University of Glasgow

The Hierarchy Problem in Non-Supersymmetric Extended Models

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NB. The results presented in this talk are still very preliminary!

Presented by:
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The Problem of a Light Higgs

The Higgs boson mass of $125.09 \pm 0.24\,\mathrm{GeV}$ is *intriguing*.

Why is it so light? The Higgs boson quantum corrections to m_H^2 vary as Λ^2, so it is natural to expect the Higgs to have a mass of the order of any "new physics" scale.

This is of course the hierarchy problem, whose most convincing solution is still Supersymmetry.

But what if Supersymmetry is not manifest at low energies?
Again the Higgs mass would naturally be pushed up to the next scale of new physics.

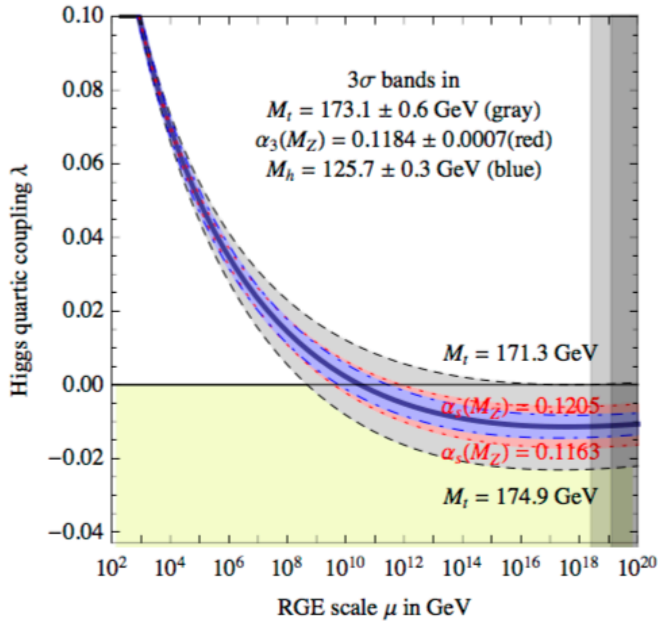
In this talk I will discuss some alternate explanations of a light Higgs boson, focusing on models with an extended Higgs sector.

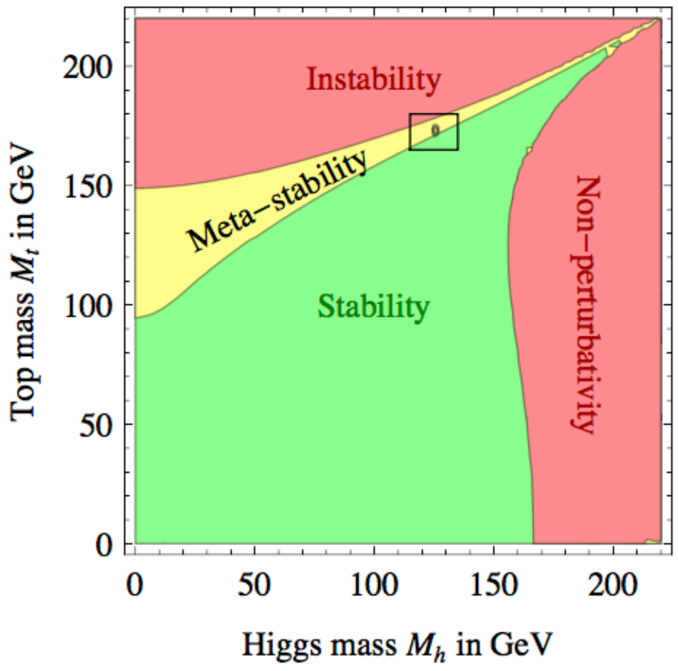
SM Higgs vacuum stability

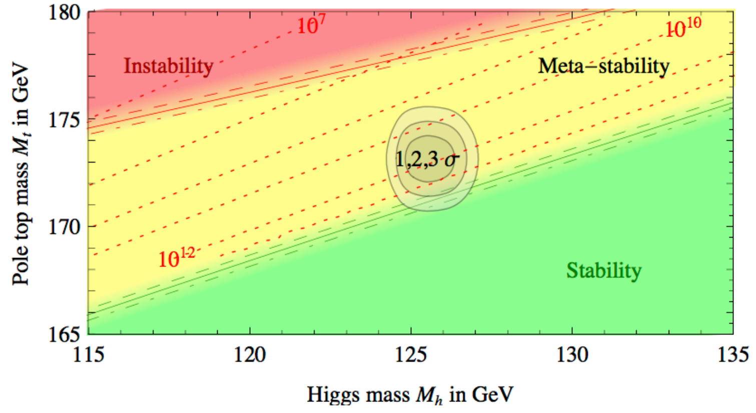
The Higgs boson mass of $125.09 \pm 0.24\,\mathrm{GeV}$ is *intriguing* for another reason.

$$V = -\mu^2|\Phi|^2 + \lambda^2|\Phi|^4$$

The quartic Higgs self coupling λ runs to 0, or even negative, at the Planck Scale. [Degrassi et al, 2013]



