Spectrum in the presence of a brane-localized mass in six dimensions

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Introduction

Symmetry in the extra-dimensions

Boundary conditions

Brane-localized masses

→ ~~Symmetry~~

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Boundary conditions
Brane-localized masses
→ ~~Symmetry~~

↓ make heavy

Exotic particles

Introduction

We study effects of the brane-mass on spectrum.

lightest mass eigenvalue

Questions

- Can the brane-mass make m_0 heavy enough?
- How much does it deform the mode functions?
- How much does m_0 depend on $\Lambda_{\rm cut}$?

cut-off scale

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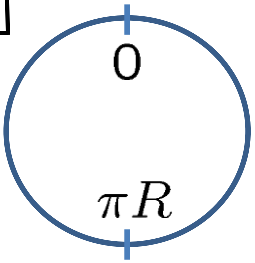
cut-off scale

5D theory

$$\mathcal{L} = -\partial^\mu \phi^* \partial_\mu \phi - \partial_y \phi^* \partial_y \phi$$

S^1

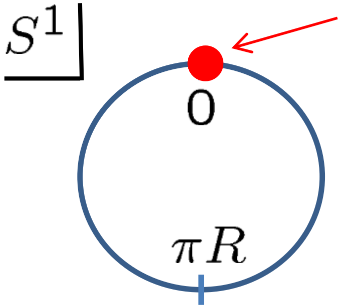


$$m_n^{(0)} = \frac{|n|}{R}$$

5D theory

dimension 1

$$\mathcal{L} = -\partial^\mu \phi^* \partial_\mu \phi - \partial_y \phi^* \partial_y \phi - \mu^2 |\phi|^2 \delta(y)$$



localized mass

change boundary condition

$$m_n^{(0)} = \frac{|n|}{R}$$

$$m_n = \frac{\mu^2}{2} \cot(m_n \pi R)$$

$$\left(\lim_{\mu \to \infty} m_n = \frac{|n + \frac{1}{2}|}{R} \right)$$

5D theory

$$m_0 < m_1 \leq m_2 \leq \cdots \leq m_N \leq \overset{\text{cut-off scale}}{\overset{\shortparallel}{\Lambda_{\text{cut}}}}$$

Mass matrix

$$M_{ab}^2 = m_a^{(0)2} \delta_{ab}$$

5D theory

$$m_0 < m_1 \leq m_2 \leq \cdots \leq m_N \leq \overset{\text{cut-off scale}}{\overset{\shortparallel}{\Lambda_{\text{cut}}}}$$

