Naturalness of the relaxion mechanism

A. Fowlie, C. Balazs, G. White, L. Marzola, and M. Raidal, (2016), arXiv:1602.03889 [hep-ph]

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Introduction to relaxion model

The relaxion mechanism: a clever new idea

Developed by Kaplan et al.[1] last year from related ideas by Abbott in the 1980s[2] and Dvali a decade ago.[3]

It's a very, very clever idea (Raman Sundrum)

It's definitely clever (Nima Arkani-Hamed)

Comparisons drawn to Dirac's large number hypothesis[4] — large ratios explained by dynamics/age of Universe.

[1] P. W. Graham et al., Phys. Rev. Lett. 115, 221801 (2015), arXiv:1504.07551 [hep-ph].

[2] L. F. Abbott, Phys. Lett. B150, 427 (1985).

[3] G. Dvali and A. Vilenkin, Phys. Rev. D70, 063501 (2004), arXiv:hep-th/0304043 [hep-th], G. Dvali, Phys. Rev. D74, 025018 (2006), arXiv:hep-th/0410286 [hep-th].

[4] P. A. M. Dirac, Nature 139, 323 (1937).

Hierarchy problem

SM generic prediction: weak scale $\sim$ Planck scale

- No symmetries protect scalar mass-squared parameter from quantum corrections
- In the SM as an EFT below the Planck scale, Higgs mass-squared parameter receives Planck scale quadratic corrections
- SM generic prediction is that $m_H^2 \approx m_0^2 + \beta M_P^2 \sim M_P^2$
- But, of course, we observe that $m_H^2 \lll M_P^2$

Hierarchy solutions

Traditional approaches:

- Protect weak scale with a symmetry/approximate symmetry — supersymmetry/pseudo-Goldstone
- No fundamental scalars near the weak scale — compositeness
- Close the gulf between the weak and Planck scales — large extra dimensions

Heretical approaches:

- Reinterpret quadratic divergences — physical naturalness/classical scale invariance, $\int \frac{d^4k}{k^2} \stackrel{!?}{=} 0$
- Nothing — fine-tuning of $m_0^2 + \beta M_P^2 \sim (100\,\text{GeV})^2$

New, *dynamical* approach — the relaxion mechanism

Utilise peculiar interplay between axion-like field and Higgs.[5]

Relaxion-dependent Higgs mass

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right)h^2 - m_b^3\langle h\rangle\cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4$$

Dynamics imply that relaxion field halts once $m_H^2 \lesssim 0$ such that (generic?) prediction

$$|m_H^2| = |\mu^2 - \kappa\langle a\rangle\phi| \lll M_P^2$$

Whilst $m_H^2 = 0$ doesn't enhance symmetry, it is a critical point in dynamics as $m_H^2 < 0$ triggers EWSB and a *backreaction*.

[5] P. W. Graham et al., Phys. Rev. Lett. 115, 221801 (2015), arXiv:1504.07551 [hep-ph].

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Backreaction to Higgs VEV

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right)h^2 - m_b^3\langle h\rangle\cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4$$

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Periodic barrier for axion field

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right) h^2 - m_b^3\langle h\rangle \cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4$$

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Linear slope for axion field

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right)h^2 - m_b^3\langle h\rangle\cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4$$

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Ordinary SM Higgs quartic

$$V = \left(\mu^2 - \kappa\langle a\rangle\phi\right)h^2 - m_b^3\langle h\rangle\cos\left(\frac{\phi}{f}\right) - m^2\langle a\rangle\phi + \lambda h^4$$

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[5] P. W. Graham et al., Phys. Rev. Lett. 115, 221801 (2015), arXiv:1504.07551 [hep-ph].

