Soft-Collinear Supersymmetry

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In collaboration with Tim Cohen and Andrew Larkoski:

"Collinear Superspace" [Phys. Rev. D 93, 125013 (2016)]
"Soft-Collinear Supersymmetry" [*in preparation*]

Motivation and Outline

Soft Collinear Effective Theory (SCET): A very powerful formalism developed to resum infrared divergences occurring in processes dominated by soft and collinear degrees of freedom.

Many Applications: heavy meson decays, LHC collisions, and even WIMP dark matter.

Can we explore the SCET limits of more general theories?
e.g. Supersymmetric theories?
What can we hope to learn by leveraging the power of SCET in these theories?

Many questions generated:

- Is the SCET limit of Super Yang-Mills self consistent?
- Chiral multiplet? $\mathcal{N} = 1$
- Gauge multiple?
- Superspace formalism?
- Subtleties of Super-Poincare invariance on the lightcone?
- Extension to $\mathcal{N} > 1$ SUSY? Supergravity? Gravity? Other?

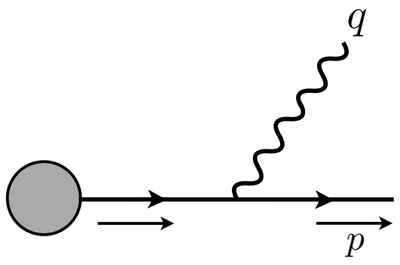
Goal for today: Introduce the basic concepts needed to formulate a consistent $\mathcal{N}=1$ SUSY-SCET YM

- Infrared structure and introduction to SCET
- Details of SCET with two component Weyl Spinors
- SUSY-SCET Basics
- I. Light Cone Gauge Theory: An example of a consistent SUSY-SCET.
- II. Light Cone Gauge Theory: Collinear Superspace

Infrared Singularities

Soft and Collinear Divergences

Amplitude for gauge boson emission off a massless left handed Weyl fermion:



$$= x^\dagger_{\dot\alpha}(p)(-ie\bar\sigma^\mu)^{\dot\alpha\alpha}\epsilon^*_\mu(q)\frac{-i(p+q)\cdot\sigma_{\alpha\dot\beta}}{2p\cdot q}\,\mathcal{M}^{\dot\beta}(p+q)$$

Diverges in the limit:

- Photon momentum is soft $q^\mu \to 0$
- Photon is emitted collinear to fermion $p\cdot q \to 0$

Soft Collinear Effective Theory:

- Developed to resum IR divergences occurring in processes dominated by soft and collinear d.o.f.
- An EFT separating physics at different momentum scales; formulated by expanding fields along a light like direction and integrating out d.of. away from the light-cone.
- Perturbative expansion possible with a small power counting parameter originating from a separation of scales.

Soft-Collinear Effective Theory (SCET)

Separation of Scales Example: Jets at Colliders

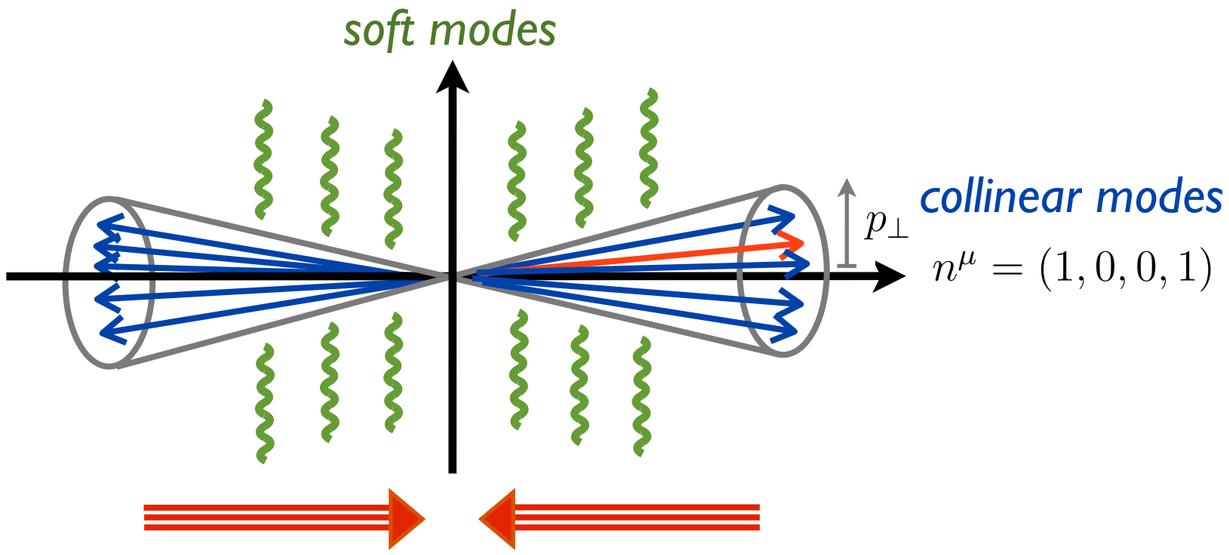
Infrared divergences in QCD leads to collimated jets and soft radiation.

Lorentz vectors in light cone coordinates $(p^+, p^-, p_\perp) = (n \cdot p, \bar{n} \cdot p, p_\perp)$

$$p^\mu = \frac{\bar{n} \cdot p}{2} n^\mu + \frac{n \cdot p}{2} \bar{n}^\mu + p_\perp^\mu$$

Choice of "anti-collinear" direction must satisfy:

$n^2 = 0 = \bar{n}^2$ and $n \cdot \bar{n} = 2$ e.g. $\bar{n}^\mu = (1, 0, 0, -1)$



Physical separation of scales:

collinear: $p^- \sim Q$ event energy

transverse: $p_\perp \sim m_J$ where $\Lambda_{QCD} \ll m_J \ll Q$

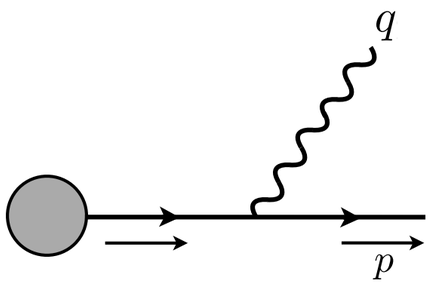
anti-collinear: $p^2 = 0 \Rightarrow p^+ \sim m_J^2/Q$

Defines a power counting parameter: $\lambda = m_J/Q \ll 1 \quad \Rightarrow \quad p_n^\mu = \left(p_n^+,\, p_n^-,\, p_{\perp n}\right) \sim Q(\lambda^2, 1, \lambda)$

Soft-Collinear Effective Theory (SCET)

Revisiting the Origin of Infrared Singularities

Factorized Amplitude:



$$= -e x^\dagger_{\dot\alpha}(p) \left[\frac{p\cdot\epsilon^*(q)}{p\cdot q}\delta^{\dot\alpha}_{\dot\beta} - \frac{(q\cdot\bar\sigma)^{\dot\alpha\alpha}(\epsilon^*(q)\cdot\sigma)_{\alpha\dot\beta}}{2p\cdot q}\right] \mathcal{M}^{\dot\beta}(p+q)$$

where $p\cdot q = n\cdot q\ \bar n\cdot p + \bar n\cdot q\ n\cdot p + p_\perp\cdot q_\perp$

Infrared singularities diverge for small power counting parameter:

Photon is emitted collinear to fermion: $p^\mu,\ q^\mu \sim (\lambda^2, 1, \lambda)$ $\Rightarrow$ $\mathcal{S}_{\text{collinear}} = \dfrac{\epsilon^*_\mu q^\mu_\perp}{2p\cdot q} \sim \dfrac{\lambda}{\lambda^2} \sim \dfrac{1}{\lambda}$

Photon momentum is soft: $q^\mu \sim (\lambda, \lambda, \lambda)$ $\Rightarrow$ $\mathcal{S}_{\text{soft}} = \dfrac{\epsilon^*_\mu q^\mu}{2p\cdot q} \sim \dfrac{\lambda}{\lambda^2} \sim \dfrac{1}{\lambda}$

SCET Lagrangian

Two Component Weyl Spinors

Theory with left handed fermions: $\mathcal{L} = i\, u^\dagger(x)\, \bar{\sigma} \cdot \partial\, u(x)$ with plane wave solutions $u(x^\mu) = \int d^3\, p\; \mathrm{x}(p)\, e^{-i\, p^\mu x_\mu}$

Boost along the collinear direction: $\mathrm{x}(p)\big|_n = \left[\sqrt{E+p_z}\left(\frac{1-\sigma_z}{2}\right) + \sqrt{E-p_z}\left(\frac{1+\sigma_z}{2}\right)\right] \bar{\sigma}_0 \xi = \left[\sqrt{\bar{n}\cdot p}\, P_n + \sqrt{n\cdot p}\, P_{\bar{n}}\right] \xi \sim \begin{bmatrix} \lambda^2 \\ \lambda \end{bmatrix}$

Collinear and anti-collinear d.o.f separated by projection operators: $\boxed{u = (P_n + P_{\bar{n}})\, u = u_n + u_{\bar{n}}}$

where: $P_n\, u_n = \frac{n\cdot\sigma}{2}\frac{\bar{n}\cdot\bar{\sigma}}{2} u_n = u_n\,;$ $\quad P_{\bar{n}}\, u_{\bar{n}} = \frac{\bar{n}\cdot\sigma}{2}\frac{n\cdot\bar{\sigma}}{2} u_{\bar{n}} = u_{\bar{n}}$

