

Renormalization-scale uncertainty in the decay rate of false vacuum

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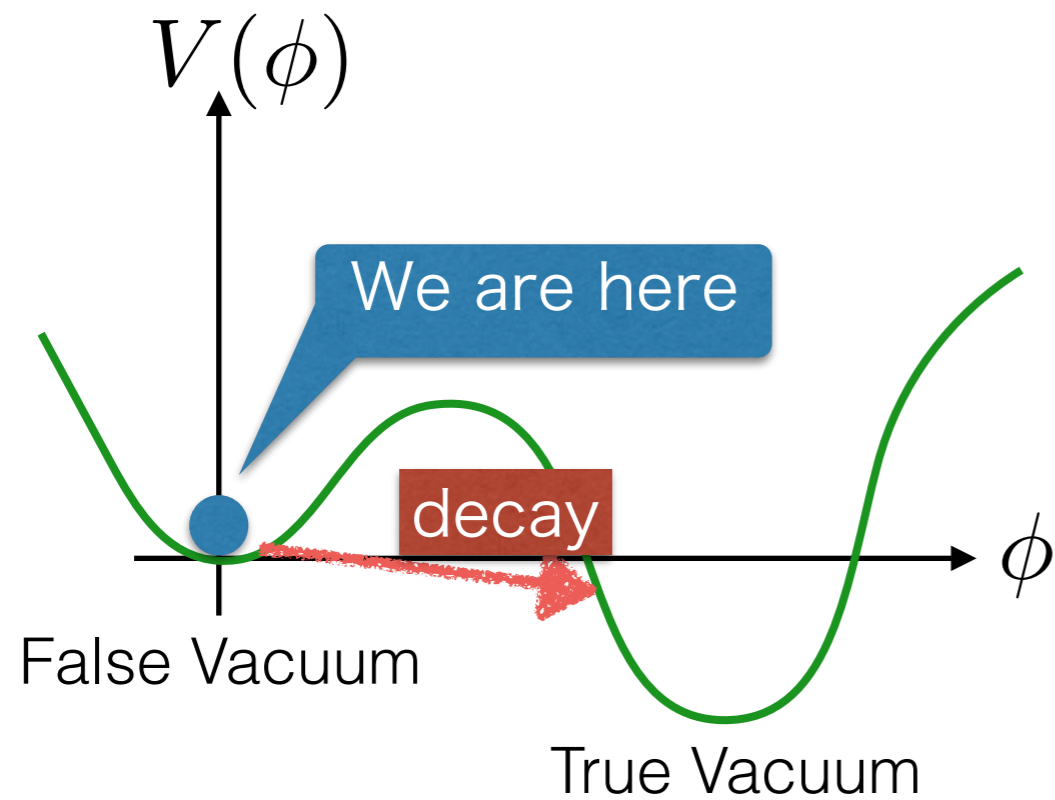
Collaborators: Motoi Endo (KEK), Takeo Moroi (UTokyo), Mihoko M. Nojiri (KEK, IPMU)

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- Introduction
- scale uncertainty and 1-loop calculation
- SM + stau system
- Summary

Introduction

Vacuum decay



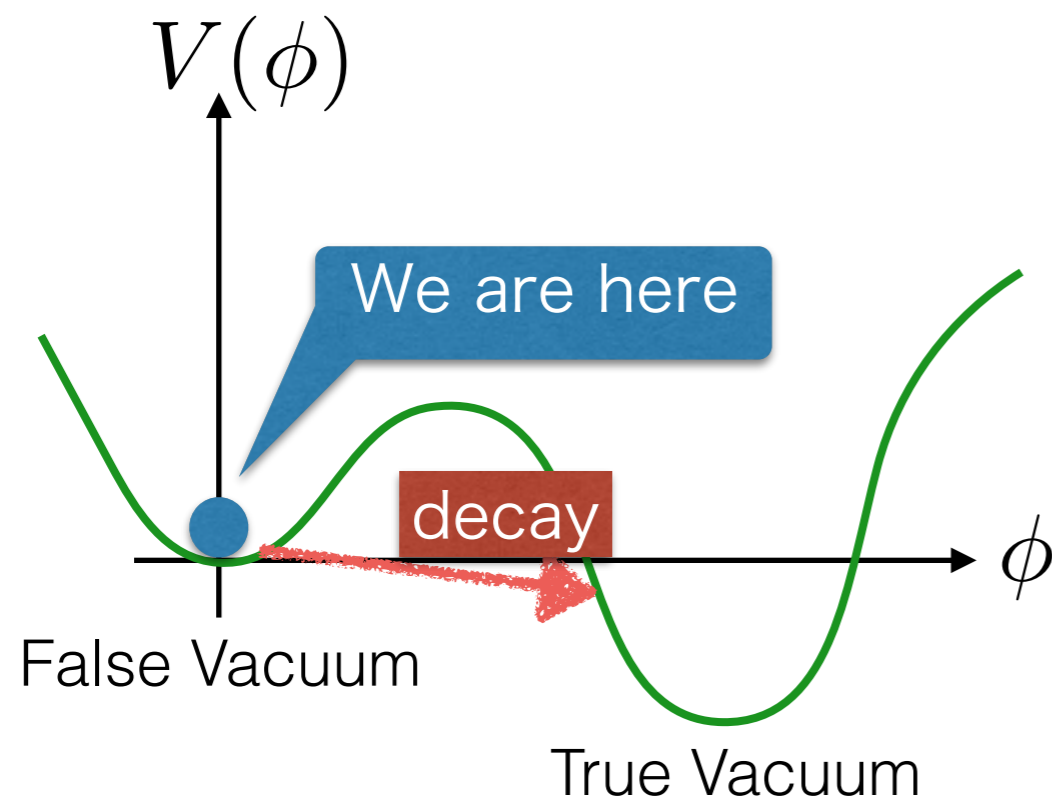
Vacuum decay

Tree level potential

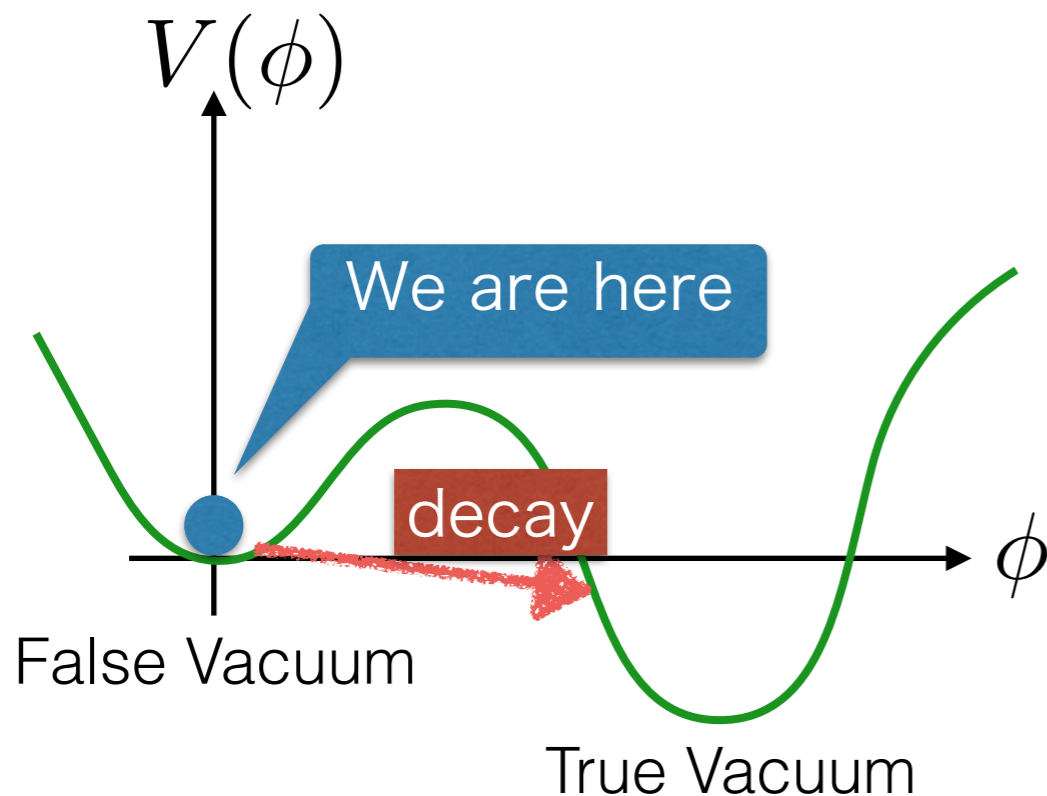
ex.) Supersymmetry

$h\tilde{t}_L\tilde{t}_R$ Higgs mass, hgg , $h\gamma\gamma$, ...

$h\tilde{l}_L\tilde{l}_R$ muon $g-2$, $h\gamma\gamma$, ...



