Signal Morphing techniques and possible application to Higgs properties measurements

Katharina Ecker on behalf of the ATLAS collaboration

Max-Planck-Institut für Physik

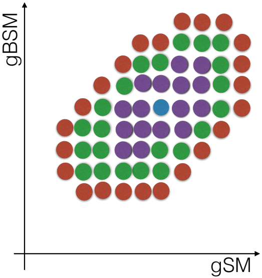
05.07.2016

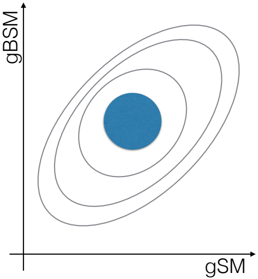
SUSY 2016, Melbourne

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Introduction

- Measurements based on formulation of a Likelihood $\mathcal{L}(x|\theta)$
- Predict distribution of observable x for each event from a composite model
  (B)SM physics model ⓧ soft physics ⓧ detector response ⓧ reconstruction

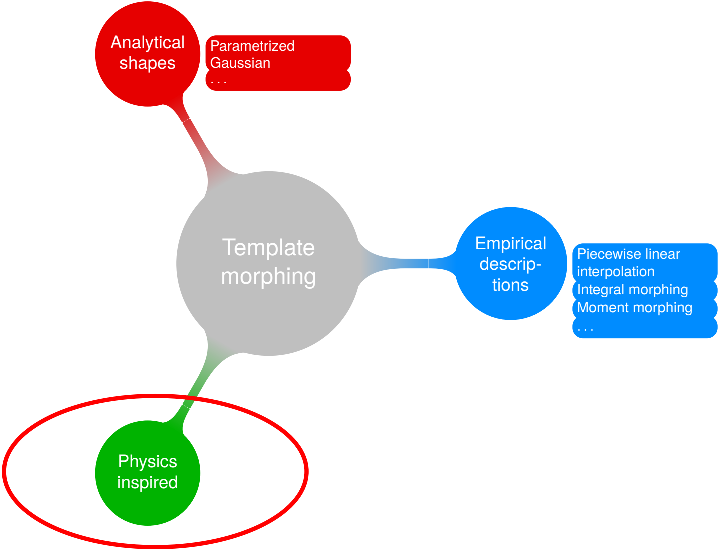




- Can we turn a grid scan into a continous function?

$\rightarrow$ **morphing** $\mathcal{L}(x|\theta = -1, 0, 1) \rightarrow \mathcal{L}(x|\theta)$

Overview of template morphing techniques

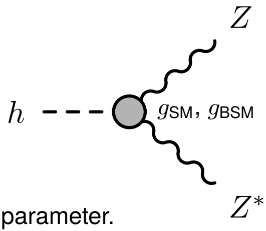


Template morphing: Physics inspired

- Applicable to physics processes in which the matrix element $|\mathcal{M}(\vec{g})|$ factorizes as follows:

$$|\mathcal{M}(\vec{g})|^2 = \underbrace{\left(\sum_{x\in p,s} g_\alpha \mathcal{O}(g_\alpha)\right)^2}_{\text{production}} \cdot \underbrace{\left(\sum_{x\in d,s} g_\alpha \mathcal{O}(g_\alpha)\right)^2}_{\text{decay}}$$

- Example:
  Beyond the Standard Model coupling parameter in interactions of the Higgs boson to Standard Model particles.



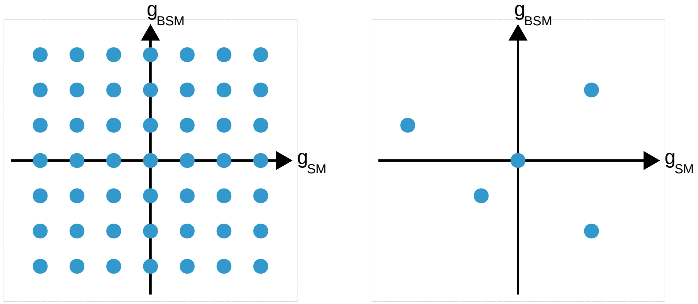
- Remark: Not applicable for scanning the mass as a free parameter.
  $\rightarrow$ Combine different morphing techniques, e.g. physics inspired morphing for coupling parameter and empirical morphing for mass parameter.