$P_{\bar{n}}\, u_n = 0\,;$ $\quad P_n\, u_{\bar{n}} = 0\,.$

For our choice of collinear/anti-collinear direction:

$n^\mu = (1,0,0,1)$ and $\bar{n}^\mu = (1,0,0,-1)$ $\Rightarrow$ $\left(\frac{n\cdot\sigma}{2}\right)_{\alpha\dot{\alpha}} = \frac{1}{2}(\sigma^0 - \sigma^3)_{\alpha\dot{\alpha}} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_{\alpha\dot{\alpha}}$ and $\left(\frac{\bar{n}\cdot\bar{\sigma}}{2}\right)^{\dot{\alpha}\alpha} = \frac{1}{2}(\bar{\sigma}_0 - \bar{\sigma}_3)^{\dot{\alpha}\alpha} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}^{\alpha\dot{\alpha}}$

$\Rightarrow$ Half the helicity states projected out: $u_n = \begin{bmatrix} 0 \\ u \end{bmatrix}$

Expanding, $u = u_n + u_{\bar{n}}$

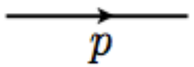
$$\mathcal{L} = i u^\dagger \bar{\sigma} \cdot \partial u \quad \Longrightarrow \quad \mathcal{L} = i\, u_n^\dagger\, \bar{\sigma} \cdot \partial\, u_n + i\, u_{\bar{n}}^\dagger\, \bar{\sigma} \cdot \partial\, u_n + i\, u_n^\dagger\, \bar{\sigma} \cdot \partial\, u_{\bar{n}} + i\, u_{\bar{n}}^\dagger\, \bar{\sigma} \cdot \partial\, u_{\bar{n}}$$

Integrate out anti-collinear modes: $u_{\bar{n}} = -\frac{\bar{n}\cdot\sigma}{2}\frac{1}{\bar{n}\cdot\partial}\left(\bar{\sigma}\cdot\partial_\perp\right) u_n$ (where $\sigma^\mu \partial_\mu = \begin{bmatrix} n\cdot\partial & \sqrt{2}\partial^* \\ \sqrt{2}\partial & \bar{n}\cdot\partial \end{bmatrix}_{\alpha\dot{\alpha}}$, and $\frac{\sigma\cdot\partial_\perp}{\sqrt{2}} \equiv \begin{bmatrix} 0 & \partial^* \\ \partial & 0 \end{bmatrix}_{\alpha\dot{\alpha}}$)

The leading order fermion Lagrangian of the EFT:

$$\boxed{\mathcal{L}_{u_n}^{(0)} = u_n^\dagger \left(i\, n\cdot\partial - \frac{\partial_\perp^2}{i\, \bar{n}\cdot\partial} \right) \frac{\bar{n}\cdot\bar{\sigma}}{2}\, u_n}$$

Non-local operators well-defined in terms of momentum eigenvalues. For instance the collinear fermion propagator in the EFT is:



$$= \frac{\bar{n}\cdot\bar{\sigma}}{2}\frac{\bar{n}\cdot p}{(n\cdot p)(\bar{n}\cdot p) + p_\perp^2}$$

Field	u_n	$u_{\bar{n}}$	u_s	ϕ_n	ϕ_s	$n \cdot A_n$	$\bar{n} \cdot A_n$	$A_{n\perp}$	A_s
Scaling	λ	λ^2	λ^3	λ	λ^2	λ^2	λ^0	λ	λ^2

Adding gauge interactions:

Including interactions with soft and collinear gauge bosons: $A^\mu = A_n^\mu + A_s^\mu$

Four vector decomposition: $A^\mu = \frac{\bar{n} \cdot A}{2} n^\mu + \frac{n \cdot A}{2} \bar{n}^\mu + A_\perp^\mu$

Gauge covariant derivatives at leading order:

$$i\, n \cdot \mathcal{D}_{n,s} = i\, n \cdot \partial + g\, n \cdot A_n + g\, n \cdot A_s \sim \mathcal{O}(\lambda^2)$$

$$i\bar{n} \cdot \mathcal{D}_n = i\, \bar{n} \cdot \partial + g\, \bar{n} \cdot A_n \sim \mathcal{O}(\lambda^0)$$

$$i\mathcal{D}_{\perp,n}^\mu = i\, \partial_\perp^\mu + g\, A_{n\perp}^\mu \sim \mathcal{O}(\lambda),$$

By analog to non-interacting fermion case:

$$\boxed{\mathcal{L} = u_n^\dagger \left(i\, n \cdot \mathcal{D}_{n,s} + i\, \bar{\sigma} \cdot \mathcal{D}_{\perp,n} \frac{1}{i\, \bar{n} \cdot \mathcal{D}_n} i\, \sigma \cdot \mathcal{D}_{\perp,n} \right) \frac{\bar{n} \cdot \bar{\sigma}}{2} u_n - \frac{1}{4} \left(F_n^{\mu\nu} \right)^2}$$

field strength

$$i\, g\, F_n^{\mu\nu} = \left[\mathcal{D}^\mu, \mathcal{D}^\nu \right]$$

Theory with scalars:

$$\boxed{\mathcal{L}_n = \phi_n^* \Box \phi_n = \phi_n^* \left(\bar{n} \cdot \partial n \cdot \partial - \partial_\perp^2 \right) \phi_n}$$

Reparametrization Invariance (RPI)

A EFT Consistency Condition: each order Lagrangian must be invariant under broken Poincare symmetry on the light cone (RPI should be restored order by order).

Poincare group:

$$\left[P_\mu, P_\nu\right] = 0\,;$$
$$\left[M^{\mu\nu}, P^\rho\right] = i\, g^{\mu\rho}\, P^\nu - i\, g^{\nu\rho}\, P^\mu\,;$$
$$\left[M^{\mu\nu}, M^{\kappa\rho}\right] = -g^{\mu\kappa} M^{\nu\rho} - g^{\nu\rho} M^{\mu\kappa} + g^{\mu\rho} M^{\nu\kappa} + g^{\nu\kappa} M^{\mu\rho}\,,$$

Rotations in the perpendicular plane and boosts along the collinear direction are unbroken

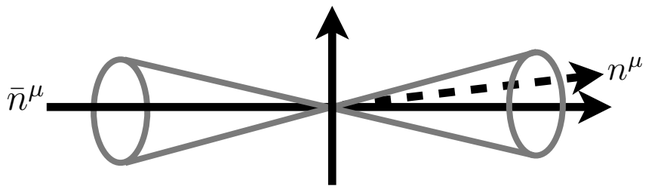
RPI:

$$R_1^\nu = \bar{n}_\mu\, M^{\mu\nu\perp}\,,$$
$$R_2^\nu = n_\mu\, M^{\mu\nu\perp}\,,$$
$$R_3 = n_\mu\, \bar{n}_\nu\, M^{\mu\nu}$$

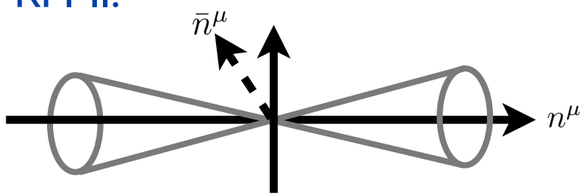
Broken Generators

We can shift collinear and anti-collinear directions and physics should unchanged as long as: $n^2 = 0 = \bar{n}^2$ and $n \cdot \bar{n} = 2$

RPI-I:



RPI-II:



RPI-I	RPI-II	RPI-III
$n_\mu \to n_\mu + \Delta_\mu^\perp$ $\bar{n}_\mu \to \bar{n}_\mu$	$n_\mu \to n_\mu$ $\bar{n}_\mu \to \bar{n}_\mu + \epsilon_\mu^\perp$	$n_\mu \to e^\alpha n_\mu$ $\bar{n}_\mu \to e^{-\alpha} \bar{n}_\mu$

(See our upcoming paper "Soft-Collinear Supersymmetry" for detailed transformations.)

SUSY and the SCET Expansion

Charges, Algebra, and Superspace in the SCET limit

$\mathcal{N}$= 1 SUSY Algebra: $\left\{Q_\alpha, Q^\dagger_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu$ where $\alpha, \dot\alpha = 1, 2$. expand on the light cone: $P_\mu = i\partial_\mu = i\frac{\bar n_\mu}{2} n\cdot\partial + i\frac{n_\mu}{2}\bar n\cdot\partial + i\partial_{\perp\mu}$

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu = 2i\begin{bmatrix} n\cdot\partial & \sqrt{2}\partial^* \\ \sqrt{2}\partial & \bar n\cdot\partial \end{bmatrix}_{\alpha\dot\alpha} \sim \begin{bmatrix} \mathcal{O}(\lambda^2) & \mathcal{O}(\lambda) \\ \mathcal{O}(\lambda) & \mathcal{O}(1) \end{bmatrix}$$

⇒ One supercharge suppressed

Charges differential operators in superspace: $Q_\alpha = i\frac{\partial}{\partial\theta^\alpha} - (\sigma\cdot\partial)_{\alpha\dot\alpha}\theta^{\dagger\dot\alpha}$

SCET expansion: $Q_2 = \left(i\frac{\partial}{\partial\theta^2} - \theta^{\dagger\dot 2}\bar n\cdot\partial - \sqrt{2}\theta^{\dagger\dot 1}\partial\right) \sim \mathcal{O}(\lambda^0)$

$$Q_1 = \left(i\frac{\partial}{\partial\theta^1} - \theta^{\dagger\dot 1} n\cdot\partial - \sqrt{2}\theta^{\dagger\dot 2}\partial^*\right) \sim \mathcal{O}(\lambda)$$