Mass matrix

$$M_{ab}^2 = m_a^{(0)2}\delta_{ab} + \frac{\mu^2}{2\pi R}$$

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Mass matrix

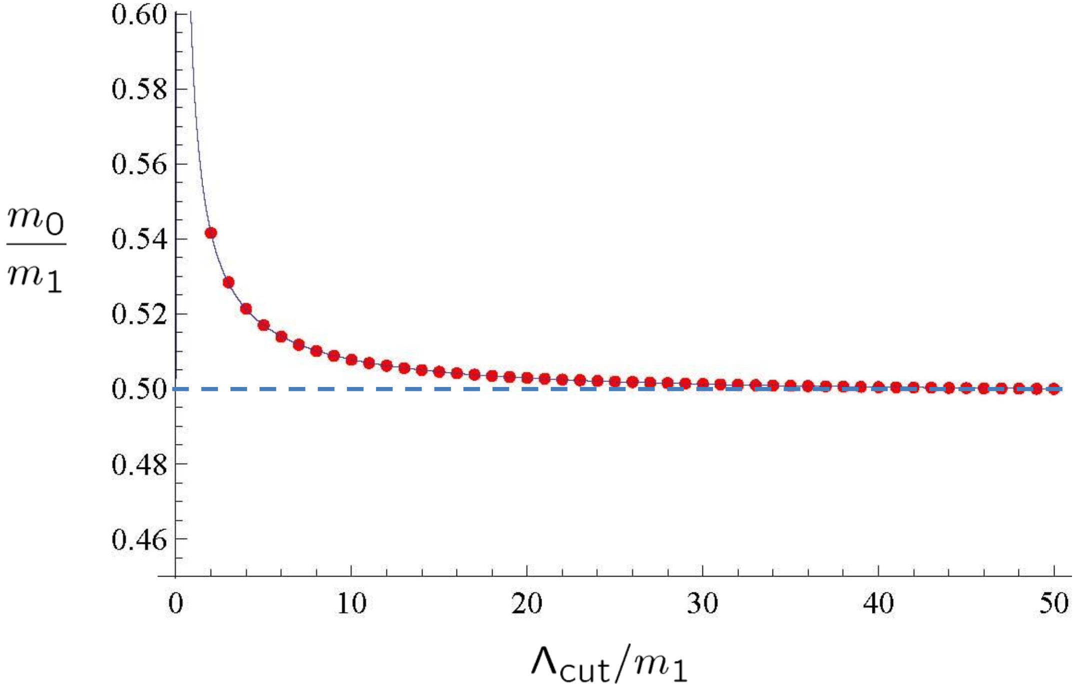
$$M_{ab}^2 = m_a^{(0)2}\delta_{ab} + \frac{\mu^2}{2\pi R}$$

$$\Big\Downarrow \ \text{diagonalize}$$

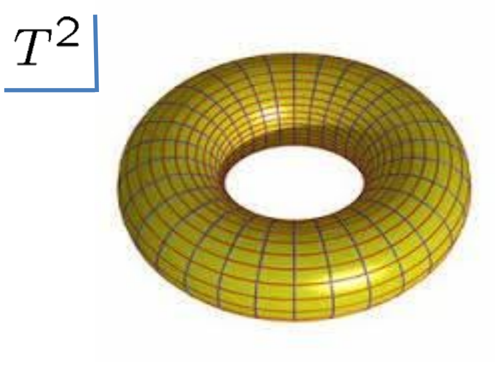
$$\begin{pmatrix} m_0(\mu) & & & \\ & m_1(\mu) & & \\ & & \ddots & \\ & & & m_N(\mu) \end{pmatrix}$$

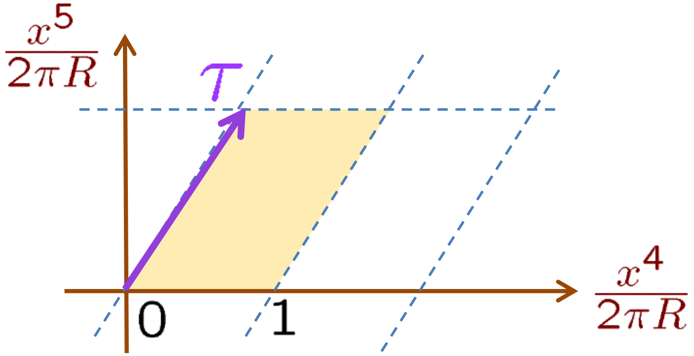
5D theory

Cut-off dependence of m_0



6D theory





dimensionless

$$\mathcal{L} = -\partial^\mu\phi^*\partial_\mu\phi - \sum_{m=4,5}\partial^m\phi^*\partial_m\phi - \mu^2|\phi|^2\delta^{(2)}(x^m)$$

