Signal Morphing techniques and possible application to Higgs properties measurements

Katharina Ecker on behalf of the ATLAS collaboration

Max-Planck-Institut für Physik

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Introduction

- Measurements based on formulation of a Likelihood $\mathcal{L}(x|\theta)$
- Predict distribution of observable $x$ for each event from a composite model $(B)SM$ physics model $\times$ soft physics $\times$ detector response $\times$ reconstruction

- Can we turn a grid scan into a continuous function?
  $\rightarrow$ **morphing** $\mathcal{L}(x|\theta = -1, 0, 1) \rightarrow \mathcal{L}(x|\theta)$
Overview of template morphing techniques

Analytical shapes

Parametrized Gaussian
...

Physics inspired

Template morphing

Empirical descriptions

Piecewise linear interpolation
Integral morphing
Moment morphing
...

Katharina Ecker (MPP)  Signal Morphing techniques  05.07.2016  3 / 17
Template morphing: Physics inspired

- Applicable to physics processes in which the matrix element $|M(\vec{g})|$ factorizes as follows:

$$
|M(\vec{g})|^2 = \left( \sum_{x \in p, s} g_\alpha O(g_\alpha) \right)^2 \cdot \left( \sum_{x \in d, s} g_\alpha O(g_\alpha) \right)^2
$$

- Example:
  Beyond the Standard Model coupling parameter in interactions of the Higgs boson to Standard Model particles.

- Remark: Not applicable for scanning the mass as a free parameter.
  → Combine different morphing techniques, e.g. physics inspired morphing for coupling parameter and empirical morphing for mass parameter.
Template morphing: Physics inspired

Idea: Event rate or kinematic distribution $T_{\text{target}}$ at arbitrary phase space point $\vec{g}_{\text{target}}$ is modelled as a linear sum of a finite number of input distributions $T_{i,\text{input}}(\vec{g}_i)$

$$T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i) \quad \text{e.g. } T = \sigma \cdot \text{BR}, \ T = \cos \theta$$

with physics inspired weight functions $w_i(\vec{g}_{\text{target}}; \vec{g}_i)$, that are polynomials in coupling parameters $\vec{g}$. 
Heart of the algorithm: \( T \propto M^2 \)

- Simplest Example: Process with two parameters in one vertex: \( g_{SM} \) and \( g_{BSM} \).

1. Distribution of a observable is proportional to the matrix element squared
\[
T(g_{SM}, g_{BSM}) \propto |M(g_{SM}, g_{BSM})|^2
\]

2. Matrix element can be factorized:
\[
M(g_{SM}, g_{BSM}) = g_{SM}O_{SM} + g_{BSM}O_{BSM}
\]
\[
T \propto |M(g_{SM}, g_{BSM})|^2 = g_{SM}^2|O_{SM}|^2 + g_{BSM}^2|O_{BSM}|^2 + 2g_{SM}g_{BSM}R(O_{SM}^*O_{BSM})
\]

3. **3 unique terms in Matrix element \( \rightarrow \) Need 3 input distributions \( T_{i,input} \) to describe arbitrary mixture \( T_{target} \):
\[
T_{target}(\vec{g}_{target}) = \sum_{i=1}^{N_{input}=3} w_i(\vec{g}_{target}; \vec{g}_i) \cdot T_{i,input}(\vec{g}_i)
\]
Calculation of weight functions

\[ T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_{i=1}^{N_{\text{input}}} w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i) \quad \text{e.g.} \quad T = \sigma \cdot \text{BR}, \quad T = \cos \theta_1 \]

- Ansatz for morphing weights:

\[ w_i = (a_{i1} \cdot g_{\text{SM}}^2 + a_{i2} \cdot g_{\text{SM}} g_{\text{BSM}} + a_{i3} \cdot g_{\text{BSM}}^2) \]

- Requirement for calculation of constants \(a_{ij}\):

\[ w_i = 1 \quad \text{and} \quad w_{j \neq i} = 0 \quad \text{if} \quad \vec{g}_{\text{target}} = \vec{g}_i \]

\( \Rightarrow \) Linear system of equations, solveable through matrix inversion.
Specific example

\((g_{SM}, g_{BSM})\)

Template 1

\((1,0)\)

\(w_1 = 1 \cdot g_{SM}^2 - 1 \cdot g_{SM} \cdot g_{BSM} + 0 \cdot g_{BSM}^2\)

Template 2

\((0,1)\)

\(w_2 = 0 \cdot g_{SM}^2 - 1 \cdot g_{SM} \cdot g_{BSM} + 1 \cdot g_{BSM}^2\)