⇒ Superspace coordinates scale: $(n\cdot x, \bar n\cdot x, x_\perp) \sim (1, 1/\lambda^2, 1/\lambda)$

recall $y^\mu = x^\mu + i\theta\sigma^\mu\theta^\dagger$

Supercharges			
$Q^1 = Q_2$	$Q^{\dagger\dot 1} = Q^\dagger_{\dot 2}$	$Q^2 = -Q_1$	$Q^{\dagger\dot 2} = -Q^\dagger_{\dot 1}$
1	1	λ	λ

Superspace Coordinates			
$\theta^1 = \theta_2$	$\theta^{\dagger\dot 1} = \theta^\dagger_{\dot 2}$	$\theta^2 = -\theta_1$	$\theta^{\dagger\dot 2} = -\theta^\dagger_{\dot 1}$
λ^{-1}	λ^{-1}	1	1

Expect not present in the EFT

We will integrate out short distance modes in the superspace formalism to show that SCET lives on a slice of superspace; "Collinear superspace"

[arXiv: 1603.09346]

Gauge Theory

The Vector Multiplet

- Infrared singularity structure known from QCD
- RPI invariant EFT Lagrangian known from QCD:

$$\mathcal{L} = u_n^\dagger \left(i\, n \cdot \mathcal{D}_{n,s} + i\, \bar{\sigma} \cdot \mathcal{D}_{\perp,n} \frac{1}{i\, \bar{n} \cdot \mathcal{D}_n} i\, \sigma \cdot \mathcal{D}_{\perp,n} \right) \frac{\bar{n} \cdot \bar{\sigma}}{2}\, u_n - \frac{1}{4} \left(F_n^{\mu\nu} \right)^2$$

- Does a supersymmetric EFT exist to describe gauge-gaugino interactions of a vector multiplet?
- Is a superspace formalism possible?

Gauge Theory

Light Cone Gauge

$$
\begin{aligned}
igF^{a\mu\nu}t^a &= \left[\mathcal{D}^\mu, \mathcal{D}^\nu\right] \\
&= \left(\frac{n_\mu}{2}\left[\bar{n}\cdot\partial, \mathcal{D}_{\perp\nu}\right] - \frac{n_\nu}{2}\left[\bar{n}\cdot\partial, \mathcal{D}_{\perp\mu}\right]\right) \\
&+ \left(\frac{1}{4}\left(n_\mu\bar{n}_\nu - n_\nu\bar{n}_\mu\right)\left[\bar{n}\cdot\partial, n\cdot\mathcal{D}\right] + \left[\mathcal{D}_{\perp\mu}, \mathcal{D}_{\perp\nu}\right]\right) \\
&+ \left(\frac{\bar{n}_\mu}{2}\left[n\cdot\mathcal{D}, \mathcal{D}_{\perp\nu}\right] - \frac{\bar{n}_\nu}{2}\left[n\cdot\mathcal{D}, \mathcal{D}_{\perp\mu}\right]\right) \\
&\sim \mathcal{O}(\lambda) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^3),
\end{aligned}
$$

The SCET Lagrangian with two fermionic d.o.f. $\mathcal{L} = u_n^\dagger \left(i\, n\cdot\mathcal{D}_{n,s} + i\,\bar{\sigma}\cdot\mathcal{D}_{\perp,n}\frac{1}{i\,\bar{n}\cdot\mathcal{D}_n} i\,\sigma\cdot\mathcal{D}_{\perp,n}\right)\frac{\bar{n}\cdot\bar{\sigma}}{2}\,u_n - \frac{1}{4}\left(F_n^{\mu\nu}\right)^2$

To explicitly see supersymmetry it is convenient to go to Light Cone Gauge (LCG), defined by the choice: $\bar{n}\cdot A = 0$

And integrate out non-propagating modes: $\frac{\delta\mathcal{L}}{\delta n\cdot A} = 0 \Rightarrow n\cdot A = -\frac{4ig}{(\bar{n}\cdot\partial)^2}u_2^* u_2 - \frac{2}{(\bar{n}\cdot\partial)^2}\mathcal{D}_{\perp\mu}\bar{n}\cdot\partial A_\perp^\mu$

Two remaining physical gauge dof make up a complex "light cone scalar" defined by: $\partial_\perp\cdot A_{n\perp} = -\partial^*\mathcal{A} - \partial\mathcal{A}^*$

- After the SCET expansion and fixing LCG, we are left with two on-shell fermionic and two on-shell bosonic degrees of freedom.
- In this gauge SUSY is manifest.
- The remaining on-shell degrees of freedom organize into a chiral supermultiplet.

Gauge Theory

The Vector Multiplet

$$u_{\bar{n}} = -\frac{\bar{n}\cdot\sigma}{2}\frac{1}{\bar{n}\cdot\partial}\left(\bar{\sigma}\cdot\partial_\perp\right)u_n$$

Integrate out modes away from light cone

$$\mathcal{L} = iu^\dagger\bar{\sigma}\cdot\partial u - \frac{1}{4}(F^{\mu\nu})^2$$

Full theory
Yang-Mills
+ adjoint fermion

Gauge Theory

The Vector Multiplet

$$\mathcal{L} = u_n^\dagger \left(i\, n \cdot \mathcal{D}_{n,s} + i\, \bar{\sigma} \cdot \mathcal{D}_{\perp,n} \frac{1}{i\, \bar{n} \cdot \mathcal{D}_n} i\, \sigma \cdot \mathcal{D}_{\perp,n} \right) \frac{\bar{n} \cdot \bar{\sigma}}{2} u_n - \frac{1}{4} \left(F_n^{\mu\nu} \right)^2$$

$$u_{\bar{n}} = -\frac{\bar{n} \cdot \sigma}{2} \frac{1}{\bar{n} \cdot \partial} \left(\bar{\sigma} \cdot \partial_\perp \right) u_n$$

Integrate out modes away from light cone

SCET
+ adjoint fermion

$$\mathcal{L} = i u^\dagger \bar{\sigma} \cdot \partial u - \frac{1}{4} (F^{\mu\nu})^2$$

Full theory
Yang-Mills
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Gauge Theory

The Vector Multiplet

$$\mathcal{L} = u_n^\dagger \left(i\, n\cdot \mathcal{D}_{n,s} + i\, \bar\sigma \cdot \mathcal{D}_{\perp,n} \frac{1}{i\, \bar n \cdot \mathcal{D}_n} i\, \sigma \cdot \mathcal{D}_{\perp,n} \right) \frac{\bar n \cdot \bar\sigma}{2}\, u_n - \frac{1}{4}\, (F_n^{\mu\nu})^2$$

$$u_{\bar n} = -\frac{\bar n \cdot \sigma}{2} \frac{1}{\bar n \cdot \partial} \left(\bar\sigma \cdot \partial_\perp \right) u_n$$

Integrate out modes away from light cone

SCET
+ adjoint fermion

Integrate out unphysical gluon polarization

$$n \cdot A_n^b = -\frac{4ig}{(\bar n \cdot \partial)^2} u_2^{*a} u_2^c (t^b)_{ac} - \frac{2}{(\bar n \cdot \partial)^2} \{ \nabla^* (\bar n \cdot \partial \mathcal{A}^b) + \nabla (\bar n \cdot \partial \mathcal{A}^{*b}) \}$$

$$\mathcal{L} = i u^\dagger \bar\sigma \cdot \partial u - \frac{1}{4} (F^{\mu\nu})^2$$

Full theory
Yang-Mills
+ adjoint fermion

Gauge Theory

The Vector Multiplet

$$\mathcal{L} = u_n^\dagger \left(i\, n\cdot \mathcal{D}_{n,s} + i\, \bar\sigma \cdot \mathcal{D}_{\perp,n} \frac{1}{i\, \bar n \cdot \mathcal{D}_n} i\, \sigma \cdot \mathcal{D}_{\perp,n} \right) \frac{\bar n \cdot \bar\sigma}{2} u_n - \frac{1}{4} \left(F_n^{\mu\nu}\right)^2$$

$$u_{\bar n} = -\frac{\bar n \cdot \sigma}{2} \frac{1}{\bar n \cdot \partial} \left(\bar\sigma \cdot \partial_\perp\right) u_n$$

Integrate out modes away from light cone

SCET
+ adjoint fermion

Integrate out unphysical gluon polarization

$$n \cdot A_n^b = -\frac{4ig}{(\bar n \cdot \partial)^2} u_2^{*a} u_2^c (t^b)_{ac} - \frac{2}{(\bar n \cdot \partial)^2} \{\nabla^* (\bar n \cdot \partial \mathcal{A}^b) + \nabla (\bar n \cdot \partial \mathcal{A}^{*b})\}$$

$$\mathcal{L} = i u^\dagger \bar\sigma \cdot \partial u - \frac{1}{4} (F^{\mu\nu})^2$$

Full theory
Yang-Mills
+ adjoint fermion

Light cone SCET
+ adjoint fermion

$$n \cdot A_n^b = 0$$

$$\partial_\perp \cdot A_{n\perp} = -\partial^* \mathcal{A} - \partial \mathcal{A}^*$$

$$\mathcal{L}_{U(1)} = \mathcal{A}^* \Box \mathcal{A}$$

Gauge Theory

The Vector Multiplet

$$\mathcal{L} = u_n^\dagger \left(i\, n\cdot \mathcal{D}_{n,s} + i\, \bar\sigma \cdot \mathcal{D}_{\perp,n} \frac{1}{i\, \bar n \cdot \mathcal{D}_n} i\, \sigma \cdot \mathcal{D}_{\perp,n} \right) \frac{\bar n \cdot \bar\sigma}{2} u_n - \frac{1}{4} (F_n^{\mu\nu})^2$$

$$u_{\bar n} = -\frac{\bar n \cdot \sigma}{2} \frac{1}{\bar n \cdot \partial} (\bar\sigma \cdot \partial_\perp)\, u_n$$