6D theory

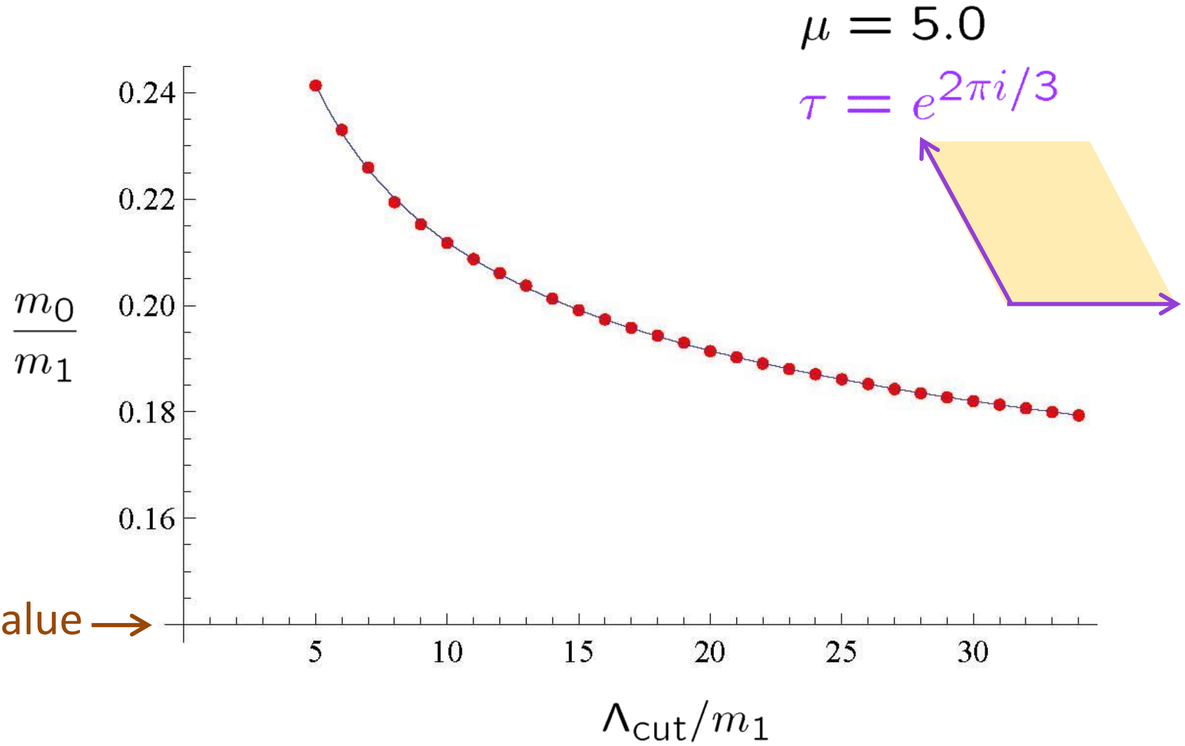
Mass matrix $$m^{(0)}_{a=(n,l)} = \frac{|n + l\tau|}{R\,\mathrm{Im}\,\tau} \quad (n, l \in \mathbb{Z})$$

$$M^2_{ab} = m^{(0)2}_a \delta_{ab} + \frac{\mu^2}{4(\pi R)^2 \mathrm{Im}\,\tau}$$