Vacuum decay



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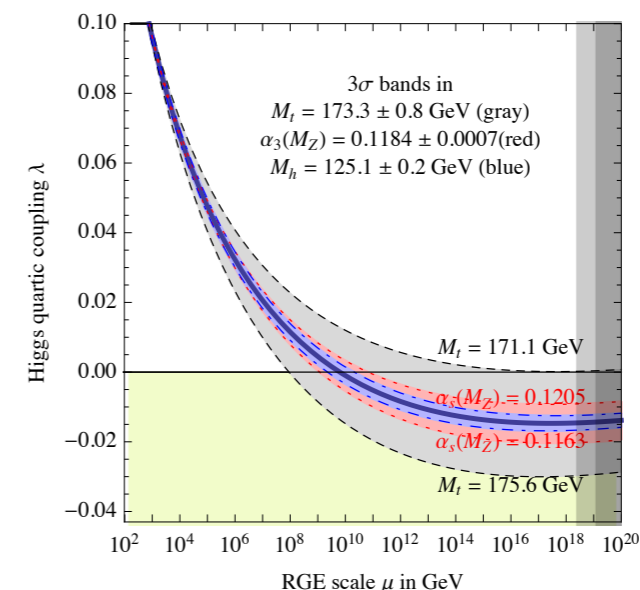
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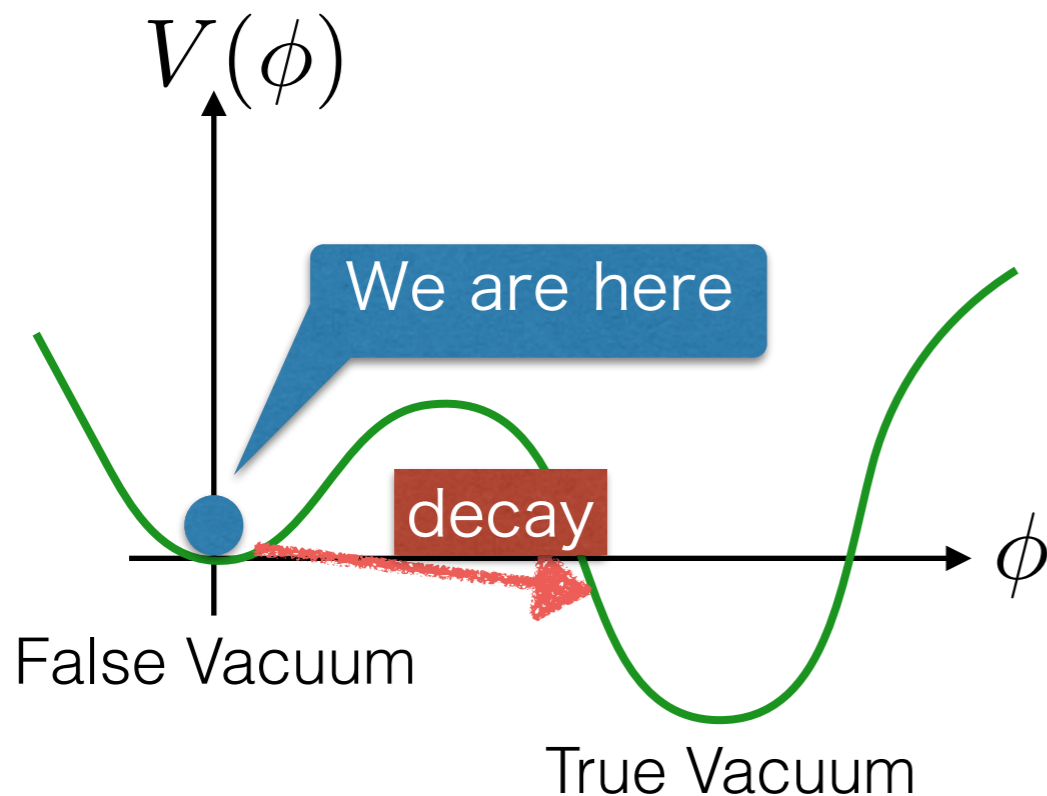
Effective potential

ex.) Standard Model



D. Buttazzo, et al.,
1307.3536/hep-ph

Vacuum decay



Tree level potential

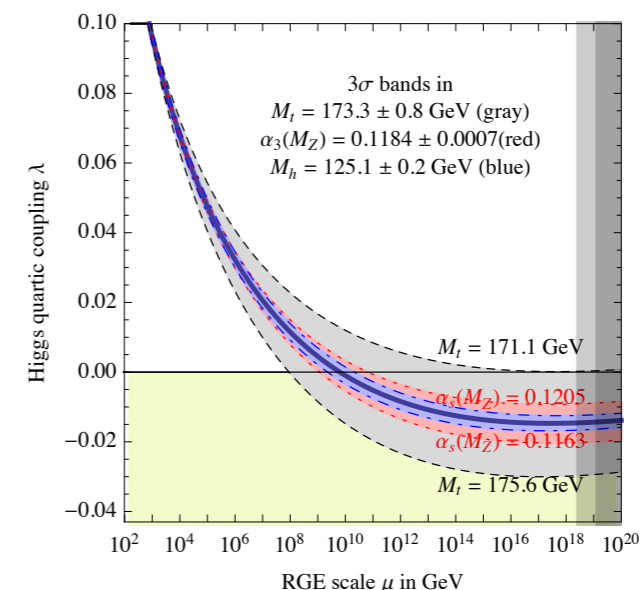
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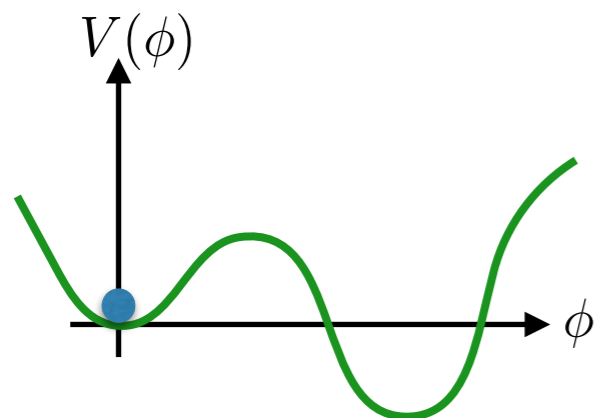
Effective potential

ex.) Standard Model



Decay rate

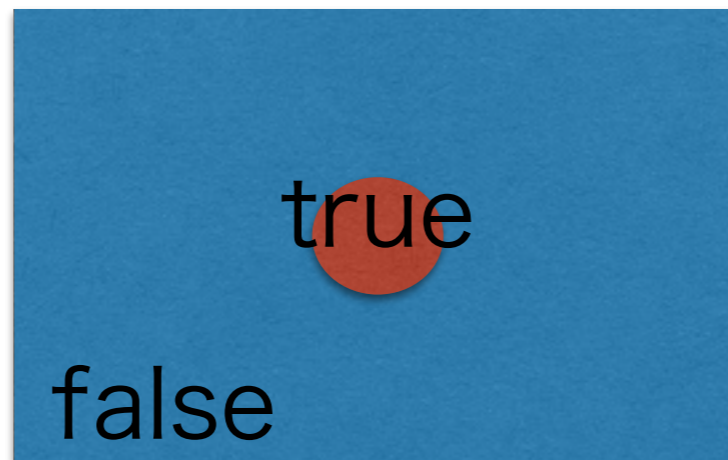
“supercooled” state



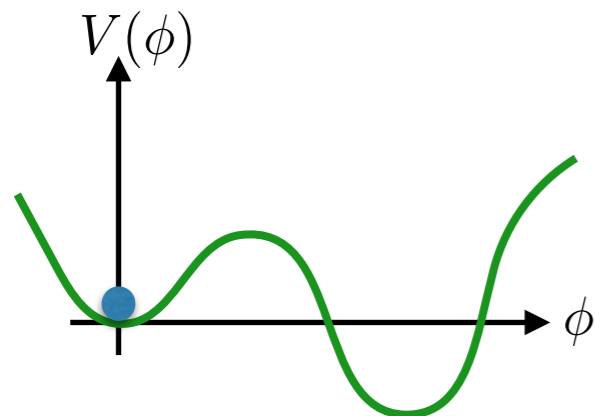
Decay rate

“supercooled” state

bubble nucleation



quantum / thermal fluctuation

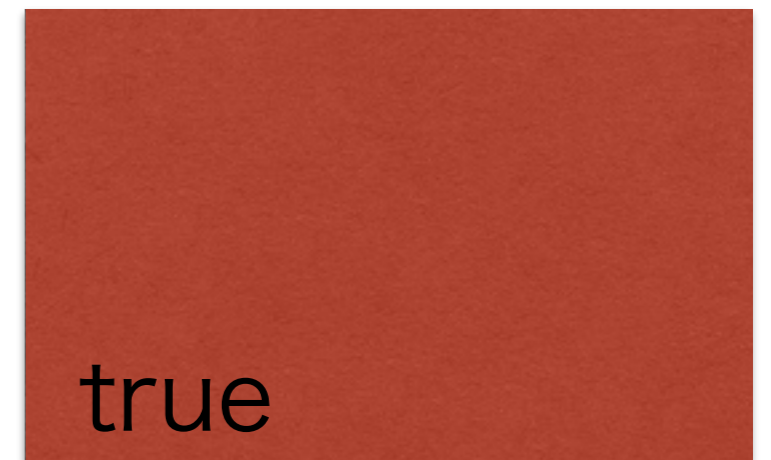
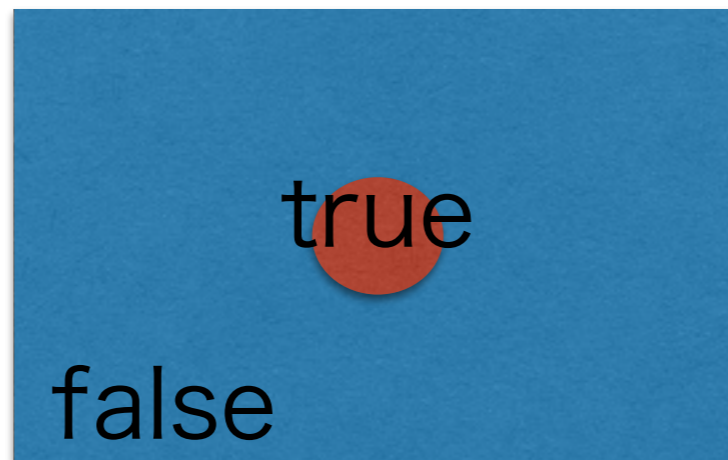


Decay rate

“supercooled” state

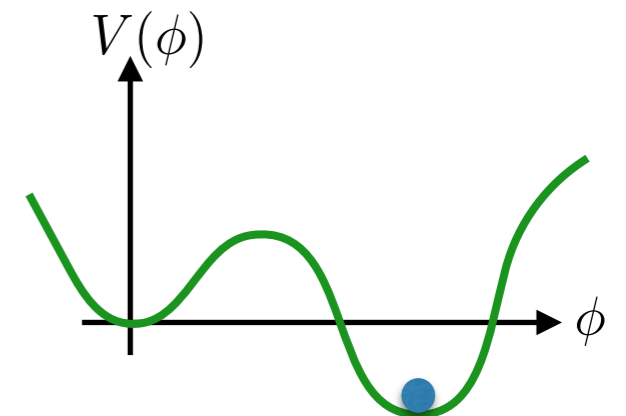
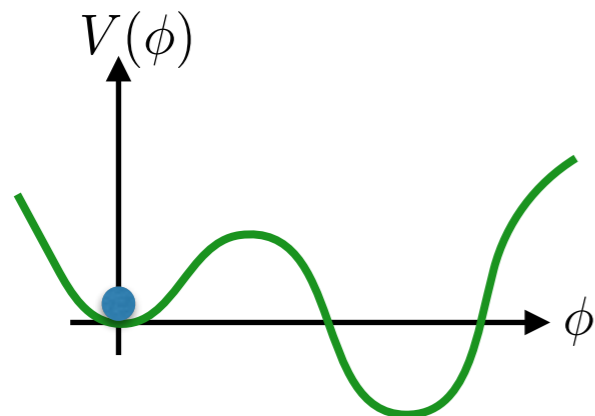
bubble nucleation