Integrate out modes away from light cone

SCET
+ adjoint fermion

Integrate out unphysical gluon polarization

$$n \cdot A_n^b = -\frac{4ig}{(\bar n \cdot \partial)^2} u_2^{*a} u_2^c (t^b)_{ac} - \frac{2}{(\bar n \cdot \partial)^2} \{ \nabla^* (\bar n \cdot \partial \mathcal{A}^b) + \nabla (\bar n \cdot \partial \mathcal{A}^{*b}) \}$$

$$\mathcal{L} = i u^\dagger \bar\sigma \cdot \partial u - \frac{1}{4} (F^{\mu\nu})^2$$

Full theory
Yang-Mills
+ adjoint fermion

Confirm SUSY

SUSY Yang-Mills
light cone EFT

$\equiv$

Light cone
SCET
+ adjoint fermion

$$n \cdot A_n^b = 0$$

$$\partial_\perp \cdot A_{n\perp} = -\partial^* \mathcal{A} - \partial \mathcal{A}^*$$

$$\mathcal{L}_{U(1)} = \mathcal{A}^* \Box \mathcal{A}$$

$$\delta_\eta u_2^a = i\sqrt{2} \eta^{\dagger \dot 2} \bar n \cdot \partial \mathcal{A}^a \quad \text{and} \quad \delta_\eta \mathcal{A}^a = \sqrt{2} \eta^2 u_2^a$$

Can show explicitly that Lagrangian is invariant under SUSY transformations

Gauge Theory

The LCG Component Lagrangian: a consistent supersymmetric EFT

$$
\begin{aligned}
\mathcal{L}_g = {} & \mathcal{A}^{*b}\Box\mathcal{A}^b - g f^{abc}\mathcal{A}^{*b}\mathcal{A}^c(\partial^*\mathcal{A}^a - \partial\mathcal{A}^{*a}) - \frac{1}{2}g^2 f^{abc}f^{ade}\mathcal{A}^{*b}\mathcal{A}^c\mathcal{A}^{*d}\mathcal{A}^e \\
& - 2g f^{abc}\left(\partial^*\mathcal{A}^b + \partial\mathcal{A}^{*b}\right)\left(\frac{1}{\bar{n}\cdot\partial}\left[\mathcal{A}^{*a}\bar{n}\cdot\partial\mathcal{A}^c + \mathcal{A}^a\bar{n}\cdot\partial\mathcal{A}^{*c}\right]\right) \\
& + 2g^2 f^{abc}f^{ebh}\left(\frac{1}{\bar{n}\cdot\partial}(u^{*a}u^c)\right)\left(\frac{1}{\bar{n}\cdot\partial}(u^{*e}u^h)\right) \\
& + 4g^2 f^{abc}f^{ebh}\left(\frac{1}{\bar{n}\cdot\partial}(u^{*a}u^c)\right)\left(\frac{1}{\bar{n}\cdot\partial}\left[\mathcal{A}^{*e}\bar{n}\cdot\partial\mathcal{A}^h + \mathcal{A}^e\bar{n}\cdot\partial\mathcal{A}^{*h}\right]\right) \\
& + \frac{3}{2}g^2 f^{abc}f^{ebh}\left(\frac{1}{\bar{n}\cdot\partial}\left[\mathcal{A}^{*a}\bar{n}\cdot\partial\mathcal{A}^c + \mathcal{A}^a\bar{n}\cdot\partial\mathcal{A}^{*c}\right]\right)\left(\frac{1}{\bar{n}\cdot\partial}\left[\mathcal{A}^{*e}\bar{n}\cdot\partial\mathcal{A}^h + \mathcal{A}^e\bar{n}\cdot\partial\mathcal{A}^{*h}\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_u = {} & i u_2^{*a}\left(\bar{n}\cdot\partial - \frac{2\partial\partial^*}{\bar{n}\cdot\partial}\right)u_2^a \\
& + 2ig f^{bac}\left[u_2^a\frac{\partial^*}{\bar{n}\cdot\partial}(u_2^{*c}\mathcal{A}^b) + u_2^{*a}\frac{\partial}{\bar{n}\cdot\partial}(\mathcal{A}^{*b}u_2^c) + u_2^{*a}u_2^c\left(\frac{1}{\bar{n}\cdot\partial}\left(\partial^*\mathcal{A}^b + \partial\mathcal{A}^{*b}\right)\right)\right] \\
& - 4ig^2 f^{bac}f^{ebh}u_2^{*a}u_2^c\left(\frac{1}{(\bar{n}\cdot\partial)^2}(u^{*e}u^h)\right) - 2ig^2 f^{eab}f^{hbc}u_2^{*a}\mathcal{A}^e\left[\frac{1}{\bar{n}\cdot\partial}(\mathcal{A}^{*h}u_2^c)\right] \\
& - 2ig^2 f^{bac}f^{ebh}u_2^{*a}u_2^c\left(\frac{1}{(\bar{n}\cdot\partial)^2}\left[\mathcal{A}^{*e}\bar{n}\cdot\partial\mathcal{A}^h + \mathcal{A}^e\bar{n}\cdot\partial\mathcal{A}^{*h}\right]\right).
\end{aligned}
$$

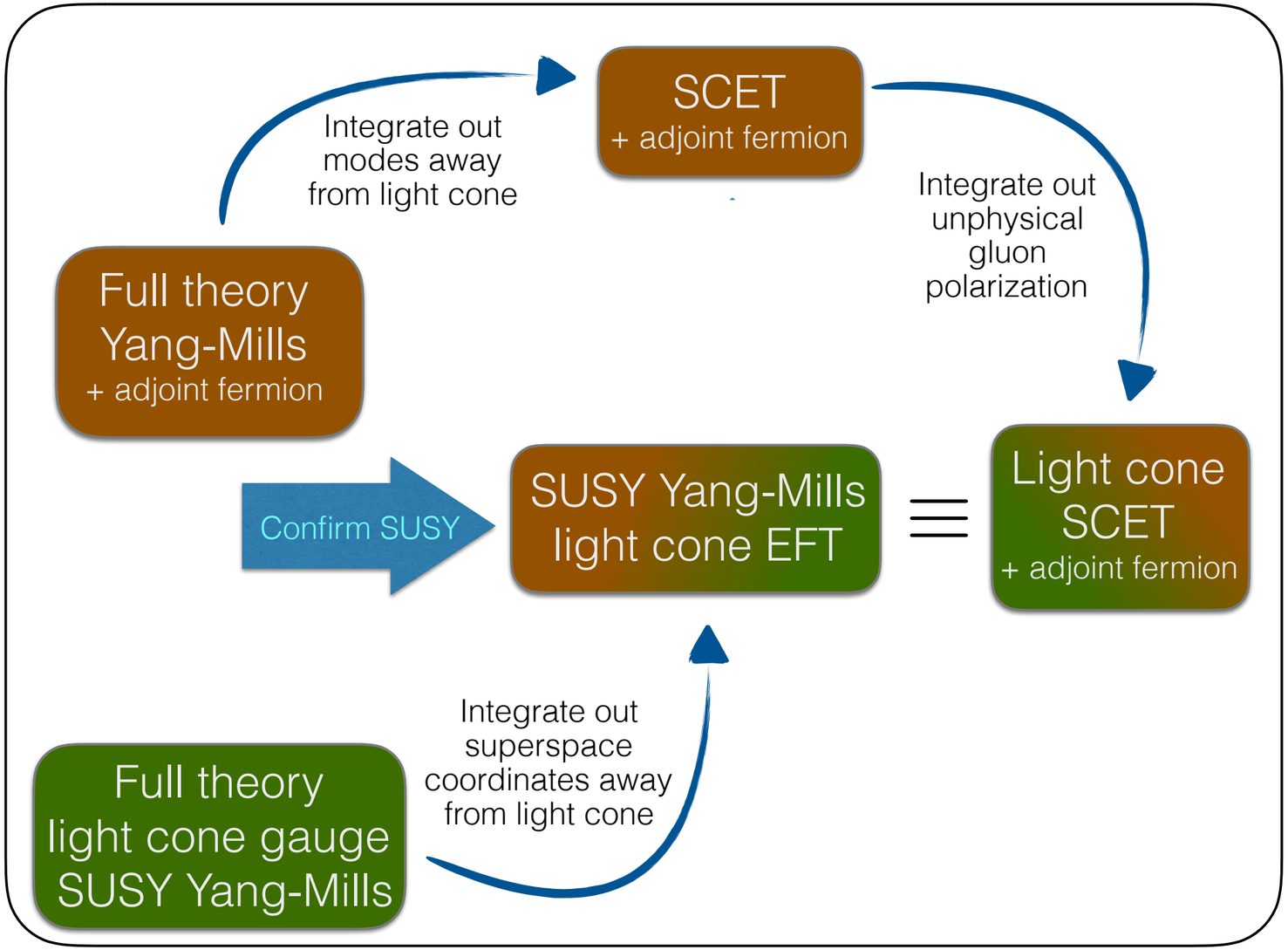
- Supersymmetric: invariant under
  $\delta_\eta u_2^a = i\sqrt{2}\eta^{\dagger\dot{2}}\bar{n}\cdot\partial\mathcal{A}^a$ and $\delta_\eta\mathcal{A}^a = \sqrt{2}\eta^2 u_2^a$
- Consistency Check: Reproduces IR singularity structure.
- Consistency Check: RPI invariant.