Template morphing: Physics inspired

Idea: Event rate or kinematic distribution T_{target} at arbitrary phase space point $\vec{g}_{\text{target}}$ is modelled as a linear sum of a finite number of input distributions $T_{i,\text{input}}(\vec{g}_i)$

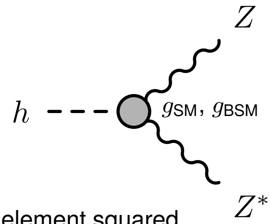
$$T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i) \qquad \text{e.g. } T = \sigma \cdot \text{BR}, \; T = \cos\theta$$

with physics inspired weight functions $w_i(\vec{g}_{\text{target}}; \vec{g}_i)$, that are polynomials in coupling parameters $\vec{g}$.



Heart of the algorithm: $T \propto \mathcal{M}^2$

- Simplest Example: Process with two parameters in one vertex: g_{SM} and g_{BSM}.



1. Distribution of a observable is proportional to the matrix element squared

$$T(g_{\mathrm{SM}}, g_{\mathrm{BSM}}) \propto |\mathcal{M}(g_{\mathrm{SM}}, g_{\mathrm{BSM}})|^2$$

2. Matrix element can be factorized:

$$\mathcal{M}(g_{\mathrm{SM}}, g_{\mathrm{BSM}}) = g_{\mathrm{SM}}\mathcal{O}_{\mathrm{SM}} + g_{\mathrm{BSM}}\mathcal{O}_{\mathrm{BSM}}$$

$$T \propto |\mathcal{M}(g_{\mathrm{SM}}, g_{\mathrm{BSM}})|^2 = g_{\mathrm{SM}}^2|\mathcal{O}_{\mathrm{SM}}|^2 + g_{\mathrm{BSM}}^2|\mathcal{O}_{\mathrm{BSM}}|^2 + 2g_{\mathrm{SM}}g_{\mathrm{BSM}}\mathcal{R}(\mathcal{O}_{\mathrm{SM}}^*\mathcal{O}_{\mathrm{BSM}})$$

3. 3 unique terms in Matrix element → Need 3 input distributions $T_{i,\text{input}}$ to describe arbitrary mixture T_{target}:

$$T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_{i=1}^{N_{\text{input}}=3} w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i)$$

Calculation of weight functions

$$T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_{i=1}^{N_{\text{input}}} w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i) \qquad \text{e.g. } T = \sigma \cdot \text{BR},\ T = \cos\theta_1$$

$\vec{g}_{\text{target}}$: variable $\qquad$ $\vec{g}_i$: fixed

- Ansatz for morphing weights:

$$w_i = (a_{i1} \cdot g_{\text{SM}}{}^2 + a_{i2} \cdot g_{\text{SM}} g_{\text{BSM}} + a_{i3} \cdot g_{\text{BSM}}{}^2)$$

- Requirement for calculation of constants a_{ij}:

$$w_i = 1 \quad \text{and} \quad w_{j \neq i} = 0 \quad \text{if} \quad \vec{g}_{\text{target}} = \vec{g}_i$$

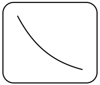
$\Rightarrow$ Linear system of equations, solveable through matrix inversion.

Specific example

(g_{SM},g_{BSM})

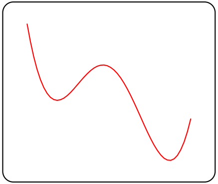
Template 1

(1,0)



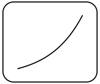
$$w_1 = 1 \cdot g_{\mathrm{SM}}{}^2 - 1 \cdot g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}} + 0 \cdot g_{\mathrm{BSM}}{}^2$$

Target



Template 2

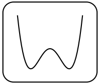
(0,1)



$$w_2 = 0 \cdot g_{\mathrm{SM}}{}^2 - 1 \cdot g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}} + 1 \cdot g_{\mathrm{BSM}}{}^2$$

Template 3

(1,1)



