>
<td>Im[$\tilde{\Delta}_{L,R}^8$]</td>
</tr>
<tr>
<td>8</td>
<td>$-2b_{78} + \frac{3}{4}g^2 U v^2 + \frac{v^2}{12} \lambda_{78}^2 + \frac{\sqrt{6}}{6}v \left(\lambda_{78} \mu_{78} + A_{78} + C_{78}\right)$</td>
<td>Im[$\tilde{\Delta}_{L,R}^{4,5,6,7}$]</td>
</tr>
<tr>
<td>8</td>
<td>$-2b_{78} + \frac{v^2}{12} \lambda_{78}^2 + \frac{\sqrt{6}}{12}v \left(3\lambda_{78} \mu_{78} + A_{78} + 3C_{78}\right)$</td>
<td>Im[$\tilde{\Delta}_C^{1,\cdots,8}$]</td>
</tr>
<tr>
<td>6</td>
<td>$-2b_{78} + \frac{v^2}{6} \lambda_{78}^2 + \frac{\sqrt{6}}{12}v \left(5\lambda_{78} \mu_{78} - A_{78} + 5C_{78}\right)$</td>
<td>Im[$\tilde{\Delta}_{L,R}^{1,2,3}$]</td>
</tr>
</tbody>
</table>