(See our upcoming paper "Soft-Collinear Supersymmetry" for a scenario where a consistent EFT cannot be constructed.)

Can we utilize the power of superspace to write this in a more simple/ transparent form?

Gauge Theory

Superspace



Superspace

A Chiral Multiplet

Full Theory: Consider a chiral multiplet: left handed Weyl fermion and complex scalar

Chirality constraint: $D_\alpha \Phi^\dagger = 0 = \bar{D}^{\dot{\alpha}} \Phi$ where $D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i(\sigma \cdot \partial)_{\alpha\dot{\alpha}} \theta^{\dagger\dot{\alpha}}$

Solution: $\Phi = \phi + u^\alpha \theta_\alpha + F \theta^\alpha \theta_\alpha$

Action: $\mathcal{L} = \int d^4\theta \Phi^\dagger \Phi = \phi^* \Box \phi + i u^\dagger \bar{\sigma} \cdot \partial u$

EFT: By supersymmetry: $\Phi = (P_n + P_{\bar{n}})\, \Phi = \Phi_n + \Phi_{\bar{n}}$

Chirality constraint: $D_2 \Phi_n^\dagger = 0 = \bar{D}_{\dot{2}} \Phi_n$ where $D_2 = \frac{\partial}{\partial \theta^2} + i\theta^{\dagger\dot{2}} (\bar{n} \cdot \partial) + i\sqrt{2} \partial \theta^{\dagger\dot{1}} \sim \mathcal{O}(\lambda^0)$ and $D_1 \sim \mathcal{O}(\lambda) \sim \bar{D}_{\dot{1}}$

Solution: $\Phi_n = e^{i\theta^{\dagger\dot{2}}\theta^2 \bar{n}\cdot\partial} \left[\phi_n\left(y^\mu\right) + \sqrt{2}\theta^2 u_2\left(y^\mu\right) \right] = \phi_n + \sqrt{2}\theta^2 u_2 + i\theta^{\dagger\dot{2}}\theta^2 \bar{n} \cdot \partial \phi_n$ * Wess Zumino Model:

$\Phi \equiv \bar{D}_{\dot{2}}\, D_1\, V_n \Big|_{\theta^1 = 0 = \theta^{\dagger\dot{1}}} = \sqrt{2}\, \mathcal{A}^* + 2\, i\, \theta^2 u^*_{n,2} + i\, \sqrt{2}\, \theta^2 \theta^{\dagger\dot{2}}\, \bar{n} \cdot \partial \mathcal{A}^*$ * Light Cone Gauge Theory:

Action: ???

(note: this is on-shell)

General Algorithm

Constructing Lagrangians in Collinear Superspace

- Find projection operators which separate the superfield into collinear and anti-collinear superfields.
- Given the superspace action for the full theory we integrate out the anti-collinear field, yielding a constraint equation.
- Motivated by the form of the constraint equation we can now guess an ansatz that solves the eom of the anti-collinear superfield. This should obey the EOM and any other constraints (chirality, reality etc.)
- Plug ansatz back into the full theory action, yielding the superspace action of the EFT.

Collinear Superspace

The Chiral Multiplet Kinetic Term

Chiral field of Wess-Zumino Model or Light Cone Gauge on-shell Vector Multiplet

1) By supersymmetry: $\Phi = (P_n + P_{\bar{n}})\,\Phi = \Phi_n + \Phi_{\bar{n}}$

Expand full theory action: $\mathcal{L} = \int d^4\theta \Big(\Phi_n + \Phi_{\bar{n}}\Big)^\dagger \Big(\Phi_n + \Phi_{\bar{n}}\Big) = \int d^4\theta \left(\Phi_n^\dagger \Phi_n + \Phi_n^\dagger \Phi_{\bar{n}} + \Phi_{\bar{n}}^\dagger \Phi_n + \Phi_{\bar{n}}^\dagger \Phi_{\bar{n}}\right)$

2) Constraint equation: $\dfrac{\delta S}{\delta \Phi_{\bar{n}}^\dagger} = 0 \Rightarrow \quad -\dfrac{1}{4} DD\Phi_{\bar{n}} - \dfrac{1}{4} DD\Phi_n = 0$

Check this reproduces expected component EOM, e.g.

$$\bar{D}_{\dot{\alpha}} DD\Phi_{\bar{n}} = -\bar{D}_{\dot{\alpha}} DD\Phi_n \Rightarrow 4i\,(\bar{\sigma}\cdot\partial)^{\dot{\alpha}\alpha}\, D_\alpha \Phi_{\bar{n}} = -4i\,(\bar{\sigma}\cdot\partial)^{\dot{\alpha}\alpha}\, D_\alpha \Phi_n$$

$$\Rightarrow u_{\bar{n}1} = \frac{\partial}{\bar{n}\cdot\partial} u_{n,2} \quad \longleftrightarrow \quad u_{\bar{n}} = -\frac{\bar{n}\cdot\sigma}{2}\frac{1}{\bar{n}\cdot\partial}\left(\bar{\sigma}\cdot\partial_\perp\right) u_n$$

3) Guess an ansatz that solves the constraint equation:

$$\boxed{\Phi_{\bar{n}} = -\Phi_n - \frac{1}{(\bar{n}\cdot\partial)\, D_1}\bar{D}^{\dot{1}} D D \Phi_n}$$

Well defined by momentum eigenvalues as seen from the Fourier Transform

$$\frac{1}{\bar{n}\cdot\partial}\psi(x) = \frac{1}{\bar{n}\cdot\partial}\int \mathrm{d}^4p\, e^{-i\,p\cdot x}\,\tilde{\psi}(p) = \int \mathrm{d}^4p\, e^{-i\,p\cdot x}\frac{1}{\bar{n}\cdot p}\,\tilde{\psi}(p)$$

Consider the super-Fourier transform e.g

$$\Phi_n(x,\theta,\theta^\dagger) = \frac{1}{(2\pi)^2}\int d^4p d^2\theta d^2\theta^\dagger \tilde{\Phi}_n(p,\eta,\eta^\dagger) e^{i\left(p^\mu x_\mu + \eta^\alpha\theta_\alpha + \eta^{\dagger\dot{\alpha}}\theta^\dagger_{\dot{\alpha}}\right)}$$

4) Plug back into Lagrangian:

$$\int d^4\theta \left(\Phi_n^\dagger\Phi_n + \Phi_n^\dagger\Phi_{\bar{n}} + \Phi_{\bar{n}}^\dagger\Phi_n + \Phi_{\bar{n}}^\dagger\Phi_{\bar{n}}\right) = \int d^4\theta \left(D^1\bar{D}\bar{D}\Phi_n^\dagger\right)\frac{1}{(\bar{n}\cdot\partial)^2\bar{D}_{\dot{1}}D_1}\left(\bar{D}^{\dot{1}}DD\Phi_n\right)$$

$$= \int d\theta_1 d\theta_2 d\theta^\dagger_{\dot{1}} d\theta^\dagger_{\dot{2}} \frac{i\Phi_n^\dagger\Box\Phi_n}{(\bar{n}\cdot\partial)\,\bar{D}_{\dot{1}}D_1} = \int d\theta^2 d\theta^{\dagger\dot{2}}\Phi_n^\dagger\frac{i\Box}{\bar{n}\cdot\partial}\Phi_n$$

$$\Rightarrow \boxed{\int d\theta^2 d\theta^{\dagger\dot{2}} i\Phi_n^\dagger\frac{\Box}{\bar{n}\cdot\partial}\Phi_n \subset i u^*_{n,2}\left(n\cdot\partial + \frac{\partial_\perp^2}{\bar{n}\cdot\partial}\right)u_{n,2} + i\phi^*\Box\phi}$$

where: $\Phi_n = \phi_n + \sqrt{2}\theta^2 u_2 + i\theta^{\dagger\dot{2}}\theta^2\bar{n}\cdot\partial\phi_n$ chiral theory

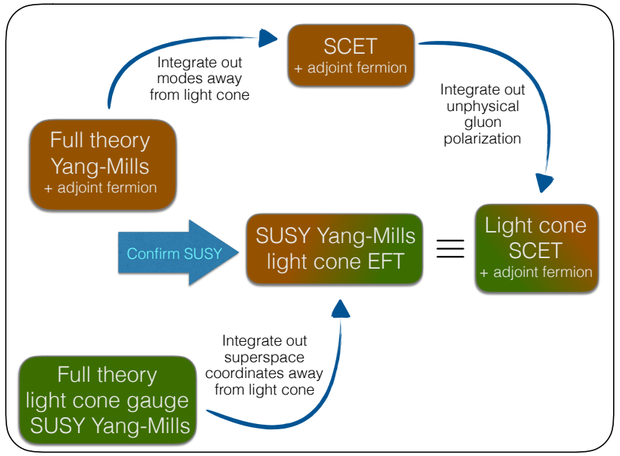
$$\Phi \equiv \bar{D}_{\dot{2}}\, D_1\, V_n\Big|_{\theta^1=0=\theta^{\dagger\dot{1}}} = \sqrt{2}\,\mathcal{A}^* + 2\,i\,\theta^2 u^*_{n,2} + i\,\sqrt{2}\,\theta^2\theta^{\dagger\dot{2}}\,\bar{n}\cdot\partial\mathcal{A}^*$$ LCG vector theory

As expected there is no dependency on the superspace coordinates with suppressed power counting scalings.