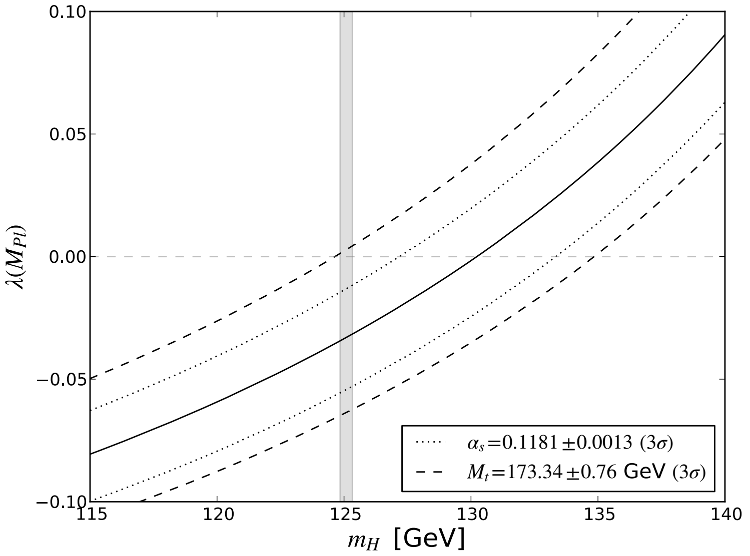


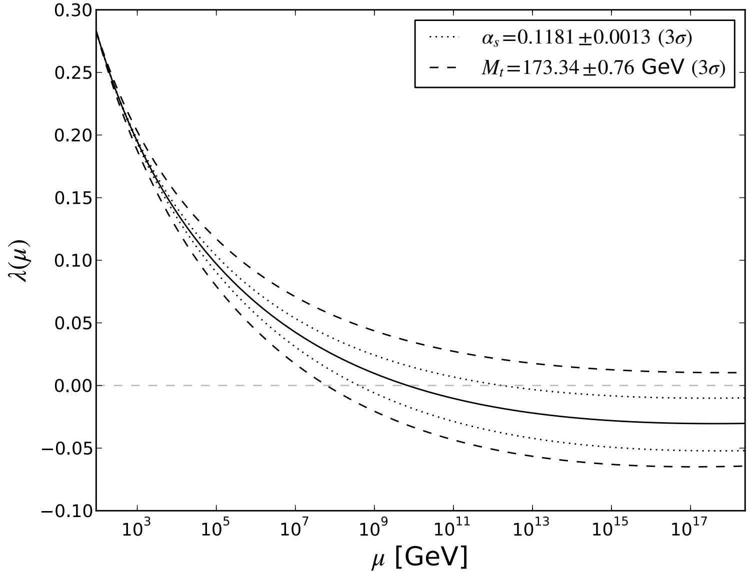


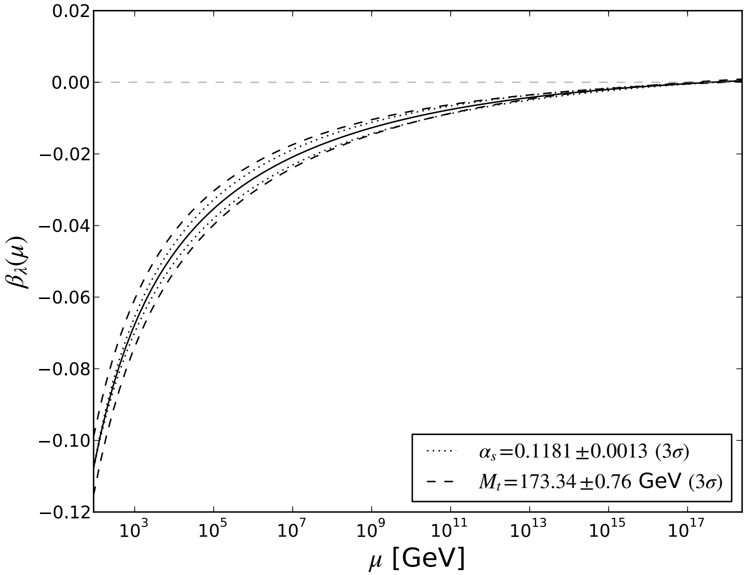
This phenomenon is very particular to a Higgs boson ≈ 125 GeV.

The slope of λ also appears to vanish at the Planck scale $\beta_\lambda(M_{\rm Pl}) \approx 0$.

Note that this is **not a fixed point**!







These plots were made at 3-loop using FlexibleSUSY [Athron et al, 2014] and Sarah [Staub et al, 2010]

Could this explain why the Higgs is light?

Asymptotic Safety

In 2010, Wetterich and Shaposhnikov predicted a Higgs boson $m_H = 126\,\mathrm{GeV}$ with "a few GeV uncertainty".

They did this by imposing "asymptotic safety" $\beta_\lambda(M_{\mathrm{Pl}}) = 0$ and saw that this also gave $\lambda \approx 0$ (as we have seen on previous slides).

$\lambda = \beta_\lambda = 0$ is not a fixed point. Gauge interactions will move one away from $\lambda = \beta_\lambda = 0$ at higher scales. Wetterich and Shaposhnikov argued that quantum gravity could tame the gauge couplings, reducing their beta functions to zero, so that $\beta_\lambda = 0$ remains stable.

However, unfortunately the corrections to the beta functions that they used [Robinson and Wilczek, 2010] appear to be gauge dependent and are in dispute.

Multiple Point Principle

Way back in 1995, Froggatt and Neilsen predicted the Higgs and top quark masses:

