Backreaction of particle production on false vacuum decay

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CL, PRD 93 (2016) 025024 [arxiv:1511.05256]

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Context

Electroweak vacuum (in)stability

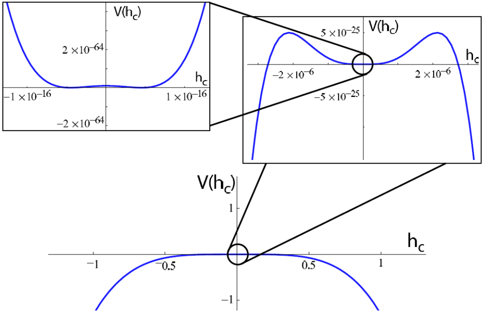
The effective (quantum-corrected) Higgs potential may become negative at high energy $\Rightarrow$ metastable electroweak vacuum

Classical Higgs potential:

$$V(h) = -\frac{1}{2}m^2h^2 + \frac{1}{4}\lambda h^4$$

Quantum corrections absorbed in the running coupling $\lambda(\mu)$:

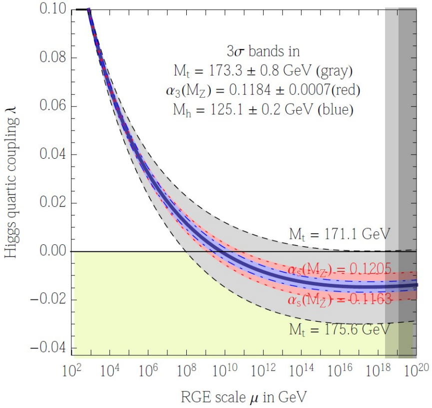
$$V(h \gg v) = \frac{1}{4}\lambda(h)h^4$$

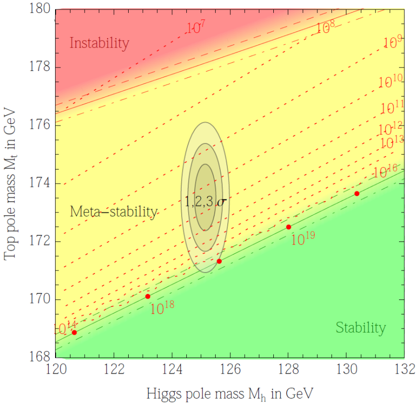


Effective Higgs potential arxiv:1307.8106

Electroweak vacuum (in)stability II

Running of the coupling λ computed by several groups at NNLO $\Rightarrow$ stability excluded at 2.8σ[1], respectively 1.3σ[2]





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[1] Buttazzo et al. [arXiv:1307.3536]

[2] Bednyakov et al. [arXiv:1507.08833]

Consequences of EW vacuum metastability

- Lifetime of the current EW vacuum is much longer than the age of the universe
- Tensions with the inflationary scenario in the early universe[3]
- Sign of new physics?
- Motivation to improve our understanding of decay processes in QFT

[3] e.g: Kobakhidze and Spencer-Smith. [arXiv:1301.2846], Espinosa et al. [arXiv:1505.04825]

False vacuum decay rate

Semiclassical decay rate in QFT

Consider a metastable scalar field: $\mathcal{L}_\sigma = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)$.

Decay rate per unit time per unit volume at leading order:[4]

$$\frac{\Gamma}{V} \propto e^{-\frac{S_B}{\hbar}}, \quad S_B = \int d\tau d^3\mathbf{x} \left[\frac{1}{2}\left(\partial_\tau\sigma_B\right)^2 + \frac{1}{2}\left(\partial_i\sigma_B\right)^2 + V(\sigma_B)\right]$$

$S_B < \infty$ only for finite volume or $\sigma_B(\tau, \mathbf{x}) \xrightarrow{|\mathbf{x}|\to+\infty} \sigma_F$:

- homogeneous bounce $\sigma_B(\tau)$:

$$\partial_\tau^2\sigma = \frac{\partial V}{\partial\sigma} \qquad \sigma(\tau) \xrightarrow{\tau\to\pm\infty} \sigma_F, \qquad \left.\frac{\partial\sigma}{\partial\tau}\right|_{\tau=0} = 0$$

- $O(4)$ symmetric bubble $\sigma_B(\rho)$ with $\rho = \sqrt{\tau^2 + \mathbf{x}^2}$:

$$\frac{d^2\sigma}{d\rho^2} + \frac{3}{\rho}\frac{d\sigma}{d\rho} = \frac{\partial V}{\partial\sigma} \qquad \sigma(\rho\to+\infty) = \sigma_F \qquad \left.\frac{d\sigma}{d\rho}\right|_{\rho=0} = 0$$

[4] Coleman, PRD 15 (1977) 2929

One-loop quantum corrections

Path integral formalism provides first-order radiative corrections:[5]

$$\frac{\Gamma_{\text{one-loop}}}{V} = \frac{S_B^2}{4\pi^2\hbar^2}\left[\frac{\det'\left(-\partial_\tau^2 - \partial_i^2 + V''(\sigma_B(\mathbf{x},\tau))\right)}{\det\left(-\partial_\tau^2 - \partial_i^2 + V''(\sigma_F)\right)}\right]^{-\frac{1}{2}} e^{-\frac{S_B}{\hbar}}$$

Divergent determinants need to be renormalized.