ground state



quantum / thermal fluctuation

bubble expansion

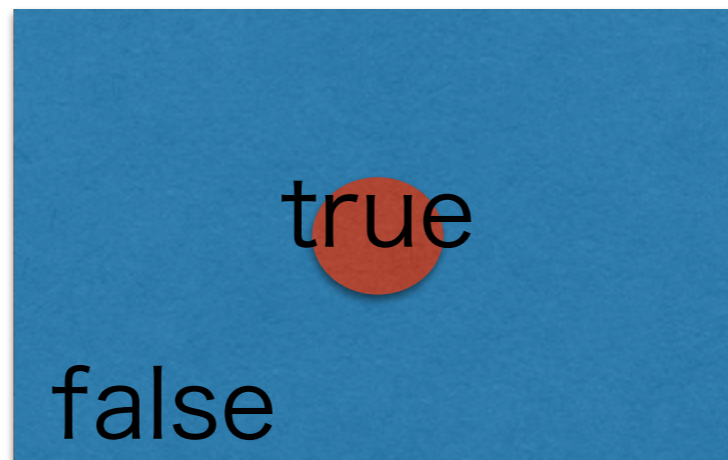


Decay rate

“supercooled” state

bubble nucleation

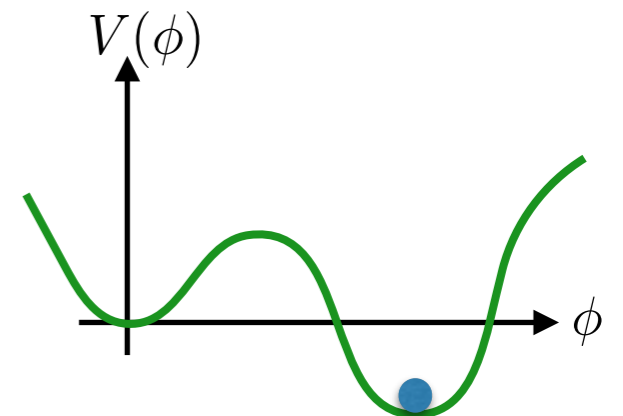
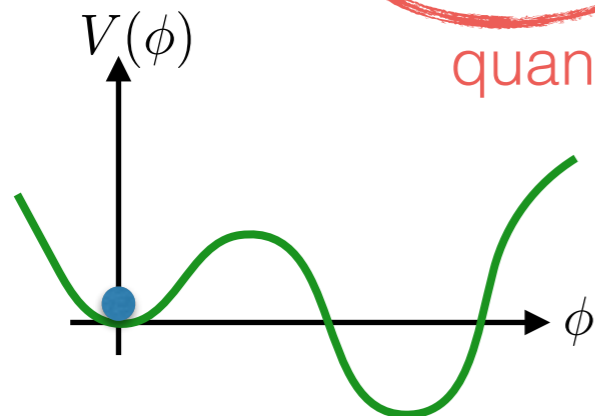
ground state



quantum / thermal fluctuation

bubble expansion

quantum jump to the same energy state



Decay rate

Bubble nucleation rate

$$\gamma = Ae^{-B}$$

“WKB” in QM

bounce action

$$B = S(\phi_B)$$

dim. = 4

1/(time*volume)

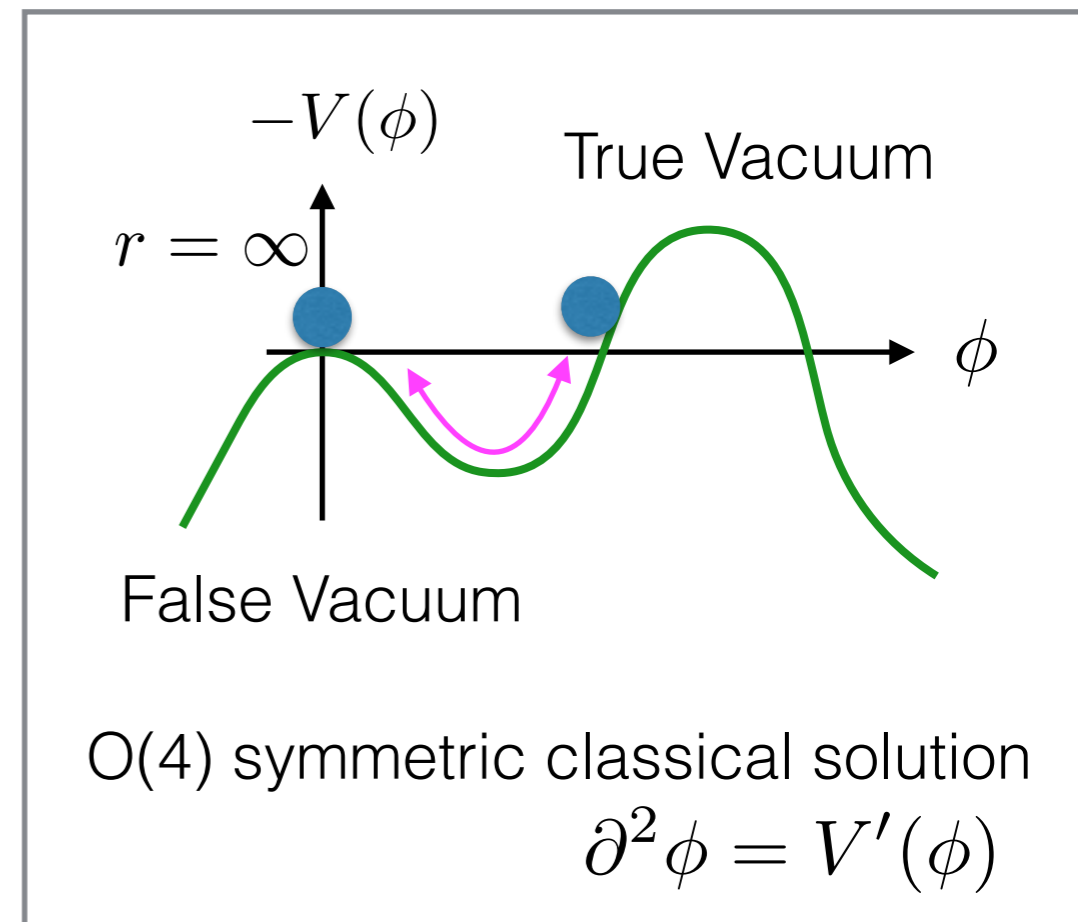
Pre-exponential factor

$$A \sim \mu^4$$

\

typical scale

bounce solution



Toy model

Potential

$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

Toy model

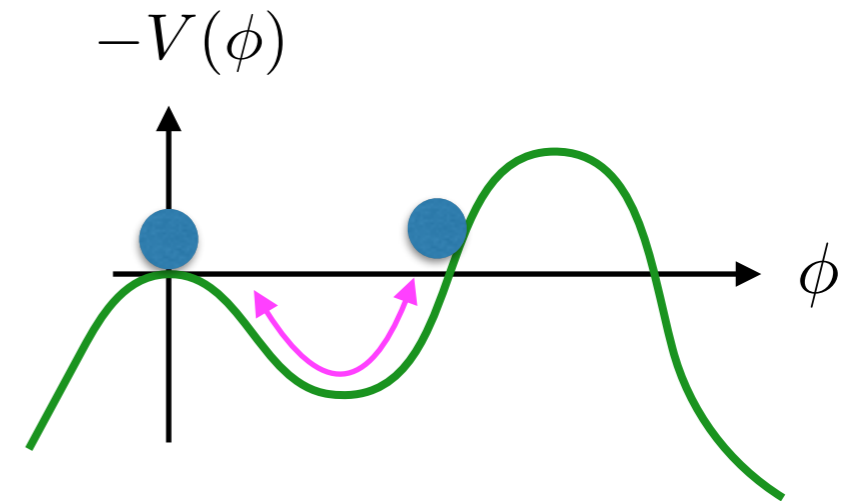
Potential

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Bounce

$$\partial^2 \phi = V'(\phi)$$



Toy model

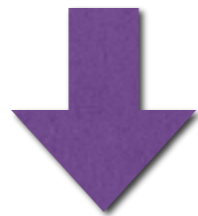
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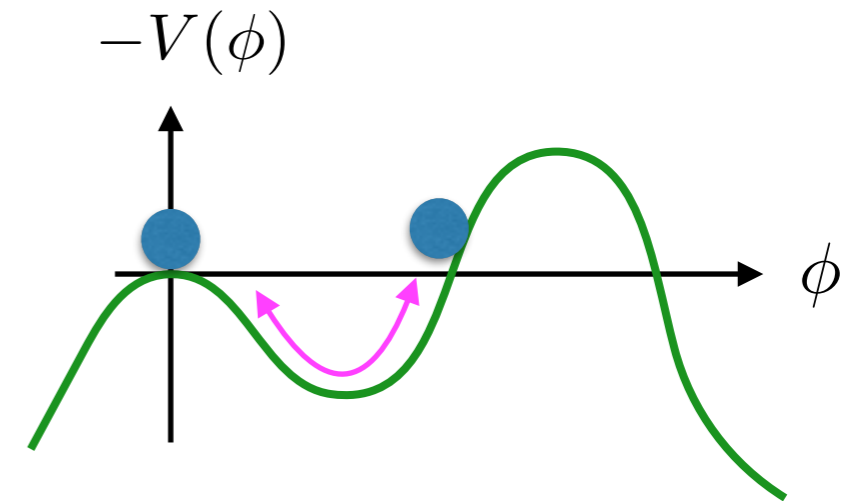
Bounce