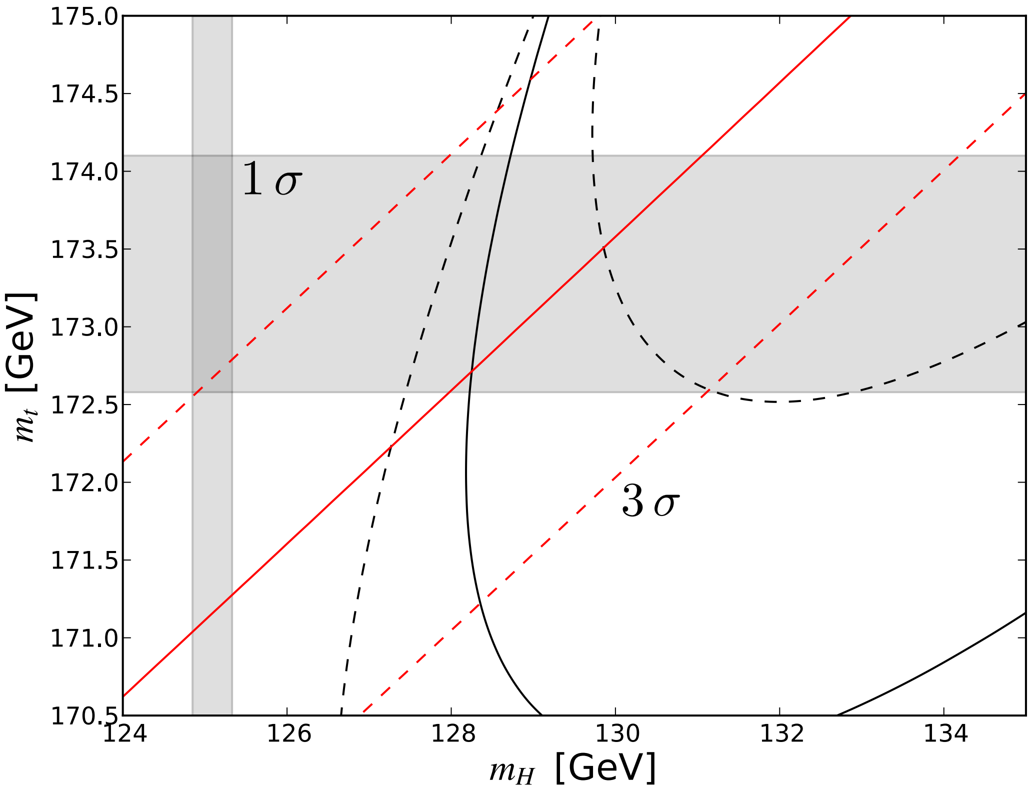
$$m_H = 135 \pm 9\,\mathrm{GeV} \qquad\qquad m_t = 175 \pm 5\,\mathrm{GeV}$$

To do this they applied the "Multiple Point Principle" which insists that there is another vacuum at the Planck scale degenerate with the Electroweak vacuum. They showed that the condition for this in the SM is again $\lambda = \beta_\lambda = 0$.

This result was only at one-loop and doesn't use the measured top mass (so the top mass above is a *prediction*).

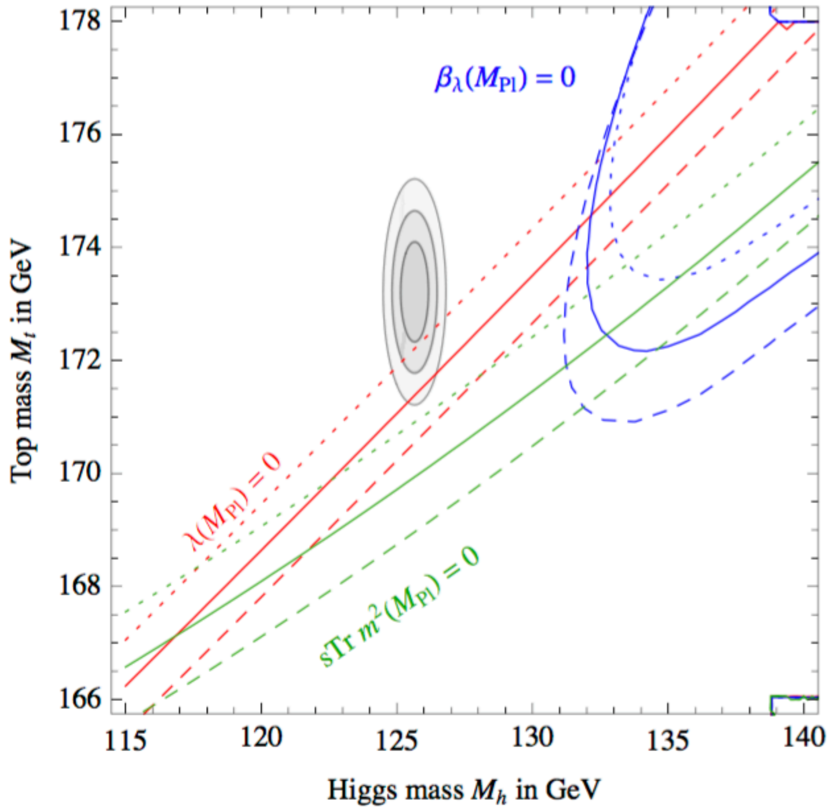
What does this "prediction" look like with more loops?

To 3-loops:



[Macauley, McDowall, DJM]