Application to the current EW vacuum:

$$S_B \sim 1956, \quad \tau_{\text{tree}} \sim 10^{613}\, T_U, \quad \tau_{1-\text{loop}} \sim 10^{655}\, T_U$$

[5] Callan and Coleman, PRD 16 (1977) 1762

Particle creation during vacuum decay

Rubakov, Nucl. Phys. B245 (1984) 481

Formalism

External (or self-excitation) scalar field ϕ in the background of the bounce:

$$\mathcal{L} = \mathcal{L}_\sigma + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2(\sigma)\phi^2$$

Quantum state $|\Psi[\sigma_B]\rangle \approx e^{-S[\sigma_B(\tau)]}\,|\phi[\sigma_B(\tau)]\rangle$, with τ-dependent Hamiltonian $H_\phi(\tau) \Rightarrow$ the vacuum state evolves to an excited state

$$|\phi(-\infty)\rangle = |O_{-\infty}\rangle \longrightarrow |\phi(\tau)\rangle = \mathcal{X}_\tau\,|O_\tau\rangle$$

for $\mathcal{X}_\tau$ a non-unitary operator (Euclidean Bogoliubov transformations).

Non-unitary operator $\mathcal{X}_\tau$

Explicit excited state expressed from the instantaneous creation-annihilation operators:

$$|\phi(\tau)\rangle = \mathcal{X}_\tau\,|O_\tau\rangle = C_\tau \exp\left(\frac{1}{2}\sum_{\alpha,\beta} D_{\alpha\beta}(\tau) a^\dagger_{\alpha,\tau} a^\dagger_{\beta,\tau}\right)|O_\tau\rangle$$

D is a symmetric real matrix given by $D = VZ^{-1}$ with

$$\begin{cases} V_{\alpha\beta}(\tau) = (\omega^\alpha_\tau \xi^\alpha_\tau, g_\beta) - (\xi^\alpha_\tau, \partial_\tau g_\beta) \\ Z_{\alpha\beta}(\tau) = (\omega^\alpha_\tau \xi^\alpha_\tau, g_\beta) + (\xi^\alpha_\tau, \partial_\tau g_\beta) \end{cases}$$

and

$$\begin{array}{lcl} \left[-\partial_i^2 + m^2(\mathbf{x},\tau)\right] \xi^\alpha_\tau(\mathbf{x}) & = & \left(\omega^\alpha_\tau\right)^2 \xi^\alpha_\tau(\mathbf{x}) \\ \left[-\partial_\tau^2 - \partial_i^2 + m^2(\mathbf{x},\tau)\right] g_\alpha(\mathbf{x},\tau) & = & 0 \end{array}$$

Number of created particles

$$N_\alpha(\tau) = \frac{\langle \phi(\tau) | a^\dagger_{\alpha,\tau} a_{\alpha,\tau} | \phi(\tau) \rangle}{\langle \phi(\tau) | \phi(\tau) \rangle} = \left(\frac{D^2}{\mathbb{1} - D^2} \right)_{\alpha\alpha}, \quad N = \sum_\alpha N_\alpha$$

At nucleation time ($\tau = 0$), a number of physical particles have been produced in each bubble.

Weak particle production and ultraviolet finiteness

- When the number of created particles is weak, the expression for D at the lowest order (in terms of $\partial_\tau m^2$) is given by

  $$D_{\alpha\beta}(\tau) = e^{-(W_\alpha(\tau)+W_\beta(\tau))} \int_{-\infty}^{\tau} d\tau' e^{\left(W_\alpha(\tau')+W_\beta(\tau')\right)} \frac{\left(\xi^\alpha_{\tau'}, \partial_{\tau'} m^2 \xi^\beta_{\tau'}\right)}{\omega^\alpha_{\tau'}+\omega^\beta_{\tau'}}$$

  with $W_\alpha(\tau) = \int_0^\tau d\tau' \omega^\alpha_\tau$.