⇓ diagonalize

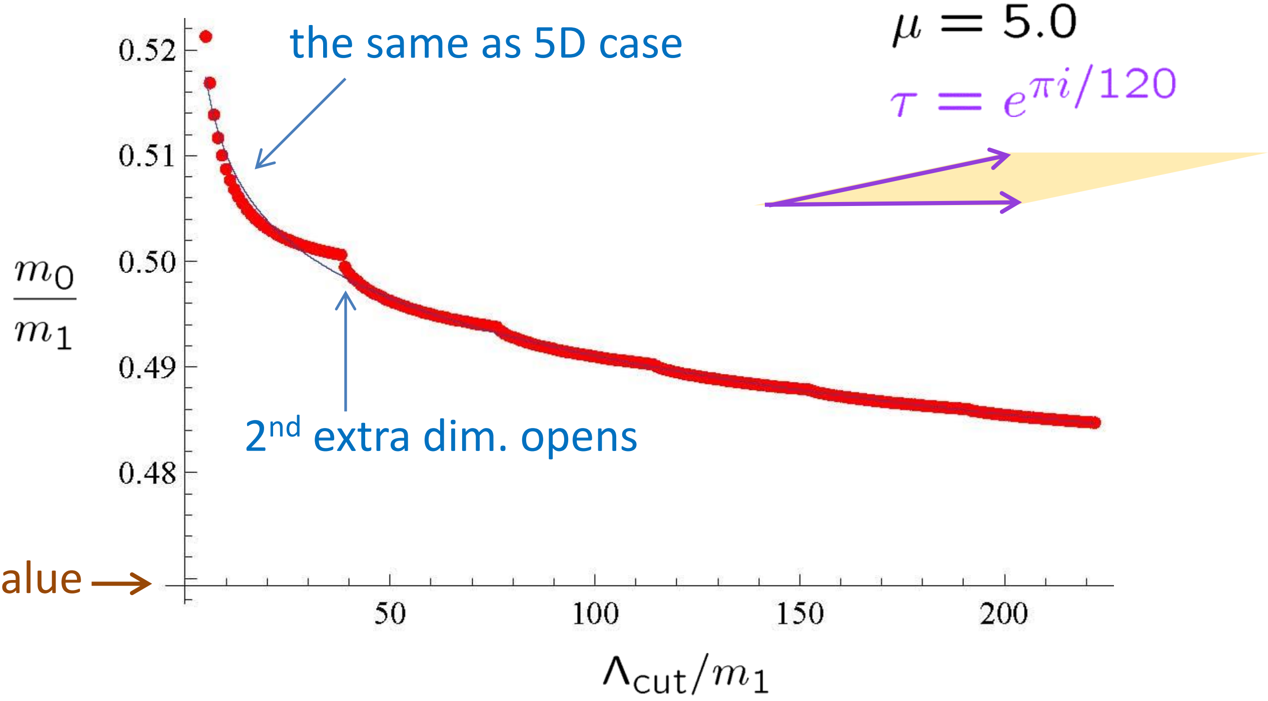
$$\begin{pmatrix} m_0(\mu) & & & \\ & m_1(\mu) & & \\ & & \ddots & \\ & & & m_N(\mu) \end{pmatrix}$$