$$\partial^2 \phi = V'(\phi)$$



Action

$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B) \right]$$



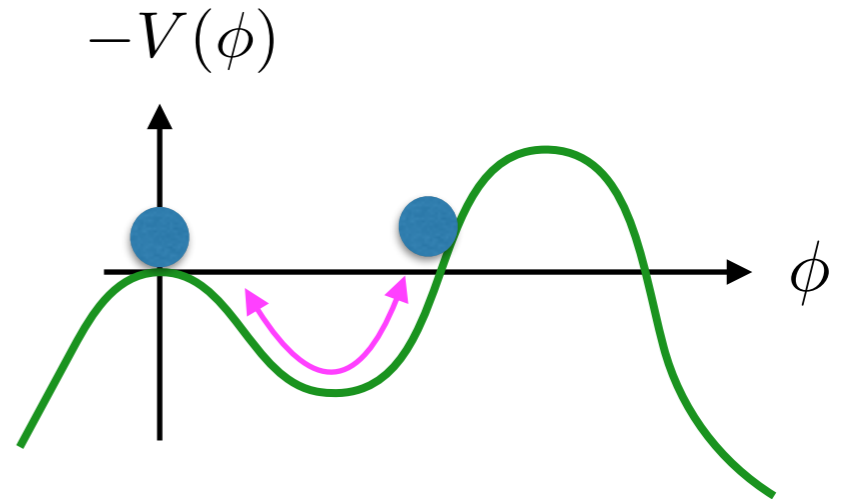
Toy model

Potential

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Decay rate

$$\gamma \simeq m^4 e^{-B}$$

Toy model

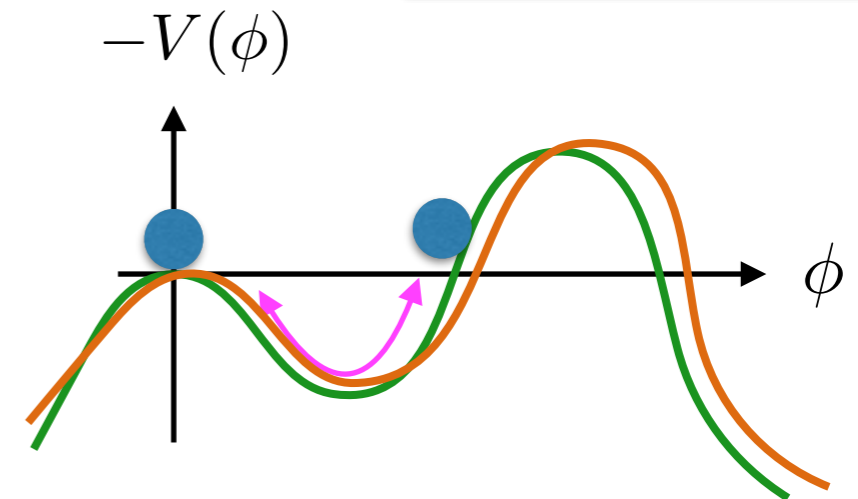
Potential

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Scale dependent

Bounce

$$\partial^2 \phi = V'(\phi)$$



Action

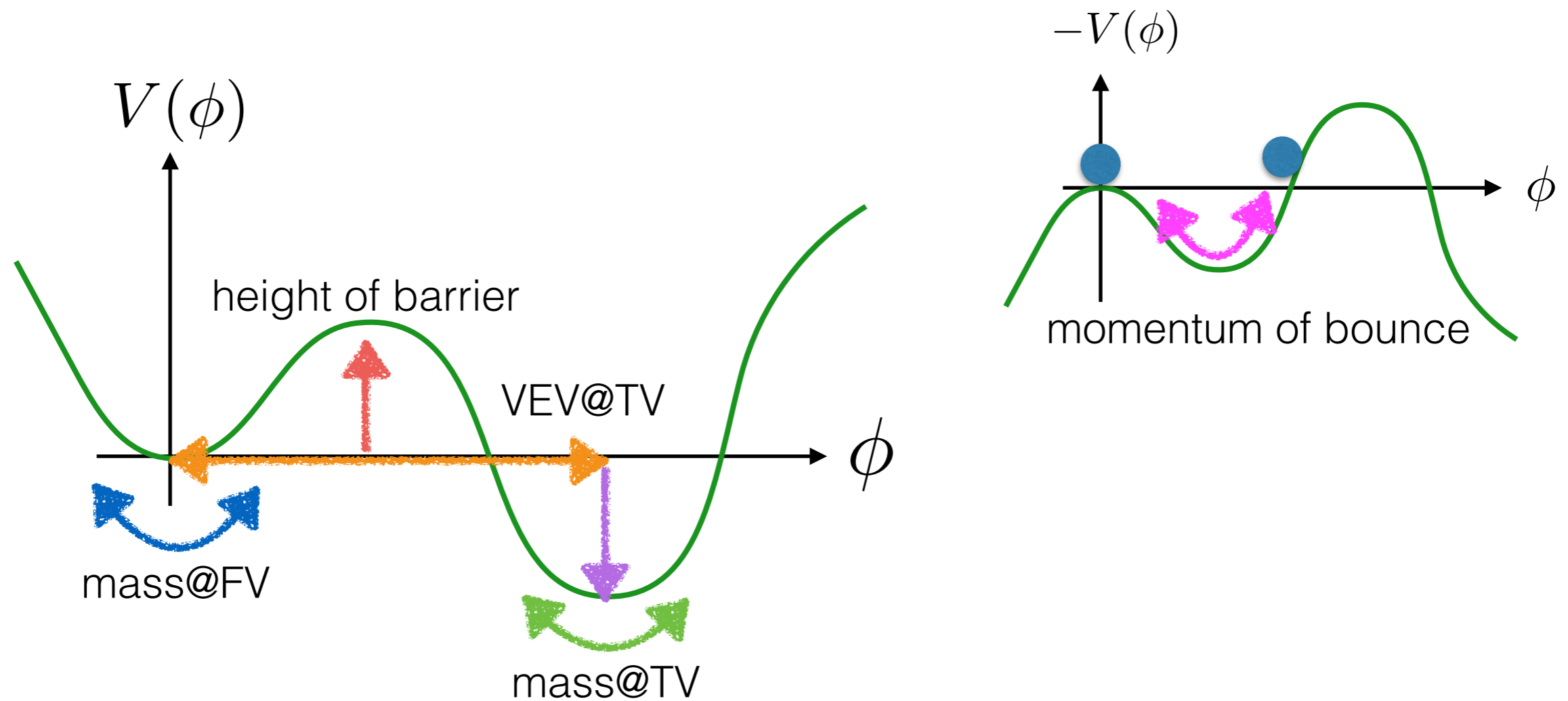
$$B = S_E(\phi_B) = \int d^4x \left[\frac{1}{2}(\partial\phi_B)^2 + V(\phi_B) \right]$$

Decay rate

$$\gamma \simeq m^4 e^{-B}$$

Scale?

Renormalization scale



There are too many relevant scales...

How large is the scale dependence?

$$V = -\bar{t}(Q)\phi + \frac{\bar{m}^2(Q)}{2}\phi^2 - \frac{\bar{A}(Q)}{2}\phi^3 + \frac{\bar{\alpha}(Q)}{8}\phi^4$$

Beta functions

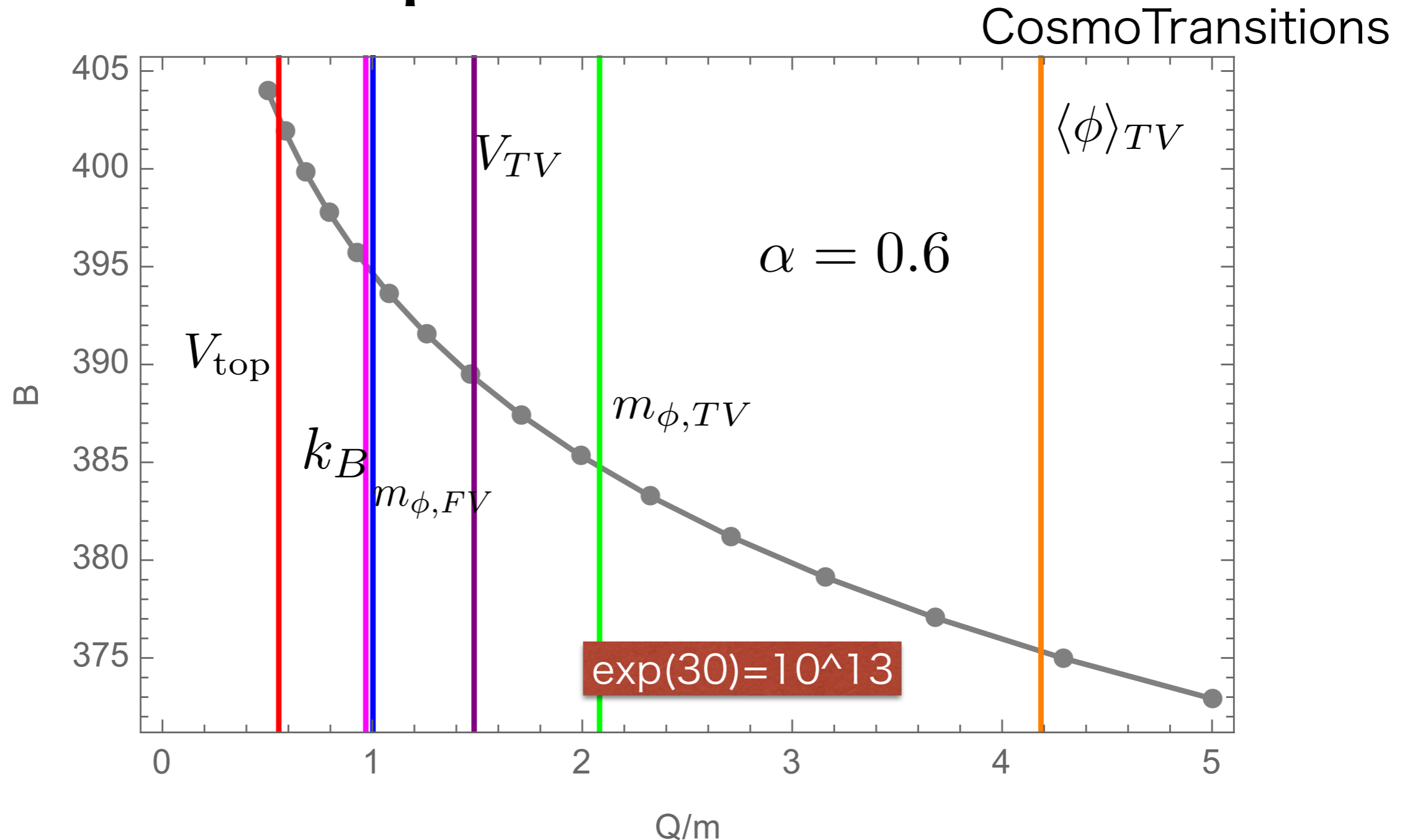
$$\beta_t = \frac{3Am^2}{16\pi^2} \quad \beta_{m^2} = \frac{3}{16\pi^2}(\alpha m^2 + 3A^2)$$
$$\beta_A = \frac{9\alpha A}{16\pi^2} \quad \beta_\alpha = \frac{9\alpha^2}{16\pi^2}$$