- $N \approx \sum_\alpha (D^2)_{\alpha\alpha} = \mathrm{Tr}(D^2)$.

- In the ultraviolet region ($\omega^\alpha_\tau \gg m$), $\xi^{\mathbf{k}}_\tau(\mathbf{x}) \propto e^{i\mathbf{k}\cdot\mathbf{x}}$ and $\omega^{\mathbf{k}}_\tau \approx k$.

  $$N_{UV} = \int d^3\mathbf{k} \left(D^2\right)_{\mathbf{kk}} \approx$$
  $$\int d^3\mathbf{k} d^3\mathbf{k}' \frac{1}{4(2\pi)^6 k k' (k+k')^4} \left| \widetilde{\partial_\tau m^2}(\mathbf{k}-\mathbf{k}', \tau) \right|^2$$

  $\Rightarrow$ finite contribution

Backreaction of particle production

CL, PRD 93 (2016) 025024 [arxiv:1511.05256]

Method

Effect of produced particles on the decay rate is computed by tracing over all external particle states.

Density operator:

$$\hat{\rho}(\tau) = e^{-2S_E[\sigma_B(\tau)]} \left(\mathcal{X}_\tau \left| O_\tau \right\rangle \left\langle O_\tau \right| \mathcal{X}_\tau^\dagger \right)$$

Reduced density operator:

$$\hat{\rho}_r(\tau) = e^{-2S_E[\sigma_B(\tau)]} \sum_{n=0}^{\infty} \sum_{\{\alpha\}_n} \frac{\left| \left\langle \{\alpha\}_n (\tau) \right| \mathcal{X}_\tau \left| O_\tau \right\rangle \right|^2}{n!}$$

Decay rate:

$$\frac{\Gamma}{V} = \frac{\hat{\rho}_r(0)}{\hat{\rho}_r(-\infty)}$$

Backreaction factor

After computation:

$$\frac{\Gamma}{V} = |C_0|^2 \frac{e^{-S_B}}{\sqrt{\mathrm{Det}\left(1 - D^2_{\tau=0}\right)}} = |C_0|^2 e^{-S_B - \frac{1}{2}\,\mathrm{Tr}\ln\left(1 - D^2_{\tau=0}\right)}$$

- Explicit factor for the backreaction of physical particles
- Weak particle production limit: $\frac{\Gamma_{\text{Ba}}}{V} \approx e^{-S_B + \frac{1}{2}N}$.
- Production of scalar particles enhances the decay rate.
- Ultraviolet finite contribution
- $|C_0|^2$ evaluated from path-integral formalism

Toy models

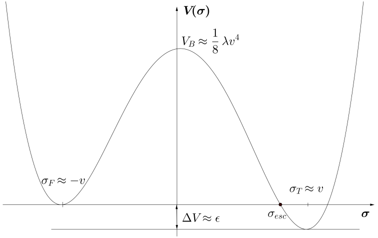
Toy model potential

Metastable potential:

$$V(\sigma) = \frac{\lambda}{8}\left(\sigma^2 - v^2\right)^2 - \frac{\epsilon}{2v}(\sigma + v)$$

For $\epsilon \ll \lambda v^4$

- $\sigma_{T/F} \approx \pm v$
- $\Delta V \ll V_B$
- $\sigma_{esc} \approx v - \frac{1}{v}\sqrt{\frac{2\epsilon}{\lambda}}$



Homogeneous bounce

Homogeneous kink solution $\sigma_B(\tau)$ in a sphere of volume V_U :

$$S_B \approx V_U \frac{4}{3}\sqrt{\lambda} v^3$$

Homogeneity $\Rightarrow$ diagonal matrix D in (k, l, m)

Mass coupling $m^2(\tau) \approx$ step function $\Rightarrow D_{kk}(\tau = 0) = \frac{\omega_+ - \omega_-}{\omega_+ + \omega_-} e^{2\omega_+ \tilde{\tau}}$

$$\omega_\pm = \sqrt{k^2 + m_\pm^2}, \quad \tilde{\tau} = -\frac{2}{v\sqrt{\lambda}} \text{arctanh}\left(1 - \sqrt{\frac{2\epsilon}{v^4 \lambda}}\right)$$

Backreaction is parameter dependent. Reasonable choice of parameters for which backreaction becomes non-negligible, e.g:

$$m_- \ll m_+, \quad \frac{\lambda v^4}{\epsilon} = 10, \quad R_U \frac{\epsilon}{\sqrt{\Lambda} v^3} = 0.1 \quad \Rightarrow \quad \frac{\Gamma}{V_U} \approx e^{-45+4.1}$$