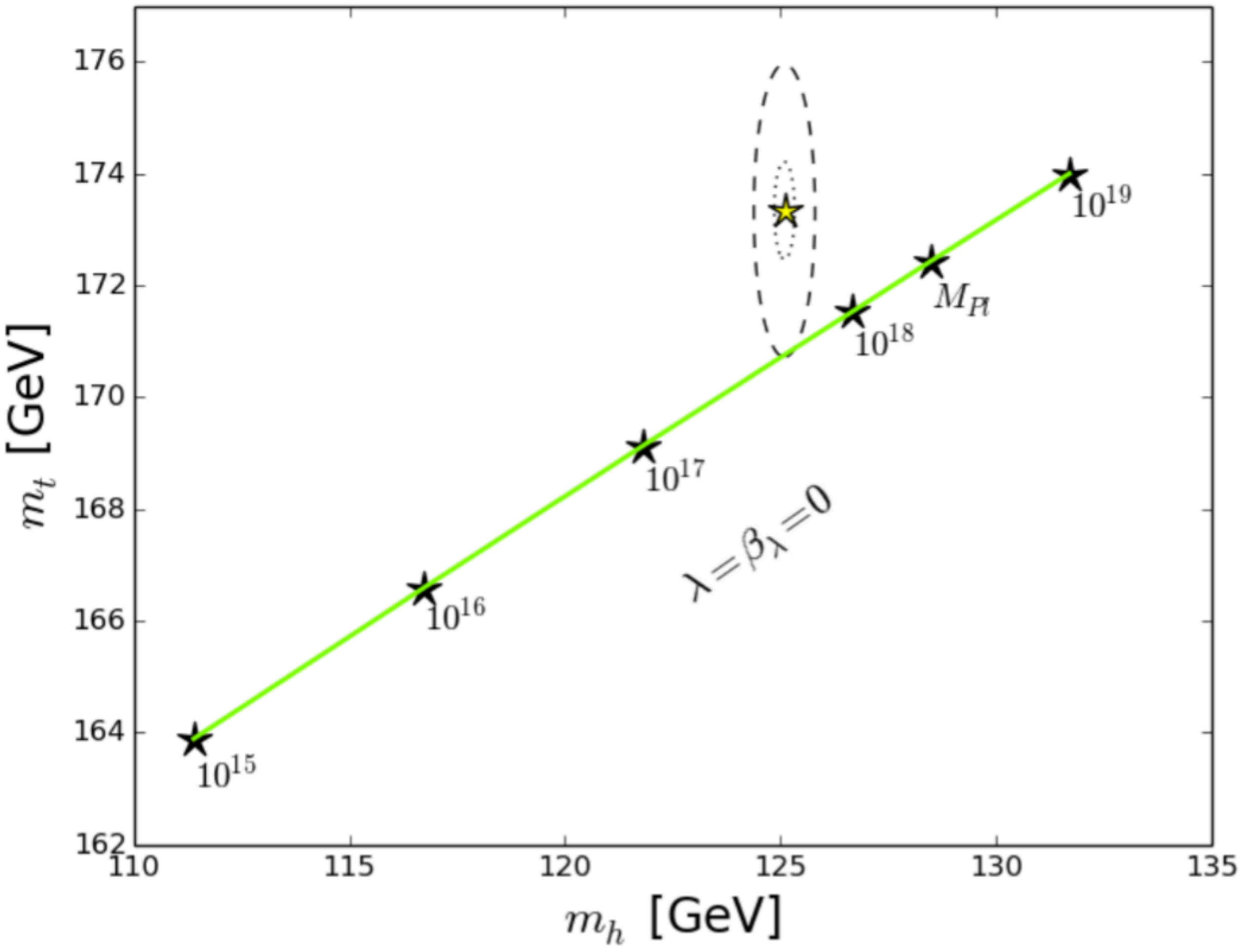
[Uses reduced Planck mass]



[Degrassi et al, 2013]

[Uses standard Planck mass]

Also note that the masses arising from $\lambda = \beta_\lambda = 0$ is very dependent on where the boundary condition is applied.



Reduced and "standard" Planck mass differ by a factor of $\sqrt{8\pi}$

Could this idea be saved?

It seems unlikely that the SM is valid all the way up to the Planck scale with no new physics.

Could asymptotic safety, or the Multiple Point Principle, be applicable to a GUT model?

For example, a non-supersymmetric Pati-Salam model $SU(4)_C \times SU(2)_L \times SU(2)_R$ within an SO(10) GUT?

New fields will alter the RGEs for λ, changing the prediction of the Higgs mass.

As a first step, we can consider what happens with extended Higgs sectors.

See also Giudice, Isidori, Salvio, Strumia (2014) for models with Total Asymptotic Freedom using "softened gravity".

A Complex Scalar Field

How does this picture change if we include new fields at the electroweak scale?

$$V(\Phi, \mathbb{S}) = -\mu^2 \left(\Phi^\dagger \Phi\right) + \lambda \left(\Phi^\dagger \Phi\right)^2 - \mu_{CS}^2 |\mathbb{S}|^2 + \lambda_{CS} |\mathbb{S}|^4 + \delta \left(\Phi^\dagger \Phi\right) |\mathbb{S}|^2 + \left(a\mathbb{S} + b\mathbb{S}^2 + c.c.\right)$$

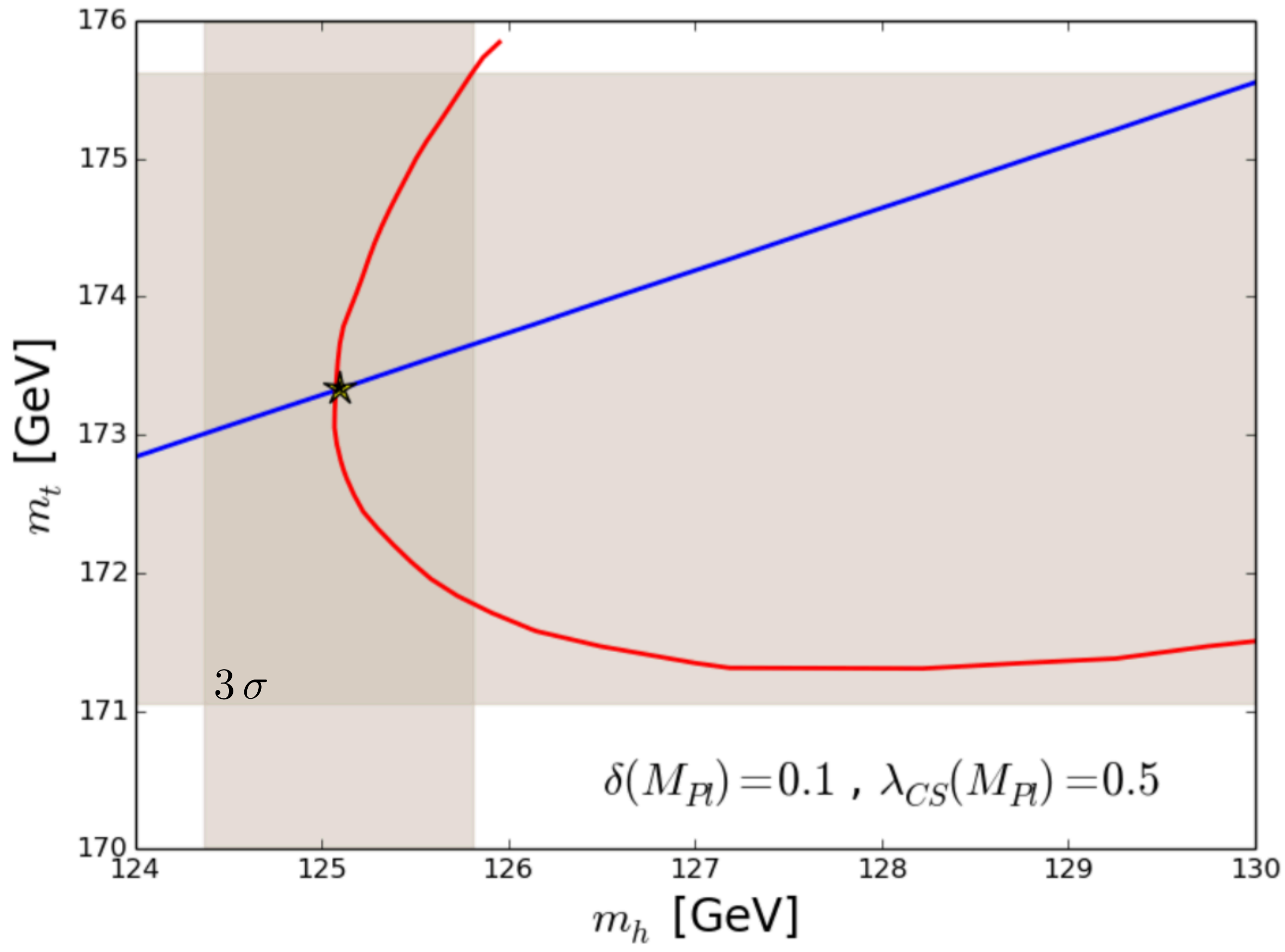
This is the sort of potential that is often used in **Higgs Portal** models to explain Dark Matter.