We can extend this to other theories following our general algorithm

Gauge Theory

In Collinear Superspace



Non-Abelian vector multiplet: $S = \int d^4x d^2\theta \mathrm{Tr}\left[\mathcal{W}^\alpha \mathcal{W}_\alpha\right] + \int d^4x d^2\bar{\theta} \mathrm{Tr}\left[\bar{\mathcal{W}}_{\dot{\alpha}} \bar{\mathcal{W}}^{\dot{\alpha}}\right]$

where $\mathcal{W}_\alpha = -\frac{i}{4}\bar{D}\bar{D}e^{-(V_n+V_{\bar{n}})}D_\alpha e^{(V_n+V_{\bar{n}})}$

Constraint equation: $\frac{\delta S}{\delta V_{\bar{n}}} = 0 \Rightarrow \; -D^\alpha \bar{D}\bar{D}D_\alpha V^e + igf^{ace}\left(D^\alpha \bar{D}\bar{D}V^c + \left[D^\alpha, V^c\right]\bar{D}\bar{D}\right)D_\alpha V^a - g^2 f^{ace} f^{adh}\left[D^\alpha, V^c\right]\bar{D}\bar{D}V^d D_\alpha V^h = 0$

$$\boxed{\begin{aligned} i\mathcal{L} = \int d\theta^2 d\bar{\theta}^{\dot{2}} \Big[& \Phi^{\dagger a}\frac{\Box}{\bar{n}\cdot\partial}\Phi^a + 2gf^{abc}\left(\Phi^a\Phi^{\dagger b}\frac{\partial^*}{\bar{n}\cdot\partial}\Phi^c - \Phi^{\dagger a}\Phi^b\frac{\partial}{\bar{n}\cdot\partial}\Phi^{\dagger c}\right) \\ & + 2g^2 f^{abc}f^{ade}\frac{1}{\bar{n}\cdot\partial}\left(\Phi^b D^\dagger_{\dot{2}}\Phi^{\dagger c}\right)\frac{1}{\bar{n}\cdot\partial}\left(\Phi^{\dagger d}D_2\Phi^e\right) \end{aligned}}$$

where $\Phi^a = \sqrt{2}\,\mathcal{A}^{*a} + 2\,i\,\theta^2 u^{*a}_{n.2} + i\sqrt{2}\,\theta^2\theta^{\dagger\dot{2}}\,\bar{n}\cdot\partial\mathcal{A}^{*a}$

- Reduces to LCG SCET component Lagrangian e.g. $i\mathcal{L}_{kin} = iu^{*a}_{\dot{2}}\frac{\Box}{\bar{n}\cdot\partial}u^a_2 + \mathcal{A}^{*a}\Box\mathcal{A}^a$
- Invariant under SUSY transformations: $\delta_\eta u^a_2 = i\sqrt{2}\eta^{\dagger\dot{2}}\bar{n}\cdot\partial\mathcal{A}^a$ and $\delta_\eta\mathcal{A}^a = \sqrt{2}\eta^2 u^a_2$

Many Questions Remain

Can we explore the SCET limits of more general theories?

- Is the SCET limit of Super Yang-Mills self consistent? (Choice of a specific frame which obscures Lorentz invariance. Possible obstruction for compatibility with extended space-time invariance.) ✔
- Formulate SCET with Weyl Fermions? ✔
- Chiral multiplet? ✔
- $\mathcal{N} = 1$ Gauge multiple? ✔
- Superspace formalism? ✔
- Subtleties of Super-Poincare invariance on the lightcone? ✔
- Operators?
- Extension to $\mathcal{N} > 1$ SUSY?
- Supergravity?
- Gravity?
- ???

Stay Tuned...

Thank You

Back ups

Super-Poincare on the Lightcone

Super-Poincare:

$$\left[Q_\alpha, P^\mu\right] = 0 = \left[Q^\dagger_{\dot\alpha}, P^\mu\right] \qquad \left[P_\mu, P_\nu\right] = 0\,;$$

$$\left[M_{\mu\nu}, Q_\alpha\right] = i(\sigma_{\mu\nu})_\alpha^{\ \beta} Q_\beta \quad \text{and} \quad \left[M_{\mu\nu}, Q^\dagger_{\dot\alpha}\right] = i(\bar\sigma_{\mu\nu})_{\dot\alpha}^{\ \dot\beta} Q^\dagger_{\dot\beta} \qquad \left[M^{\mu\nu}, P^\rho\right] = i\, g^{\mu\rho} P^\nu - i\, g^{\nu\rho} P^\mu\,;$$

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \qquad \left[M^{\mu\nu}, M^{\kappa\rho}\right] = -g^{\mu\kappa} M^{\nu\rho} - g^{\nu\rho} M^{\mu\kappa} + g^{\mu\rho} M^{\nu\kappa} + g^{\nu\kappa} M^{\mu\rho}$$

$$R_1^\nu = \bar n_\mu\, M^{\mu\nu\perp}\,,$$
$$R_2^\nu = n_\mu\, M^{\mu\nu\perp}\,,$$
$$R_3 = n_\mu\, \bar n_\nu\, M^{\mu\nu}$$

$$M^{\mu\nu} = \begin{bmatrix} 0 & K^1 & K^2 & K^3 \\ -K^1 & 0 & -J^3 & J^2 \\ -K^2 & J^3 & 0 & J^1 \\ -K^3 & -J^2 & -J^1 & 0 \end{bmatrix}.$$

$$\left[R_1^\nu, n\cdot P\right] = 2\, i\, P_\perp^\nu\,, \qquad \left[R_2^\nu, n\cdot P\right] = 0\,, \qquad \left[R_3, n\cdot P\right] = -2\, i\, n\cdot P\,,$$
$$\left[R_1^\nu, \bar n\cdot P\right] = 0\,, \qquad \left[R_2^\nu, \bar n\cdot P\right] = 2\, i\, P_\perp^\nu\,, \qquad \left[R_3, \bar n\cdot P\right] = 2\, i\, \bar n\cdot P\,,$$
$$\left[R_1^\nu, P_\perp^\rho\right] = -i\, g_\perp^{\nu\rho}\, \bar n\cdot P\,, \qquad \left[R_2^\nu, P_\perp^\rho\right] = -i\, g_\perp^{\nu\rho}\, n\cdot P\,, \qquad \left[R_3, P_\perp^\rho\right] = 0\,.$$

$$[R_1^\nu, Q_1] = -i\bar a^\nu Q_2 \qquad [R_1^\nu, Q_2] = 0$$
$$[R_2^\nu, Q_1] = 0\,, \qquad [R_2^\nu, Q_2] = -i a^\nu Q_1$$
$$[R_3, Q_1] = -iQ_1\,, \qquad [R_3, Q_2] = 0$$

$$\left[R_1^\nu, R_2^\rho\right] = \bar n_\mu\, n_\kappa \left[M^{\mu\nu\perp}, M^{\kappa\rho\perp}\right] = -2\, M^{\nu\perp\rho\perp} + g_\perp^{\nu\rho}\, R_3 = \pm(2\, J_3 + R_3)$$

scaling: $R_1^\nu \sim \lambda^{-1}\,,$ $\qquad R_2^\nu \sim \lambda\,,$ $\qquad R_3 \sim 1$

Reparametrization Invariance (RPI)

Transformation of fields

Projections should be invariant $P_n u_n = u_n$ e.g $\frac{n\cdot\sigma}{2}\frac{\bar{n}\cdot\bar{\sigma}}{2}u_n \xrightarrow[\text{RPI-I}]{} \left(\frac{n\cdot\sigma}{2}\frac{\bar{n}\cdot\bar{\sigma}}{2}+\frac{\Delta^\perp\cdot\sigma}{2}\frac{\bar{n}\cdot\bar{\sigma}}{2}\right)u_n = u_n$

i.e. Full theory spinor should be RPI invariant

RPI-I: rotates in back the helicity component which has been integrated out

$$u_{n,\alpha} = \begin{bmatrix} 0 \\ u_2 \end{bmatrix} \xrightarrow[\text{RPI-I}]{} \begin{bmatrix} \frac{1}{2}(\Delta_\perp\cdot\sigma)_{1\dot{2}}u_2 \\ u_2 \end{bmatrix}$$

RPI-II: rescaling of fermion field

$$u_{n,\alpha} = \begin{bmatrix} 0 \\ u_2 \end{bmatrix} \xrightarrow[\text{RPI-II}]{} \begin{bmatrix} 0 \\ \left(1+\frac{1}{2}(\epsilon_\perp\cdot\sigma)_{2\dot{1}}\frac{1}{\bar{n}\cdot D}(D_\perp\cdot\bar{\sigma})^{\dot{1}2}\right)u_2 \end{bmatrix}$$

RPI-III: boost that leaves the fermion field invariant

$$\mathcal{L}_u = iu_n^\dagger\frac{\Box}{\bar{n}\cdot\partial}\frac{\bar{n}\cdot\bar{\sigma}}{2}u_n \xrightarrow[\text{RPI-III}]{} u_n^\dagger\frac{\Box}{e^\alpha\bar{n}\cdot\partial}\frac{e^\alpha\bar{n}\cdot\bar{\sigma}}{2}u_n\,;$$

Summary:

	RPI-1	RPI-2	RPI-3
$n\to$	$n+\Delta_\perp$	n	$e^\alpha n$
$\bar{n}\to$	$\bar{n}$	$\bar{n}+\epsilon_\perp$	$e^{-\alpha}\bar{n}$
$u_n\to$	$\left(1+\frac{1}{4}\sigma\cdot\Delta_\perp\bar{n}\cdot\bar{\sigma}\right)u_n$	$\left(1+\frac{\epsilon^\perp\cdot\sigma}{2}\frac{1}{i\bar{n}\cdot\partial}i\partial_\perp\cdot\bar{\sigma}\right)u_n$	u_n
$n\cdot A\to$	$n\cdot A+\Delta_\perp\cdot A$	$n\cdot A$	$e^\alpha n\cdot A$
$\bar{n}\cdot A\to$	$\bar{n}\cdot A$	$\bar{n}\cdot A+\epsilon_\perp\cdot A$	$e^{-\alpha}\bar{n}\cdot A$
$A_{\perp\mu}\to$	$A_{\perp\mu}-\frac{1}{2}\Delta_{\perp\mu}\bar{n}\cdot A-\frac{\bar{n}_\mu}{2}\Delta_\perp\cdot A_\perp$	$A_{\mu\perp}-\frac{\epsilon_{\mu\perp}}{2}n\cdot A-\frac{n_\mu}{2}\epsilon_\perp\cdot A$	$A_{\mu\perp}$
$\phi_n\to$	ϕ_n	ϕ_n	ϕ_n

Gauge Theory

Deriving Abelian Theory in Collinear Superspace

Abelian vector multiplet $S = \int d^4x d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \int d^4x d^2\theta^\dagger \bar{\mathcal{W}}_{\dot\alpha} \bar{\mathcal{W}}^{\dot\alpha}$

1) By supersymmetry $V = V^\dagger = P_n V + P_{\bar n} V = V_n + V_{\bar n}$ $\Rightarrow$ $\mathcal{W}_\alpha = -\frac{i}{4} \bar{D}\bar{D} D_\alpha \left(V_n + V_{\bar n} \right)$

where in LCG:

$$V_n = -\theta^1 \theta^{\dagger \dot 1} n \cdot A_n - \sqrt{2} \left(\theta^1 \theta^{\dagger \dot 2} \mathcal{A}_n^* + \theta^2 \theta^{\dagger \dot 1} \mathcal{A}_n \right) + 2\, i\, \theta^1 \theta^2 \theta^{\dagger \dot 2} u^*_{n,\dot 2} - 2\, i\, \theta^{\dagger \dot 1} \theta^{\dagger \dot 2} \theta^2 u_{n,2}$$

$$V_{\bar n} = -\theta^1 \theta^{\dagger \dot 1} n \cdot A_{\bar n} - \theta^2 \theta^{\dagger \dot 2} \bar n \cdot A_{\bar n} - \sqrt{2} \left(\theta^1 \theta^{\dagger \dot 2} \mathcal{A}_{\bar n}^* + \theta^2 \theta^{\dagger \dot 1} \mathcal{A}_{\bar n} \right) + 2\, i\, \theta^1 \theta^2 \theta^{\dagger \dot 1} u^*_{\bar n, \dot 1} - 2\, i\, \theta^{\dagger \dot 1} \theta^{\dagger \dot 2} \theta^1 u_{\bar n, 1}$$

2) Constraint equation $\frac{\delta S}{\delta V_{\bar n}} = 0 \Rightarrow \left(-16\, \Box + 4\, i\, D^\alpha (\sigma \cdot \partial)_{\alpha\dot\alpha} \bar{D}^{\dot\alpha} \right) (V_n + V_{\bar n}) = 0$

Gauge Theory

Deriving Abelian Theory in Collinear Superspace

3) Ansatz solution $$V_{\bar{n}} = -V_n - \frac{1}{\bar{n}\cdot\partial D_2\,\bar{D}_1\,D_1}\bar{D}_{\dot{2}}\,DD\left(\bar{D}_{\dot{2}}\,D_1\,V_n\right) - \frac{1}{\bar{n}\cdot\partial\,\bar{D}_{\dot{2}}\,D_1\,\bar{D}_{\dot{1}}}D_2\,\bar{D}\bar{D}\left(D_2\,\bar{D}_{\dot{1}}\,V_n\right)$$

note: on-shell LCG dof can be written in terms of chiral and anti-chiral superfields

e.g. $$\Phi \equiv \bar{D}_{\dot{2}}\,D_1\,V_n\Big|_{\theta^1=0=\theta^{\dagger\dot{1}}} = \sqrt{2}\,\mathcal{A}^* + 2\,i\,\theta^2 u^*_{n,2} + i\,\sqrt{2}\,\theta^2\theta^{\dagger\dot{2}}\,\bar{n}\cdot\partial\mathcal{A}^*$$

4) Plug back into Lagrangian $$S = -2\int d^4x d^2\theta d^2\theta^\dagger\,\bar{D}_{\dot{1}}D^\alpha(V_n+V_{\bar{n}})\bar{D}_{\dot{2}}D_\alpha(V_n+V_{\bar{n}}) + \text{h.c.}$$

$$\Rightarrow \quad \mathcal{L} = \int \mathrm{d}^4\theta\left(\frac{1}{\bar{n}\cdot\partial\,\bar{D}_{\dot{1}}}D^1\,\bar{D}\,\bar{D}\,(D_2\bar{D}_{\dot{1}}V_n)\right)\left(\frac{1}{\bar{n}\cdot\partial\,D_1}\bar{D}^{\dot{1}}\,D\,D\,(\bar{D}_{\dot{2}}D_1V_n)\right)$$

$$= \int \mathrm{d}\theta^2\,\mathrm{d}\theta^{\dagger\dot{2}}\,\mathrm{d}\theta^{\dagger\dot{1}}\,\mathrm{d}\theta^1\,\frac{1}{D_1\,\bar{D}_{\dot{1}}}\left(D^1\,\bar{D}\bar{D}\,(D_2\bar{D}_{\dot{1}}V_n)\right)\frac{1}{(\bar{n}\cdot\partial)^2}\left(\bar{D}^{\dot{1}}\,DD\,(\bar{D}_{\dot{2}}D_1V_n)\right)$$

$$= \int \mathrm{d}\theta^2\,\mathrm{d}\theta^{\dagger\dot{2}}\,\Phi^\dagger_n\,\frac{\bar{D}\bar{D}D_2\bar{D}_{\dot{2}}DD}{(\bar{n}\cdot\partial)^2}\Phi_n = \int \mathrm{d}\theta^2\,\mathrm{d}\theta^{\dagger\dot{2}}\,\Phi^\dagger_n\frac{i\,\Box}{\bar{n}\cdot\partial}\Phi_n \quad\subset\quad i\,u^*_{n,2}\left(n\cdot\partial + \frac{\partial^2_\perp}{\bar{n}\cdot\partial}\right)u_{n,2} + \mathcal{A}^*\Box\mathcal{A}$$

Interacting Chiral Multiplet in Superspace

$$\mathcal{L} = \int d^4\theta \left(\Phi_n^\dagger \Phi_n + \Phi_n^\dagger \Phi_{\bar{n}} + \Phi_{\bar{n}}^\dagger \Phi_n + \Phi_{\bar{n}}^\dagger \Phi_{\bar{n}}\right) + \int d^2\theta W\left(\Phi_n, \Phi_{\bar{n}}\right) + \text{h.c.}$$

1) By supersymmetry: $\Phi = \left(P_n + P_{\bar{n}}\right)\Phi = \Phi_n + \Phi_{\bar{n}}$ superpotential $W = m\Phi_n\Phi_{\bar{n}} + \frac{y}{3}\left(\Phi_n^2\Phi_{\bar{n}} + \Phi_{\bar{n}}^2\Phi_n\right)$

2) Constraint equation: $\frac{\delta S}{\delta \Phi_{\bar{n}}^\dagger} = 0 \Rightarrow -\frac{1}{4}DD\Phi_{\bar{n}} - \frac{1}{4}DD\Phi_n + m^*\Phi_n^\dagger + \frac{y^*}{3}\left(\Phi_n^{\dagger 2} + 2\Phi_{\bar{n}}^\dagger\Phi_n^\dagger\right) = 0$

3) Guess an ansatz: $\Phi_{\bar{n}} = -\Phi_n + \frac{\bar{D}^{\dot{1}}DD}{4i(\bar{n}\cdot\partial)D_1}\Phi_n - m^*\frac{\bar{D}^{\dot{1}}}{i(\bar{n}\cdot\partial)D_1}\Phi_n^\dagger - \frac{2}{3}y^*\frac{\bar{D}^{\dot{1}}}{i(\bar{n}\cdot\partial)D_1}\Phi_n^\dagger,$

4) Plug back into Lagrangian:

we get interaction terms such as: $|y|^2\int d\theta^2 d\theta^{\dagger\dot{2}}\frac{1}{\bar{n}\cdot\partial}(\Phi_n D_2\Phi_n)\frac{1}{\bar{n}\cdot\partial}(\Phi_n^\dagger\bar{D}_{\dot{2}}\Phi_n^\dagger) \to |y^2|\phi_n^* u_{n,2}^*\frac{1}{i\bar{n}\cdot\partial}u_{n,2}\phi_n.$

Such interaction terms actually result in a breakdown of the EFT (helicity flipping interactions violate RPI and have the wrong IR structure).

The single flavor interacting WZ model does not have a consistent collinear EFT at each order.