Renormalization conditions

$$@ Q = m$$

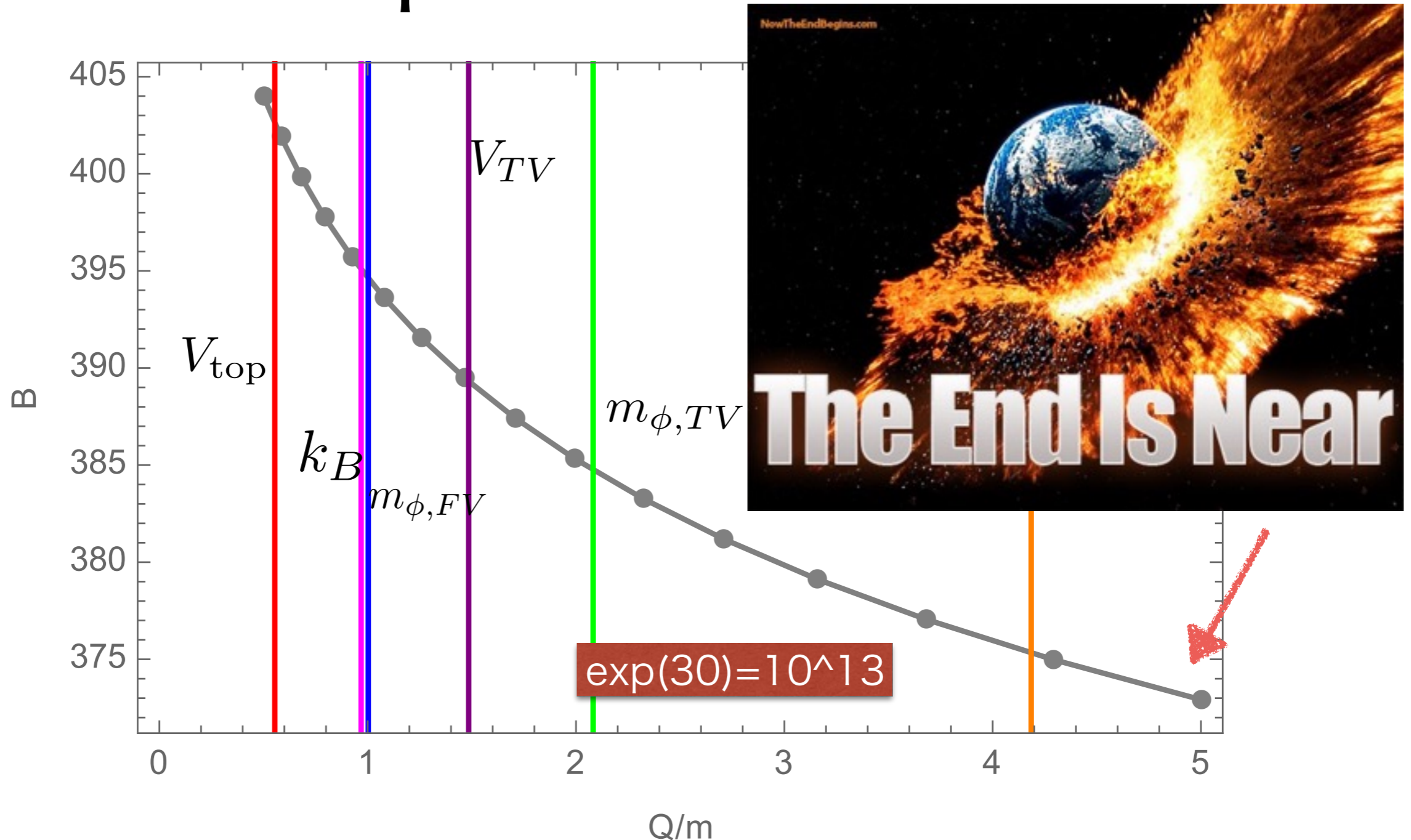
$$\bar{m}^2(m) = m^2, \quad \bar{A}(m) = m, \quad \bar{t}(m) = 0, \quad \bar{\alpha}(m) = \alpha$$

How large is the scale dependence?



can be much larger in a realistic model (top loop)

How large is the scale dependence?



can be much larger in a realistic model (top loop)

1-loop calculation

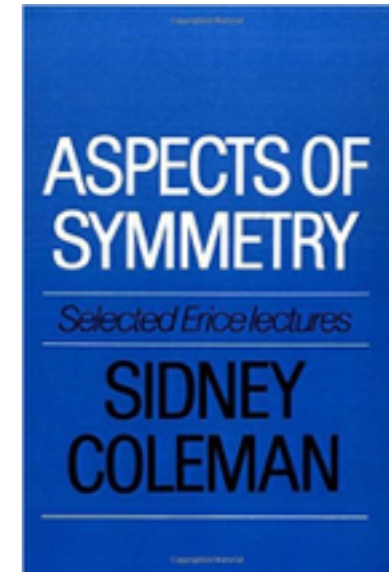
Pre-exponential factor

$$\gamma = Ae^{-B}$$

fluctuations around the bounce



$$A = \frac{B^2}{4\pi^2} \left(\frac{\det' S''|_{\text{Bounce}}}{\det S''|_{\text{False}}} \right)^{-1/2}$$



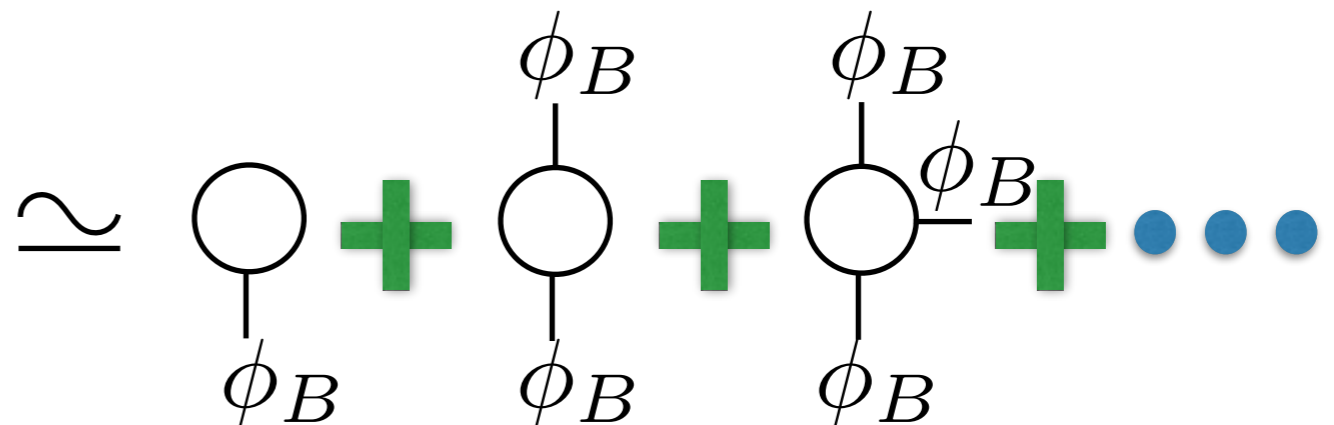
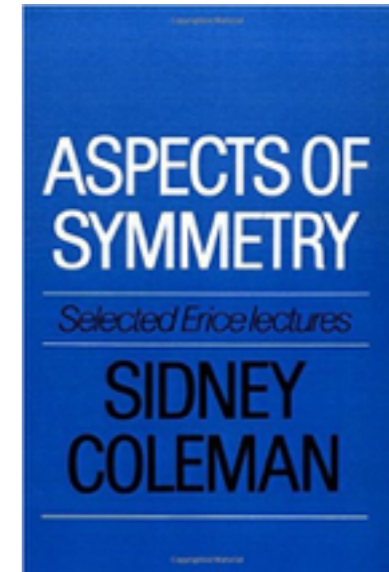
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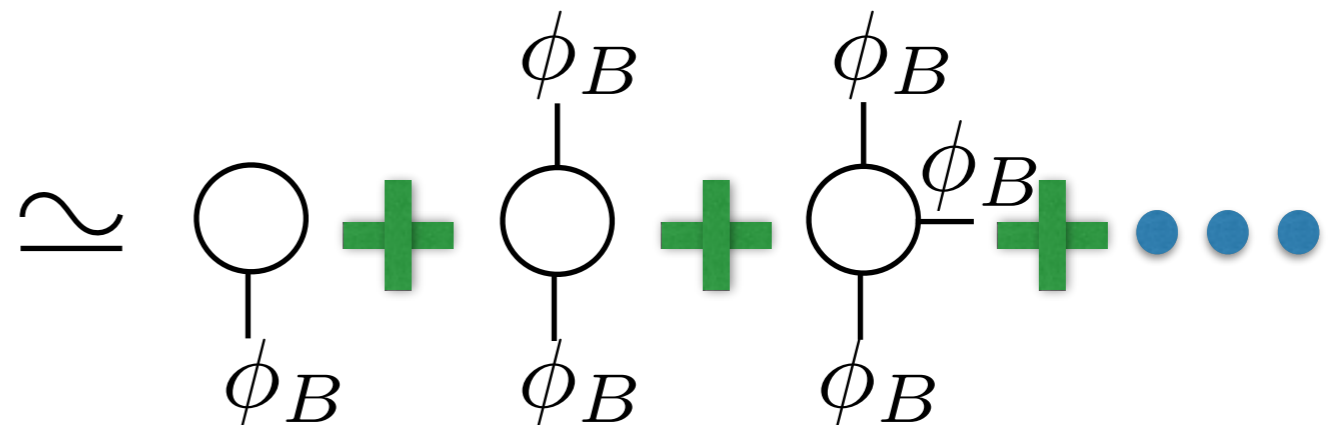
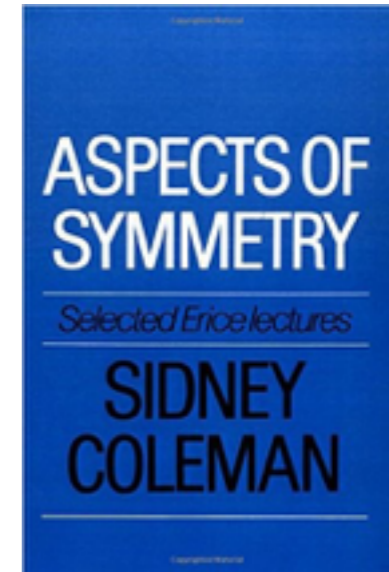
Pre-exponential factor