How the relaxion dynamics insure $m_H \lll M_P$

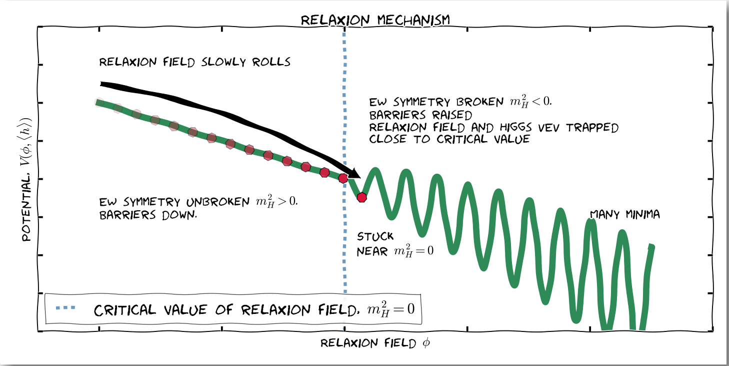

Figure 1: Higgs mass is field dependent. Relaxion field dynamics halt at $m_H \lll M_P$.

Require extra ingredient: Hubble friction, so that relaxion field dissipates energy and cannot surmount barriers.

Approximate solution for dynamics

The potential involves a cosine: the tadpoles are transcendental and have no closed-form solutions.

- OK: just write squiggles $\sim$
- Better: find intervals bounding solutions to tadpoles and solve numerically
- Better still: solve the dynamics by evolving initial conditions

We chose middle approach. Sidesteps issues about prior for initial conditions (e.g. constructing a Liouville measure[6]).

Assume that, because of Hubble friction, ultimately field always stuck in first minima. This may not be true.[7]

[6] G. W. Gibbons et al., Nucl. Phys. B281, 736 (1987).

[7] J. Jaeckel et al., Phys. Rev. D93, 063522 (2016), arXiv:1508.03321 [hep-ph].

Find intervals by graphing

Solving tree-level tadpoles, we find

$$\sin(\phi/f) = \frac{f\kappa\langle a\rangle}{m_b^3}\left(\frac{m^2/\kappa + \langle h\rangle^2}{\langle h\rangle}\right)$$

For phenomenologically viable points, confirm literature expressions for weak scale

$$\langle h\rangle \approx f\frac{m^2\langle a\rangle}{m_b^3}$$

and that $\theta_{\text{QCD}} \approx \pi/2$.

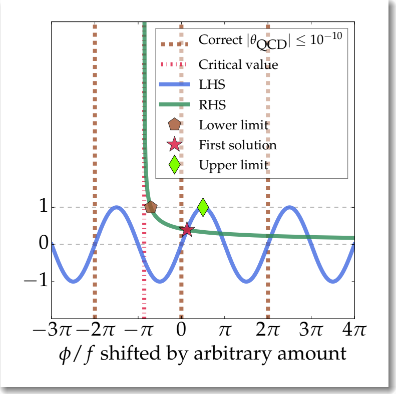

Figure 2: Tadpole equations. Frequency actually much higher, such that $\theta_{\text{QCD}} \gg 0$.

First epoch of inflation

Slowly-rolling fields generate Hubble friction during relaxion mechanism. Assume that it satisfies a constraint on the vacuum energy,

$$H^2 M_P^2 < \frac{\mu^2 m^2}{\kappa}$$

that classical beats quantum behaviour,

$$H^3 < m^3 \langle a \rangle$$

and that

$$H \lesssim m_b$$

This results in a constraint upon the relaxion parameters,

$$\sqrt{\frac{\mu^2 m^2}{\kappa}} < M_P \min(m_b, m^{2/3} \langle a \rangle^{1/3})$$

We impose this latter constraint, but assume that an acceptable Hubble constant is generated at no cost.

Second epoch of inflation

The first epoch of inflation does not satisfy constraints on density fluctuations $\delta\rho/\rho$. We require a second epoch of inflation that generates our cosmological observables.

The second epoch of inflation must satisfy

$$H \lesssim m_b$$

to prevent destruction of the periodic barriers. Fine-tuned as typically $H \ggg M_W$.

Calculate inflationary observables measured by Planck/BICEP, n_S, A_S, and r, with η and ϵ slow-roll parameters.