Cut-off dependence of m_0



limit value →

Cut-off dependence of *m*$_0$



the same as 5D case

2nd extra dim. opens

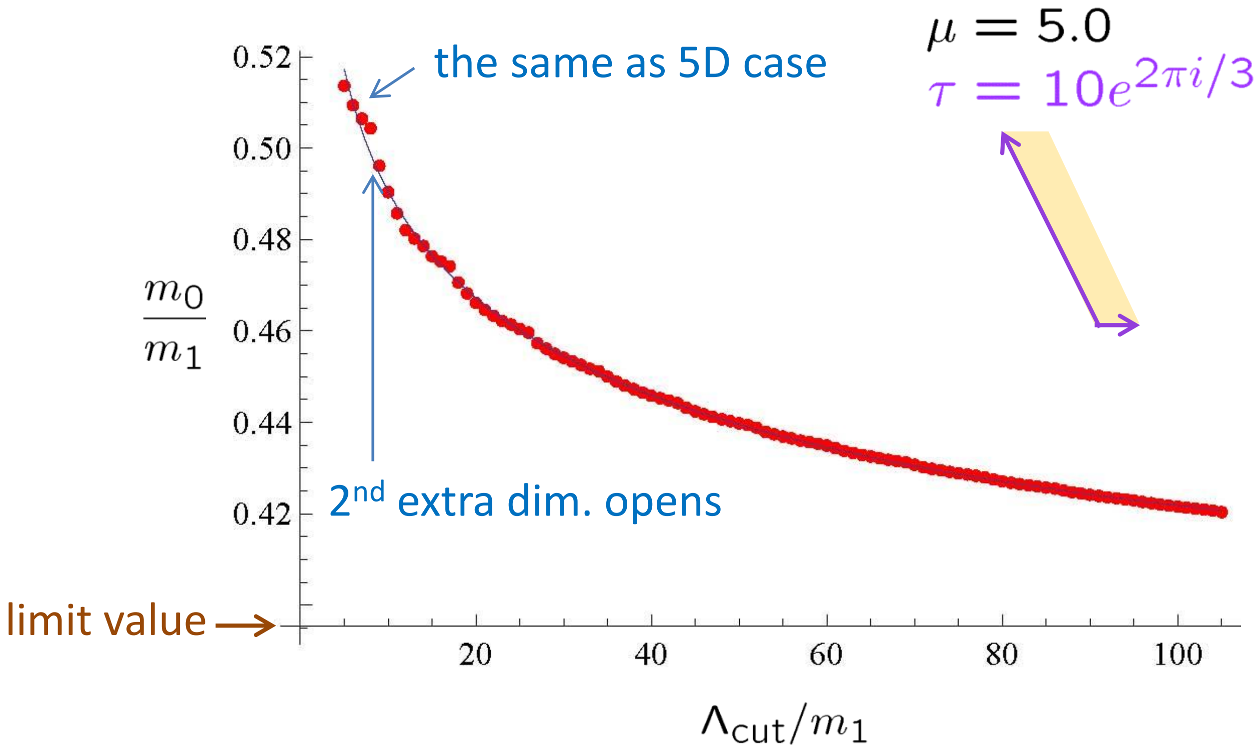
limit value →

$\mu = 5.0$

$\tau = e^{\pi i/120}$

$$\text{At } \Lambda_{\text{cut}} = 15 m_1, \quad \frac{m_0}{m_1} \simeq \lim_{\Lambda_{\text{cut}}\to\infty} \frac{m_0}{m_1} \times \begin{cases} 1.40 & (\tau = e^{2\pi i/3}) \\ 1.07 & (\tau = e^{\pi i/120}) \end{cases}$$