For simplicity, here I set $a = b = 0$. This gives an extra U(1) symmetry, so includes a massless Goldstone boson, but we will ignore this for now.

Need to make sure that the vacuum is stable: $\lambda > 0,\ \lambda_{\rm CS} > 0,\ \delta > -2\sqrt{\lambda\lambda_{\rm CS}}$

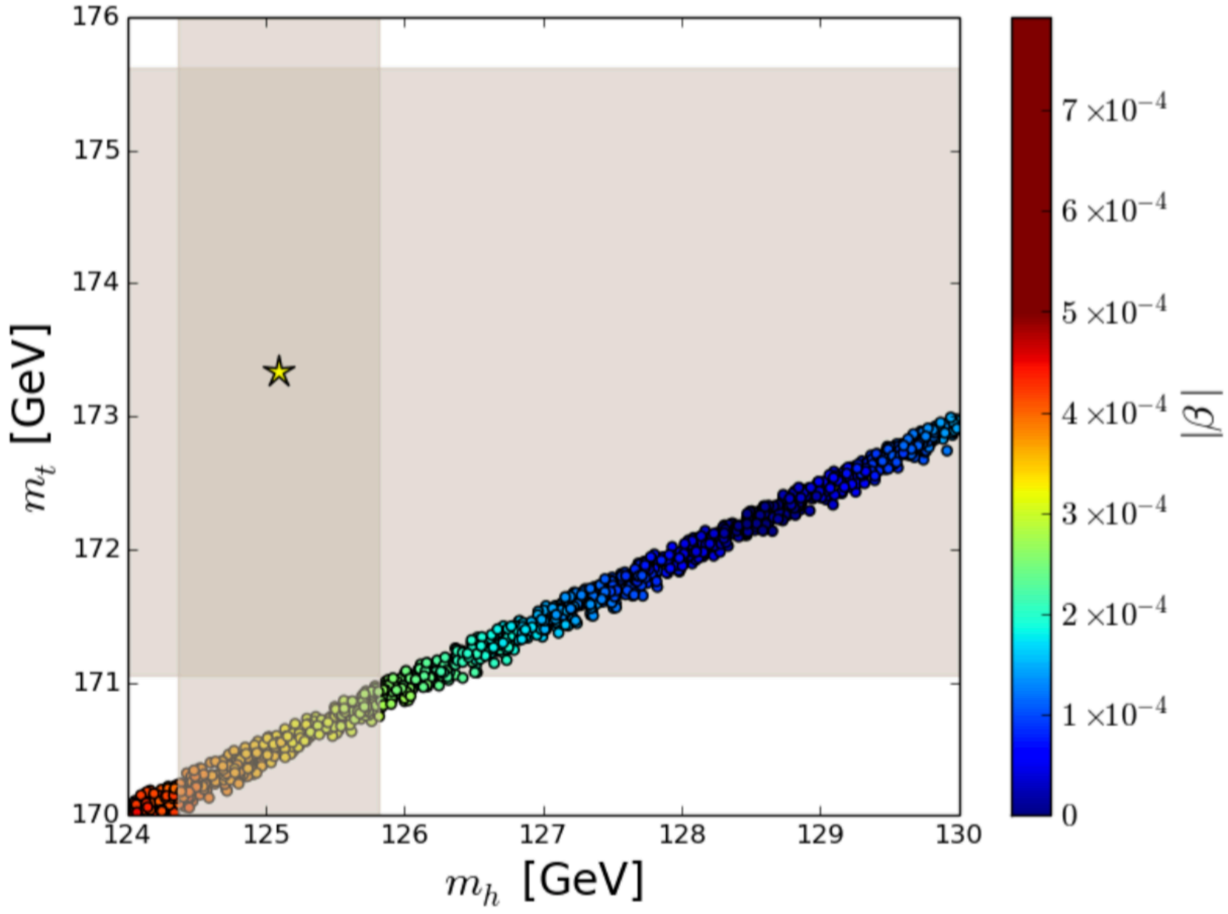
Stability is trivial at M_{Pl} if $\lambda \approx 0$ but needs to be true at all scales.