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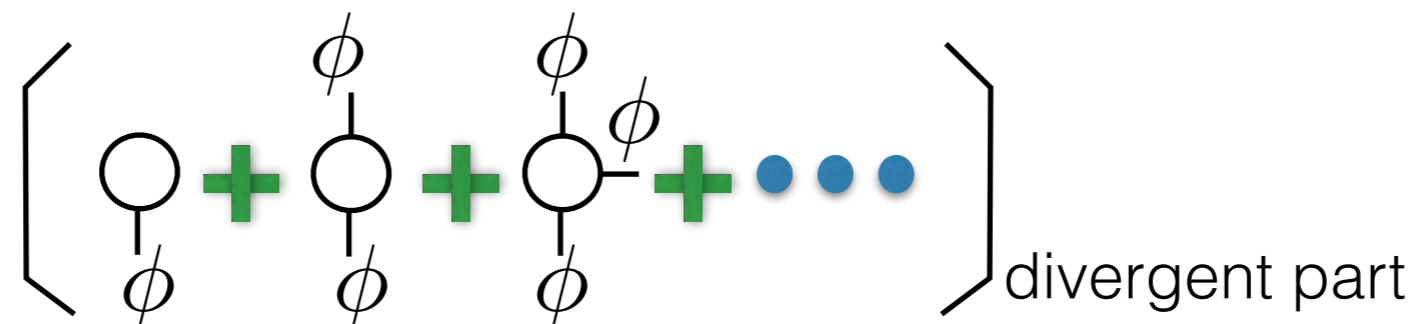
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cf.) RGE is related to



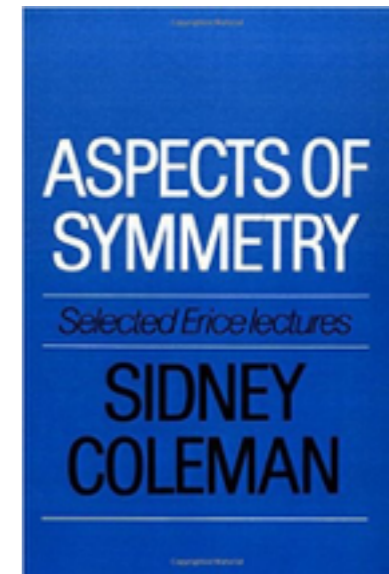
Pre-exponential factor

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fluctuations around the bounce

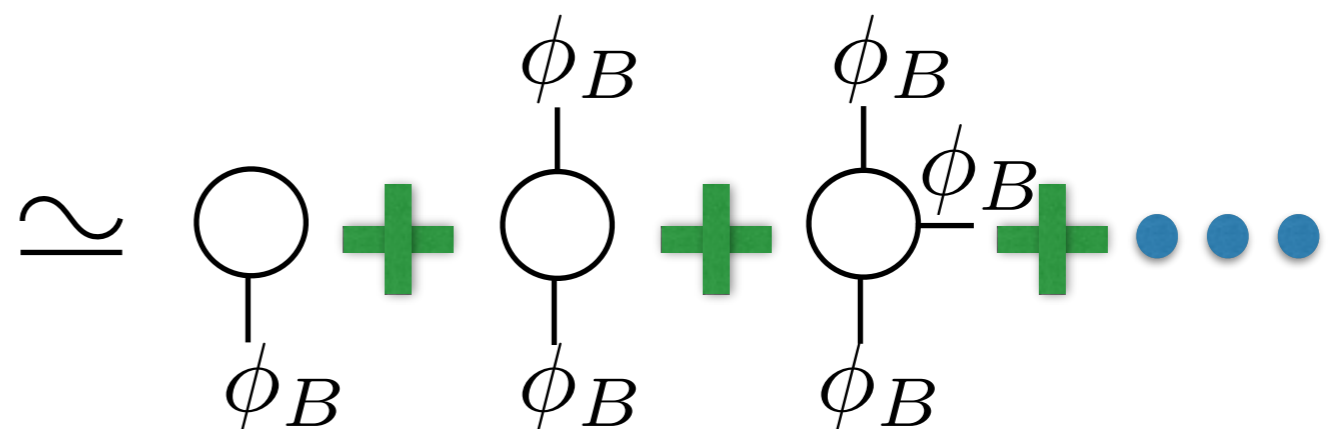


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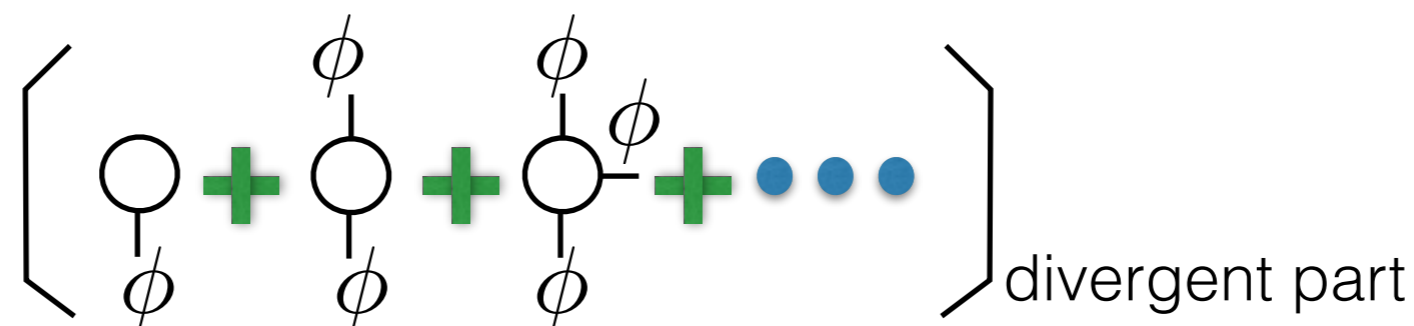


Expectation

cancellation of
the scale dependence
@1-loop



cf.) RGE is related to



Theorem

$$\gamma = Ae^{-B}$$

function of the bounce solution

$$\ln A^{-2} = \ln \frac{\det [-\partial^2 + m_0^2 + \delta\hat{W}]}{\det [-\partial^2 + m_0^2]}$$

Theorem (Dirichlet BC) (J. H. van Vleck, '28; R. H. Cameron and W. T. Martin, '45; ...)

The ratio of the determinant of differential operators,

$$L_j = -\frac{d^2}{dx^2} + R_j(x) \quad \text{on } I = [0, 1] \quad \text{w/ Dirichlet BC}$$

$$f(0) = 0, \quad f(1) = 0$$

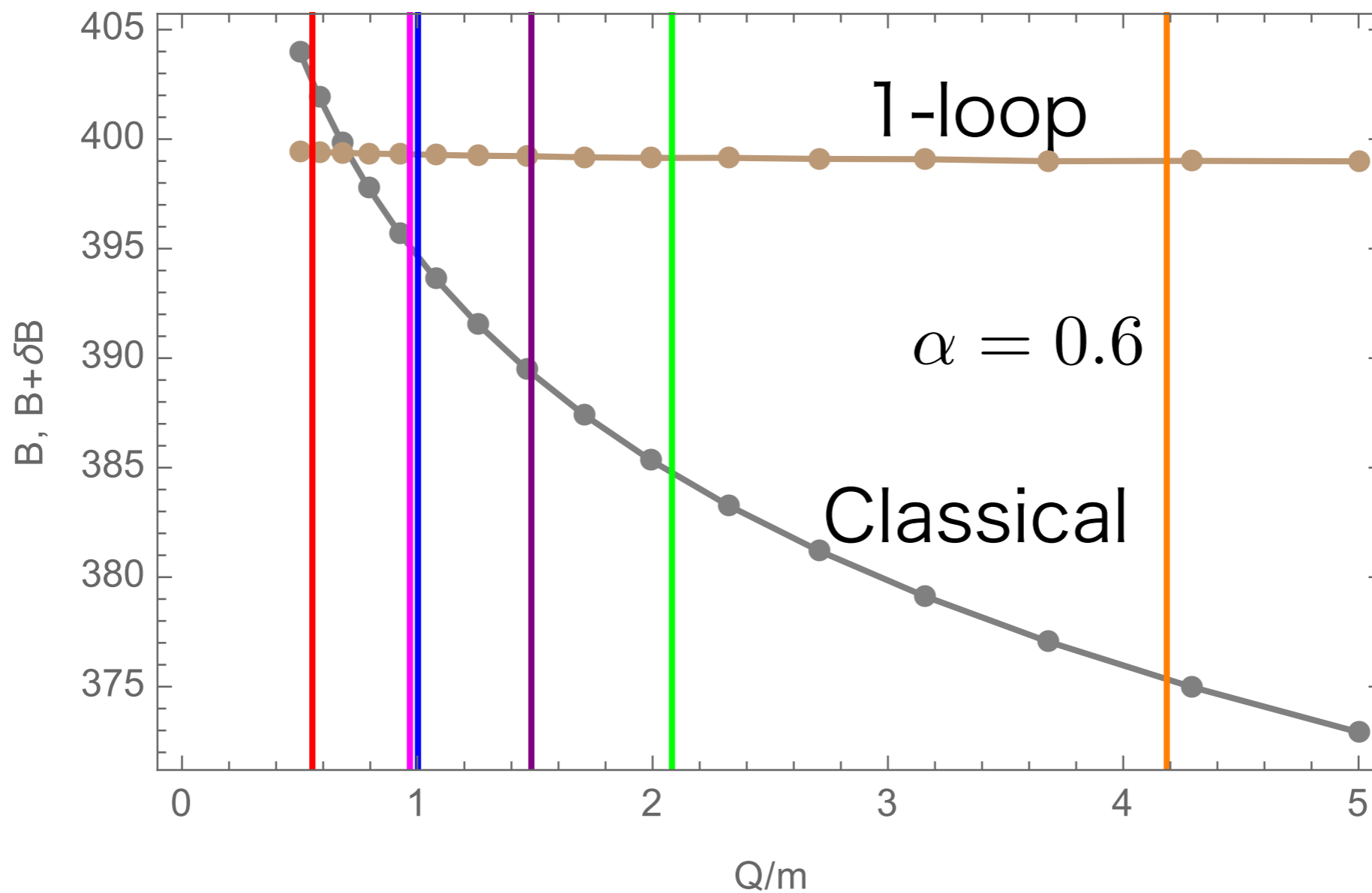
is given by the ratio of the solutions of differential equations

$$\frac{\text{Det}L_1}{\text{Det}L_0} = \frac{y_1(1)}{y_0(1)} \quad L_j y_j(x) = 0$$