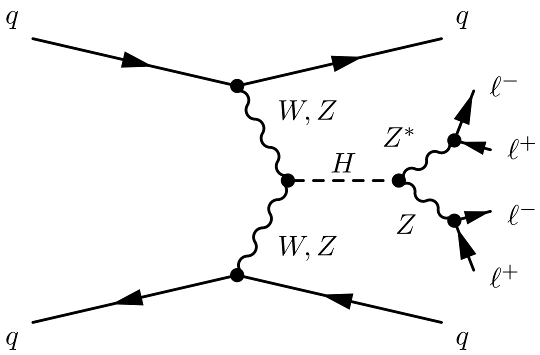
$$w_3 = 0 \cdot g_{\mathrm{SM}}{}^2 + 1 \cdot g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}} + 0 \cdot g_{\mathrm{BSM}}{}^2$$

$$T_{\text{target}}(g_{\mathrm{SM}}, g_{\mathrm{BSM}}) = \underbrace{(g_{\mathrm{SM}}{}^2 - g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}})}_{=w_1} T_{i,\text{input}}(1,0) + \underbrace{(g_{\mathrm{BSM}}{}^2 - g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}})}_{=w_2} T_{i,\text{input}}(0,1) + \underbrace{g_{\mathrm{SM}} \cdot g_{\mathrm{BSM}}}_{=w_3} T_{i,\text{input}}(1,1)$$

$\Rightarrow$ Set of input templates can be arbitrarily chosen as long as linear system of equations can be solved.

Validation

- Validation is shown on vector boson fusion produced Higgs decays in four leptons:



- Observable: Angle between leading jets $\Delta\Phi_{\mathrm{jj}}$, that is sensitive to Beyond the Standard Model contributions in the HVV-coupling structure.
- Comparison of distribution from simulated Monte Carlo sample and morphing output.

Theoretical framework

- Example: Study non Standard Model admixtures in HZZ vertex.
- Theory: Interaction of scalar and pseudo-scalar states with Z-bosons and gluons (*Higgs characterisation framework*, arXiv:1306.6464):

SM CP-even,tree-level
BSM CP-even
BSM CP-odd

$$\mathcal{L}_0^V = \Big\{ c_\alpha \kappa_{\mathrm{SM}} \left[\frac{1}{2} g_{\mathrm{HZZ}} Z_\mu Z^\mu \right] - \frac{1}{4} \left[c_\alpha \kappa_{\mathrm{Hgg}} g_{\mathrm{Hgg}} G^a_{\mu\nu} G^{a,\mu\nu} + s_\alpha \kappa_{\mathrm{Agg}} g_{\mathrm{Agg}} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \right] - \frac{1}{4} \frac{1}{\Lambda} \left[c_\alpha \kappa_{\mathrm{HZZ}} Z_{\mu\nu} Z^{\mu\nu} + s_\alpha \kappa_{\mathrm{AZZ}} Z_{\mu\nu} \tilde{Z}^{\mu\nu} \right] - \frac{1}{\Lambda} c_\alpha \left[\kappa_{\mathrm{H\partial Z}} Z_\nu \partial_\mu Z^{\mu\nu} + h.c.) \right] \Big\} \mathcal{X}_0$$

α = CP mixing angle
κ = HC coupling parameter
g = coupling strength SM or MSSM
Λ = cut-off energy
$c_\alpha = \cos(\alpha)$
$s_\alpha = \sin(\alpha)$

- **CP violation:** Mixture of CP even and CP odd.
- $\vec{g} = \{ c_\alpha \kappa_{\mathrm{SM}}, c_\alpha \kappa_{\mathrm{Hgg}}, s_\alpha \kappa_{\mathrm{Agg}}, c_\alpha \kappa_{\mathrm{Hzz}}, s_\alpha \kappa_{\mathrm{Azz}}, c_\alpha \kappa_{\mathrm{H\partial Z}} \}$

Validation

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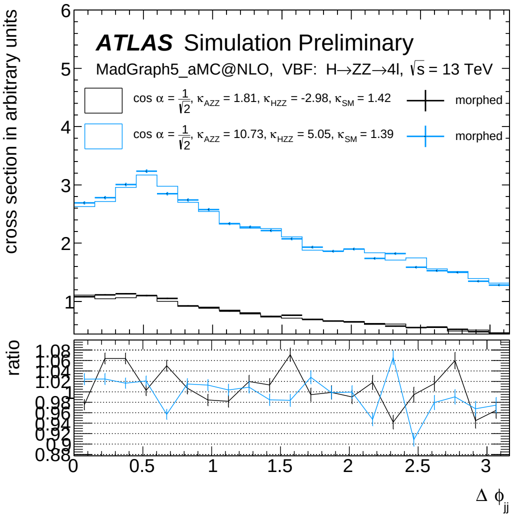


- Comparison of two MC generated samples with the morphing output.