It is quite easy to find values of λ_{CS} and δ that give the right Higgs and top masses at low energies.



But this seems a bit like "cheating" because we have not imposed our boundary condition on λ_{CS}.

If we set $\beta_{\lambda_{CS}} = 0$ too then we can still do pretty well. (Here, we have still let δ vary.)



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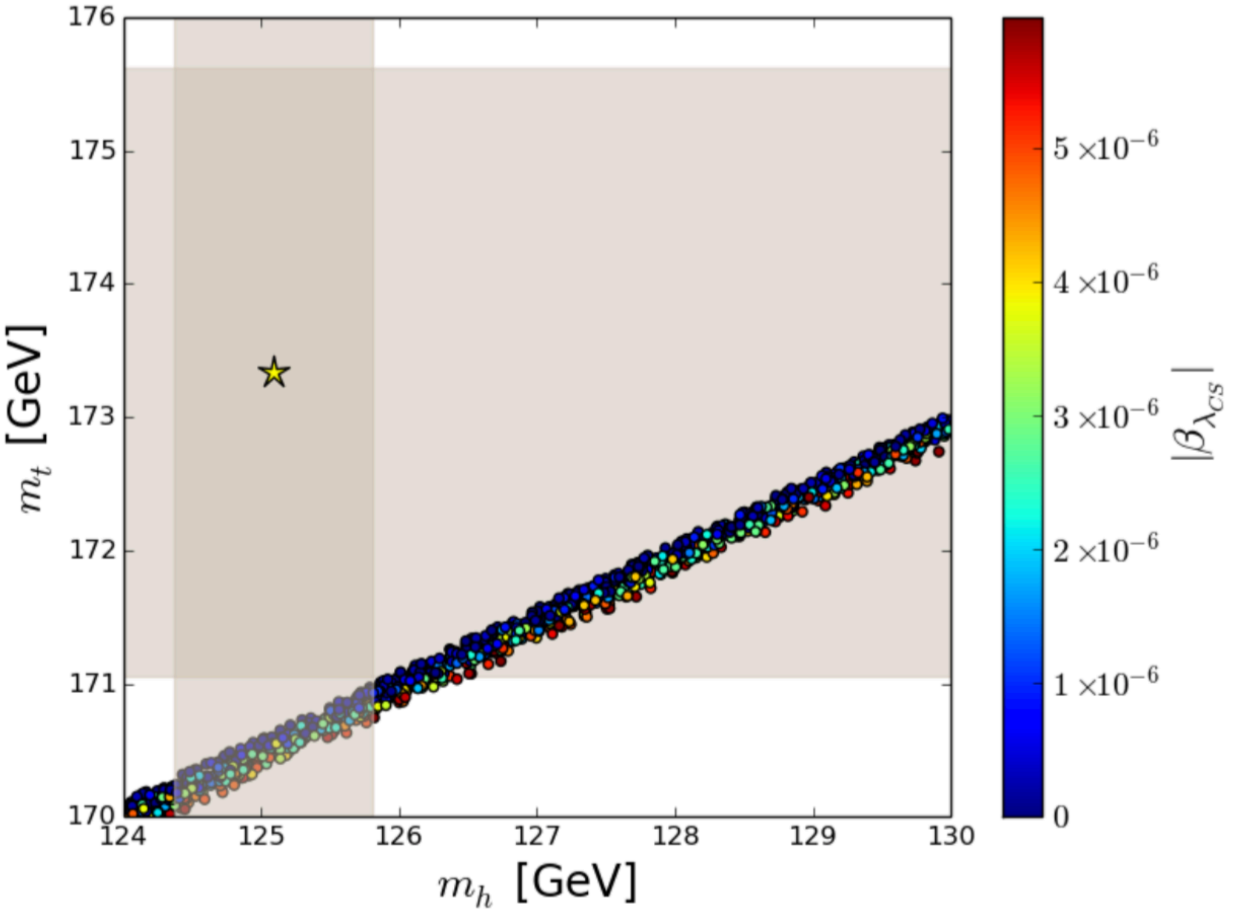


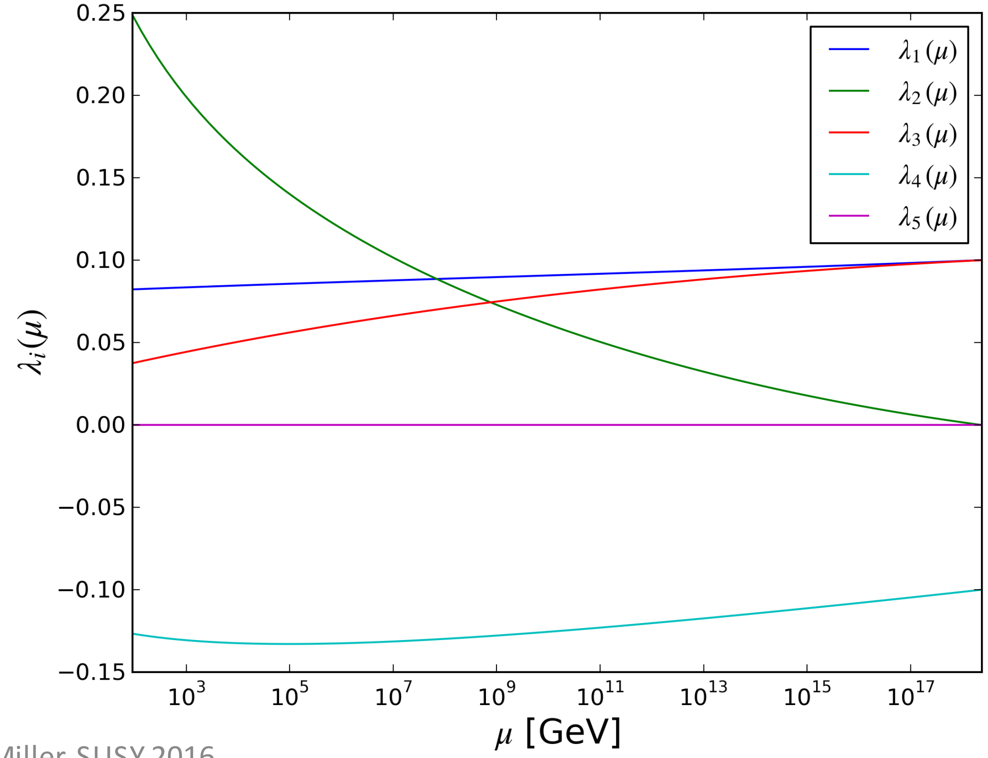
A Two-Higgs Doublet Model (Type II)