Thin-wall bubble

Nucleation of a thin-wall bubble with radius $R^\star = \frac{2\sqrt{\lambda}v^3}{\epsilon}$:

$$S_B = \frac{8}{3}\pi^2 \frac{\lambda^2 v^{12}}{\epsilon^3} \gg 1$$

Mass coupling $\approx$ step function at the bubble wall $r_\star = \sqrt{R^{\star 2} - \tau^2}$:

$$m^2(r,\tau) = \begin{cases} m_-^2 & \text{if} \quad \tau < -R^\star \\ m_+^2 + (m_-^2 - m_+^2)\Theta(r - r_\star(\tau)) & \text{if} \quad -R^\star \leq \tau \leq 0 \end{cases}$$

Non-diagonal matrix D. At leading order, backreaction $\approx \frac{1}{2}\,\mathrm{Tr}(D^2)$ computed in the weak limit.

Dominant contribution when $|m_+^2 - m_-^2| \gg (R^\star)^{-2}$:

$$\mathrm{Tr}\left(D^2\right) \approx \begin{cases} 8.4 \cdot 10^{-3}, & m_- \ll (R^\star)^{-1} \ll m_+ \\ 4.7 \cdot 10^{-5}, & m_+ \ll (R^\star)^{-1} \ll m_- \end{cases}$$

$\Rightarrow$ parametrically suppressed backreaction

Conclusion

Outline

- Explicit factor for backreaction of scalar particles on the decay rate (in flat space-time)
- Decay rate enhancement and UV finiteness
- Negligible contribution on the thin-wall bounce but significant impact for some parameters of the homogeneous bounce.

Future investigations

- Application to the Higgs potential
- Inclusion of fermion and vector fields
- Behaviour in curved space-time and during early universe

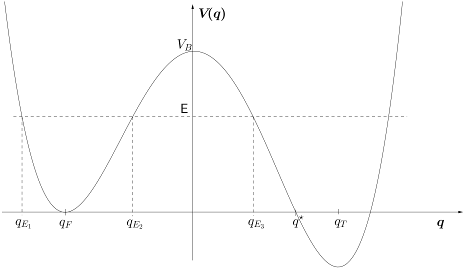
Backup slides

Semiclassical decay rate in QM

Particle of mass m trapped in the **false vacuum** of a potential.

Solution of Schrödinger equation at lowest order in $(-i\hbar) \Rightarrow$

$$\Gamma \propto e^{-\frac{2}{\hbar}\int_{q_F}^{q^\star}\sqrt{2mV(q)}dq}$$



The exponential factor corresponds to the Euclidean action evaluated along the bounce trajectory: $S_E|_{q_B} = 2\int_{q_F}^{q_\star}\sqrt{2mV(q)}dq$.

$$\tau = i\,t \qquad S_E = -iS = \int d\tau \left[\frac{1}{2}m\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]$$

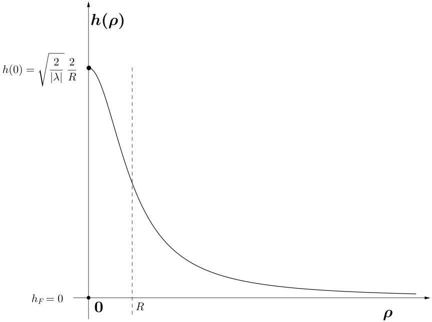
Higgs potential - semiclassical decay rate

Approximation at high energy scale: $V(h) \approx \frac{1}{4}\lambda h^4$ with $\lambda < 0$.

- scale-invariant bounce:

  $$h_B(\rho) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{\rho^2+R^2}$$

- $S_B = \frac{8\pi^2}{3|\lambda|}$



One-loop corrections fix R and the renormalization scale for λ.[6] For the central values of the SM:[7]

$$R_M \sim 1.87 \cdot 10^{-17}\ \text{GeV}^{-1} \quad \lambda(1/R_M) = -0.01345 \quad S_B = 1956.54$$

[6] Isidori et al. [arXiv:hep-ph/0104016]
[7] Branchina et al. [arXiv:1408.5302]

Higgs potential - backreaction (in progress)

- Self-excitation since h is the only scalar field of the SM

$$m^2(\rho) = V''(\rho) = -\frac{24R^2}{(\rho^2 + R^2)^2}$$

- Negative mass-coupling because of the concavity of the potential.
- First attempt in the weak particle production limit. D is given in terms of $\eta_p(r)$ such that:

$$\left[-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} - \frac{24R^2}{(r^2+\tau^2+R^2)^2}\right]\eta_p = (\omega_p)^2\,\eta_p$$

- It reduces to a confluent Heun's equation whose properties will be investigated in a subsequent work.