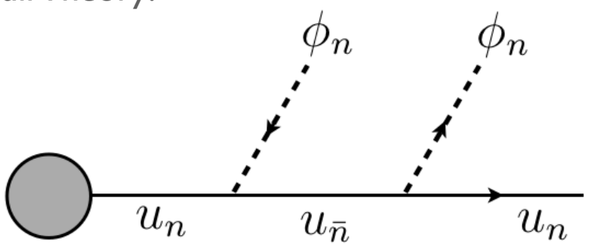
Problems alleviated when by adding multiple flavors and gauge interactions. Can still proceed with $\mathcal{N}$ = 1, $\mathcal{N}$ > 1 SUSY.

The Collinear Wess-Zumino Model

1 ⟶3 Splitting:

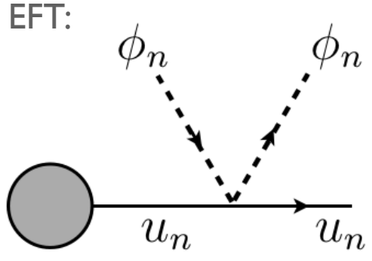
IR singularities from double collinear scalar emission

Full Theory:



$\Rightarrow$

EFT:



Integrating out anti-collinear modes:

$$u_{\bar{n},\alpha} = (\sigma\cdot\partial_\perp)_{\alpha\dot{\alpha}}\frac{1}{\bar{n}\cdot\partial}\left(\frac{\bar{n}\cdot\bar{\sigma}}{2}\right)^{\dot{\alpha}\beta}u_{n,\beta} + y^*\phi_n^*\frac{1}{i\bar{n}\cdot\partial}\left(\frac{\bar{n}\cdot\sigma}{2}\right)_{\alpha\dot{\beta}}u_n^{\dagger\dot{\beta}}$$

$$\Rightarrow \quad \mathcal{L}_n^{(0)} = -\phi_n^*\Box\phi_n + u_n^\dagger\left(in\cdot\partial - \frac{\partial_\perp^2}{i\bar{n}\cdot\partial}\right)\frac{\bar{n}\cdot\bar{\sigma}}{2}u_n - \frac{|y|^2}{4}|\phi_n|^4 - |y|^2\phi_n^*u_n^\dagger\frac{1}{i\bar{n}\cdot\partial}\frac{\bar{n}\cdot\bar{\sigma}}{2}u_n\phi_n$$

(recall we also found this interaction term via the superspace derivation)

Supersymmetric under: $\delta\phi_n = \eta u_n\,, \qquad \delta u_n = -\frac{n\cdot\sigma}{2}\eta^\dagger(i\bar{n}\cdot\partial)\phi_n\,,$

Interaction term leads to problems...

The Collinear Wess-Zumino Model

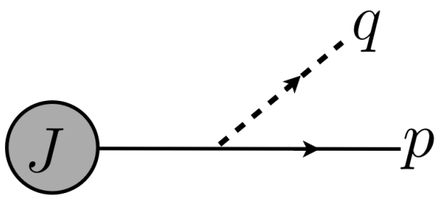
IR singularities from Yukawa Interaction

Determine if amplitudes are singular by power counting external momentum as either soft or collinear.

1 ⟶2 Splitting:

Has collinear but no soft IR singularity

Scalar emission from fermion current:



$$= y\bar{u}(p)\frac{\not{q}}{2p\cdot q}\not{J} \Rightarrow y\frac{\bar{u}(p)v(q)}{2p\cdot q}\bar{v}(q)\not{J}$$

Wess-Zumino Model:

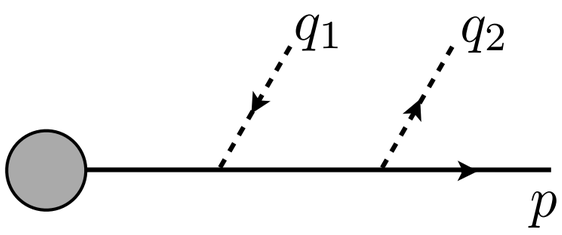
$$\mathcal{L} = \int d^4\theta \Phi^\dagger\Phi + \int d^2\theta\left(m\Phi^2 + \frac{y}{3}\Phi^3\right) + \text{h.c.} = -\phi^*\Box\phi - m^2|\phi|^2 + iu^\dagger\bar{\sigma}^\mu\partial_\mu u - \frac{m}{2}uu - \frac{m^*}{2}u^\dagger u^\dagger$$
$$-\frac{ym^*}{2}\phi^\dagger\phi^2 - \frac{y^*m}{2}\phi\phi^{\dagger 2} - \frac{|y|^2}{4}|\phi|^4 - \left(\frac{y}{2}\phi uu + \frac{y^*}{2}\phi^\dagger u^\dagger u^\dagger\right)$$

Two spin states required for IR divergence from one-two spitting.
Single Chiral multiplet in the EFT has only one spin state.

Problems with the Wess-Zumino Model

IR Structure

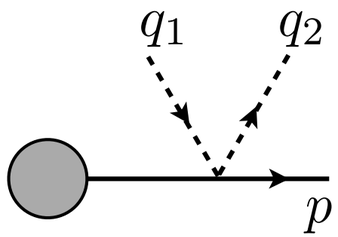
Full Theory with collinear momentum:



$$= u^\dagger(p)\left(-i\frac{y}{2}\right)\frac{-i(p+q_2)\cdot\bar{\sigma}}{2p\cdot q_2}\left(-i\frac{y^*}{2}\right)\frac{i(p+q_1+q_2)\cdot\sigma}{(p+q_1+q_2)^2}$$

$$\sim -\frac{|y|^2}{4}u^\dagger(p)\left[1+\frac{\bar{n}\cdot(q_1+q_2)}{\bar{n}\cdot p}\right]\frac{1}{(p+q_1+q_2)^2}$$

Effective Theory:



$$= u^\dagger(p)(-i|y|^2)\frac{1}{\bar{n}\cdot(p+q_2)}\frac{\bar{n}\cdot\bar{\sigma}}{2}i\frac{n\cdot\sigma}{2}\frac{\bar{n}\cdot(p+q_1+q_2)}{(p+q_1+q_2)^2}$$

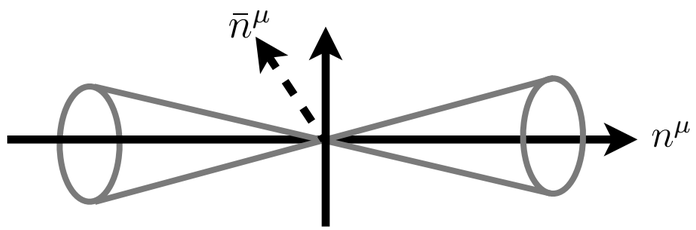
$$= |y|^2u^\dagger(p)\left[1+\frac{\bar{n}\cdot q_1}{\bar{n}\cdot(p+q_2)}\right]\frac{1}{(p+q_1+q_2)^2}\,.$$

Problems with the Wess-Zumino Model

Reparametrization Invariance

RPI-II:

$$n \to n$$
$$\bar{n} \to \bar{n} + \epsilon_\perp$$
$$u_n \to \left(1 + \frac{\epsilon^\perp \cdot \sigma}{2} \frac{1}{i\bar{n}\cdot\partial} i\partial_\perp \cdot \bar{\sigma}\right) u_n$$
$$\phi_n \to \phi_n$$



$$u_n^\dagger \phi_n^* \frac{1}{i\bar{n}\cdot\partial} \frac{\bar{n}\cdot\bar{\sigma}}{2} \phi_n u_n \to u_n^\dagger \left(1 + i\partial_\perp \cdot \bar{\sigma}_\perp \frac{1}{i\bar{n}\cdot\partial} \frac{\epsilon_\perp \cdot \sigma_\perp}{2}\right) \phi_n^* \left(\frac{1}{i\bar{n}\cdot\partial} - \frac{i\epsilon_\perp \cdot \partial_\perp}{(i\bar{n}\cdot\partial)^2}\right)$$
$$\times \left(\frac{\bar{n}\cdot\bar{\sigma}}{2} + \frac{\epsilon_\perp \cdot \bar{\sigma}_\perp}{2}\right) \phi_n \left(1 + \frac{\epsilon_\perp \cdot \sigma_\perp}{2} \frac{1}{i\bar{n}\cdot\partial} i\partial_\perp \cdot \bar{\sigma}_\perp\right) u_n$$
$$= u_n^\dagger \phi_n^* \frac{1}{i\bar{n}\cdot\partial} \frac{\bar{n}\cdot\bar{\sigma}}{2} \phi_n u_n + u_n^\dagger i\partial_\perp \cdot \bar{\sigma}_\perp \frac{1}{i\bar{n}\cdot\partial} \frac{\epsilon_\perp \cdot \sigma_\perp}{2} \phi_n^* \frac{1}{i\bar{n}\cdot\partial} \frac{\bar{n}\cdot\bar{\sigma}}{2} \phi_n u_n$$
$$- u_n^\dagger \phi_n^* \frac{i\epsilon_\perp \cdot \partial_\perp}{(i\bar{n}\cdot\partial)^2} \frac{n\cdot\bar{\sigma}}{2} \phi_n u_n + u_n^\dagger \phi_n^* \frac{1}{i\bar{n}\cdot\partial} \frac{\bar{n}\cdot\bar{\sigma}}{2} \phi_n \frac{\epsilon_\perp \cdot \sigma_\perp}{2} \frac{1}{i\bar{n}\cdot\partial} i\partial_\perp \cdot \bar{\sigma}_\perp u_n \neq 0$$

Solve by adding additional flavors generating 1 to 2 splitting interactions in the EFT and alleviate these problems. Before doing so lets study gauge theory.