Propagation of statistical uncertainties

- Morphing function per bin

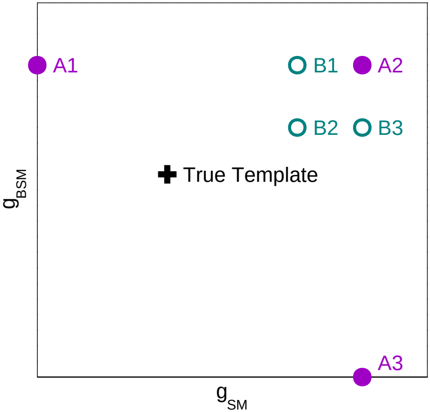
$$T^{\text{bin}}_{\text{target}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T^{\text{bin}}_{i,\text{input}}(\vec{g}_i)$$

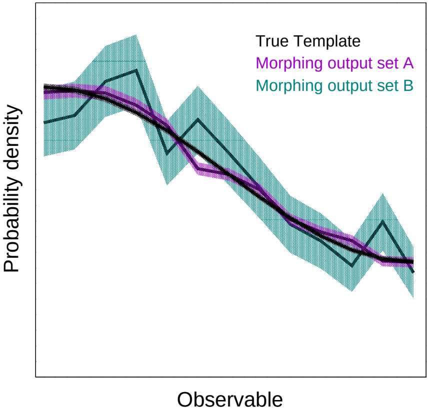
- The total propagated statistical uncertainty is

$$\Delta T^{\text{bin}}_{\text{target}} = \sqrt{\sum_i w_i^2(\vec{g}_{\text{target}}; \vec{g}_i) N^{\text{bin}}_{\text{MC},i,\text{input}}(\vec{g}_i) \cdot (\sigma_{i,\text{input}}(\vec{g}_i) \cdot \mathcal{L}/N_{\text{MC,i,input}})^2}$$

- Statistical uncertainty $\Delta T^{\text{bin}}_{\text{target}}$ is highly dependent on
  1. Configuration of input templates $\vec{g}_i$
  2. Target parameter space $\vec{g}_{\text{target}}$

Propagation of statistical uncertainties: Choice of set of input templates





- Increase of statistical uncertainty when parameter configuration of target template is not well represented by the input template set.

Challenge: Optimal input template set

- Starting point:
  1. Set of couplings $\vec{g}$.
  2. Target phase space.
- Goal: Find input template set with
  1. robust morphing performance.
  2. minimal statistical uncertainty ΔT_{target} in target phase space.
- Non-trivial problem: *Constrained optimization*.
- On-going study on how to choose good input template sets.

Number of input templates

- The number of input templates N is the number of unique terms in $|\mathcal{M}(\vec{g})|^2$, i.e. is dependent on the number of modelled couplings

$$T(\vec{g}) \propto |\mathcal{M}(\vec{g})|^2 = \underbrace{\left(\sum_{i=1}^{n_p+n_s} g_i \mathcal{O}_i\right)^2}_{\text{production vertex}} \cdot \underbrace{\left(\sum_{j=1}^{n_d+n_s} g_j \mathcal{O}_j\right)^2}_{\text{decay vertex}}$$

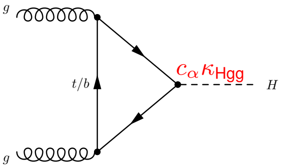
- n_s: Number of shared couplings in production and decay vertex.
- n_p: Number of couplings only in production.
- n_d: Number of couplings only in decay.

$$N = \quad f(n_s, n_p, n_d).$$

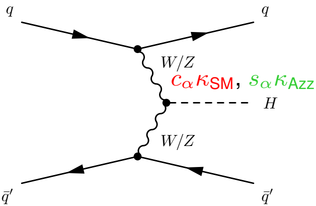
Example: Number of input templates N for modelling CP-violation in gluon fusion and vector boson fusion produced $H \to ZZ^* \to 4\ell$ decays