$$V(\Phi_1,\Phi_2) = m_{11}^2\Phi_1^\dagger\Phi_1 + m_2^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) + \lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2$$
$$+\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2 + \lambda_4\Phi_1^\dagger\Phi_2\Phi_2^\dagger\Phi_1 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \left(\Phi_2^\dagger\Phi_1\right)^2\right]$$
$$+\lambda_6\Phi_1^\dagger\Phi_1\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) + \lambda_7\Phi_2^\dagger\Phi_2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right)$$

Set $\lambda_5 = \lambda_6 = \lambda_7 = 0$ for simplicity (Z_2 symmetry).

λ_2 is analogous to λ in the SM, so we set this to zero at M_{Pl} and let the others vary.

See also Froggatt, Laperashvili, Nevzorov, Nielsen, Sher (2006) for a discussion of the MPP in a 2HDM.



At the Planck Scale:

$\lambda_1 \in [0, 0.5]$

$\lambda_2 = 0$

$\lambda_3 \in [0, 0.5]$

$\lambda_4 \in [-\lambda_3, 0.5]$

$m_{12} = 200\,\mathrm{GeV}$

Also:

$\tan\beta = 2$

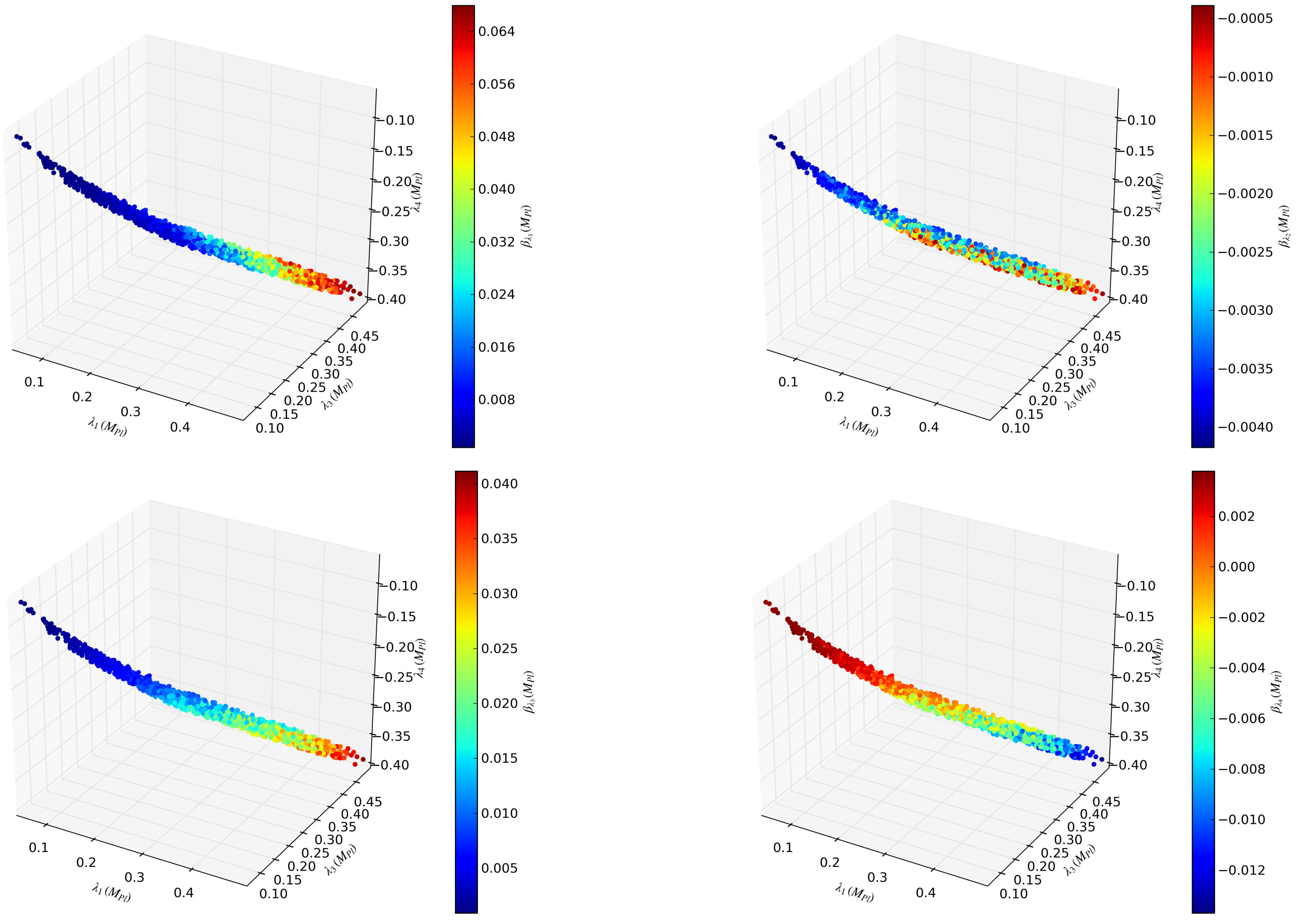
$m_t = 173.34\,\mathrm{GeV}$

$\alpha_s = 0.1181$

Apply vacuum stability conditions at all scales [Branco et al, 2011]

$$
\begin{aligned}
&\text{VSC 1:} && \lambda_1 \geq 0 \\
&\text{VSC 2:} && \lambda_2 \geq 0 \\
&\text{VSC 3:} && \lambda_3 + \sqrt{\lambda_1\lambda_2} \geq 0 \\
&\text{VSC 4:} && \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1\lambda_2} \geq 0
\end{aligned}
$$