Bayesian approach to fine-tuning/naturalness

Built/solved relaxion model. Does it *actually* solve fine-tuning problem? Is it less fine-tuned than SM?

What is fine-tuning?

Something to do with QFT? EFT? Quadratic divergences? Barbieri-Giudice? Aesthetic? A lot of confusion.

Bayesian statistics is a *unique* logical framework for quantifying the plausibility of a model.[8]

Includes an automatic Occam's razor/penalty for fine-tuning.

Anything that was correct/logical about old-fashioned fine-tuning arguments is automatically included in Bayesian statistics. Everything that wasn't, isn't.

[8] H. Jeffreys, (Oxford University Press, 1939), E. T. Jaynes, (Cambridge University Press, 2003).

What do you calculate?

Calculate the Bayesian evidence for each model under consideration

$$p(D \mid M) = \int p(D \mid M, \boldsymbol{p}) \cdot p(\boldsymbol{p} \mid M) \prod \mathrm{d}\boldsymbol{p}$$

Probability of data given point in model (likelihood).

Probability of point given model (prior). *Somewhat* subjective, though should reflect knowledge or ignorance about parameters.

Compare the evidences in a so-called Bayes-factor:

$$p(D \mid M_b) / p(D \mid M_a) \propto p(M_b \mid D) / p(M_a \mid D)$$

which is proportional to the posterior odds. May not agree with frequentist methods, even with *informative* data.[9]

[9] D. V. Lindley, Biometrika 44, 187–192 (1957), M. S. Bartlett, Biometrika 44, 533–534 (1957).

Why Bayesian evidence captures fine-tuning in one slide[10]

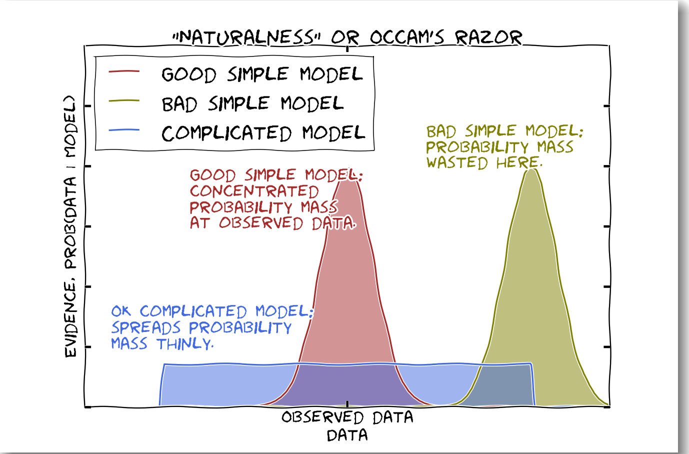

Figure 3: Bayesian evidence captures old-fashioned ideas about FT. SM is a bad simple model: concentrates probability at $M_Z \sim M_P$.

[10] D. Mackay, (1992).

Bayes-factors (i.e. naturalness) of SM vs. relaxion model

What we calculated: models/priors

We looked at the SM + mixed inflation

$$V = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda_\sigma \sigma^4.$$

A canonical model of the weak scale and inflation.

Two relaxion models: QCD relaxion and non-QCD relaxion augmented with a renormalizable inflationary potential,

$$V = m_3^3 \sigma + \frac{1}{2} m_2^2 \sigma^2 + \frac{1}{3} m_1 \sigma^3 + \frac{1}{4} \lambda_\sigma \sigma^4.$$

A general model of inflation that could satisfy $H \lesssim m_b$.

We picked non-informative priors for the Lagrangian parameters (typically logarithmic, since we are ignorant of their scale).

What we calculated: data/likelihood functions

We considered $M_Z \approx 90\,\text{GeV}$, constraints on the axion decay constant and θ_{QCD}, and the inflationary observables r, n_S, and A_S measured by Planck/BICEP.

The likelihood functions were Gaussians or step-functions. We added data one by one to assess their impact.

All scalar masses received Planck-scale quadratic corrections $\Delta m^2 \sim M_P^2$.