$$y_j(0) = 0, \quad y_j'(0) = 1$$

Result

$$\gamma = Ae^{-B} \equiv m^4 e^{-B-\delta B}$$



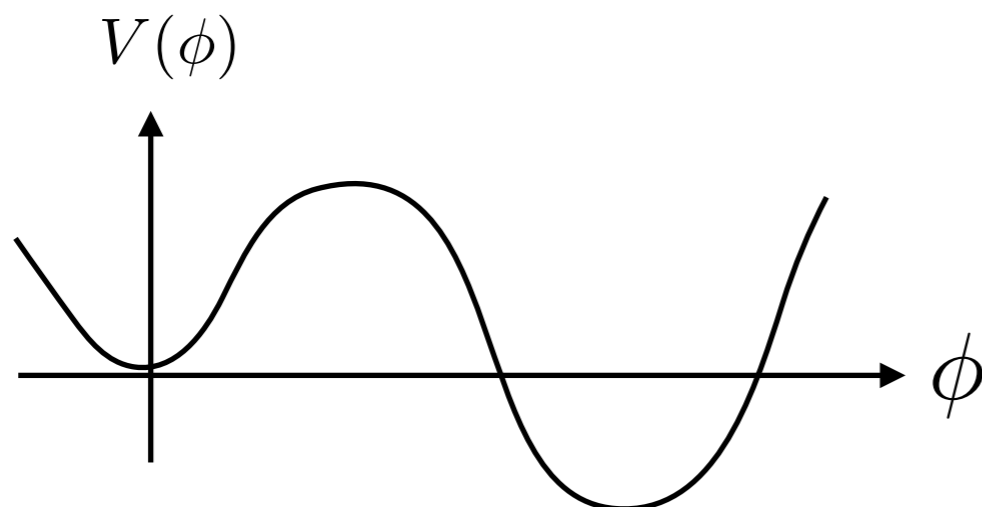
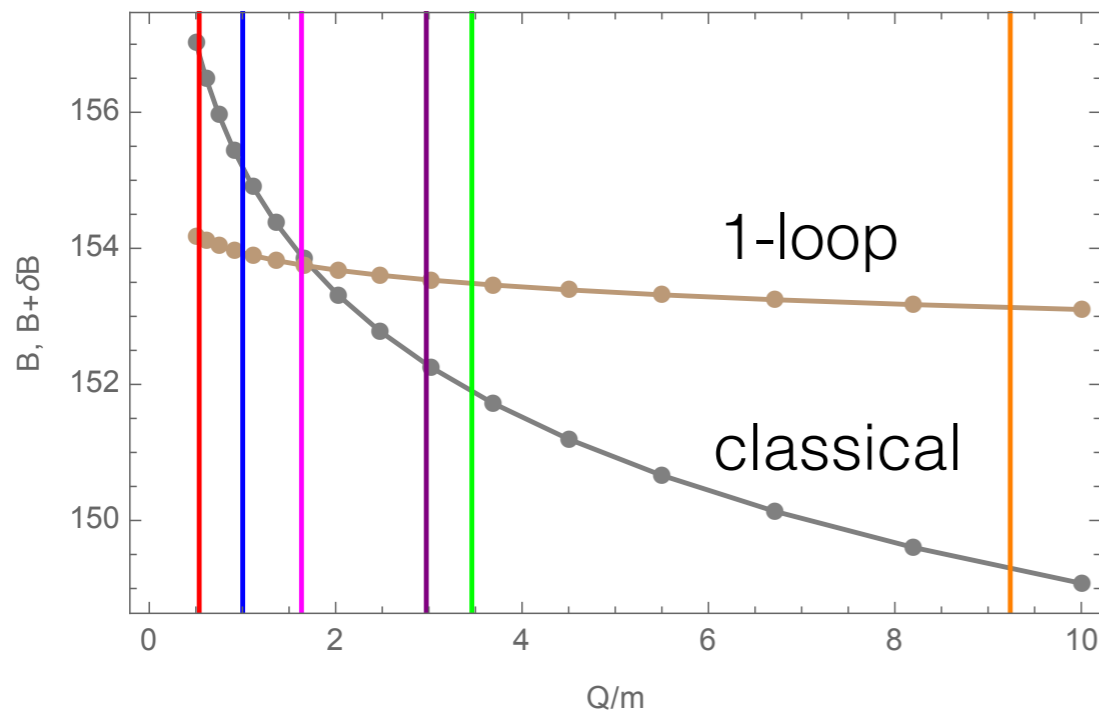
Very good



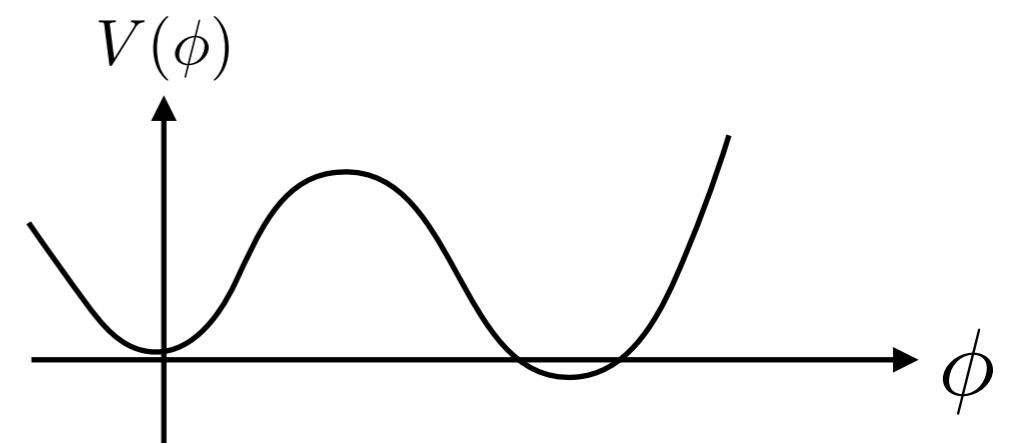
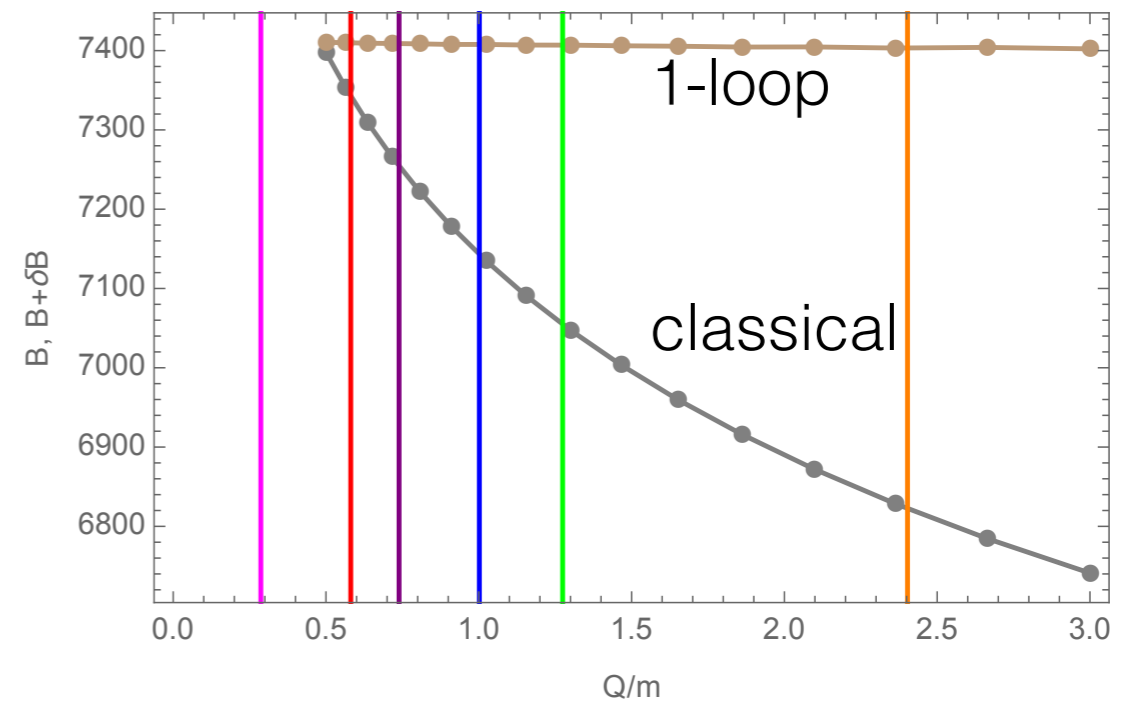
Results

$$V = \frac{m^2}{2}\phi^2 - \frac{A}{2}\phi^3 + \frac{\alpha}{8}\phi^4$$

$\alpha = 0.3$



$\alpha = 0.9$



SM + stau system

Light stau

Stau can be light

$$m_{\tilde{\tau}} > 103.5 \text{ GeV (LEP)}$$

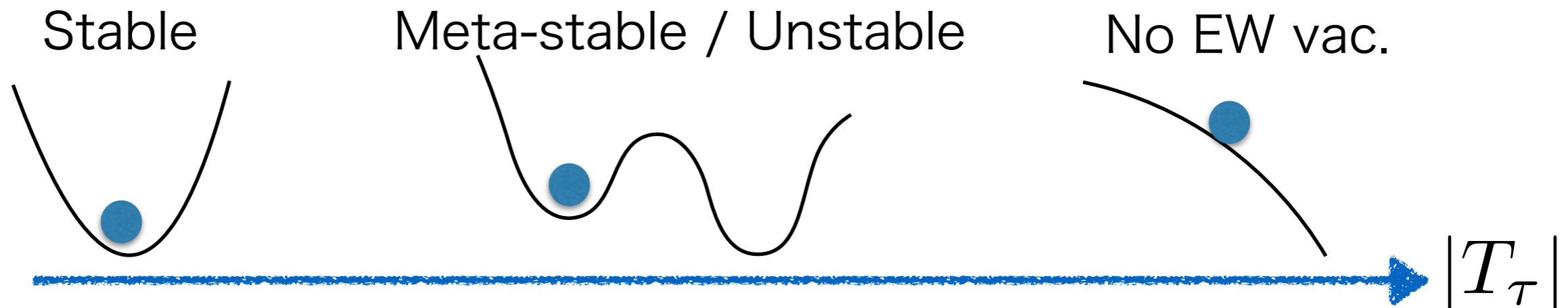
$h \gamma \gamma$ coupling, co-annihilation with bino, ...