Cut-off dependence of m_0



the same as 5D case

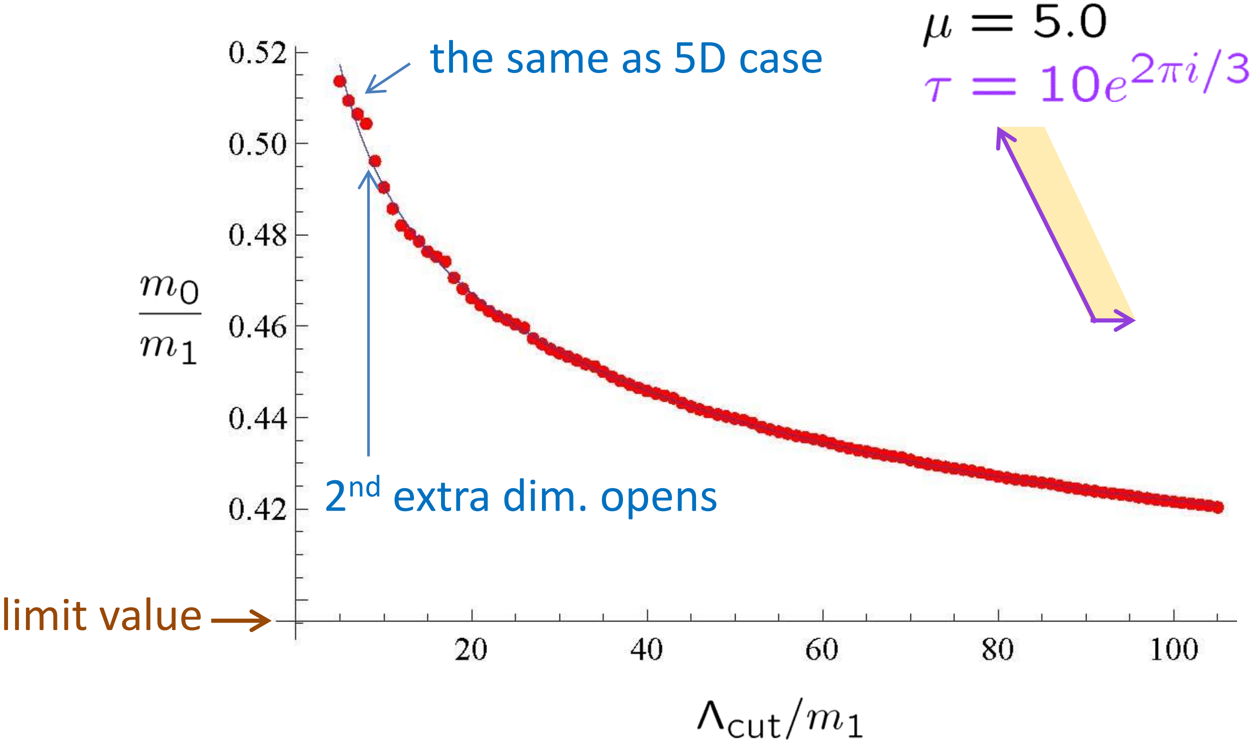
$\mu = 5.0$

$\tau = 10e^{2\pi i/3}$

2nd extra dim. opens

limit value

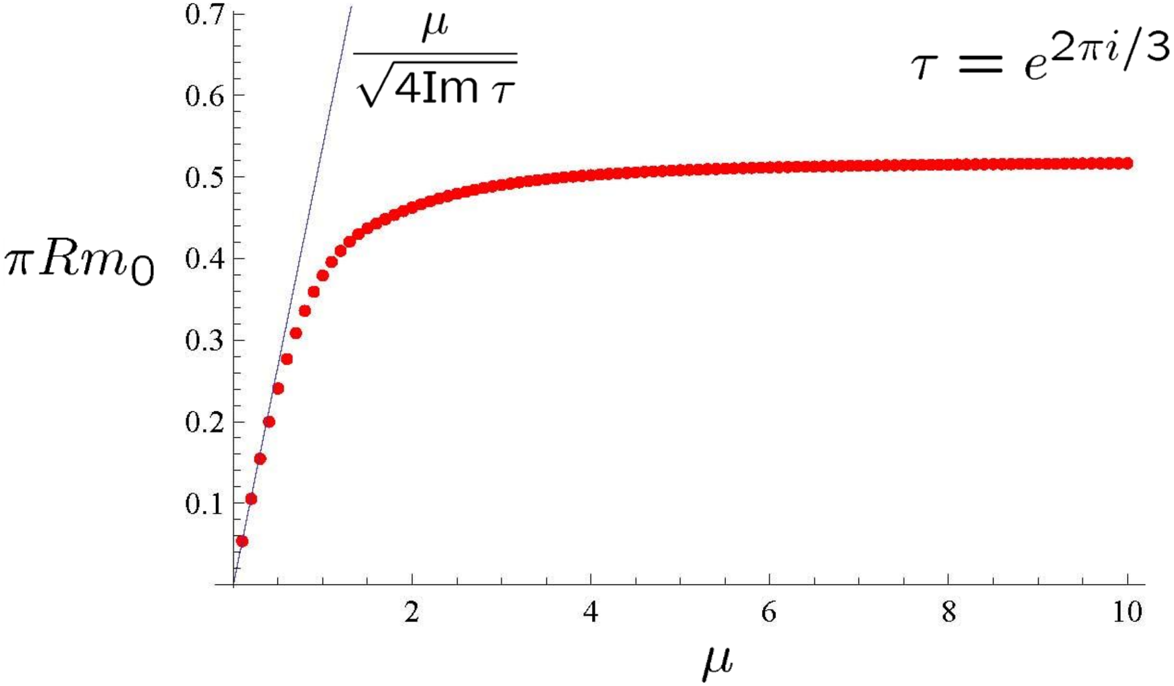
Cut-off dependence of m_0



In the following, we take a limit $\Lambda_{\rm cut} \to \infty$.

μ-dependence of m_0

$$M_{ab}^2 = m_a^{(0)2} + \frac{\mu^2}{4(\pi R)^2 \mathrm{Im}\,\tau}$$



Approximate expression of m_0

- In the limit of $\begin{cases} \arg\tau \to 0, \pi & \text{(squashed torus)} \\ \quad\text{or} & \\ |\tau| \to 0, \infty & \text{(long, thin torus)} \end{cases}$,

  the spacetime approaches to 5D.