We calculated the evidence, $p(D\,|\,M)$, for each model with `MultiNest`.[11]

[11] F. Feroz et al., Mon. Not. Roy. Astron. Soc. 398, 1601–1614 (2009), arXiv:0809.3437 [astro-ph], F. Feroz et al., (2013), arXiv:1306.2144 [astro-ph.IM].

Experimental data

Parameter	Measurement
M_Z	91.1876 ± 0.0021 GeV
f_a	$f_a \gtrsim 10^9$ GeV
$\lvert\theta_{\mathsf{QCD}}\rvert$	$\lvert\theta_{\mathsf{QCD}}\rvert \lesssim 10^{-10}$
r	$r < 0.12$ at 95%
n_s	0.9645 ± 0.0049
$\ln(10^{10} A_s)$	3.094 ± 0.034

Table 1: Data included in our evidences, $p(D \mid M)$.

For table of priors, see full paper.[12]

[12]A. Fowlie et al., (2016), arXiv:1602.03889 [hep-ph].

SM vs. relaxion: Applying no constraints (not even inequalities)

SM vs. QCD relaxion

- SM wastes probability near Planck scale
- Relaxion model makes a broad prediction, much greater at correct weak scale
- Z-mass vastly favours relaxion models in Bayes-factor
- Compare with Fig. 3

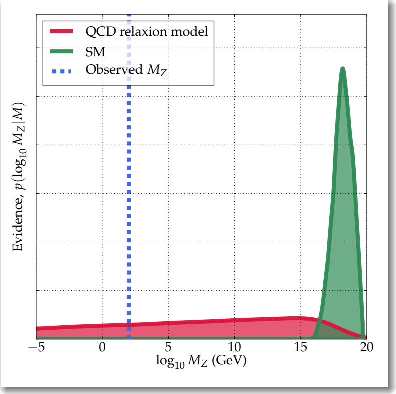

Figure 4: Evidence of model as function of $\log_{10} M_Z$

SM vs. relaxion: Adding data...

Weak scale + physicality conditions + inflation

- QCD and non-QCD relaxion *much worse* than SM
- Bayes-factors favour SM by many orders of magnitude
- Relaxion model destroyed by fine-tuned inflation and constraints on Hubble parameter during relaxation
- SM + scalar-field inflation 10^{24} times more plausible than relaxion model (after seeing all data)
- Hierarchies introduce enormous factors in Bayes-factors

SM vs. relaxion: in numbers

Data-set	M_Z only	All data
Evidence of SM + inflation $\cdot$ GeV	10^{-34}	10^{-53}
Evidence of non-QCD relaxion $\cdot$ GeV	10^{-4}	10^{-77}
Evidence of QCD relaxion $\cdot$ GeV	10^{-4}	$\lll 10^{-81}$

Table 2: Bayesian evidences for SM and relaxion models.

Many more numbers in paper.

Considering only M_Z, relaxion models favoured, but with all data, relaxion models much worse than SM.

The evidence for the QCD relaxion model is effectively zero, as it makes a bad prediction for $|\theta_{\text{QCD}}|$.

Caveats

Bayes-factors *valid only for models under consideration* and may not apply to your favourite relaxion + inflation model

These Bayes-factors are *not* the final word for relaxion models in general

But any claims that more complicated relaxion models improve naturalness should be accompanied by calculations of Bayes-factors (or follow similar qualitative reasoning)

Conclusions

Naturalness of the relaxion mechanism = unnatural

- First statistical analysis of relaxion model.
- Bayesian statistics includes *automatic* penalties for fine-tuning/naturalness. *No cheating. Nothing by hand. No Barbieri-Giudice fine-tuning measures.*
- Found that, all told, relaxion models were much less plausible than SM + single-field inflation.
- Problems with unusual cosmology.
- Arguably overlooked issues with relaxion model that would further damage its plausibility.
- Strictly speaking, conclusions applicable only to simple relaxion models under consideration, though results are not promising for relaxion models in general.

Questions?

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