But, the potential may become unstable towards the stau direction

$$V = T_{\tau} (H^{\dagger} \tilde{\ell}_L \tilde{\tau}_R^* + h.c.) + m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 + m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 + \dots$$

$$T_{\tau} = y_{\tau} (A_{\tau} - \mu \tan \beta)$$

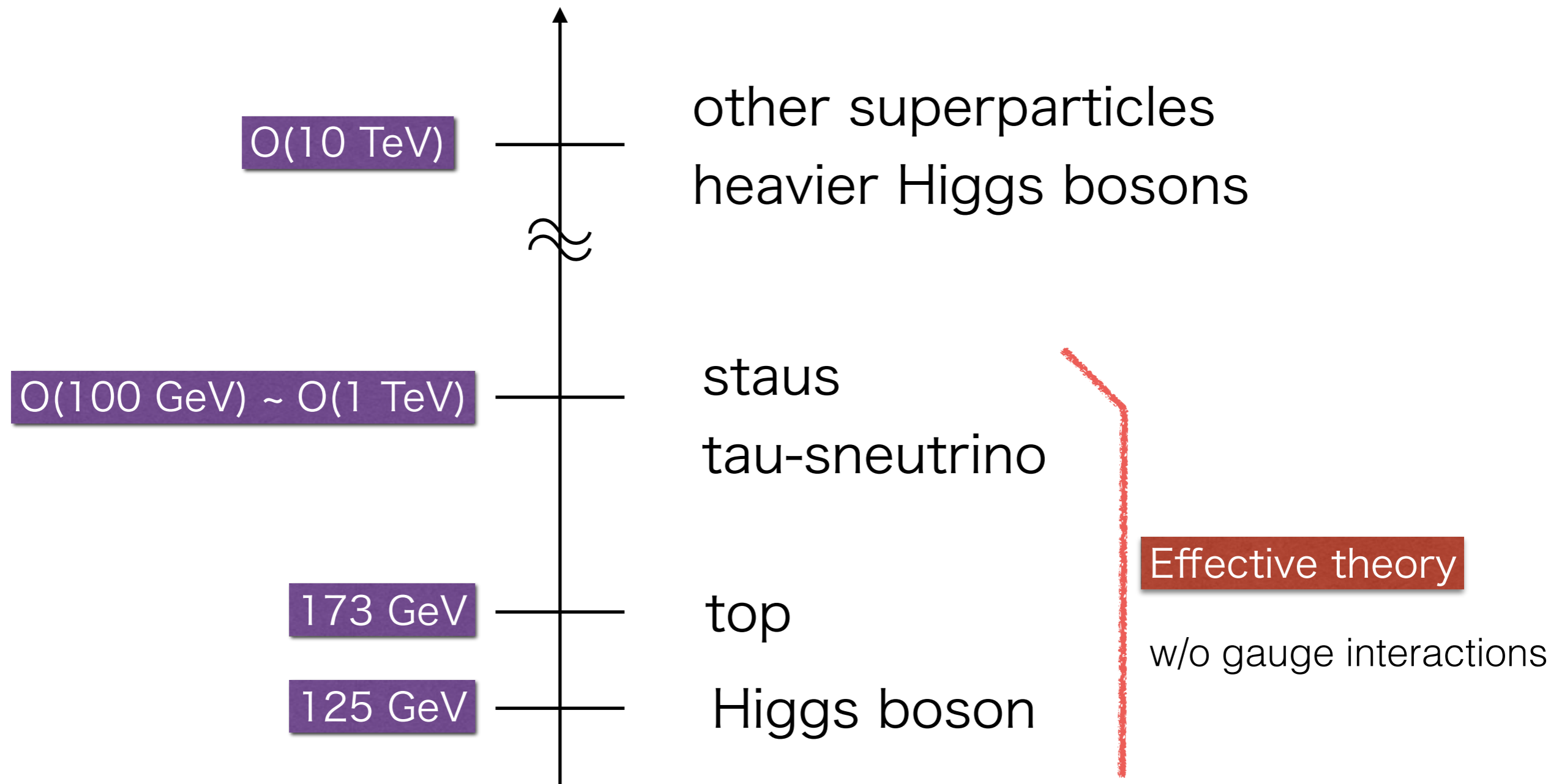
$$\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$$



Spectrum

For simplicity,

we assume only the staus are light



Effective theory

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \mathcal{L}_{\text{kin}} - y_t(H q_L t_R^c + \text{h.c.}) - m_H^2 |H|^2 - \frac{1}{4} \lambda_H |H|^4 \\ & - m_{\tilde{\ell}_L}^2 |\tilde{\ell}_L|^2 - m_{\tilde{\tau}_R}^2 |\tilde{\tau}_R|^2 - T_\tau (H^\dagger \tilde{\ell}_L \tilde{\tau}_R^* + \text{h.c.}) - \frac{1}{4} \kappa^{(1)} |\tilde{\ell}_L|^4 - \frac{1}{4} \kappa^{(2)} |\tilde{\tau}_R|^4 \\ & - \frac{1}{4} \lambda^{(1)} |H|^2 |\tilde{\ell}_L|^2 - \frac{1}{4} \lambda^{(2)} |H^\dagger \tilde{\ell}_L|^2 - \frac{1}{4} \lambda^{(3)} |H|^2 |\tilde{\tau}_R|^2 - \frac{1}{4} \kappa^{(3)} |\tilde{\ell}_L|^2 |\tilde{\tau}_R|^2, \end{aligned}$$

Boundary conditions

EW scale

$$y_t = \frac{M_t}{v},$$

$$m_H^2(M_t) = -\frac{1}{2} M_h^2,$$

$$\lambda_H(M_h) = \frac{M_h^2}{2v^2},$$

stau mass

$$m_{\tilde{\tau}} \equiv m_{\tilde{\ell}_L} = m_{\tilde{\tau}_R} = 250 \text{ GeV},$$

$$T_\tau = 300 \text{ GeV}.$$

SUSY scale (10TeV)

$$\lambda^{(1)}(M_{\text{SUSY}}) = (g^2 + g'^2) \cos 2\beta,$$

$$\lambda^{(2)}(M_{\text{SUSY}}) = 4y_\tau^2 - 2g^2 \cos 2\beta,$$

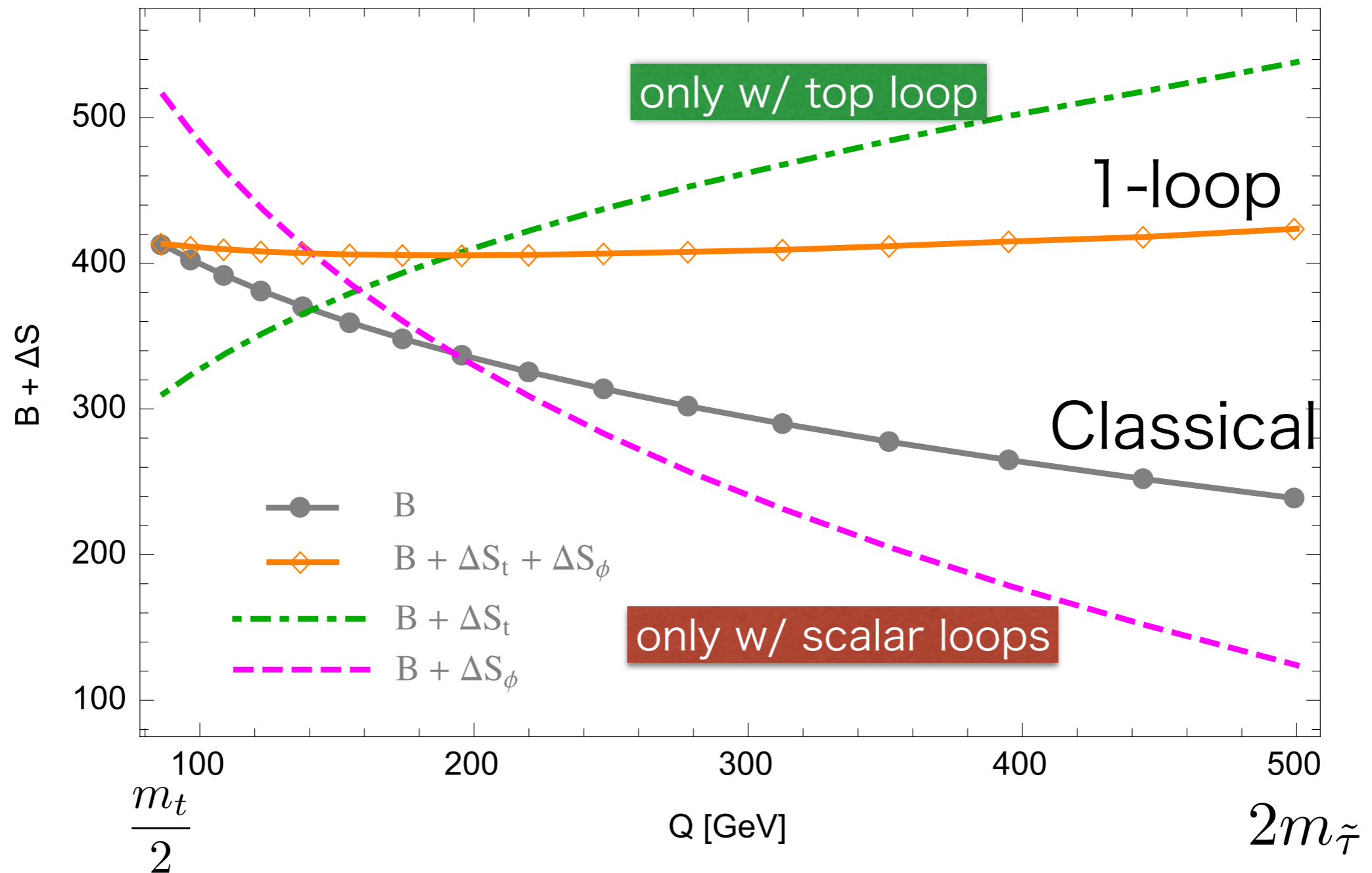
$$\lambda^{(3)}(M_{\text{SUSY}}) = 4y_\tau^2 - 2g'^2 \cos 2\beta,$$

$$\kappa^{(1)}(M_{\text{SUSY}}) = \frac{1}{2}(g^2 + g'^2),$$

$$\kappa^{(2)}(M_{\text{SUSY}}) = -\kappa^{(3)}(M_{\text{SUSY}}) = 2g'^2,$$

$$\tan \beta = 20$$

Result



Summary

- The bubble nucleation rate has often been estimated without calculating the pre-exponential factor.
- This estimate involves uncertainty in the renormalization scale, which, we showed, results in $O(10\%)$ uncertainty in the exponent of the bubble nucleation rate.
- To reduce the uncertainty, we explicitly calculated the pre-exponential factor and showed that it is greatly reduced.
- Scalars and fermions have already been implemented, but the gauge bosons are now ongoing.
Our paper on the gauge independence of the gauge determinant will appear soon.