  Namely,

$$\lim_{\mu\to\infty} m_0 \to \lim_{\mu\to\infty} \frac{m_1}{2} \equiv \frac{\bar{m}_1}{2}$$

Approximate expression of *m_0*

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  Namely,

$$\lim_{\mu\to\infty} m_0 \to \lim_{\mu\to\infty} \frac{m_1}{2} \equiv \frac{\bar{m}_1}{2}$$

- Since m_a is a function on the torus,

$$m_a\left(\mu; -\frac{1}{\tau}\right) = |\tau| m_a(\mu;\tau)$$

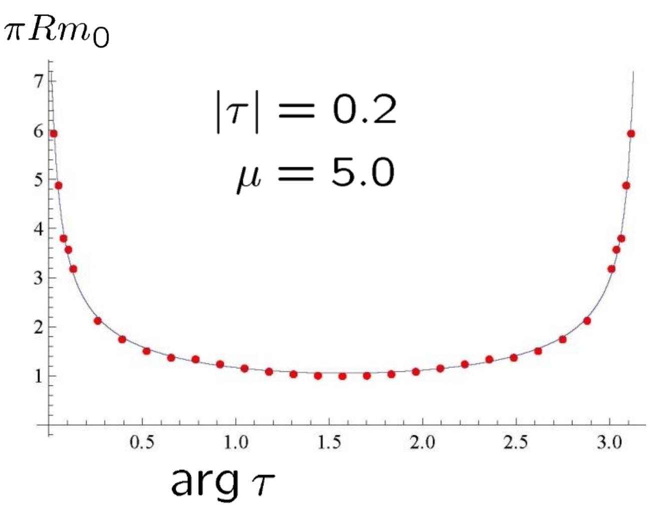
Approximate expression of *m*$_0$

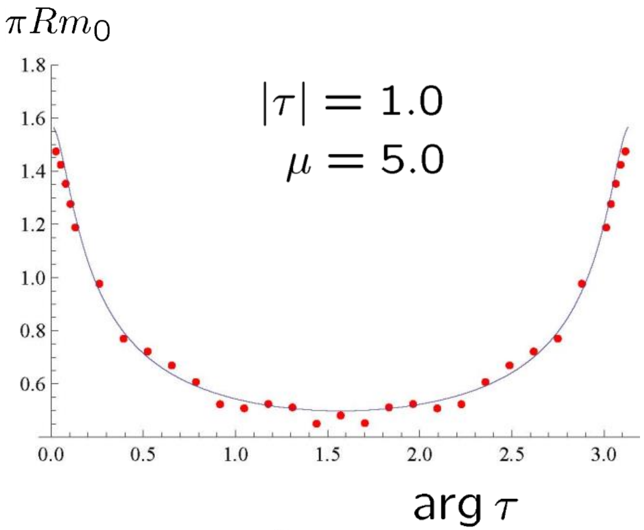
$$\begin{cases} \lim_{\mu\to\infty} m_0 = \dfrac{\bar{m}_1}{2} \\ m_a\left(\mu; -\dfrac{1}{\tau}\right) = |\tau| m_a(\mu;\tau) \end{cases}$$