Template 3

\((1,1)\)

\(w_3 = 0 \cdot g_{SM}^2 + 1 \cdot g_{SM} \cdot g_{BSM} + 0 \cdot g_{BSM}^2\)

\(T_{\text{target}}(g_{SM}, g_{BSM}) = (g_{SM}^2 - g_{SM} \cdot g_{BSM}) T_{i, \text{input}}(1, 0) + (g_{BSM}^2 - g_{SM} \cdot g_{BSM}) T_{i, \text{input}}(0, 1) + g_{SM} \cdot g_{BSM} T_{i, \text{input}}(1, 1)\)

\(= w_1 + w_2 + w_3\)

\(\Rightarrow\) Set of input templates can be arbitrarily chosen as long as linear system of equations can be solved.
Validation

- Validation is shown on vector boson fusion produced Higgs decays in four leptons:

  \[ W, Z \to H, Z^* \to q\ell^+\ell^- q\ell^+\ell^- \]

- Observable: Angle between leading jets \( \Delta \Phi_{jj} \), that is sensitive to Beyond the Standard Model contributions in the HVV-coupling structure.

- Comparison of distribution from simulated Monte Carlo sample and morphing output.
Theoretical framework

- Example: Study non Standard Model admixtures in $HZZ$ vertex.

$$L_V^0 = \left\{ c_\alpha \kappa_{\text{SM}} \left[ \frac{1}{2} g_{HZZ} Z_\mu Z^\mu \right] \right.$$  
$$- \frac{1}{4} \left[ c_\alpha \kappa_{Hgg} g_{Hgg} G^a_{\mu \nu} G^a_{\mu \nu} + s_\alpha \kappa_{Agg} g_{Agg} G^a_{\mu \nu} \tilde{G}^a_{\mu \nu} \right]$$  
$$- \frac{1}{4} \frac{1}{\Lambda} \left[ c_\alpha \kappa_{HZZ} Z_\mu Z^\mu + s_\alpha \kappa_{AZZ} Z_\mu Z^\mu \tilde{Z}^\mu \right]$$  
$$- \frac{1}{\Lambda} c_\alpha \left[ \kappa_{H \partial Z} Z_\nu \partial^\nu Z^\mu + h.c. \right] \right\} \chi_0$$

- **CP violation**: Mixture of CP even and CP odd.

$$\tilde{g} = \{ c_\alpha \kappa_{\text{SM}}, c_\alpha \kappa_{Hgg}, s_\alpha \kappa_{Agg}, c_\alpha \kappa_{HZZ}, s_\alpha \kappa_{AZZ}, c_\alpha \kappa_{H \partial Z} \}$$
Validation

Comparison of two MC generated samples with the morphing output.
Propagation of statistical uncertainties

- Morphing function per bin

\[ T_{\text{target}}^{\text{bin}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}^{\text{bin}}(\vec{g}_i) \]

- The total propagated statistical uncertainty is

\[ \Delta T_{\text{target}}^{\text{bin}} = \sqrt{\sum_i w_i^2(\vec{g}_{\text{target}}; \vec{g}_i) N_{\text{MC},i,\text{input}}^{\text{bin}}(\vec{g}_i) \cdot \left(\sigma_{i,\text{input}}(\vec{g}_i) \cdot L / N_{\text{MC},i,\text{input}}\right)^2} \]

- Statistical uncertainty \( \Delta T_{\text{target}}^{\text{bin}} \) is highly dependent on
  1. Configuration of input templates \( \vec{g}_i \)
  2. Target parameter space \( \vec{g}_{\text{target}} \)
Increase of statistical uncertainty when parameter configuration of target template is not well represented by the input template set.
Challenge: Optimal input template set

- Starting point:
  1. Set of couplings $\tilde{g}$.
  2. Target phase space.

- Goal: Find input template set with
  1. robust morphing performance.
  2. minimal statistical uncertainty $\Delta T_{\text{target}}$ in target phase space.

- Non-trivial problem: *Constrained optimization*.