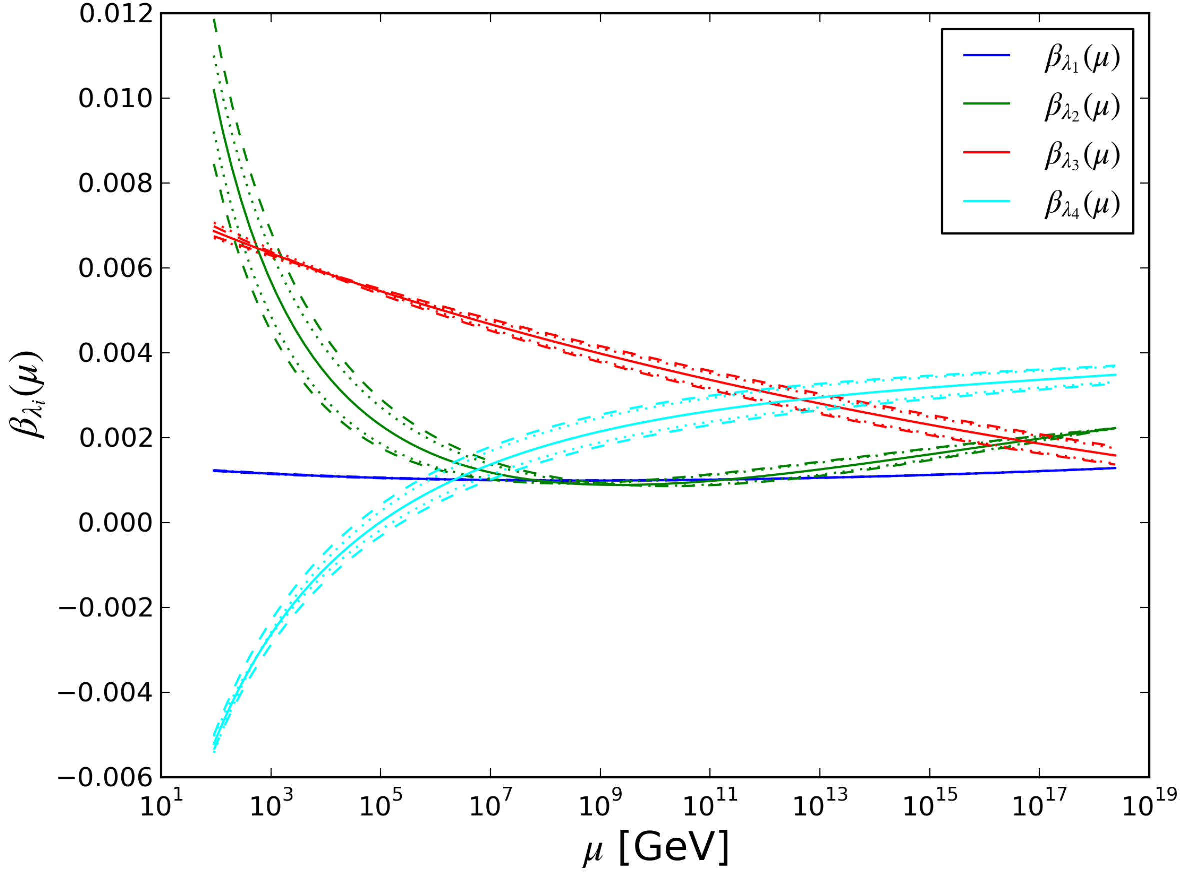
Scan over $\lambda_{1,3,4}$ and throw away any points that fail the stability conditions or give m_h outside 124 – 126 GeV.



λ1 (MPl)
λ3 (MPl)
λ4 (MPl)
βλ1 (MPl)
βλ2 (MPl)
βλ3 (MPl)
βλ4 (MPl)
0.1 0.2 0.3 0.4
0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45
−0.10 −0.15 −0.20 −0.25 −0.30 −0.35 −0.40
0.064 0.056 0.048 0.040 0.032 0.024 0.016 0.008
−0.0005 −0.0010 −0.0015 −0.0020 −0.0025 −0.0030 −0.0035 −0.0040
0.040 0.035 0.030 0.025 0.020 0.015 0.010 0.005
0.002 0.000 −0.002 −0.004 −0.006 −0.008 −0.010 −0.012

The β_λ don't quite all go to zero at the same point (and recall that this is not a fixed point).

However, this is close enough to be tantalizing!



Next Steps:

- Set $\beta_{\lambda_i} = 0$ exactly at M_{Pl} (letting λ_i vary) and see what our *prediction* for m_h is.

- Try including extra matter, such as vector-like quarks.

- Construct a non-supersymmetric GUT model (e.g. Pati-Salam, SO(10)) and examine predictions for m_h.

Conclusions

The recently measured Higgs boson mass is peculiar because it leads to $\lambda \approx \beta_\lambda \approx 0$ at the Planck scale.

Is this a coincidence or the sign of some deeper principle? Note that this is not a fixed point.

Within the SM, setting $\lambda = \beta_\lambda = 0$ at the Planck scale gives a Higgs mass that is a wee bit too high (though still rather startling).

What would this boundary condition give in a GUT model?

As a first step, we have looked at models with extended Higgs sectors.

The SM with an extra singlet, and a 2HDM both do encouragingly well at describing the Higgs mass, though the extra degrees of freedom prevent this from being a *prediction*.