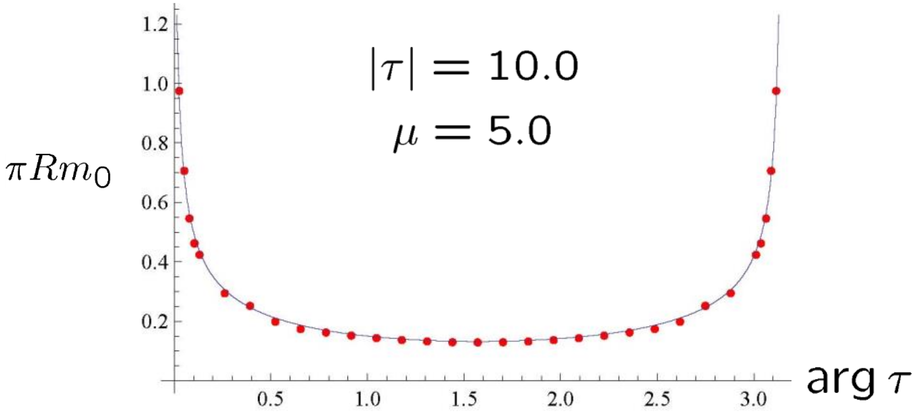
Approximate formula

$$\lim_{\mu\to\infty} m_0 \simeq \frac{\sqrt{\sin\{\arcsin((\pi R\bar{m}_1)^2 \mathrm{Im}\,\tau)}}{2\pi R\sqrt{\mathrm{Im}\,\tau}}$$

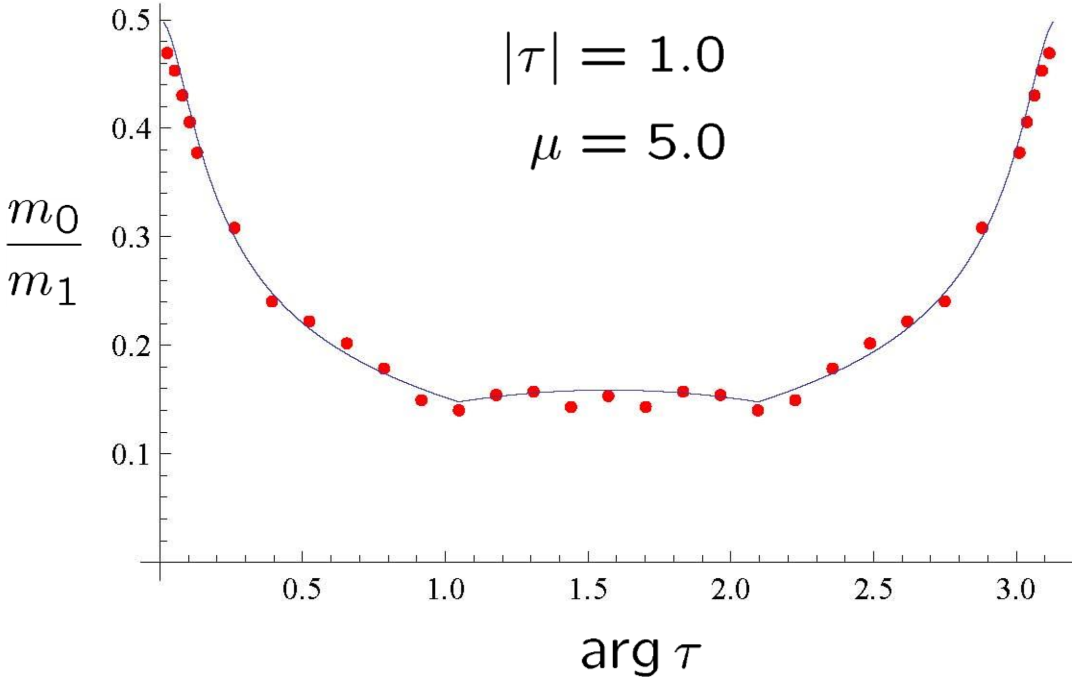
Approximate expression of m_0







Ratio to KK scale



Summary

- We investigate the spectrum of a 6D scalar theory in the presence of a brane-localized mass term.
- The cut-off dependence is non-negligible.
- An approximate expression of m_0 is found.
- $m_0/m_{\rm KK}$ is much smaller than the 5D case.

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- We investigate the spectrum of a 6D scalar theory in the presence of a brane-localized mass term.
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Future works

- Calculate the deformed mode functions
- Discuss more general set-ups
  (more than one brane-masses, magnetic fluxes, …)