Production:

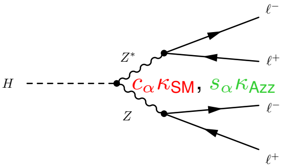
a) Gluon fusion (ggF)



b) Vector boson fusion (VBF)



Decay: $H \to 4\ell$



Number of input templates N_{input}:

	n_s	n_p	n_d	N_{input}
a) ggF	0	1	2	3
b) VBF	2	0	0	5

Summary

- Goal: Continuous signal model in a multidimensional parameter space $\vec{g}$.
- Phycis inspired template morphing (ATL-PHYS-PUB-2015-047) predicts event rate or differential distributions via the weighted sum of a finite number of input templates:

$$T_{\text{target}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T_{i,\text{input}}(\vec{g}_i)$$

- The weight functions w_i are polynomials in coupling parameters and their construction is inspired by the structure of $|\mathcal{M}(\vec{g})|^2$.
- The number of input templates is equal to the number of unique terms in $|\mathcal{M}(\vec{g})|^2$, i.e. it is dependent on the number of couplings present in production and decay vertices.
- Challenge: Find optimal set of input templates with robust performance and minimal statistical uncertainty on the morphing output template.

Backup

Propagation of statistical uncertainties

- Morphing function per bin

$$T^{\text{bin}}_{\text{target}}(\vec{g}_{\text{target}}) = \sum_i w_i(\vec{g}_{\text{target}}; \vec{g}_i) \cdot T^{\text{bin}}_{i,\text{input}}(\vec{g}_i)$$

- For one input distribution, the bin content is calculated as follows

$$T^{\text{bin}}_{i,\text{input}}(\vec{g}_i) = N^{\text{bin}}_{\text{MC},i,\text{input}}(\vec{g}_i) \cdot \sigma_{i,\text{input}}(\vec{g}_i) \cdot \mathcal{L}/N_{\text{MC},i,\text{input}}$$

with an uncertainty of

$$\Delta T^{\text{bin}}_{i,\text{input}}(\vec{g}_i) = \sqrt{N^{\text{bin}}_{\text{MC},i,\text{input}}(\vec{g}_i)}$$

- The total propagated statistical uncertainty is

$$\Delta T^{\text{bin}}_{\text{target}} = \sqrt{\sum_i w_i^2(\vec{g}_{\text{target}}; \vec{g}_i) N^{\text{bin}}_{\text{MC},i,\text{input}}(\vec{g}_i) \cdot (\sigma_{i,\text{input}}(\vec{g}_i) \cdot \mathcal{L}/N_{\text{MC,i,input}})^2}$$

- Statistical uncertainty $\Delta T^{\text{bin}}_{\text{target}}$ is highly dependent on
  1. Configuration of input templates $\vec{g}_i$
  2. Target parameter space $\vec{g}_{\text{target}}$

Number of input templates

- The number of input templates N is the number of unique terms in $|\mathcal{M}(\vec{g})|^2$, i.e. is dependent on the number of modelled couplings

$$T(\vec{g}) \propto |\mathcal{M}(\vec{g})|^2 = \underbrace{\left(\sum_{i=1}^{n_p+n_s} g_i \mathcal{O}_i\right)^2}_{\text{production vertex}} \cdot \underbrace{\left(\sum_{j=1}^{n_d+n_s} g_j \mathcal{O}_j\right)^2}_{\text{decay vertex}}$$

$$\begin{aligned} N = \;& \frac{n_p\,(n_p+1)}{2} \cdot \frac{n_d\,(n_d+1)}{2} + \binom{4+n_s-1}{4} \\ & + \left(n_p \cdot n_s + \frac{n_s\,(n_s+1)}{2}\right) \cdot \frac{n_d\,(n_d+1)}{2} \\ & + \left(n_d \cdot n_s + \frac{n_s\,(n_s+1)}{2}\right) \cdot \frac{n_p\,(n_p+1)}{2} \\ & + \frac{n_s\,(n_s+1)}{2} \cdot n_p \cdot n_d + (n_p + n_d) \binom{3+n_s-1}{3}. \end{aligned}$$

- n_s: Number of shared couplings in production and decay vertex.
- n_p: Number of couplings only in production.
- n_d: Number of couplings only in decay.