- On-going study on how to choose good input template sets.
The number of input templates $N$ is the number of unique terms in $|\mathcal{M}(\vec{g})|^2$, i.e. is dependent on the number of modelled couplings:

$$T(\vec{g}) \propto |\mathcal{M}(\vec{g})|^2 = \left( \sum_{i=1}^{n_p+n_s} g_i \mathcal{O}_i \right)^2 \cdot \left( \sum_{j=1}^{n_d+n_s} g_j \mathcal{O}_j \right)^2$$

- $n_s$: Number of shared couplings in production and decay vertex.
- $n_p$: Number of couplings only in production.
- $n_d$: Number of couplings only in decay.

$$N = f(n_s, n_p, n_d).$$
Example: Number of input templates $N$ for modelling CP-violation in gluon fusion and vector boson fusion produced $H \rightarrow ZZ^* \rightarrow 4\ell$ decays

Production:

a) Gluon fusion (ggF)

\[ t/b \quad g \rightarrow C_{\alpha} K_{Hgg} \]

b) Vector boson fusion (VBF)

\[ q \rightarrow W/Z \quad C_{\alpha} K_{SM}, S_{\alpha} K_{Azz} \]

Decay: $H \rightarrow 4\ell$

\[ \ell^- \quad H \rightarrow Z^* \quad \ell^+ \]

Number of input templates $N_{input}$:

<table>
<thead>
<tr>
<th></th>
<th>$n_s$</th>
<th>$n_p$</th>
<th>$n_d$</th>
<th>$N_{input}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ggF</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>b) VBF</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>
• **Goal:** Continuous signal model in a multidimensional parameter space $\vec{g}$.

• **Phycis inspired template morphing** (ATL-PHYS-PUB-2015-047) predicts event rate or differential distributions via the weighted sum of a finite number of input templates:

\[
T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i)
\]

• The weight functions $w_i$ are polynomials in coupling parameters and their construction is inspired by the structure of $|\mathcal{M}(\vec{g})|^2$.

• The number of input templates is equal to the number of unique terms in $|\mathcal{M}(\vec{g})|^2$, i.e. it is dependent on the number of couplings present in production and decay vertices.

• **Challenge:** Find optimal set of input templates with robust performance and minimal statistical uncertainty on the morphing output template.
Backup
Propagation of statistical uncertainties

- Morphing function per bin

\[ T_{\text{target}}^{\text{bin}}(\bar{g}_{\text{target}}) = \sum_{i} w_i(\bar{g}_{\text{target}}; \bar{g}_i) \cdot T_{i,\text{input}}^{\text{bin}}(\bar{g}_i) \]

- For one input distribution, the bin content is calculated as follows

\[ T_{i,\text{input}}^{\text{bin}}(\bar{g}_i) = N_{\text{MC},i,\text{input}}^{\text{bin}}(\bar{g}_i) \cdot \sigma_{i,\text{input}}(\bar{g}_i) \cdot L/N_{\text{MC},i,\text{input}} \]

with an uncertainty of

\[ \Delta T_{i,\text{input}}^{\text{bin}}(\bar{g}_i) = \sqrt{N_{\text{MC},i,\text{input}}^{\text{bin}}(\bar{g}_i)} \]

- The total propagated statistical uncertainty is

\[ \Delta T_{\text{target}}^{\text{bin}} = \sqrt{\sum_{i} w_i^2(\bar{g}_{\text{target}}; \bar{g}_i) N_{\text{MC},i,\text{input}}^{\text{bin}}(\bar{g}_i) \cdot \sigma_{i,\text{input}}(\bar{g}_i) \cdot L/N_{\text{MC},i,\text{input}}^2} \]

- Statistical uncertainty \( \Delta T_{\text{target}}^{\text{bin}} \) is highly dependent on
  1. Configuration of input templates \( \bar{g}_i \)
  2. Target parameter space \( \bar{g}_{\text{target}} \)
Number of input templates

- The number of input templates $N$ is the number of unique terms in $|\mathcal{M}(\vec{g})|^2$, i.e. is dependent on the number of modelled couplings

$$T(\vec{g}) \propto |\mathcal{M}(\vec{g})|^2 = \left( \sum_{i=1}^{n_p+n_s} g_i \mathcal{O}_i \right)^2 \cdot \left( \sum_{j=1}^{n_d+n_s} g_j \mathcal{O}_j \right)^2$$

$$N = \frac{n_p(n_p+1)}{2} \cdot \frac{n_d(n_d+1)}{2} + \frac{4+n_s-1}{4}$$

$$+ \left( n_p \cdot n_s + \frac{n_s(n_s+1)}{2} \right) \cdot \frac{n_d(n_d+1)}{2}$$

$$+ \left( n_d \cdot n_s + \frac{n_s(n_s+1)}{2} \right) \cdot \frac{n_p(n_p+1)}{2}$$

$$+ \frac{n_s(n_s+1)}{2} \cdot n_p \cdot n_d + (n_p + n_d) \left( \frac{3+n_s-1}{3} \right).$$

- $n_s$: Number of shared couplings in production and decay vertex.
- $n_p$: Number of couplings only in production.
- $n_d$: Number of couplings only in decay.